

An Abstract Multi-Rate Method for Vehicle Dynamics Simulation

James Critchley, and Paramsothy Jayakumar

Multibody.org, and US Army TARDEC

Copyright © 2012 SAE International

ABSTRACT

The design of vehicles increasingly challenges existing cost, weight, durability, and handling regimes. This challenge is further compounded by pressure to decrease or limit the duration of the design cycle. The simulation of vehicle dynamic behavior commonly applies just rigid, or better rigid and linear flexibility models to predict motions and determine load cases. However, as the boundaries of materials are pushed these are becoming insufficient to accurately predict behavior. Alternatively, complete nonlinear finite element representations of vehicle dynamics are always possible but are presently infeasible for the support of a single design under virtual test, not to mention several design iterations. To address these issues, a novel abstract multi-rate simulation method is outlined which is designed to exploit the richness of available model in the vehicle dynamics domain. The method relies on the availability of a virtual continuum of modeling fidelities and uses the fast executing low fidelity models to seed increasingly high fidelity models which execute concurrently in different regions of the time domain. As a result, discontinuities will appear in the states time-histories, and the method must then validate (or invalidate) the discontinuities as being possible states given the chaotic nature of the higher frequency components in the system.

INTRODUCTION

Simulation of vehicle dynamic loads in military, off-road, and/or durability environments is used for virtual design verification of all modern vehicles. Vehicle dynamic simulation, also called multibody dynamic simulation, is a key piece of the product durability process which is critical to reducing the number of prototypes tested. Reductions in prototyping in turn reduces the development cost and time before a vehicle design can move to production. Today these types of performance prediction capabilities are being pushed beyond their limits as manufacturers seek to simultaneously reduce weight and cost while extending performance levels and reducing the design cycle time.

Lighter weight parts constructed with less material or with new materials means that interactions which were previously negligible or coarsely characterized are now relevant and

require significant modeling details. Chief among these are material nonlinearities and large deformations in flexible components. These considerations are properly handled in the context of finite element analysis but such an approach to vehicle and multibody dynamics is infeasible because the size of the system and the time scales involved. In other words, simulations will not complete until long after the allocated time for the design verification activity has lapsed.

There are two directions any proposed solution can take. The first is to seek out new modeling methods tailored to the specific class of models under consideration. At present, the modeling approaches employed in traditional multibody commercial software do not incorporate such details because they are designed for rigid body systems and subsequently extended to accommodate small deformation via linear flexibility. The traditional approach has worked well for the past three decades because the modeling task truncates frequencies which do not contribute significantly to the solution. In short, the solution directly benefits from the experience of the analyst and extraneous details are not simulated.

In the flexible multibody systems literature there are three modeling approaches which are being actively pursued. These methods are: i. co-simulation which runs both multibody and flexible body models concurrently and applies a gluing algorithm strategy to share information; ii. implementing new multibody relationships within existing finite element methods; and iii. new finite element methods implemented within existing multibody methods [1 Shabana]. Each of these approaches has numerous proponents and the relative performance of each solution depends on the type of problem being solved and this means that the various industries are actively pushing for different solutions.

It is interesting to note that these three solutions are pursued specifically to deal with the approximation error in beam, plate, and shell element type models. Such elements are not iso-parametric and therefore introduce errors in element inertias undergoing finite rotations. Three dimensional solid elements however are iso-parametric and where such models are required, the large deformation flexible multibody dynamics problem is automatic and exact. In general, the suspension components under consideration are solid three

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE 01 JUN 2013		2. REPORT TYPE		3. DATES COVERED	
4. TITLE AND SUBTITLE An Abstracts Multi-Rate Method for Vehicle Dynamics Simulation				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) James Critchley; Paramsothy Jayakumar				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U.S. Army TARDEC, 6501 East Eleven Mile Rd, Warren, MI, 48397-5000				8. PERFORMING ORGANIZATION REPORT NUMBER #23479	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited.					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT The design of vehicles increasingly challenges existing cost, weight, durability, and handling regimes. This challenge is further compounded by pressure to decrease or limit the duration of the design cycle. The simulation of vehicle dynamic behavior commonly applies just rigid, or better rigid and linear flexibility models to predict motions and determine load cases. However, as the boundaries of materials are pushed these are becoming insufficient to accurately predict behavior. Alternatively, complete nonlinear finite element representations of vehicle dynamics are always possible but are presently infeasible for the support of a single design under virtual test, not to mention several design iterations. To address these issues, a novel abstract multi-rate simulation method is outlined which is designed to exploit the richness of available model in the vehicle dynamics domain. The method relies on the availability of a virtual continuum of modeling fidelities and uses the fast executing low fidelity models to seed increasingly high fidelity models which execute concurrently in different regions of the time domain. As a result, discontinuities will appear in the states time-histories, and the method must then validate (or invalidate) the discontinuities as being possible states given the chaotic nature of the higher frequency components in the system.					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES 9	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

dimensional objects and while some may be appropriately modeled with beams, plates, and shells, it is expected that solid three dimensional elements will be required.

