

| REPORT DOCUMENTATION PAGE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |             |                                |                               | Form Approved<br>OMB No. 0704-0188                         |                                           |
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| <p>The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to the Department of Defense, Executive Service Directorate (0704-0188). Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.</p> <p><b>PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ORGANIZATION.</b></p>                                                                                                                    |             |                                |                               |                                                            |                                           |
| 1. REPORT DATE (DD-MM-YYYY)<br>04/01/2005                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |             | 2. REPORT TYPE<br>Final report |                               | 3. DATES COVERED (From - To)<br>04/01/2005 05/31/2009      |                                           |
| 4. TITLE AND SUBTITLE<br>UNCERTAINTY MODELING IN THE ANALYSIS AND DESIGN OF DAMAGE TOLERANT COMPOSITE STRUCTURAL SYSTEMS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |             |                                |                               | 5a. CONTRACT NUMBER<br>FA9550-05-1-0140                    |                                           |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |             |                                |                               | 5b. GRANT NUMBER                                           |                                           |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |             |                                |                               | 5c. PROGRAM ELEMENT NUMBER                                 |                                           |
| 6. AUTHOR(S)<br>P. Hajela and G. Dvorak                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |             |                                |                               | 5d. PROJECT NUMBER                                         |                                           |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |             |                                |                               | 5e. TASK NUMBER                                            |                                           |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |             |                                |                               | 5f. WORK UNIT NUMBER                                       |                                           |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br>Rensselaer Polytechnic Institute<br>110 8th street<br>Troy, NY 12180-3590                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |             |                                |                               | 8. PERFORMING ORGANIZATION<br>REPORT NUMBER                |                                           |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)<br>Air Force Office of Scientific Research<br>875 North Randolph Street,<br>Arlington, VA 22203                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |             |                                |                               | 10. SPONSOR/MONITOR'S ACRONYM(S)                           |                                           |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |             |                                |                               | 11. SPONSOR/MONITOR'S REPORT<br>N AFRL-OSR-VA-TR-2013-0944 |                                           |
| 12. DISTRIBUTION/AVAILABILITY STATEMENT<br>Distribution Statement A: Approved for public release. Distribution is unlimited.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |             |                                |                               |                                                            |                                           |
| 13. SUPPLEMENTARY NOTES                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |             |                                |                               |                                                            |                                           |
| <b>14. ABSTRACT</b><br><p>The final stage of the proposed research was completed during this performance period. The focus of this work was on integrating the composite damage analysis programs into a framework where they could be effectively used for design optimization. Critical to this phase of the work was a problem formulation that took into consideration the multiscale nature of the damage process, and fully accounted for the residual life in the composite structure in a post-failure (local failure mode). A system effectiveness approach was developed as a logical framework for the design problem formulation. This approach fully accounts for the probabilities of going from one state of structural damage to another. The problem formulation also allows for creating objective criteria that include system reliability and residual life in the structure. The approach was demonstrated using simple composite design problems.</p> |             |                                |                               |                                                            |                                           |
| 15. SUBJECT TERMS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |             |                                |                               |                                                            |                                           |
| 16. SECURITY CLASSIFICATION OF:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |             |                                | 17. LIMITATION OF<br>ABSTRACT | 18. NUMBER<br>OF<br>PAGES                                  | 19a. NAME OF RESPONSIBLE PERSON           |
| a. REPORT                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | b. ABSTRACT | c. THIS PAGE                   |                               |                                                            | 19b. TELEPHONE NUMBER (Include area code) |

**Performance Report**

**FA 9550-05-1-0140**

**Uncertainty Modeling in the Analysis and Design of Damage Tolerant  
Composite Structural Systems**

By

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20130919094

## Summary Statement

This document summarizes some of the key finds of the AFOSR research project. The report is divided into four sections, each addressing the major new contributions that resulted from the work

