

Leakage and paralysis in ancilla-assisted qubit measurement: Consequences for topological error correction in superconducting architectures

Joydip Ghosh

E-mail: joydip.ghosh@gmail.com

Department of Physics and Astronomy, University of Georgia, Athens, Georgia 30602, USA

Austin G. Fowler

E-mail: austingfowler@gmail.com

Centre for Quantum Computation and Communication Technology, School of Physics, The University of Melbourne, Victoria 3010, Australia

John M. Martinis

E-mail: martinis@physics.ucsb.edu

Department of Physics, University of California, Santa Barbara, California 93106, USA

Michael R. Geller

E-mail: mgeller@uga.edu

Department of Physics and Astronomy, University of Georgia, Athens, Georgia 30602, USA

Abstract.

Although topological error-correcting codes offer a promising paradigm for fault-tolerant quantum computation, their robustness in the presence of leakage to non-computational states is unclear. Here we explore the signature and consequences of leakage errors on ancilla-assisted Pauli operator measurement in superconducting devices. We consider a realistic coupled-qutrit model and simulate the repeated measurement of a single σ^z operator. Typically, a data-qubit leakage event manifests itself by producing a “noisy” ancilla qubit that randomly reads $|0\rangle$ or $|1\rangle$ from cycle to cycle. Although the measurement operation is compromised, the presence of the leakage event is apparent and detectable. However, there is also the possibility of a less typical but more dangerous type of leakage event, where the ancilla becomes *paralyzed*, rendering it oblivious to data-qubit errors for many consecutive measurement cycles and compromising the fault-tolerance. Certain dynamical phases associated with the entangling gate determine which type of leakage event will occur in practice. Leakage errors occur in most qubit realizations and our model and results are relevant for many stabilizer-based error correction protocols.

Report Documentation Page			Form Approved OMB No. 0704-0188		
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE 04 JUN 2013		2. REPORT TYPE		3. DATES COVERED 00-00-2013 to 00-00-2013	
4. TITLE AND SUBTITLE Leakage and paralysis in ancilla-assisted qubit measurement: Consequences for topological error correction in superconducting architectures			5a. CONTRACT NUMBER		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S)			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) University of California, Santa Barbara, Department of Physics, Santa Barbara, CA, 93106			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)			10. SPONSOR/MONITOR'S ACRONYM(S)		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S)		
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT Although topological error-correcting codes offer a promising paradigm for fault- tolerant quantum computation, their robustness in the presence of leakage to non- computational states is unclear. Here we explore the signature and consequences of leakage errors on ancilla-assisted Pauli operator measurement in superconducting devices. We consider a realistic coupled-qutrit model and simulate the repeated measurement of a single σ_z operator. Typically, a data-qubit leakage event manifests itself by producing a "noisy" ancilla qubit that randomly reads $0\rangle$ or $1\rangle$ from cycle to cycle. Although the measurement operation is compromised, the presence of the leakage event is apparent and detectable. However, there is also the possibility of a less typical but more dangerous type of leakage event, where the ancilla becomes paralyzed rendering it oblivious to data-qubit errors for many consecutive measurement cycles and compromising the fault-tolerance. Certain dynamical phases associated with the entangling gate determine which type of leakage event will occur in practice. Leakage errors occur in most qubit realizations and our model and results are relevant for many stabilizer-based error correction protocols.					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT Same as Report (SAR)	18. NUMBER OF PAGES 12	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

1. Introduction

Topological quantum error-correcting codes, such as surface and toric codes, are attracting attention because of their high error thresholds and realistic designs that only require nearest-neighbor interactions [1, 2, 3, 4, 5, 6]. While the robustness of standard fault-tolerant topological codes to discrete Pauli errors is a subject of active research [6, 7, 8, 9], the effect of leakage to non-computational states still remains an open question and is relevant for most quantum computing architectures. Understanding the effect of such leakage errors is important for superconducting qubits not only because higher energy states $|2\rangle, |3\rangle, \dots$ are present, as is the case with most other qubit realizations, but also because they can be utilized to implement two-qubit entangling operations such as the Strauch controlled- σ^Z (CZ) gate [10, 11]. One way to suppress the effects of leakage is to adopt the topological cluster-state approach [2, 3, 12], where each qubit is repeatedly initialized, operated on by gates, and measured: This approach systematically removes leakage errors from all qubits in the array, at the cost of some additional operations. Another approach might be to use a stabilizer-based topological error-correcting code for *qudits*, and theoretical progress has been made in this direction [13, 14, 15, 16, 17, 18, 19].