As an alternative to exact multibody constraints, one might also consider large deformation and large rotation explicit FEA solvers. These solutions are very easy to run in parallel because they employ penalty methods for constraints [2 Hallquist]. Such formulations also add fictitious dynamics which are a function of the discretization and time-step which is not desirable.

The alternative to new modeling methods is a new simulation scheme addressing the run-time issues of highly detailed models. These types of improvements typically come from parallel computer implementations and tailored integration methods. Here no new modeling technology is introduced and instead the manner in which the solution is calculated or propagated through time is altered.

Parallel computing for the solution of multibody problems is often restricted to the function evaluation of implicit and explicit integration schemes [3 Featherstone][4 Critchley]. As simulation details are increased, the frequency of the dynamic interactions also increases and requires smaller step sizes. This means that even in the presence of a theoretically optimal logarithmic $O(\log N)$ solution time per function evaluation, models of increased details will still require several orders of magnitude more run-time than the lower fidelity multibody simulations of today.

Multi-rate solvers offer the possibility of larger outer time steps on the order of current systems with de-coupled (parallel) time-steps for resolution of higher frequency dynamics. These methods are commonly realized in multibody dynamics as subsystem partitioning methods which allows for parallel computing of the subsystems [5 Arnold]. A typical multi-rate integration scheme applies a manual or automated partitioning scheme to identify directions in the solution space which are slowly changing. These motions are integrated over large time periods then used as a given solution in the integration of the fast dynamics at finer intervals (often concurrently). The low frequency result is then recomputed where the slow moving results are known via interpolation and checked for convergence.

A final option is State-Time simulation [6 Anderson]. This method formulates dynamics problems as nonlinear finite element problems with shape functions which span the spacial (coordinate) and temporal dimensions. The method is sparse and massively parallel (in state and time) but the nonlinear finite element approach is not guaranteed to converge in general. And in the presence of discontinuities such as coulomb friction, iterative solution convergence is likely to degenerate into sequential time stepping.

In contrast to the existing multi-rate literature, we propose a new multi-rate scheme which employs successive modeling fidelity abstractions. In this method distinct models of the

Page 2 of 9

same system are proposed to resolve the trajectory of multibody and vehicle dynamic solutions at different time scales in a massively parallel computer environment. The solution is referred to as “abstract multi-rate”, because the only required commonality of the solution across the various timescales is the physical system, not the mathematical model (differential equations) or even physical model (modeling assumptions). The solution exploits the availability of various low, intermediate, and high fidelity modeling representations of vehicle dynamics (often available in other multibody domains) and the ability to automatically generate an arbitrary model via a direct linkage to design geometries.

To facilitate the development, a brief review of multi-rate methods is first given and followed by a summary of the multigrid methods which we seek to emulate. Next, the new abstract multi-rate method and its theoretical application to a vehicle dynamic simulation is described. Open questions, challenges, and future work are then summarized in the conclusions.

MULTI-RATE SIMULATION METHODS

Multi-rate simulation is motivated by two observations pertaining to slow and fast dynamics contained within a single solution. The first is that a small subset of fast dynamics (high frequency) in a system causes the entire solution to proceed with small time steps. These small steps result in wasted operations which calculate the slow moving solution components to values of precision much smaller than the required local truncation errors. The second observation is that the dynamics of the fast subset are more easily calculated when the values of the slow subset are known. Combining these observations results in the fast calculation of high frequency dynamics without the loss of stated accuracy.

Multi-rate schemes are most easily described and implemented in the context of two-rate methods where a macro step is applied for slow dynamics and a micro step is used for the fast dynamics. However, the term multi-rate applies most generally to implementations where each component of the solution may be integrated using its own local rate.

The typical multi-rate method for non-linear systems begins with an accepted initial state at time t_0 and applies a prediction procedure to identify slow dynamics throughout the interval $t_0 + t_M$ where t_M is the macro step size. This prediction procedure can be as simple as a linear or higher order extrapolation from the initial state or any other method such as an approximate numerical integration. With an assumed trajectory for the slow dynamics, the fast dynamics can now be integrated sequentially in n steps of size t_μ over the interval $t_0 + nt_\mu$ where $nt_\mu = t_M$.