- a. Section I describes the progress in the development of the transformation field analysis (TFA) based progressive damage model, referred to as the micro-TFA model, for laminated composites.
- b. Section II describes the uncertainty propagation across different scales of the model, and how it can be used to predict degraded performance of the composite with a defined probability. It also illustrates the need for specialized techniques to handle uncertainty in predicting reliability, as well-known distributions of uncertainty in basic material properties may yield quite varied distributions in composite strength.
- c. Section III addresses one issue of incorporating this analysis model in a design optimization environment – that of the high cost of analysis and the need to create surrogate representations of the exact damage progression analysis. A new approach referred to as a Local Matamodeling-Based Uncertainty Characterization (LMUC) for reducing the computational cost of performing Monte Carlo simulations on the micro-TFA model is proposed.
- d. In Section IV, a design problem formulation based on a state transition approach is introduced, which allows for the handling of multiple failure modes in a rational manner. This methodology termed as a system effectiveness approach, models designer preference as to acceptability of degraded performance, and is used to develop optimal designs. A comparison of these designs against those obtained from a more widely used competing risk methodology provides insight into the advantages of the new approach.

## Personnel

The following personnel were involved in the project at various stages of the program.

Drs. P. Hajela and G. Dvorak, Professors, MANE Department, Rensselaer Polytechnic Institute

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Mr. Varun Sakalkar, Doctoral Student, MANE Department, Rensselaer Polytechnic Institute (May 15, 2007 – present; expected graduation is February 2010)

## **Publications**

The following major publications resulted from the work, two are pending journal reviews and possible publication in the archival literature.

A. Mullur, P. Hajela, and Yehia Bahei-El-Din "Uncertainty Management in Design Optimization of Coupled Systems," Proceedings of the 11<sup>th</sup> AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Portsmouth, Virginia, September 2006.

P. Hajela and A. Mullur, "Modeling and Management of Uncertainties in Multidisciplinary Design Optimization," Plenary Lecture, 2nd LNCC Meeting on Computational Modeling, Petropolis, Brazil, August 2006.

P. Hajela and A. Mullur, "Uncertainty Modeling in Structural Design: Recent Developments," P. Hajela and A. A. Mullur, Innovation in Computational Structures Technology, eds. B.H.V. Topping, G. Montero, and R. Montenegro, pp. 51-74, Saxe-Coburg Publications, Kippen, UK, 2006.

P. Hajela and S. Vittal, "Optimal Design in the Presence of Modeling Uncertainties," ASCE Journal of Aerospace Engineering, pp. 204-216, Vol. 19, No. 4, October 2006.

A. Mullur, Y. Bahei-El-Din, P. Hajela, J. Peters and G. Dvorak, "Nondeterministic Modeling of Progressive Failure in Laminated Composites", proceedings of the 47th AIAA/ASME/ASCE/AHS/ASC SDM Conference, Newport, Rhode Island, May 2006.

R. Khire, P. Hajela, and Yehia Bahei-El Din, "Handling Uncertainty Propagation in Laminated Composites Through Multiscale Modeling of Progressive Failure," proceedings of the 48<sup>th</sup> AIAA/ASME/ASCE/AHS SDM Conference, April 22-25, 2007, Honolulu, Hawaii.

P. Hajela and V. Sakalkar, "Uncertainty Modeling in Composite Structural Design," presented at the 7th ASMO UK Engineering Design Optimization Conference, Bath, UK, July 2, 2008.

V. Sakalkar and P. Hajela, "Nondeterministic decomposition based structural design optimization", proceedings of the 49<sup>th</sup> AIAA/ASME/ASCE/AHS SDM Conference, Schaumburg, Illinois, 2008.

P. Hajela, A. Mullur, and V. Sakalkar, "Multidisciplinary Analysis and Design: Tools for Uncertainty Modeling, Journal of Aerospace Sciences and Technologies, Vol 61, No. 1, pp 240-251, February 2009.

V. Sakalkar and P. Hajela, "State Transition Approach to Reliability Based Design of Composite Structures", to appear in proceedings of the 50<sup>th</sup> AIAA/ASME/ASCE/AHS SDM Conference, Palm Spring, California, 2009

The complete report was too big to upload to the website and has been e-mailed to the Program Manager. Only the final section of the report that deals with new contributions for the year are included in this segment.