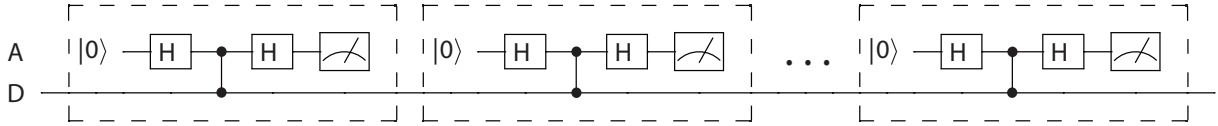


Figure 1. Protocol for ancilla-assisted σ^z measurement. Here “A” is the ancilla qubit and “D” the data qubit. Each cycle (dashed box) consists of a reset of the ancilla to the $|0\rangle$ state, a Hadamard gate H on the ancilla, a CZ gate, and another Hadamard followed by ancilla readout in the diagonal basis. The readout result is recorded and the cycle is repeated indefinitely. The data qubit never gets measured or reset.

In this work, however, we consider the standard stabilizer-based approach for qubits—but applied to three-level qutrits—and regard any population transfer to the $|2\rangle$ state of the ancilla or data qutrit as a potential error. We investigate the origin and signature of such leakage errors for an ancilla-assisted measurement of the data qutrit, identify a potentially dangerous regime where data errors are invisible to the ancilla, and discuss its consequences for topological error correction.

Figure 1 shows the circuit for our protocol. Let’s review how this works in the ideal limit: Initially, the data qutrit D is assumed to be in some pure qubit (not qutrit) state

$$|\psi_D\rangle = a|0\rangle + b|1\rangle, \quad (1)$$

while the ancilla A is initialized to $|0\rangle$. We perform the gate operations shown, record the measurement outcome, reset the ancilla to $|0\rangle$, and repeat this cycle many times. Throughout this work, the Hadamard gate (H) is assumed to be ideal and to act as the

identity on the third level of the qutrit,

$$H \equiv \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

The Hadamards and **CZ** combine to produce a controlled-NOT (**CNOT**) gate that copies the data qubit to the ancilla, but here we implement this **CNOT** with the gates shown in Fig. 1 because we believe that the **CZ** gate can be implemented in superconducting architectures with very high fidelity [11]. For an initial D state (1), the state of the system after the second **H** gate is, in the $|AD\rangle$ basis,

$$a|00\rangle + b|11\rangle. \quad (3)$$

Thus, in the absence of any errors, the readout projects the data qubit into the observed eigenstate of the ancilla. And once the data qubit is projected to a computational basis state, it remains there forever.

In this work we study the effects of intrinsic gate errors and decoherence on this process. The remainder of the paper is organized as follows: In Sec. 2 we describe our physical model and consider ancilla-assisted measurement in the presence of decoherence. The non-ideal **CZ** gate is discussed in Sec. 3.1. Leakage errors and ancilla paralysis are discussed in Sec. 3.2. We discuss the implications of our results for the design of error-corrected superconducting quantum computers in Sec. 4.

2. Coupled qutrit model

In this section we describe our model, and for a warm-up, show how the ancilla-assisted measurement protocol works with ideal gates, but in the presence of decoherence.

2.1. Model

The Hamiltonian for a pair of capacitively coupled transmon or phase qutrits is given by

$$H(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & 2\epsilon_1 - \eta_1 \end{bmatrix}_{q_1} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & 2\epsilon_2 - \eta_2 \end{bmatrix}_{q_2} + gY \otimes Y, \quad (4)$$

where

$$Y \equiv \begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i\sqrt{2} \\ 0 & i\sqrt{2} & 0 \end{bmatrix}. \quad (5)$$

Qutrit 1 is the ancilla qutrit and qutrit 2 is the data qutrit. In (5) we have assumed harmonic qutrit eigenfunctions. The time-dependence of the Hamiltonian (4) is embedded in the qubit frequencies ϵ_1 and ϵ_2 ; the Hadamard gates are implemented with microwaves via terms not shown in (4). For the **CZ** gate protocol, we assume the