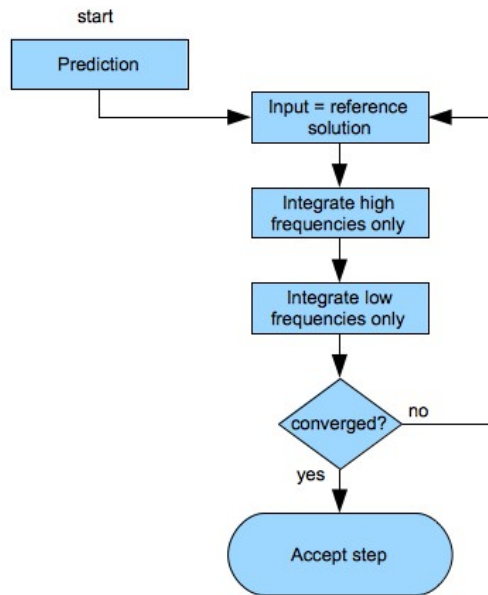


Figure 1: multi-rate flowchart

Once the high frequency dynamics are computed, the entire system is integrated over the macro step where the micro contributions are now fixed. The relative change in the macro approximation is then used as an acceptance criteria for the step. Should the criteria fail, the process restarts using the new macro step result as the prediction for the next micro step integration. The procedure is iterative and, under the proper conditions, convergence to the desired tolerance is rapid. The algorithm flowchart for this scheme is shown in figure 1.

Convergence is rapid when a proper partitioning of micro and macro timescales components is used. By definition a low frequency motion can at most be weakly coupled to a high frequency motion, otherwise it would have high frequency state dynamics. However the opposite does not hold and small changes in the low frequency motion can have a large influence on the high frequency motion [7 Chen].

The partitioning of micro and macro timescales in multibody and vehicle systems is usually prescribed by the analyst and based largely on mass, inertia, and subsystems observed to be largely independent. It is also possible to apply monitoring of local truncation errors within the integration method to dynamically adjust the timescales of the micro and macro steps. Such an approach can adaptively select the best time scale for each state (coordinate) based on the current system configuration.

In multibody dynamics, slow dynamics take the form of prescribed motions in the calculation of the fast dynamics. This should be viewed as a significant simplification for articulated systems because a specified motion decouples the equations of motion for all attached elements resulting in smaller and independent equations which are amenable for an efficient parallel computer simulation. However this is not the case in general as it is possible for the kinematic constraints

associated with the low frequency dynamics to transmit energy and loads across the system (between high frequency components) effectively re-coupling them rendering such a multi-rate scheme unstable [5 Arnold]. Thus a complete system simulation is required at the smallest step-size to maintain stability in general for multibody systems.

It is also interesting to note that proper multi-rate application involves identification of fast and slow modes via an eigen decomposition. The coordinates subsequently used in the rate separated solution would then be associated with eigenfunctions and not in general the coordinates of the system as modeled. Such a method offers very little in the way of simplifying the solution of the multibody problem as all mechanical degrees of freedom remain active and coupled in the high frequency solution. Finding an eigen decomposition of the nonlinear multibody problem which remains valid over large periods of time also proves problematic in the general case.

Given the discussion of two-rate methods, it is not difficult to envision a true multi-rate scheme employing a tiered approach where several independent time scales are spanned. In the multibody domain Arnold presents one such method based on a parallel execution of subsystems [5 Arnold]. The method uses independent local subsystem integrators (operating at different rates) made to exchange interface data at predetermined synchronization points (times).

A common thread in all multi-rate schemes is the presence of a single macro outermost step size. This outermost step size means that the solution is intended to sequentially march through the time domain and ultimately limits the total concurrency of the solution.

As a high level summary, a multi-rate method applied to vehicle dynamics is realized as three stages. First the low fidelity gross system dynamics are estimated. These estimates then enable higher frequency motions to be evaluated independently. And lastly, the combined system dynamics are checked for validity. Any method which performs these steps will be termed an “abstract multi-rate” method.

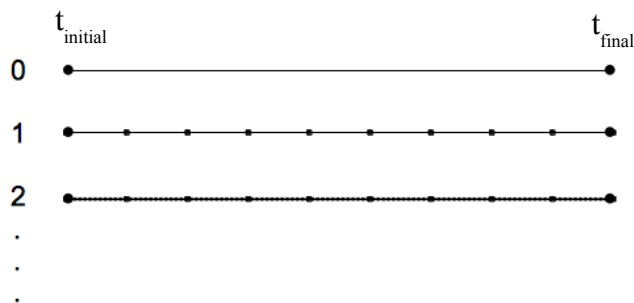


Figure 2: Recursive time domain partitioning

MULTIGRID SIMULATION METHODS

In contrast to the sequential temporal nature of a multi-rate method, multigrid methods decompose the entire solution domain. The new abstract multi-rate method will have more in common with these schemes so a brief review will be given. For additional information the reader should consult computational linear algebra texts such as [8 Demmel].

Multigrid methods are generally applied to boundary value problems. For one dimensional problems, the spatial domain is partitioned recursively into subdomains based on the first (smallest) eigenvalues and the associated eigenvectors. The use of eigenvalues and eigenvectors indicates that multigrid is a linear equation solver however the adaptation to nonlinear problems is applied in practice and adaptation is quite easy as the solution is already iterative, although stability is not guaranteed.