#### Section IV. System Reliability Assessment

A structural system composed of more than one structural element may also include multiple modes of failure, such as those resulting from large deflections, or from buckling, shear, and corrosion, among others. Evaluation of the reliability of such a system involves multiple limit state functions, and obtaining independent failure probability estimates for each constraint. Component-level reliabilities can be estimated using the techniques standard analytical techniques (FORM, SORM etc) or Monte Carlo simulations. The final system reliability is likely to be a function of the probabilities for the individual modes of failure, and any correlation between the different modes. The failure time of a system with two or more failure modes can be modeled with a series-system or competing risk model. Each risk is like a component in a series system. When one component fails, the system (i.e., product) fails. Each unit has a potential failure time associated with each failure mode. The observed failure time is the minimum of these individual potential failure times. Most generic system reliability techniques, such as the failure mode approach, may not be adequate for large-scale aerospace systems, where multiple engineering teams work concurrently on different aspects of the design. Instead, a dedicated approach that caters to their specific needs may need to be developed. A state transition approach to evaluate system reliability or effectiveness is proposed in this context. This approach takes into consideration the probabilities of a system transitioning from one failure state to another in a systematic manner.

The state transition method is an alternative approach for estimating risk in a system with multiple failure modes (limit states). The concept is loosely patterned on ideas developed in the military operations research community in the field of system effectiveness analysis. The approach is particularly significant in that it can compute the probability of a system existing in different states (failure modes) as a function of system operation history and failure mode relationship. This makes it possible to develop accurate risk models and "manage" the life of the system in a manner that maximizes usage and minimizes risk. In the context of composite design using TFA, each failure mode corresponds to progressive damage in the various constituent plies. Effectiveness is defined as a product of availability (A), the dependability (D), and the capability (C) of the system under consideration, and expressed mathematically as the following matrix product.

$$E = \{A_1 \quad A_2 \quad A_3 \quad \dots \quad A_i\} \begin{bmatrix} D_{11} & D_{12} & \dots & D_{1j} \\ D_{21} & D_{22} & \dots & D_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ D_{i1} & D_{i2} & \dots & D_{ij} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_j \end{bmatrix} \quad (5)$$

In Eq. 5, 'E' is the System effectiveness and is a measure of the extent to which a system will perform its mission; it is function of the probability to survive various states of damage, as well as the consequence of arriving in a particular state (condition). 'A' is the availability, a measure of the system condition at the start of a mission. For the purpose of the system degradation, it is the probability that a system is put into service in various conditions ranging from new (full life), to a fully degraded state with no life.  $A_i$  is the

probability of a system starting its mission state. In context of composites it can be the probability of a particular ply to be available at any point of time during its service life. 'D' is the dependability indicated by a transition matrix, where elements of the matrix  $d_{ij}$  represent the probability of a system starting in state 'i' and ending in state 'j'. 'C' is the capability, a measure of the system's ability to achieve a mission objective. For system-lifing applications, a system with full life will complete its desired objective that may involve supporting given loads or surviving applied temperature gradients.

In the context of a composite design problem, each of the failure modes like the ones described in Fig. 2 can be included in the design problem. The feasibility of this approach can be illustrated using a composite design problem involving two failure modes as follows.

**Mode I:** The failure strain at the point of first failure ( $\epsilon^{f1}$  in Fig. 2) does not exceed an allowable value (0.5%).

**Mode II:** The failure strain at the point of second failure ( $\epsilon^{f2}$  in Fig. 2) does not exceed an allowable value (0.75%).

The suitability of system effectiveness approach in probabilistic optimization is illustrated through a comparative study of deterministic and nondeterministic design optimization solutions as follows.

**Case A:** Deterministic optimization formulation - the composite design problem is first formulated as a deterministic multiobjective optimization problem that seeks to minimize the weight of the laminate and maximize the modulus of elasticity ( $\bar{E}_{11}$ ) in the direction of unidirectional in-plane loading (11-direction from Fig. 2). The design variables are chosen as the volume fraction of the fiber material ( $c_r$ ) and the ply angle  $\phi$  in a  $[0, +/\phi, 90]_s$  laminate. The design problem statement is explicitly stated as follows.