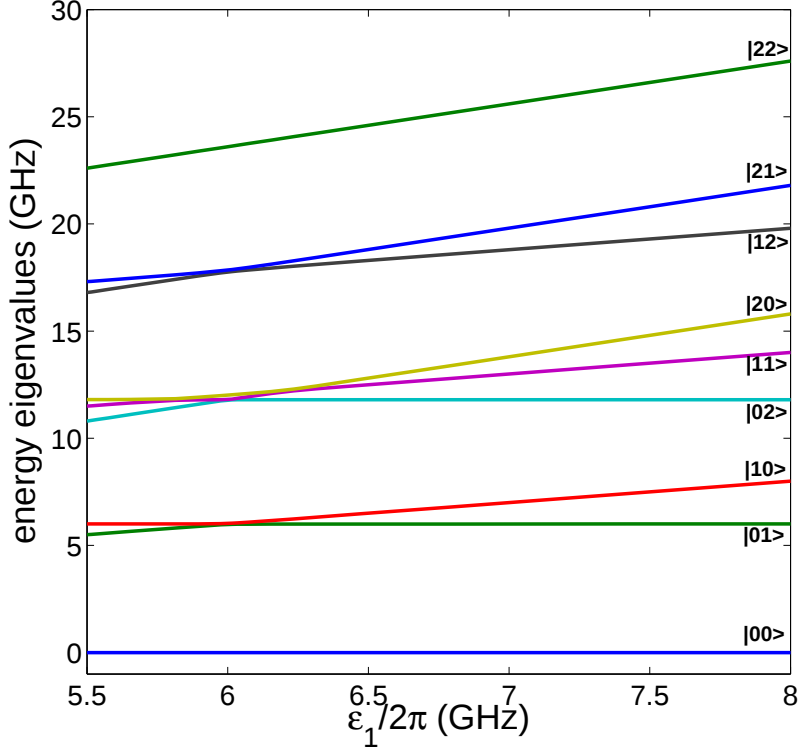


Figure 2. (Color online) Energies of various levels, in the $|AD\rangle$ basis, as a function of $\epsilon_1/2\pi$. Here $\epsilon_2/2\pi = 6$ GHz, the coupling strength is $g/2\pi = 25$ MHz, and $\eta_1/2\pi = \eta_2/2\pi = 200$ MHz.

frequency of the data qubit to be fixed at 6 GHz, while the ancilla's frequency is varied. The anharmonicities $\eta_i/2\pi$ are assumed to be equal, frequency-independent, and fixed at 200 MHz. Figure 2 shows the energies of several relevant eigenstates as a function of ϵ_1 , with $\epsilon_2/2\pi = 6$ GHz and coupling strength $g/2\pi = 25$ MHz. Note that the only anticrossing at $\epsilon_1 = \epsilon_2 + \eta_1$ ($\epsilon_1/2\pi = 6.2$ GHz in Fig. 2) is between the $|11\rangle$ and $|20\rangle$ channels; we use this anticrossing for our CZ gate [10, 11].

The CZ gate, both ideal and non-ideal, is parameterized in this work via its *generator*. A generator of any unitary matrix U is defined as a Hermitian matrix S such that $U = e^{iS}$. For a two-qutrit system, the generator of the ideal CZ gate is a Hermitian matrix S , whose matrix representation in the basis

$$|AD\rangle = \{ |00\rangle, |01\rangle, |02\rangle, |10\rangle, |11\rangle, |12\rangle, |20\rangle, |21\rangle, |22\rangle \} \quad (6)$$

is

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \pi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_4 \end{bmatrix}. \quad (7)$$

Note that within the computational subspace, e^{iS} acts as a standard CZ gate, while non-computational basis states acquire phases $e^{i\xi_i}$. We emphasize that any extension of an ideal CZ gate to qutrits is dependent on the assumed model and gate protocol. For the Strauch CZ gate, auxiliary σ^z rotations on the ancilla and data qubits nullify the phases acquired by the $|01\rangle$ and $|10\rangle$ channels. Since we use the anticrossing between $|11\rangle$ and $|20\rangle$, they acquire a phase of angle π . We assume that the gate is in the adiabatic regime, and that the parameters ξ_i are dynamical phases, which can then be expressed as

$$\begin{aligned} \xi_1 &\approx - \int_0^{t_{\text{gate}}} E_{02} dt = - \int_0^{t_{\text{gate}}} (2\epsilon_2 - \eta_2) dt, \\ \xi_2 &\approx - \int_0^{t_{\text{gate}}} E_{12} dt = - \int_0^{t_{\text{gate}}} (2\epsilon_2 - \eta_2) dt - \int_0^{t_{\text{gate}}} \epsilon_1 dt, \\ \xi_3 &\approx - \int_0^{t_{\text{gate}}} E_{21} dt = - \int_0^{t_{\text{gate}}} \epsilon_2 dt - \int_0^{t_{\text{gate}}} (2\epsilon_1 - \eta_1) dt, \\ \xi_4 &\approx - \int_0^{t_{\text{gate}}} E_{22} dt = - \int_0^{t_{\text{gate}}} (2\epsilon_2 - \eta_2) dt - \int_0^{t_{\text{gate}}} (2\epsilon_1 - \eta_1) dt. \end{aligned} \quad (8)$$

Here t_{gate} is the operation time for the CZ gate (including auxiliary z rotations), and E_{ij} is the energy of eigenstate $|ij\rangle$, shown in Fig. 2. To keep our analysis general we do not assume specific values for the ξ_i . They depend on the details of the CZ gate implementation but remain fixed throughout a given experiment or simulation (unless one changes t_{gate} or the pulse shape). As we will explain below, the difference

$$\theta \equiv \xi_2 - \xi_1 = - \int_0^{t_{\text{gate}}} \epsilon_1 dt \quad (9)$$

determines if the ancilla becomes paralyzed during a leakage event. Note that θ can be varied during an experiment by changing the gate time.