Each subdomain contains a subset of the original problem and is capable of computing the local residual error and solving for the local states which minimize it (a process referred to as smoothing). Operators are also provided between grid representations for the *interpolation* of a coarse grid solution on the fine grid and *restriction* of the fine grid result into the coarse grid.

The multigrid solution procedure begins with a complete solve on the coarsest grid (a trivial operation). Interpolation to and smoothing on the next finer grid then provides an improved local solution in each of the finer grid's subdomains. The multiple subdomains are then combined by a restriction step to the coarse grid and the coarse grid states are subsequently smoothed. The smoothed coarse grid solution is then interpolated again on the fine grid and solved. Having reduced the error in the two level approximation, a third level can be interpolated and the result smoothed similarly across all multigrid levels recursively following the same procedure. This procedure is summarized in the algorithm flowchart of figure 3.

For linear problems, convergence occurs in a constant number of iterations independent of problem size. Solution can then be realized in large parallel computer environment in $O(\log N)$ time where N is the number of sub domains in the finest grid

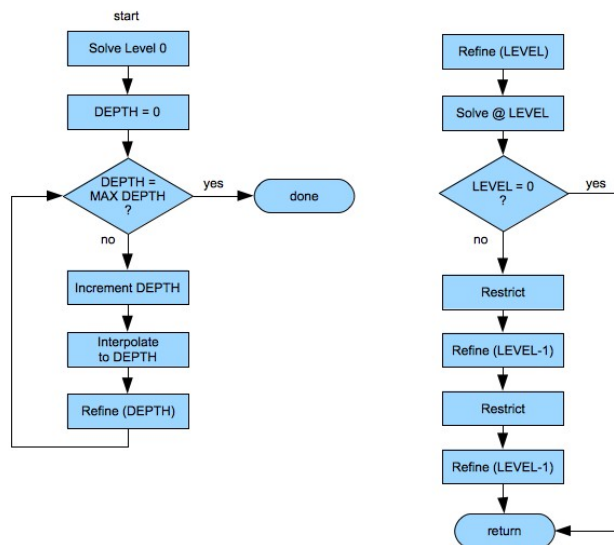


Figure 3: Multigrid algorithm

representation. Rapid convergence is observed because the domains communicate interactions on the hierarchical grid resulting in system wide propagations which are much faster than other iterative nearest neighbor methods [8 Demmel].

When relating a single multi-rate step to a complete multigrid solution, the principles are observed to be physically equivalent. The coarse shape functions of the multigrid are heavily weighted in the solution and only small adjustments are applied due to the shorter wave length behaviors. However, owing to the initial formulation being a linear boundary value problem the macro step is stable even when it spans the entire domain of the solution. In the vehicle dynamics problem, this would be equivalent to taking a single macro multi-rate step from initial to final time.

TODO: There are time domain multigrid approaches. This should be mentioned here with citations.

Douglas has provided the extension of the multigrid to virtually any problem, not just those derived from discretized PDEs by providing a generalized problem statement, iteration procedure, and convergence criteria [9 Douglas1]. This "abstract multigrid" method requires only that the problem be defined as any sequence of linear algebra problems ($A^{(i)}x^{(i)} = b^{(i)}$) which approximate the real problem ($Ax = b$) [10 Douglas2]. The abstract multigrid solution is then constructed using only restriction and interpolation matrices which map the solutions of adjacent domains into one another.

THE ABSTRACT MULTI-RATE METHOD

In considering a new method, we seek to construct a multi-rate solution which behaves like the multigrid method. Such a method will decompose the entire solution time domain at successive levels of detail (frequency content) in the solution.

The coarse grain decompositions must be easy and fast to solve and provide a firm basis for beginning independent simulations of shorter duration within each subdomain. And the resulting solution must meet convergence criteria relevant to the system under study.

Successive Levels of Detail

Echoing the statement of Douglas, “a sequence of (linear) problems are formed which approximate the *real problem*” [10 Douglas2], one can recognize that the *real problem* of vehicle dynamics is given by the physical problem statement. This is the input CAD geometry and proving ground, not the final discretization previously viewed as a complete multibody finite element model of significant detail. From this abstract perspective one can recognize that approximations truncated at nearly every level of frequency interaction are readily available to vehicle dynamics analysts. The multi-objective nature of vehicle dynamics simulation means that these models are not simply part of a rich history, they remain in active use through out the industry and may be fine tuned to predict or match any observed behavior.

In the catalog of models supporting vehicle dynamics, we begin with simplified faster than real-time models such as those employed in first order design optimization tools [TODO: citations]. Increasing in detail one encounters those models supporting hard real-time requirements as for controls development, hardware in the loop testing, and operator evaluation or training [TODO: citations]. Next are vehicle models of modest detail which support off-line analysis of vehicle handling, virtual kinematics and compliance assessments, and the like [TODO: citations]. Traditional detailed durability loads analysis models follow with stiff bushing elements and truncated linear flexible component models [TODO: citations]. Last are the full finite element models which at present can only be executed over short time spans and per the earlier discussion are the topic of on-going research [TODO: citations]. Once a finite element discretization is introduced, models of increased details and increasing frequency content are readily generated. For the purpose of tiered fidelity one may begin with a course mesh and decrease mesh size in appropriate increments.

