Mario Zampolli · **Alessandra Tesei** · **Finn B. Jensen**

Finite element models for the solution of underwater acoustics problems

Abstract Acoustic waves are used in SONAR systems for the detection and classification of objects which may be swimming in the water column, proud on the seafloor, or buried below the sediment. In some applications, such as low frequency detection and classification of structurally complex objects immersed in a fluid, signal processing techniques must be aided by a-priori model based knowledge of the target echo. Because objects of interest in practical applications have a complex geometry and a detailed internal structure, consisting of elastic materials and fluid partitions, mathematical techniques capable of dealing with generic 3-D geometries, and capable of coupling different physical domains (i.e. the solid and the fluid domains) must be used. For these reasons, the Finite Element (FE) Method is the technique of choice for many researchers in the field. A FEMlab application for computing the scattering from undersea targets is presented, and guidelines for the discretization of the computational domain in the presence of structure-borne guided elastic waves are given. It is found that the guided elastic waves impose strict requirements on the meshing of the elastic structure *and* of the surrounding water volume. The presented results are relevant also to other areas of acoustics, where one is interested in modeling scattering or radiation from elastic structures and within elastic structures. Such areas include non destructive testing, vehicle noise prediction and control, acoustic transducer design, MEMS, architectural acoustics, and medical acoustics.

Keywords acoustics · structural acoustics · resonances · Lamb waves

1 Introduction

The modeling of scattering and radiation from complex 3-D structures is a challenging computational task. This paper discusses an example of scattering from a submerged void cylindrical shell with hemispherical endcaps (see Fig. 1), which exhibits an apparently peculiar convergence behavior. It is found that the FE convergence behavior described in Section 2 is determined by the need of discretizing short-wavelength guided elastic waves (called Lamb waves [1], [2], [3]), which are present in elastic structures of finite thickness [4], [5]. Capturing the Lamb wave resonances in the frequency domain is crucial for determining the time-domain echo scattered by a target.

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Section 3 presents discretization guidelines which ensure FE convergence at the Lamb wave resonances, and hence enable the researcher to synthesize the time domain echoes by use of the inverse Fourier transform. For verification, the FEMlab computational results are compared to results obtained from a non commercial structural acoustics finite-element tool [7], developed at the NATO Undersea Research Centre. The reference tool [7] is based on a hierarchic polynomial shape function development framework [8]. A verification study, in which the reference tool [7] has been successfully compared to other standard tools used in the underwater acoustics community has been published in [4]. In particular, the reference solutions shown in this paper have been verified in [4]. To obtain the results presented here, both tools, FEMlab and the reference tool, have received the same structured mesh as an input.

Future work is identified and conclusions are given in Section 4. The results show that the FEMlab multiphysics tool has the capability for modeling complex structural acoustic scattering, propagation and radiation phenomena.

2 Problem description, model and convergence issues

Figure 1.a shows the cylindrical target made of steel (modeled as isotropic elastic solid), which is immersed in water. The water domain surrounding the target (Fig. 1.a and 1.b) is modeled with the 3-D acoustics application mode of FEMlab, and the target is modeled with the 3-D solid stress-strain application mode of FEMlab. Details on how the physics modes in the fluid and solid subdomains are set up can be found in the FEMlab Structural Mechanics Module documentation [6]. The steel shell is 1mm thick, with hemispherical endcaps of outer radius $a = 9.08$cm. The end-to-end length of the cylinder is $L = 60$cm. The inside of the cylinder is considered to