2.2. Ancilla-assisted measurement with decoherence

As shown in Fig. 1, each measurement cycle consists of ancilla initialization, three gate operations, and ancilla readout. Assuming ideal gates, the data qutrit after the first cycle is projected to a computational $|0\rangle$ or $|1\rangle$ state depending on the observed state of the ancilla [recall (3)]. In the absence of any errors, the measurement outcome of the ancilla remains unaltered thereafter. However, the situation is different in the presence of decoherence.

In order to model the effects of decoherence on the measurement outcomes of the ancilla, we assume that the readout and reset operations are instantaneous, while the Hadamard and CZ gates take 10 and 25 ns respectively. We also assume that amplitude damping is the only source of decoherence, in which case the single-qutrit Kraus matrices can be written as

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\lambda_1} & 0 \\ 0 & 0 & \sqrt{1-\lambda_2} \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & \sqrt{\lambda_1} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 & 0 & \sqrt{\lambda_2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (10)$$

For an operation of time duration Δt ,

$$\lambda_m = 1 - e^{-m \Delta t / T_1}. \quad (11)$$

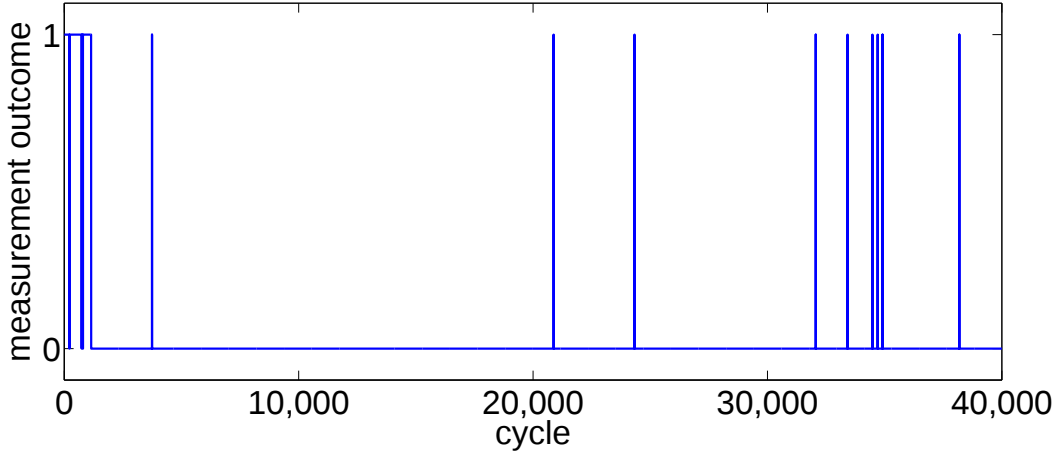


Figure 3. (Color online) Simulated repeated readout of the ancilla qutrit in the presence of amplitude damping. Single peaks, upward or downward, indicate errors on the ancilla. Data errors result in steps; an example is shown near cycle 1000. In this simulation we assume $T_1 = 40 \mu\text{s}$, $T_2 = 2T_1$, and $t_{\text{cycle}} = 45 \text{ ns}$.

We simulate the ancilla-assisted measurement protocol for an ideal CZ gate but in the presence of decoherence, for 40,000 consecutive cycles, and Fig. 3 shows a typical outcome. The duration t_{cycle} of each complete measurement cycle is 45 ns (one CZ gate plus two Hadamards). Initially, the data qutrit is in state $|1\rangle$, and a single downward peak denotes an error on the ancilla. Near the 1000th cycle the data qutrit relaxes to $|0\rangle$ due to decoherence, and once in the ground state it stays there forever. The remaining

upward peaks are caused by decoherence on the ancilla qutrit. Since the ancilla gets reset at the end of every cycle, such errors are manifested as single peaks. Note that if the initial state of the two-qutrit system is inside the computational subspace, it does not leak to non-computational states and therefore Fig. 3 is insensitive to the values of the ξ_i .