Employing a single “coarse” and “fine” mesh solution as shown in figure 4, the successive levels of detail in each model, spans frequency content and/or run-time scales of approximately one order of magnitude each.

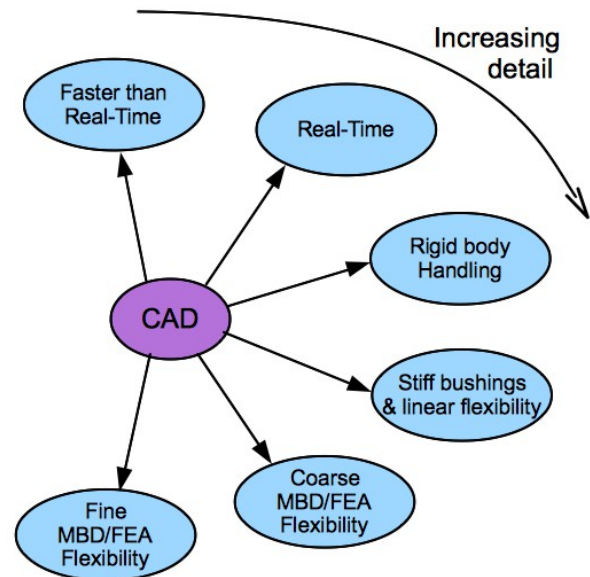


Figure 4: Common vehicle dynamics models

Recent innovations in automated analysis allow the analyst to completely automate the construction of detailed multibody models from input CAD geometry. The process applies abstract relationships which are associated with labels attached to parts, assemblies, design features, and geometries to construct entire families of executable models without user intervention [11 Panthaki]. Where an automated modeling tool is employed, the vehicle dynamics analyst is expected to maintain the simulation virtual proving ground and the tedious CAD model transcription tasks are replaced with the role of supporting the adaptation of automated output capabilities for several tiers of modeling fidelity.

After the initial investment of setting up these simulation templates, common geometry changes within a vehicle design result in no additional work for the analyst to generate new simulation models (and simulations). This type of direct linkage to computational geometry (CAD models) is required to support the construction of a hierarchy of models employing different simplifying assumptions and not derived from a common mathematical model.

Time Domain Partitioning

Considering a hypothetical vehicle dynamics simulation which may run for 100 seconds. The resulting partitioning and

model type	# subdomains	grid size (s)	# of DoF	step size (s)	run time (s)
Real-time	1	100	~25	1e-2	~25
Handling	10	10	~100	1e-3	~25
Linear flex	100	1	~500	1e-4	~25
Course FEA	1000	0.1	~4,000	1e-5	~100
Fine FEA	10000	0.01	~160,000	1e-6	~400

Table 1: Hierarchy of typical models.

hypothetical run times per grid are shown in Table 1. The partitioning results in what is commonly referred to as the “embarrassingly parallel” problem at each level of the grid and results in constant time, $O(1)$, in the presence of scalable parallel resources. Note that the geometric expansion of the number of degrees of freedom in the finite element domain can be compensated with the addition of subdomain local processors and so maintain a relatively constant hypothetical run-time (4 processors on the coarse FEA and 16 processors on the fine mesh FEA).

A single iteration of a solution flowing from the coarse model to fine model can then be accomplished in constant time $O(L)$ in the presence of $O(10^L)$ processors, where L is the number of levels in the algorithm (counted from 0).

Interpolation Mappings

As in the multi-rate and multigrid methods, the interpolation operation used here will map solutions obtained in the coarse approximation to a next level finer approximation. Since each subdomain of the grid is a complete dynamic simulation, the mapping is only required to generate initial conditions for the simulation. This step is a trivial operation in the multibody modeling domain but a well thought out domain specific modeling implementation will be critical to the success of the method.

As the objectives of interpolation are the initial conditions for other simulations, interpolation in the time domain can be removed by aligning the subdomains with the output steps in the coarse method. The remaining task of interpolating the states of one multibody system onto another more detailed one is almost as simple and has three basic cases.

In the coarsest grids it is possible to see vehicle systems with fewer moving parts than are indicated in the CAD geometry. Such models might use kinematics and compliance data to simulate vehicle motions without any suspension linkages and limit the use of bodies to only the chassis and wheels (or unsprung mass). An increase in detail from this point can employ the known body positions and resolve new coordinate values from any number of kinematic solvers such as those already contained in most industrial multibody packages and specifically designed for this task.

The next common case is the expansion from suspension models employing generalized coordinates (e.g. perfect constraints), to stiff bushing type models requiring full descriptor form (six degrees of freedom per body). The construction of valid absolute coordinates from such models is trivial and the replacement is automatic.