$$\begin{aligned}
 & \text{Min : Weight} \\
 & \text{Max : } \bar{E}_{11} \\
 & \text{s.t.} \\
 & \epsilon^{f1} \leq \epsilon^{allow1} \\
 & \epsilon^{f2} \leq \epsilon^{allow2} \\
 & 0.2 \leq c_r \leq 0.9 \\
 & 30 \leq \phi \leq 60, \text{Integer}
 \end{aligned} \tag{6}$$

This is a design problem with both continuous and integer design variables. A genetic algorithm (GA) based optimization solution was implemented in this problem given the mixed nature of the design variables and expected multimodalities in the design space. The GA based approach required a very large number of function evaluations and the use of the RBF approximation was critical to this implementation. This formulation gives the Pareto optimal set of solutions shown in Fig. 7, and which will be used for comparison with nondeterministic cases.

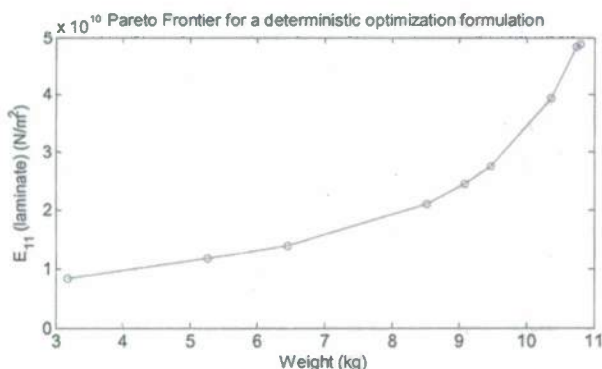


Figure 7. Pareto optimal solutions for the

**Case B:** Nondeterministic optimization formulation (reliability constraint) - the composite optimization problem is formulated as a non deterministic multiobjective optimization problem seeking to minimize the mean weight ( $\text{Weight}^{\mu}$ ) of the laminate while also maximizing the mean axial modulus of

elasticity ( $\bar{E}_{11}^\mu$ ) of the laminate. The design variables are the volume fraction of the fiber material and the ply angle  $\phi$  in a  $[0, +/-\phi, 90]$ s laminate. The design problem statement is explicitly stated as follows.

Min:  $Weight^\mu$

Max:  $\bar{E}_{11}^\mu$

s.t.

$R_{system} \geq R_{min}$ ; where  $R_{system} = (1 - P_I)(1 - P_{II}) = (1 - P_f(\bar{\epsilon}^{f1} \leq \bar{\epsilon}^{allow1}))(1 - P_f(\bar{\epsilon}^{f2} \leq \bar{\epsilon}^{allow2}))$

$0.2 \leq \bar{c}_r^\mu \leq 0.9$

$30 \leq \phi \leq 60, \text{Integer}$

(7)

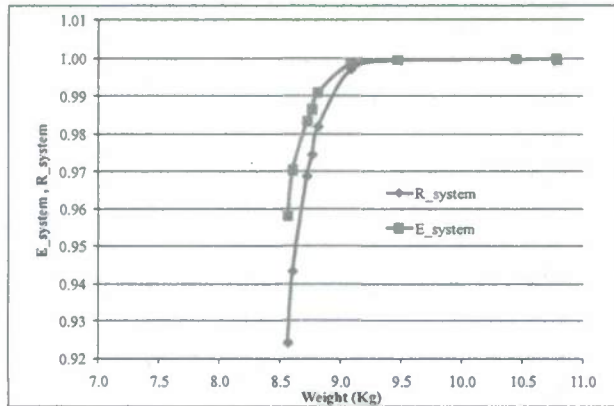


Figure 8. Pareto solution for Case B formulation

Here,  $\bar{c}_r$  is a normal random variable  $\bar{c}_r = N(\bar{c}_r^\mu, 0.01)$  with its mean value ( $\bar{c}_r^\mu$ ) as the design variable in the optimization problem. The allowable failure strains follow a normal distribution, with values as  $\bar{\epsilon}^{allow1} = N(0.5, 0.05)$ ;  $\bar{\epsilon}^{allow2} = N(0.75, 0.075)$ . The system reliability (using Eq. 19) and system effectiveness (using Eq. 15) are computed for each of the solutions on the Pareto front. The results obtained are presented in Table 2 and also shown on Fig.8.