3. Non-ideal CZ gate

In this section, we first discuss how a non-ideal CZ gate is parameterized and then investigate its action on the ancilla-assisted qubit measurement.

3.1. Parameterization of the non-ideal CZ gate

Let us first give a brief review of the dominant intrinsic error mechanisms that are relevant for the Strauch CZ gate [11]; the Hadamards are always assumed to be ideal [see (2)]. The CZ gate of Strauch *et al.* [10] is performed by using the anticrossing between the $|11\rangle$ and $|20\rangle$ states at $\epsilon_1 = \epsilon_2 + \eta_1$. Although the other states are detuned from each other at this anticrossing point, a small amount of nonadiabatic population transfer is unavoidable, and these nonadiabatic excitations dominate the intrinsic gate errors. These errors can be thought of as producing a second unitary matrix whose generator S' can be parameterized, in the basis (6), as

$$S' = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \zeta_1 & 0 & i\chi_1 e^{i\phi_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i\chi_2 e^{i\phi_2} & 0 & 0 & 0 & 0 \\ 0 & -i\chi_1 e^{-i\phi_1} & 0 & \zeta_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i\chi_2 e^{-i\phi_2} & 0 & \zeta_3 & 0 & i\chi_3 e^{i\phi_3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i\chi_4 e^{i\phi_4} & 0 \\ 0 & 0 & 0 & 0 & -i\chi_3 e^{-i\phi_3} & 0 & \zeta_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i\chi_4 e^{-i\phi_4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (12)$$

The complete non-ideal CZ gate is

$$U_{\text{CZ}} = e^{i(S+S')}, \quad (13)$$

where S is the generator (7) of the ideal CZ gate. The parameters χ_i and ζ_i in (12) are small, while the angles ϕ_i take arbitrary values between 0 and 2π . ζ_1 and ζ_2 parameterize the errors occurring during pre and post σ^z rotations, and ζ_3 and ζ_4 denote the controlled-phase error for the $|11\rangle$ and $|20\rangle$ channels. In our simulations we assume $\chi_i = \zeta_i = 10^{-2}$ for all $i = 1, \dots, 4$. Because population transfer probability scales with $|\chi_i|^2$, our choice of parameters bounds the intrinsic gate errors to about 10^{-4} .

3.2. Leakage events and ancilla paralysis

The CZ gate (13) produces, on any $|11\rangle$ input component, a small amplitude of $|02\rangle$ (the amount determined by χ_2) and $|20\rangle$ (determined by χ_3). A $|20\rangle$ component either

results in the possibility of an ancilla readout of $|2\rangle$ —if the readout protocol distinguishes $|1\rangle$ and $|2\rangle$ —or the possibility of an isolated ancilla error if it does not. Neither case compromises fault-tolerance. The parameter χ_2 is responsible for data qubit leakage events. By a leakage *event* we mean a near-unity population of the data $|2\rangle$ state.

The principal mechanism producing a leakage event is the abrupt, nonlinear transformation on the data qutrit induced by the ancilla measurement. We denote these transformations by $\mathbf{T}_0$, $\mathbf{T}_1$, and $\mathbf{T}_2$, where the subscript corresponds to the ancilla readout result. Repeatedly measuring the ancilla applies a random sequence of the $\mathbf{T}$ maps to the data qutrit.

For the model, gate implementation, and parameter values considered in this work, the map $\mathbf{T}_0$ is primarily responsible for the observed leakage events. Although the general form of $\mathbf{T}_0$ is quite complex, it is possible to construct a simple special case of it that exhibits the essential features. To do this we choose simplified parameter values

$$\begin{aligned}\xi_1 &= \pi, \\ \phi_i &= 0, \\ \zeta_i &= 0, \\ \chi_3 &= 0, \\ \chi_4 &= 0,\end{aligned}\tag{14}$$

and calculate the action of the non-ideal measurement circuit on an arbitrary data qutrit state