The final case comes with the introduction of (or increase in) flexibility. This is a true spatial interpolation where rigid bodies become flexible bodies with zero deformation, and refined meshes receive node values interpolated from the coarse representation.

Such interpolation procedures are easy to implement but will clearly fall short of meaningful initial conditions. Even well constructed initial conditions typically require a settling time,

and there is much which can be done in specific domains to intelligently minimize these effects. Returning to vehicle dynamics and the models previously described, the analyst has many options.

Modeling transitions involving the coarsest grids may do well to match the potential energy stored in a deflected suspension rather than or in addition to (via weighting) the exact location of the wheel in the course model.

A generalized coordinate formulation being interpolated to full descriptor form can solve a nonlinear dynamic equilibrium problem for the deflections of the bushings given the accelerations of all bodies. Such a procedure smooths the transition from the infinite stiffness of a constraint to the compliant bushing which would otherwise have excessive and unrealistic initial vibration amplitudes.

The addition of flexibility to initial conditions can similarly benefit from intelligent preprocessing via the quasi-static multibody flexibility solution. More sophisticated initial conditions might take what is understood to be a typical energy content and smear it across the vibration modes of the bodies, and so on.

It is further recommended that the interpolation process be allowed to consume an additional 5-10% of subdomain runtime. This allocation is intended to provide an overlap with the adjacent (earlier) subdomain so that discontinuities caused by newly created initial conditions can be damped out before the formal subdomain is simulated. Indeed one can implement a scheme where the system is dynamically relaxed from coarse to fine representation over this period. A solution which may be prove easier to automate than the earlier methods.

Restriction Mappings

A restriction mapping passes information from fine to coarse grids. In the new abstract method, the fine grain solution contains all of the details of the multibody solution. As such it is possible that the solution can be obtained in a single iteration of the grid, requiring no restriction steps. Such conditions are the objective of the analyst because in order to obtain a different result from the procedure, one or more coarse models or interpolation procedures must change.

The automation of the restriction procedure is highly specialized to the model and problem domains. However there are many examples of such automation and in an arbitrary domain restriction can follow a standard parameter identification method. Vehicle dynamics models at all levels of fidelity are routinely tuned to observations to support hardware in the loop investigations and increased accuracy of analysis [TODO: cite several examples here.]. These same methods can be used to automatically update lower fidelity (coarse) models to more accurately represent the system.

An automated restriction mapping can even be allowed to identify model parameters uniquely within each subdomain resulting in a pseudo localized linearization or fuzzy logic type performance where the local operating conditions feedback into the model definition.

In any case the restriction procedure has benefits beyond the enabling of the new abstract multi-rate method. Low fidelity models which are shown to accurately reproduce the truncated system dynamics can be made available to other engineers such as those responsible for ride and handling, controls, testing, and throughout the enterprise for systems engineering.

It should also be pointed out that in the worst case, the restriction step is a point of intensive iterative effort for the analyst to manually produce a new model of sufficient quality. For this reason the method is initially targeting established subdomains.

Convergence Criteria

As an abstract method, the exact convergence criteria is also abstract. It is expected that adherence to a residual based on tight state continuity will result in instability and degenerate to sequential time marching within the high fidelity result (e.g. no improvement). This is to say that multi-rate methods applied to nonlinear systems are known to be unstable in general for the very large time steps proposed here, so this should not be the objective.

Instead the new method appeals to the features of complex dynamic systems when defining a convergence criteria. Dynamic systems are inherently chaotic and such chaos manifests itself at different timescales. Given a reference solution for the high frequency ringing of a flexible suspension part, the introduction of small changes to the model parameters, the excitation, or the numerical integration method will result in a solution 180 degrees out of phase in a very short time. Similar reasoning holds at larger timescales for chassis and wheel-hop motions, and smaller vibrations. It is the objective of the convergence criteria to limit the existence of adverse unmodeled behaviors thereby allowing discontinuous trajectories to be properly considered as equivalent to a continuous solution.

In a good vehicle design, the chaotic behaviors are expected to be limited to their local timescales. However, not all chaotic behaviors are confined to a single timescale. Systems can exhibit emergent behaviors where a regular chaotic behavior at one level produces an ordered effect at another. Nonlinear resonances can also slowly gather energy in a mode of motion, only to explosively transfer it to another mode with catastrophic consequences [12 Nayfeh Mook Marshal].

The analyst will need to familiarize themselves with such nonlinear dynamics behaviors and seek them out when developing and applying convergence criteria. Conversely, where such features are the design intent, they would be predicted in the low fidelity models. The lack of convergence would then indicate the absence of a desired response.

Each time subdomain (such as those depicted in Table 1) must be carefully selected by the analyst to allow for random and ordered effects to manifest themselves in the local solution. This can be verified for example by solving the subdomain a second time from slightly different initial conditions.