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14. ABSTRACT
Acoustic waves are used in SONAR systems for the detection and classification of objects which may be swimming in the water column, proud on the seafloor or buried below the sediment. In some applications, such as low frequency detection and classification of structurally complex objects immersed in a fluid, signal processing techniques must be aided by a-priori model based knowledge of the target echo. Because objects of interest in practical applications have a complex geometry and a detailed internal structure, consisting of elastic materials and fluid partitions, mathematical techniques capable of dealing with generic 3-D geometries, and capable of coupling different physical domains (i.e. the solid and the fluid domains) must be used. For these reasons the Finite Element (FE) Method is the technique of choice for many researchers in the field. A FEMlab application for computing the scattering from undersea targets is presented, and guidelines for the discretization of the computational domain in the presence of structureborne guided elastic waves are given. It is found that the guided elastic waves impose strict requirements on the meshing of the elastic structure and of the surrounding water volume. The presented results are relevant also to other areas of acoustics, where one is interested in modeling scattering or radiation from elastic structures and within elastic structures. Such areas include non destructive testing, vehicle noise prediction and control acoustic transducer design, MEMS, architectural acoustics and medical acoustics.

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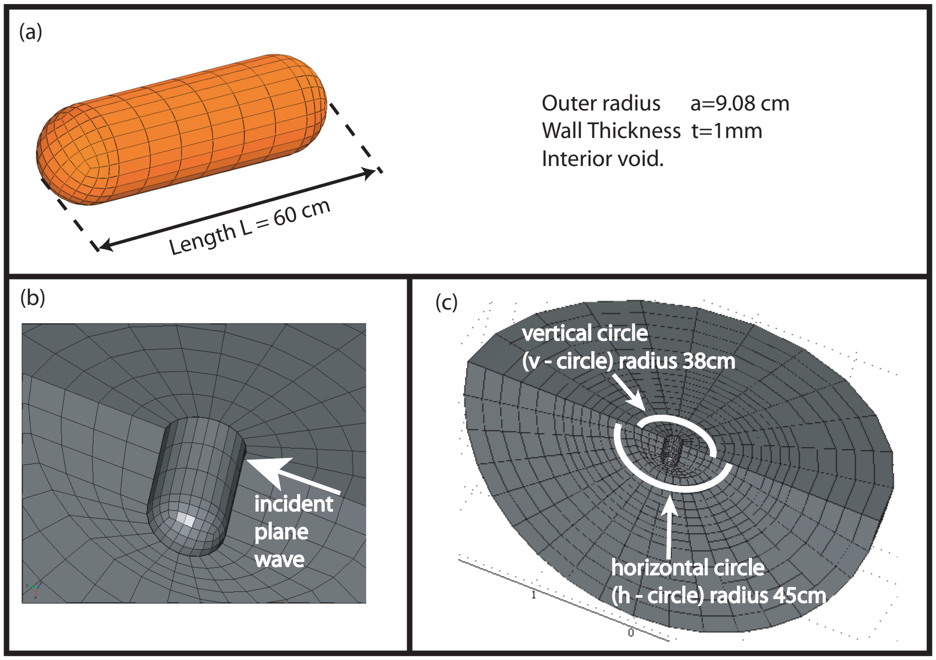

Fig. 1 (a) Dimensions of the empty cylindrical shell. (b) Details of the FEMlab structured mesh in the vicinity of the target, with incident plane wave denoted by arrow. (c) Full mesh, with vertical and horizontal circles (labeled "v-circle" and "h-circle" respectively) on which the acoustic near field is sampled.

be a vacuum, and hence the natural stress-free conditions for the free faces of solid finite elements hold on the interior surface of the cylinder. A plane wave, described by a known function $p_{\text{inc}} = \exp(-iky)$, where k is the acoustic wavenumber, is incident on the cylinder from broadside along the direction indicated by the arrow in Fig. 1.b.

