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An Analysis of Martian Satellite Photographic Observations of 1967[1]

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Photographic observations of the Martian satellites were made at the opposition of 1967 with the Naval Observatory's 61-inch astrometric reflector. A small partially transparent metallic film filter was used to diminish the light from Mars in order that a measurable image for the planetary disk as well as for the satellites could be obtained. The plates were reduced by the method of plate constants using positions for the faint background stars determined from astrographic field plates. The random mean error of these observations was estimated to be not greater than $\pm 0''.10$.

The main result of the orbital adjustment is a +2° correction to the zero of mean longitude for Phobos. This confirms the findings of Wilkins (1970) and is compatible with the results of the Mariner 9 observations. The scale of the orbits of both satellites gave accordant values for the mass of Mars and the combined value of 30 99 500 ± 2800 (m.e.) is in good agreement with modern determinations.

The mean error for Deimos derived from the residuals after solution is $\pm 0''.11$, which agrees well with the observational error and indicates no large systematic error in either the theory or the observations. For Phobos, however, the residual error, $\pm 0''.19$, is twice the expected observational error. The implications of this discrepancy are discussed.

INTRODUCTION

In his analysis of all available observations of the Martian satellites Wilkins (1964, 1967, 1968) could neither confirm nor reject the secular accelerations in the longitudes of Phobos and Deimos found by Sharpless (1945) 20 years earlier. Although a sizable acceleration term was obtained in his formal solution, Wilkins was reluctant to accept its validity because the standard error of the solution had increased to $\pm 0''.55$ from the expected value of $\pm 0''.3$ for a solution using only the observations from a given opposition, telescope and observer (visual). The anomalous increase in the residuals was attributed to either an incomplete theory or systematic observational errors, or to both.

While a reinvestigation of the theory was in order, and has in fact been completed by Sinclair (1972), there are at least two indications that a large portion of the anomalous residuals are due to the observations. First, if the orbital models are too simple, it is expected that the effects would be more noticeable for Phobos than for Deimos. This is not the case; the standard error for Deimos is a little larger than that for Phobos. Second, the scales of the orbits of the two satellites have never been compatible with *one* mass for Mars nor with modern determinations of the mass.

Late in 1966 a satellite-observing program was reinitiated at the Naval Observatory with the primary aim of obtaining high-precision observations of the Martian moons. To date, the Martian satellites have been observed at four oppositions beginning with that of 1967. This paper discusses the 1967 observations.

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14. ABSTRACT

Photographic observations of the Martian satellites were made at the opposition of 1967 with the Naval Observatory's 61-inch astrometric reflector. A small partially transparent metallic film filter was used to diminish the light from Mars in order that a measurable image for the planetary disk as well as for the satellites could be obtained. The main result of the orbital adjustment is a +2 deg correction to the zero of mean longitude for Phobos. The scale of the orbits of both satellites gave accordant values for the mass of Mars, and the combined value is in good agreement with modern determinations. The mean error for Deimos derived from the residuals after solution is plus or minus 0.11 sec which agrees well with the observational error and indicates no large systematic error in either the theory or the observations. For Phobos however, the residual error, plus or minus 0.19 sec, is twice the expected observational error. The implications of this discrepancy are discussed.

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OBSERVATIONAL WORK

The superiority of the photographic technique for the Galilean satellites had been demonstrated early in the century. Its failure for the moons of Mars was due to the proximity of the satellites to the bright primary and to the inferior emulsions of that time. Although Sharpless made a considerable effort to photograph the moons during the oppositions of 1939, 1941, and 1943, he had very little success. Good photographs of the satellites were not obtained until 1954 by Kuiper (1961) and others. It is evident that a long-focus instrument is needed to diminish the intensity of the halo and that a moderate *f*-ratio is necessary to insure as short exposures as possible without undue loss of astrometric precision. The Naval Observatory's 61-inch astrometric reflector has these qualifications in addition to having astrometric stability and was therefore used to obtain the 1967 series.