As a first approach, each subdomain in Table 1 will be allowed to continue simulation beyond its boundary and cover an additional two subdomains. This provides both a comparison from different but related initial conditions and an extended continuity check. In the comparison of the three trajectories, properties of forced excitations can be seen to align in overall amplitude and character while random vibrations are expected to be fully developed and sampled.

Commonalities in the movement of energy across the solution (at any fidelity level) are expected to appear as trends which lag each other in the three numerical solutions available on any subdomain. Unmodeled energy movement would appear in such output as sawtooth waves because of the periodic resets at the subdomain boundaries. A well developed domain modeling suite is required to automatically detect such monotonic discontinuities and alert the analyst. Preferably automation can include the subsequent adjustment of the energy distribution in initial conditions to realize a zero mean normal distribution of the discontinuities after only a few iterations.

These are the proposed convergence criteria, for an initial implementation. In later investigations and implementations, more rigorous probabilistic estimators can be employed. Methods such as the unscented (nonlinear) Kalman filter [TODO citation], particle filters [TODO citation], direct sensitivity (Jacobian) based approaches, and other similar techniques can be applied to formulate the characteristics of the distributions which arise in simulation through the propagation small errors such as integrator accuracy. These methods introduce a significant computational burden, however all estimators are based on observations and therefore may run concurrently with the solution. In practice the overlapping simulation scheme may prove to be the dominant workhorse method and the other techniques may be used to spot check specific or random regions in the solution.

In summary, a solution which does not converge has unmodeled (and therefore unexpected) behaviors. The role of the analyst is to provide better models and procedures for updating model parameters to achieve a convergent multi-rate simulation.

Application Summary

A general framework has been described which is intended for intensive development and domain specialization. As such, this abstract multi-rate method as proposed leaves many open questions. The application to vehicle dynamics evaluations is the subject of on-going investigations however it is of benefit to outline in more detail the proposed initial implementation and the theoretical result.

The first application will involve the first three levels of the abstract method. It will employ a real-time model, a handling model, and a stiff bushing type model referred to as a "loads model." The objective will be the recovery of a proper load-time history as required for durability estimation as described

in [TODO: cite Jayakumar, Purushothaman, Critchley, Data, Pisipati].

The performance bounds of these three models are very well understood. The real-time model predicts with high confidence the general location of the vehicle on terrain subject to driver (automated) controls. The handling model provides small adjustments to the speed and path tracking predictions of the real-time model, and accurately predicts the motion of suspension components. The role of the loads model is only to provide the accurate loads required by the durability process.

Given 100 seconds of rough off-road terrain as depicted previously in table 1, it is not uncommon for the loads model (termed “linear flex” in the table) to run in approximately 10000 seconds (or 40 minutes). The multibody model is relatively simple (small) and executes at very high frequencies and as such only a small amount of parallel computing via conventional methods is feasible to obtain additional speed increases.

Application of the abstract multi-rate method to this system would first automatically generate a real-time model from the design (CAD) data. Such a simulation would execute in approximately 25 seconds (the real-time model is actually a good bit faster than real-time) and the 100 seconds of data are used to construct 10 sets of initial conditions for cascading to the handling model.

The handling model is generated next and supplied 10 initial chassis locations and speeds at time values of 0.0s, 9.0s, 19.0s, and so on. Note that the initial conditions are 10% in advance of the actual evaluation intervals (subdomains) to allow for settling of the initial conditions. These chassis locations (and speeds) combined with the terrain under the vehicle allow the multibody solver to provide plausible initial conditions for the previously un-modeled linkages via common non-linear initial condition iteration.

The handling model then runs independently on 10 parallel processors (or workstations) taking a total time of approximately 27.5 seconds. However, these simulations are extended an additional two subdomain time intervals for comparison purposes and run approximately 77.5 seconds. The difference between the handling model output and the real-time model are compared. In a mature design development activity these models should match to the expected level of accuracy. If the two model results are not within tolerance, then an iteration whereby the real-time model is more accurately characterized is undergone and the cycle restarts.

Assuming convergence on the initial pass, the algorithm has now provided a sufficiently accurate handling simulation in 105 seconds which would have otherwise taken 250 seconds. Having deployed 10 processors in this effort, it is apparent that resources have been wasted (e.g. the simulation ideally would have taken only 25 seconds), but this data is more properly viewed as statistical sampling and provides added value to the

Page 8 of 9

analyst beyond the typical deterministic single evaluation [TODO: citation].

Following the process again, the loads model is automatically generated from the CAD data and initial conditions are provided from the detailed handling model. The loads model executes independently in parallel on 100 processors and adds an additional 77.5 seconds of user wait time. Here again, the handling model is well understood and is expected to have converged on the first attempt but additional refinement of the handling model is possible with iterations. Should the model require refinement, or if more accurate models are desired, it is useful to point out that the recursive model restriction as described in multigrid are not required and the real-time model can be refined directly from the data in the loads model. This is because at every level, a complete vehicle dynamics problem is solved. It is also useful to point out that even in simultaneous reduction across all levels of the scheme, each level has an order of magnitude more parallel processors available to it perform automated optimizations.