The discretized field variable is scattered pressure, p_{scat}, which is defined by the relation

$$p = p_{\text{scat}} + p_{\text{inc}} \,, \tag{1}$$

where p is the total acoustic pressure in the fluid medium. The total field p, as well as the incident field p_{inc}, both satisfy the Helmholtz equation

$$(\nabla^2 + k^2)p = 0 \,, \; k = \frac{2\pi f}{c} \tag{2}$$

(where k is the wavenumber, f is the frequency, and c is the soundspeed of water). From the linearity of the Helmholtz equation, it follows that also p_{scat} satisfies Eq.(2), and thus it is only necessary to solve for p_{scat}, instead of solving for $p_{\text{scat}} + p_{\text{inc}}$. Hence, Eq. 2 can be written with p_{scat} instead of p, and the known incident pressure p_{inc} is applied via the boundary conditions at the interface between the solid and the fluid. This approach makes it possible to obtain generally more accurate results compared to those obtained by solving for the total field p. Physically, the scattered field is the quantity more closely related to the monostatic echo. To model the scattered field in FEMlab, one simply uses the setup provided by the FEMlab acoustics mode, as described in [6]. The only difference compared to the approach described in [6] is that the fluid/solid subdomain coupling conditions must take p_{inc} and the normal derivative of p_{inc} with respect to the coupling surface into account. Furthermore, the incident field must be set to zero in the spherical radiation BC's of FEMlab, which are applied on the spherical boundary of the computational domain (Fig.1.c) to approximate the radiation condition for p_{scat}. The radiation BC boundary is located approximately at a distance of 4 acoustic wavelengths from the cylindrical target.

Because the problem is symmetric with respect to two planes passing through the origin and containing the wavenumber vector of the incident plane wave, with one such plane being perpendicular to the cylinder axis, and the other one being perpendicular to the cylinder radius, it is sufficient to model only one quarter of the geometry, as shown in Fig. 1.c. The symmetry of the solution is enforced using homogeneous Neumann BC's on the intersections of the symmetry planes with the spherical computational fluid domain, and using zero normal elastic displacement conditions (Dirichlet type) on the two surface strips of finite thickness obtained by intersecting the cylindrical target (elastic domain) with the symmetry planes.

Computations are performed at three different frequencies, namely $f_1 = 2.5$kHz, $f_2 = 2.895$kHz and $f_3 = 3.5$kHz. The frequency f_2 corresponds to the location of a Lamb wave mode resonance [4]. Examples of the FEMlab output obtained for computations at a frequency of 3.5kHz are presented in Fig. 2. The quantity of interest for the present study is the target strength (TS):

$$TS = 20\log_{10}(R\,|p_{\text{scat}}|) \,, \tag{3}$$

where R is the observation range, *i.e.* the radius of one or the other circular path described in Fig. 1.c. TS is measured in decibels [dB].