The early success of the photographic technique in satellite astrometry was due largely to an orbital correction technique in which only intersatellite measures are used. In this scheme the observed and computed positions of a pair of satellites relative to each other are used to correct the parameters of both orbits simultaneously. Thus, the problem of making measurements on an overexposed primary is eliminated. This method however is not without drawbacks. The most obvious one is that there are now twice as many unknowns in each conditional equation to determine from the observations. This alone will degrade the precision of the corrections but, in addition to this, statistical correlations between the corrections to the parameters of both satellites will further confound the results. Another problem is encountered when the observations for one satellite are more accurate than those for the other or when the theory for one of the satellites is more complete than that for the other. This is the case for the Martian moons—the observational and theoretical problems are greater for Phobos than for Deimos. The result of an orbital adjustment for Deimos from observations relative to Phobos would not lead to an improvement in its orbit. Furthermore, neither the $(O-C)$'s nor the residuals after solution would give an indication as to whether the problem was observational, theoretical, or for that matter, which satellite was in error.

A more serious complication encountered in the use of intersatellite positions was pointed out by H. Struve (Laves, 1938). Struve showed that for evenly distributed observations the eccentricity and apse, as determined from intersatellite positions, have only 1/30 the weight of those obtained from planet–satellite positions. This is important since, if the eccentricity and apse are not well determined, then neither is the semimajor axis; thus the mass of the primary is also poorly determined.

It is clear from these arguments that observations of the satellites of Mars should be measurements relative to the primary. In order to obtain a measurable image of the primary a technique suggested to the writer by L. W. Frederick (personal communication, 1966) was used: A small partially transparent metallic film filter "spot" was deposited by evaporation onto the center of a GG14 filter. The metallic film, which is chromium or nichrome, transmits neutrally, is microscopically thin, and does not affect the resolution. Its optical density is about 3.0, which equalizes the intensity in the image of the primary to that in the satellites. In the observing procedure the small metallic film filter is placed over the primary. [For additional details and photographs see Pascu (1967)]. The plates are 103aJ in combination with the above-mentioned Schott GG14 filter.

For the 1967 opposition, 69 positions were obtained for Phobos and 78 for Deimos, on six nights and distributed over three weeks. The plates were reduced by the method of plate constants using the faint background stars. Positions for these faint stars were obtained from astrographic field plates.

The precision of the observations was estimated by two methods: First, a comparison of independent reductions of the direct and reverse measurements—accounting for the systematic portion—

gave a mean error of one satellite position with respect to Mars as ±0″085. This method also showed that the position of Mars could be obtained with the same precision as that of its satellites. The second method is more direct and involves the comparison of $(O-C)$'s for two exposures separated by only a few minutes. It is assumed here that the variation in two consecutive exposures is due to observational error only. In this way the mean error of one observation for Phobos was found to be ±0″08 and for Deimos ±0″10.

ORBITAL ADJUSTMENT

In the classical model, introduced by Struve (1898), the satellites move in Keplerian orbits which are inclined at a constant angle to the Laplacian or fixed plane. The nodes of the orbits on the Laplacian plane regress at a constant rate while the longitude of the apse increases at approximately the same rate. The orbital model is defined by the eleven parameters listed in Table I together with their initial values. These values are taken from Wilkins (1964) except for e, K, and π_R which are from Sinclair (1972).

In polar coordinates the conditional equations for the differential correction procedure are

$$\sum_{i=1}^{n} \frac{\partial p}{\partial \epsilon_i} \Delta\epsilon_i = (O-C)_p,$$

$$\sum_{i=1}^{n} \frac{\partial s}{\partial \epsilon_i} \Delta\epsilon_i = (O-C)_s,$$

where $\Delta\epsilon_i$ is the correction to the ith element, $\partial p/\partial\epsilon_i$ and $\partial s/\partial\epsilon_i$ are the partial derivatives of the position angle p and separation s with respect to the ith element, and n is the number of parameters of the model. The right-hand members, or the $(O-C)$'s in s and p are simply the differences between the observed and computed (predicted) values of the codinates. Analytical expressions for the partials have been derived previously (Pascu 1972).