$$|\psi_D\rangle = a|0\rangle + b|1\rangle + c|2\rangle.\tag{15}$$

We find (in the $|\text{AD}\rangle$ basis) that

$$\begin{aligned}a|00\rangle + b|01\rangle + c|02\rangle &\rightarrow |0\rangle \otimes \left[\left(\frac{a}{2} + \frac{a}{2} \cos \chi_1 + \frac{b}{2} \sin \chi_1 \right) |0\rangle \right. \\ &+ \left(\frac{b}{2} \cos \chi_1 - \frac{a}{2} \sin \chi_1 - \frac{b}{2} \cos \chi_2 - \frac{c}{2} \sin \chi_2 \right) |1\rangle + \left(\frac{c}{2} + \frac{b}{2} \sin \chi_2 \right. \\ &\left. \left. - \frac{c}{2} \cos \chi_2 \right) |2\rangle \right] + |1\rangle \otimes \left[\left(\frac{a}{2} - \frac{a}{2} \cos \chi_1 - \frac{b}{2} \sin \chi_1 \right) |0\rangle + \left(\frac{b}{2} \cos \chi_1 \right. \right. \\ &\left. \left. - \frac{a}{2} \sin \chi_1 + \frac{b}{2} \cos \chi_2 + \frac{c}{2} \sin \chi_2 \right) |1\rangle - \left(\frac{c}{2} - \frac{b}{2} \sin \chi_2 + \frac{c}{2} \cos \chi_2 \right) |2\rangle \right].\end{aligned}\tag{16}$$

An ancilla readout result of $|0\rangle$ then induces the map $\mathbf{T}_0$ given by

$$\begin{aligned}a &\rightarrow a' = \frac{a + a \cos \chi_1 + b \sin \chi_1}{\sqrt{\mathcal{N}}}, \\ b &\rightarrow b' = \frac{b \cos \chi_1 - a \sin \chi_1 - b \cos \chi_2 - c \sin \chi_2}{\sqrt{\mathcal{N}}}, \\ c &\rightarrow c' = \frac{c + b \sin \chi_2 - c \cos \chi_2}{\sqrt{\mathcal{N}}},\end{aligned}\tag{17}$$

where

$$\begin{aligned}\mathcal{N} &\equiv |a + a \cos \chi_1 + b \sin \chi_1|^2 + |b \cos \chi_1 - a \sin \chi_1 - b \cos \chi_2 - c \sin \chi_2|^2 \\ &+ |c + b \sin \chi_2 - c \cos \chi_2|^2.\end{aligned}\tag{18}$$

Using (17) we find that in the limit $\chi_1 = 0$ and $\chi_2 \rightarrow 0$ the data qutrit prepared in the $|1\rangle$ state transforms as

$$\mathbf{T}_0 |1\rangle = |2\rangle. \quad (19)$$

Our simulations confirm that the dominant mechanism for producing a leakage event is the process (19).

Once leaked, the data qutrit remains in the $|2\rangle$ state (for many cycles) until it either undergoes a nonadiabatic “reverse-leakage” transition or it relaxes back to the computational subspace. The behaviour of the ancilla during a leakage event depends on the values of ξ_1 and ξ_2 in (7). While the data qubit is in the $|2\rangle$ state, the two-qutrit system is restricted to the subspace spanned by

$$\{|02\rangle, |12\rangle\}, \quad (20)$$

because the $|22\rangle$ state is decoupled and remains unoccupied. In this subspace, the CZ gate (13) acts as

$$\exp \left[i \begin{pmatrix} \xi_1 & 0 \\ 0 & \xi_2 \end{pmatrix} \right], \quad (21)$$

and therefore performs a z rotation on the ancilla by an angle (9). The Hadamards in Fig. 1 convert this to an x rotation [see (9)]

$$e^{-i(\theta/2)\sigma^x} \quad (22)$$

acting on the initial ancilla state $|0\rangle$. Therefore, during a leakage event, while the data qubit is locked in the $|2\rangle$ state, the state of the ancilla after every cycle is

$$\cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} |1\rangle, \quad (23)$$

and upon measurement the ancilla qubit reads $|0\rangle$ with probability $\cos^2(\theta/2)$.

For example, if

$$\theta \bmod \pi = \frac{\pi}{2}, \quad (24)$$

we will observe random ancilla outcomes with equal probabilities for observing $|0\rangle$ and $|1\rangle$. This type of leakage event is simple to detect (and possibly correct). However, if

$$\theta \bmod \pi = 0, \quad (25)$$

then the ancilla will always read $|0\rangle$, cycle after cycle, giving no indication of the data error and thereby compromising fault-tolerance. We refer to this dangerous phenomena as ancilla *paralysis*.