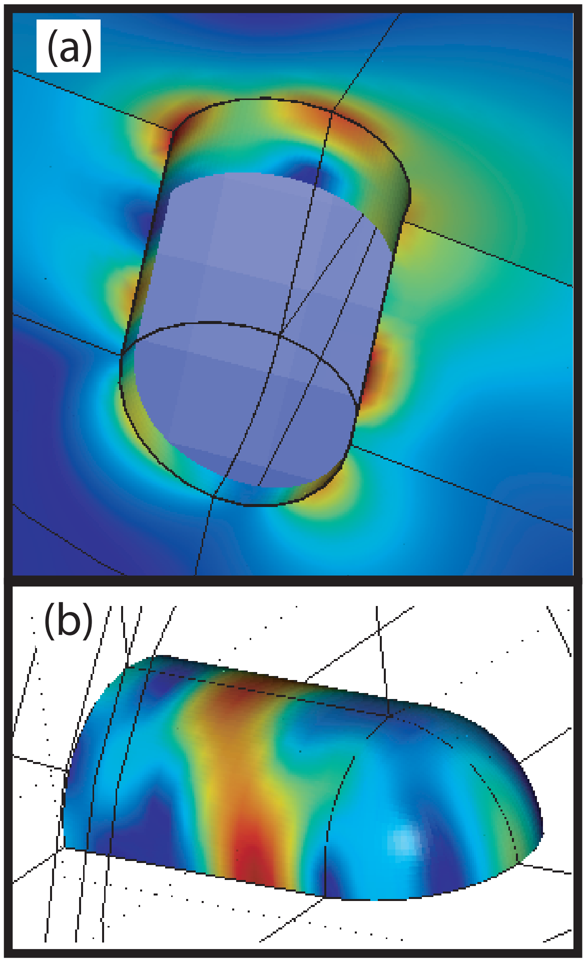

Fig. 2 FEMlab results at $f = 3.5$kHz. (a) magnitude of acoustic pressure in the near field, (b) von Mises stress on the surface of the elastic target.

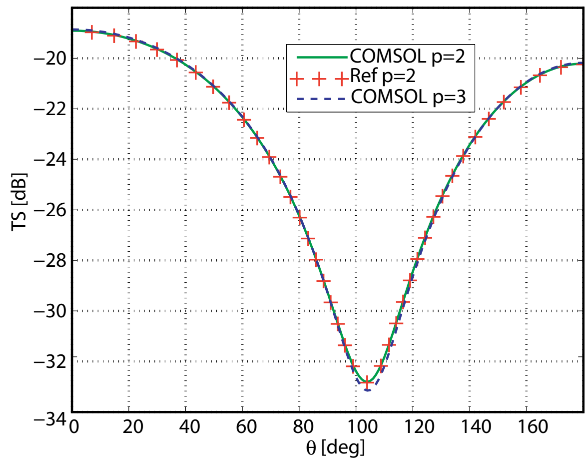

Fig. 3 Results on the v-circle at $f = 2.5$kHz. Agreement between “COMSOL p=2” (solid) result and the “Ref p=2” (crosses) is so close that the two curves cannot be distinguished on the plot.

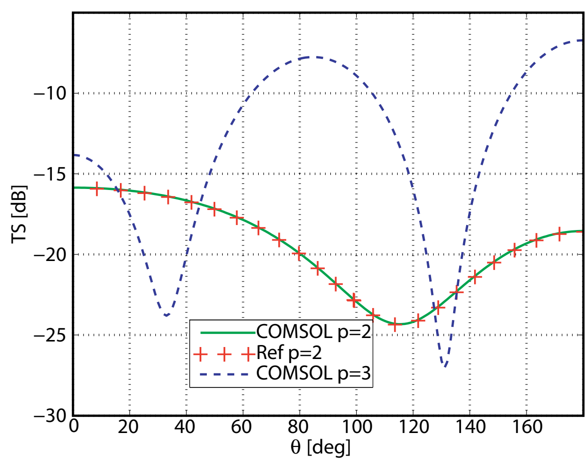

Fig. 4 Results on the v-circle at resonance $f = 2.895$kHz. The p=2 curves appear to miss completely the resonant behavior of the structure.

To make an accurate comparison between the different results possible, the magnitude of acoustic scattered pressure $|p_{\text{scat}}(\theta)|$ is sampled on the two circles defined in Fig. 1.c, and labeled “v-circle” and “h-circle” respectively, with θ denoting the angle between the point of impact of the plane wave and the observation point.

The results obtained from the mesh of Fig. 1 at $f = f_1 = 2.5$kHz are presented in Fig. 3. Here and in what follows, the label “COMSOL” denotes results obtained with FEMlab, and the label “Ref” denotes results obtained with the reference tool [7]. The use of quadratic polynomial shape functions is represented by the label “p=2” in the plots, and the use of cubic shape functions is represented by the label “p=3”. Figure 3 shows that the convergence is not completely reached when using quadratic polynomial shape functions. At the critical resonance frequency $f = 2.895$kHz, a complete lack of convergence is evident in Fig. 4. For p=2, the COMSOL result and the reference result agree closely, but they fail completely in capturing the resonant behavior described by the p=3 curve. As the frequency is increased to 3.5 kHz, the convergence improves again (Fig. 5), indicating that the FE convergence is related to a wave phenomenon which is predominant at the resonance frequency. Spurious reflections from the spherical boundary on which the radiation BC’s are applied are not evident in the results, as can be seen from the curves at 2.5 kHz and at 3.5 kHz, which suggests that the radiation conditions are performing well.