The end result is that the 10000 second simulation is accomplished in approximately 182.5 seconds using 100 parallel processors. The resulting simulation possess discontinuities in the states, particularly at the boundaries of the coarsest subdomains, but this is irrelevant from a durability loads perspective. The magnitudes of the loads extracted from the discontinuous time-history (actually there will be three sets of loads) will be compared via a rain-flow type method to a statistical sampling of the deterministic result.

The application to nonlinear material properties and large deformation increases the potential for massively parallel implementations and even more dramatic speed increases. It is also likely that it is here that the analyst will encounter the greatest challenges while developing superior low fidelity predictions through a thorough understanding of the system.

SUMMARY/CONCLUSIONS

Based heavily on the multi-rate and multigrid literature, a new abstract multi-rate method has been described. The method is expected to solve the current vehicle dynamics loads analysis problem two orders of magnitude faster when employing a small to modest parallel computer cluster and provides a framework for dramatic speed increases with models of increased detail (the subject of future work).

The abstract multi-rate scheme exploits established modeling automation capabilities to employ the most relevant modeling approximations as determined by domain specialists. This method should not be confused with a gluing algorithm nor is it an alternative to current investigations into large deformation and nonlinear material multi-flexible-body formulations. Instead it is capable of accelerating the throughput of arbitrary models and benefits from modeling advances at all levels.

Abstract multi-rate promotes the application and mutual improvement of all vehicle dynamics modeling technologies. In contrast to disparate models supporting activities across the enterprise, the deployment of an abstract multi-rate method benefits both the detailed CAE and domain experts (vehicle controls, etc.) with a source of verified and trace-able models at every level of fidelity.

For the analyst, the abstract multi-rate concept requires that they develop a detailed understanding of the underlying nonlinear dynamics phenomena and become intimately familiar with approximation and modeling methods in adjacent domains. The objective of any long standing abstract multi-rate architecture is to develop a rigorous process for moving the relevant details under study into faster models, relegating the intensive highly detailed modeling technologies into a verification role.

REFERENCES

1. Shabana, A., Bauchau, O. and Hulbert, G. "Integration of Large Deformation Finite Element and Multibody System Algorithms," *Journal of Computational and Nonlinear Dynamics*, 2007, doi: 10.1115/1.2756075.
2. Hallquist, John O. "Ls-dyna theoretical manual." California, Livermore Software Technology Corporation (1998).
3. Featherstone, Roy. "A divide-and-conquer articulated-body algorithm for parallel $O(\log(n))$ calculation of rigid-body dynamics. Part 1: Basic algorithm." *The International Journal of Robotics Research* 18, no. 9 (1999): 867-875.
4. Critchley, J. H., and K. S. Anderson. "A parallel logarithmic order algorithm for general multibody system dynamics." *Multibody System Dynamics* 12, no. 1 (2004): 75-93.
5. Arnold, M. "Multi-rate time integration for large scale multibody system models." In *IUTAM Symposium on Multiscale Problems in Multibody System Contacts*, pp. 1-10. Springer Netherlands, 2007, doi: DOI: 10.1007/978-1-4020-5981-0_1.
6. Anderson, Kurt S., and Mojtaba Oghbaei. "Dynamic Simulation of Multibody Systems Using a New State-Time Methodology." *Multibody System Dynamics* 14, no. 1 (2005): 61-80.
7. Chen, Jingjia, and Mariesa L. Crow. "A variable partitioning strategy for the multirate method in power systems." *Power Systems, IEEE Transactions on* 23, no. 2 (2008): 259-266, doi: 10.1109/TPWRS.2008.919319.
8. Demmel, James W. *Applied numerical linear algebra*. Society for Industrial Mathematics, 1997.
9. Douglas, Craig C. "Parallel multilevel and multigrid methods." *SIAM News* 25, no. 3 (1992): 14-15. doi: 10.1.1.75.3815.
10. Douglas, Craig C. "Implementing abstract multigrid or multilevel methods." In *NASA CONFERENCE PUBLICATION*, pp. 127-127. NASA, 1993.
11. Panthaki, Malcolm. "Concurrent engineering to the rescue: Do we have the software tools to support it?." In *Optical Engineering+ Applications*, pp. 70710G-70710G. International Society for Optics and Photonics, 2008.
12. Nayfeh, A. H., D. T. Mook, and L. R. Marshall. "NONLINEAR COUPLING OF PITCH AND ROLL MODES IN SHIP MOTIONS." *Journal of Hydronautics* 7, no. 4 (1973). doi: 10.2514/3.62949

ACKNOWLEDGMENTS

This material is based upon work supported by the United States Army under Contract No. W56HZV-12-C-0219.