Figure 4 shows the readout values generated from the sequential measurements of the ancilla qubit for different choices of θ , including all error process contained in the non-ideal CZ gate (13). While we observe random oscillations for larger values of θ , no such signature is present for $\theta = 0$. In order to quantify the paralysis of the ancilla we define a metric W , which is the average spacing—number of cycles—between consecutive readouts of $|1\rangle$. In the absence of decoherence, we can estimate it [see (23)] as

$$W = \csc^2(\theta/2), \quad (26)$$

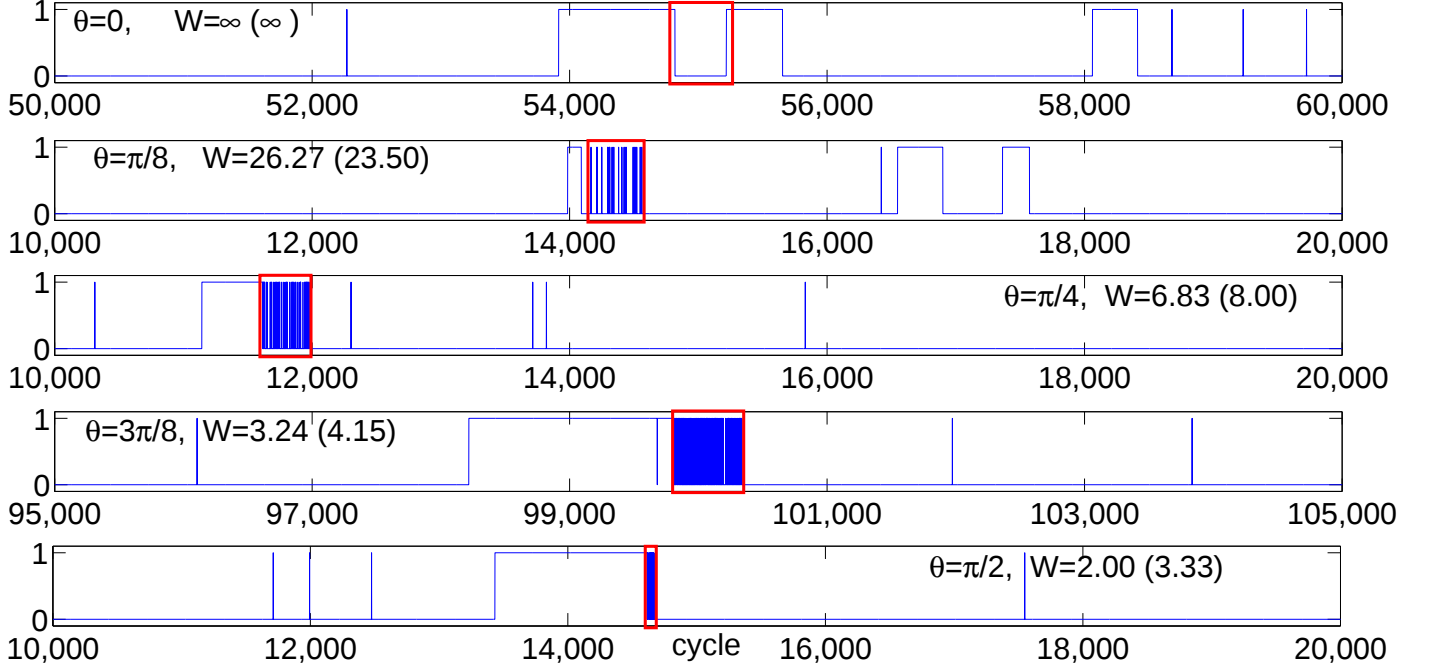


Figure 4. (Color online) Simulated sequential measurements of the ancilla qubit. The readout values $|0\rangle$ or $|1\rangle$ are shown as a function of measurement cycle number. Red rectangles signify leakage events, where the data $|2\rangle$ state probability is close to unity. Random ancilla oscillations during the leakage events are observed except when $\theta \approx 0$. Two values of W are given for each trace: the theoretical value from (26) and a value, shown in parentheses, numerically computed from the simulation. The simulations assume $T_1 = 40 \mu\text{s}$, $T_2 = 2T_1$, $\chi_i = \zeta_i = 10^{-2}$ for all $i = 1, \dots, 4$, and random values of phase angle parameters consistent with the indicated values of θ .

which agrees well with the numerical simulations.

The detectability of a leakage event depends on whether W is small enough to be observed in the presence of a background value W^* resulting from decoherence (and possibly other errors). For example, in the simulations of Fig. 4, which have $T_1 = 40 \mu\text{s}$ and $T_2 = 2T_1$, the average spacing between ancilla $|1\rangle$ peaks away from the leakage events is 2381 cycles, which is not too far from the crude theoretical estimate

$$W^* \approx \frac{2T_1}{t_{\text{cycle}}} = 1778, \quad (27)$$

using $t_{\text{cycle}} = 45 \text{ ns}$. The estimate in (27) can be derived from the Pauli twirling approximation for qubit decoherence [6], which predicts σ^x and σ^y errors on the ancilla with probability $p_x = p_y = t_{\text{cycle}}/4T_1$, leading to a total bit-flip probability $p_x + p_y$ of $t_{\text{cycle}}/2T_1$. We can use (26) to estimate the critical value of θ separating the region of dangerous ancilla paralysis and that of ordinary leakage, namely

$$\theta^* = 2 \csc^{-1} \sqrt{W^*} \approx 2 \csc^{-1} \left(\sqrt{\frac{2T_1}{t_{\text{cycle}}}} \right), \quad (28)$$

which is $\theta^* = 0.04$ in the simulations reported here. CZ gates with $\theta \bmod \pi < \theta^*$ are susceptible to undetectable leakage events.