From observations obtained in such a brief interval of time (three weeks), corrections to the motion in longitude and the motions of the node and apse cannot be determined. Furthermore, the small motion of the node during these observations indicates that large correlations will exist between the corrections to J_1 and j and the corrections to θ_0 and N_1. In order to derive as much information as possible

TABLE I

ASSUMED INITIAL VALUES FOR THE ORBITAL PARAMETERS OF THE SATELLITES OF MARS (EQUINOX AND EQUATOR OF 1950.0)

Epoch JED		Phobos (2414600.5)	Deimos (2414800.5)
1. a	Apparent semimajor axis at unit distance	12″91	32″36
2. e	Eccentricity	0.0184	0.0020
3. n	Daily mean motion in longitude	1128°8443	285°16192
4. l_0	Mean longitude at epoch	227°1	333°87
5. N_1	Longitude of node of fixed plane on equator	46°9	46°40
6. J_1	Inclination of fixed plane to equator	37°57	36°64
7. j	Inclination of orbital plane to fixed plane	0°9	1°80
8. θ_0	Argument of node of orbital plane on fixed plane	80°	358°3
9. K	Daily motion of node of orbit on fixed plane	−0°4354	−0°01813
10. π_0	Longitude of pericenter at epoch	211°	300°
11. π_R	Daily motion of pericenter	+0°4345	+0°01814

TABLE II

Epoch JED	Phobos 2414600.5	Deimos 2414800.5
a	12″927 ± 0″011	32″341 ± 0″010 (m.e.)
m^{-1a}	3097700 ± 7000	3099900 ± 3000
e	0.0225 ± 0.0008	0.0026 ± 0.0004
Δl_0	+2°10 ± 0°1	—
π_0	—	341° ± 5°
rms $(O-C)$before solution	± 0″31	± 0″14
rms $(O-C)$ after solution	± 0″19	± 0″11
Estimated obs. mean error	± 0″09	± 0″09
Combined mass	3099500 ± 2800	
Fly-by mass (Mariner IV) (Null 1969)	3098708 ± 9	

[a] Mass of Mars in reciprocal solar units.

from the observations and to test the reliability of the results a number of solutions were made. These trial solutions began with corrections to a, e, l_0, N_1, J_1, j, θ_0, and π_0 as unknowns. Unknowns that were indeterminate or highly correlated were then dropped in order that a convergent solution be obtained.

For Phobos, the root mean square (rms) $(O-C)$ before solution is ±0″31. The first solution, involving corrections to the eight parameters mentioned above, diverged since the corrections to j and θ_0 were indeterminate and the correction to π_0, while significant with respect to its error, was too large to be handled by a linear approximation. In order to obtain a better starting value for π_0, nine additional observations from 1969 and 1971 were added. While the accuracy of these observations is not as good as those of 1967, their weight in the solution for $\Delta\pi_0$ is greater. Corrections of −9° for π_0 and −13° for θ_0 were obtained from a convergent solution using these additional observations and including corrections to all eleven parameters of the orbit. A convergent solution was obtained for the 1967 observations with corrections to a, e, l_0, N_1, and J_1 using the corrected values for π_0 and θ_0. Significant corrections were obtained for a, e, and l_0 only and are given in Table II.

For Deimos the situation is different. The rms $(O-C)$ before solution for the 1967 observations is only ±0″14. In the orbital adjustment only corrections to a, e, and π_0 were determinate and the solution with these three parameters converged rapidly, giving the values in Table II.

CONCLUSIONS

There are a number of conclusions which can be drawn from the results presented in Table II concerning the accuracy of these observations. First, the good agreement between the recomputed rms $(O-C)$ after solution and the estimated observational error for Deimos indicates that there is no significant systematic error in the measured position of the primary [that is, there is no center-of-figure error (Pascu, 1975)]. Second, the agreement of the values of the mass of the primary, obtained independently from the orbital scales of each satellite, and the fly-by value indicates that there are no significant effects due to the photographic emulsion or to the halo of the planet.

The correction of +2° to the longitude of Phobos confirms the value found by Wilkins (1970) for 1967 and shows that, despite claims to the contrary, Earth-based observations can and have in fact detected the longitude error for Phobos. A longitude error of this magnitude tends to support the secular acceleration theory for Phobos although conclusive evidence of this can only come from a program of concentrated observation over a period of 10 to 25 years.

The fact that the residuals after solution (recomputed $(O-C)$'s) were comparable to the observational error for Deimos suggests that Struve's theory is sufficient to describe its motion even with the considerable increase in observational accuracy. On the other hand, the somewhat larger than expected value for the residuals after solution for Phobos means either that one of the highly correlated parameters (for the 1967 observations only) of the theory needs a substantial adjustment or that, in fact, these observations are precise enough to warrant the application of a more sophisticated theory, such as that of Sinclair (1972).

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