Table 2 Pareto solution for Case B formulation

| Volume Fraction | Ply angle | Weight | Laminate Axial Stiffness | $R_{\text{system}}$ | $E_{\text{system}}$ |
|-----------------|-----------|--------|--------------------------|---------------------|---------------------|
| 0.898           | 57        | 10.780 | 4.979E+10                | 1.000               | 1.000               |
| 0.898           | 56        | 10.775 | 4.949E+10                | 1.000               | 1.000               |
| 0.870           | 57        | 10.445 | 4.114E+10                | 1.000               | 1.000               |
| 0.789           | 57        | 9.465  | 2.766E+10                | 1.000               | 1.000               |
| 0.757           | 56        | 9.081  | 2.456E+10                | 0.997               | 0.999               |
| 0.734           | 56        | 8.811  | 2.278E+10                | 0.982               | 0.991               |
| 0.730           | 55        | 8.763  | 2.248E+10                | 0.975               | 0.986               |
| 0.727           | 57        | 8.720  | 2.224E+10                | 0.969               | 0.983               |
| 0.717           | 57        | 8.604  | 2.158E+10                | 0.943               | 0.970               |
| 0.714           | 55        | 8.563  | 2.135E+10                | 0.924               | 0.958               |

**Case C:** Nondeterministic optimization formulation (effectiveness constraint) – this case involves the solution of a multiobjective optimization problem as before, with the addition of system effectiveness based constraints. The problem formulation is as follows.

$$\begin{aligned}
 & \text{Min : } Weight^{\mu} \\
 & \text{Max : } \bar{E}_{11}^{\mu} \\
 & \text{s.t.} \\
 & E_{\text{system}} \geq E_{\min} \\
 & 0.2 \leq \tilde{c}_r^{\mu} \leq 0.9 \\
 & 30 \leq \phi \leq 60, \text{Integer}
 \end{aligned} \tag{8}$$

This problem formulation is the same as in case (B) except that reliability constraint is replaced by effectiveness constraint. The results for this problem formulation are summarized in Table 3 and depicted graphically in Fig. 9.

**Table 3 Pareto solutions for Case C formulation**

| Volume Fraction | Ply angle | Weight | Laminate Axial Stiffness | $R_{\text{system}}$ | $E_{\text{system}}$ |
|-----------------|-----------|--------|--------------------------|---------------------|---------------------|
| 0.899           | 56        | 10.784 | 4.977E+10                | 1.000               | 1.000               |
| 0.896           | 56        | 10.755 | 4.886E+10                | 1.000               | 1.000               |
| 0.880           | 55        | 10.563 | 4.358E+10                | 1.000               | 1.000               |
| 0.863           | 56        | 10.354 | 3.925E+10                | 1.000               | 1.000               |
| 0.777           | 57        | 9.330  | 2.648E+10                | 0.999               | 0.999               |
| 0.730           | 57        | 8.763  | 2.250E+10                | 0.975               | 0.987               |
| 0.728           | 57        | 8.741  | 2.237E+10                | 0.972               | 0.985               |
| 0.715           | 57        | 8.584  | 2.147E+10                | 0.938               | 0.967               |
| 0.710           | 56        | 8.517  | 2.111E+10                | 0.915               | 0.956               |
| 0.695           | 57        | 8.337  | 2.021E+10                | 0.825               | 0.909               |

#### 4.1 Discussion

In Tables 2 and 3, it is evident that the reliability is bounded between 0.825 and 1.0, and effectiveness is bounded between 0.909 and 1.0. For the same design on the Pareto front, the reliability is always lower than effectiveness. This is due to the fact that competing risk model emphasizes both failure modes equally whereas the state transition approach places less emphasis on the first failure mode. Thus, the

effectiveness metric provides a more realistic estimate of the true system capability and is based on the acceptance by the designer that the residual load carrying capacity of the structure continues to be significant. This is also demonstrated in the second formulation with the effectiveness constraint. The designs generated in Tables 2 and 3 are in general lighter than the designs generated by the reliability based approach. In this case study, first failure strain was considered a benign failure, i.e., from operational perspective a laminate with initial strain failure has still completed a part of its mission. This "risk reduction" is captured by the effectiveness-based formulation and is finally reflected in the optimal designs produced.