4. Conclusion

We have studied the basic ancilla-assisted measurement circuit in the presence of leakage errors, and identified a rare but potentially dangerous ancilla paralysis effect that could compromise the error-detecting ability of a stabilizer measurement operation. Whether or not undetectable paralysis will occur depends on the difference (9) of phase angles produced by the CZ gate. Although fault-tolerance is compromised with either type of leakage event, the ability to detect such an event might allow one to reset the affected qubit to recover from it. We note that the value of θ^* is likely to be larger in a multi-qubit Pauli measurement because the cycle time is longer (the background value W^* is larger). Our results suggest that leakage be addressed either at the hardware level, by periodically removing any $|2\rangle$ state probability, or by using an architecture such as the 2D topological cluster code, where every qubit gets measured during the error-correction cycle. In addition, it is of course advantageous to adjust $\theta \bmod \pi$ to a safe value near $\pi/2$.

Acknowledgments

This research was funded by the US Office of the Director of National Intelligence (ODNI), Intelligence Advanced Research Projects Activity (IARPA), through the US Army Research Office grant No. W911NF-10-1-0334. All statements of fact, opinion or conclusions contained herein are those of the authors and should not be construed as representing the official views or policies of IARPA, the ODNI, or the US Government. Part of this work was carried out while M.G. was a Lady Davis Visiting Professor in the Racah Institute of Physics at Hebrew University, Jerusalem.

References

- [1] Dennis, E., Kitaev, A., Landahl, A., and Preskill, J. (2002) *Journal of Mathematical Physics* **43**(9), 4452–4505.
- [2] Raussendorf, R. and Harrington, J. (2007) *Phys. Rev. Lett.* **98**, 190504.
- [3] Raussendorf, R., Harrington, J., and Goyal, K. (2007) *New Journal of Physics* **9**(6), 199.
- [4] Fowler, A. G., Stephens, A. M., and Groszkowski, P. (2009) *Phys. Rev. A* **80**, 052312.
- [5] Fowler, A. G., Mariantoni, M., Martinis, J. M., and Cleland, A. N. (2012) *Phys. Rev. A* **86**, 032324.
- [6] Ghosh, J., Fowler, A. G., and Geller, M. R. (2012) *Phys. Rev. A* **86**, 062318.
- [7] Fowler, A. G. (2012) *Phys. Rev. Lett.* **109**, 180502.
- [8] Fowler, A. G., Whiteside, A. C., and Hollenberg, L. C. L. (2012) *Phys. Rev. A* **86**, 042313.
- [9] Fowler, A. G., Whiteside, A. C., and Hollenberg, L. C. L. (2012) *Phys. Rev. Lett.* **108**, 180501.
- [10] Strauch, F. W., Johnson, P. R., Dragt, A. J., Lobb, C. J., Anderson, J. R., and Wellstood, F. C. (2003) *Phys. Rev. Lett.* **91**, 167005.
- [11] Ghosh, J., Galiutdinov, A., Zhou, Z., Korotkov, A. N., Martinis, J. M., and Geller, M. R. (2013) *Phys. Rev. A* **87**, 022309.

- [12] Fowler, A. G., Whiteside, A. C., McInnes, A. L., and Rabbani, A. (2012) *Phys. Rev. X* **2**, 041003.
- [13] Gottesman, D. (1999) *Chaos, Solitons & Fractals* **10(10)**, 1749 – 1758.
- [14] Hostens, E., Dehaene, J., and De Moor, B. (2005) *Phys. Rev. A* **71**, 042315.
- [15] Bullock, S. S. and Brennen, G. K. (2007) *Journal of Physics A: Mathematical and Theoretical* **40(13)**, 3481.
- [16] Jafarizadeh, M. A., Najarbashi, G., Akbari, Y., and Habibian, H. (2008) *The European Physical Journal D* **47**, 233–255.
- [17] Chen, X., Zeng, B., and Chuang, I. L. (2008) *Phys. Rev. A* **78**, 062315.
- [18] Looi, S. Y. and Griffiths, R. B. (2011) *Phys. Rev. A* **84**, 052306.
- [19] Gheorghiu, V. (2011) *arXiv:1101.1519*.