At a first glance, the behavior of the FE solutions appears to be rather peculiar, since the elements used in the mesh discretize the surface of the cylinder with as many as 18 elements per acoustic wavelength $\lambda_{\text{water}} = c/f$, with $c = 1500$m$/$s being the soundpeed in the surrounding water. This appears to be in contrast with the common practice of discretizing the domain with 6 quadratic (p=2) elements per λ_{water} (see for example [9]). Such meshing guidelines are justified by the analysis of the approximation of sinusoids of a given wavelength using the polynomial shape functions. As shown below, the guideline is valid also in the case considered here, provided that, instead of limiting the attention only to λ_{water} and to evanescent waves which decay rapidly with distance from the target, one identifies the wavelength of all the relevant wave phenomena occurring in the fluid loaded structure of finite thickness.

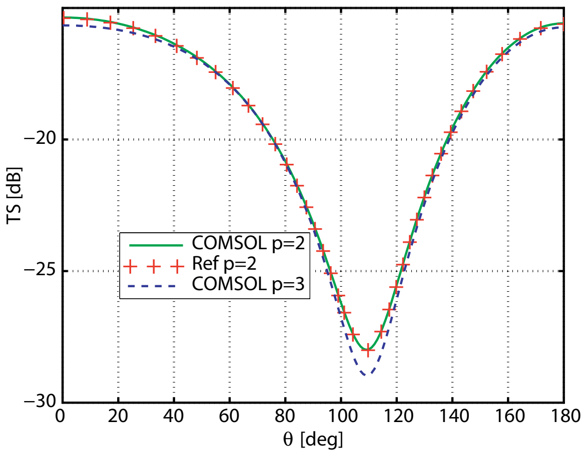

Fig. 5 Results on the v-circle at frequency $f = 3.5$kHz. The disagreement between the p=2 curves and the p=3 curve is not as evident as in the 2.895 kHz resonance case.

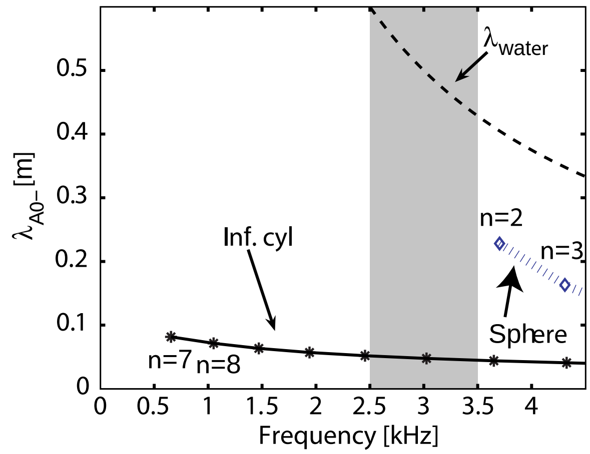

Fig. 6 Wavelengths of subsonic asymmetric A_0^- Lamb waves with modal resonance orders *n* in the frequency band of interest (area shaded in grey) for a spherical shell (diamonds), and for a cylinder (asterisks). Dashed line shows wavelength w.r.t. soundspeed in water.

3 Lamb wave based discretization

The convergence behavior of the FE model can be explained by noticing that the shortest wavelength in the problem is not the wavelength $\lambda_{\text{water}} = c/f$ computed with respect to the soundspeed in the water $c = 1500\,\text{m/s}$. An analysis of the elastic waves present in structures of finite thickness according to [2] reveals that the smallest wavelength encountered in the problem is the one associated with the asymmetric subsonic Lamb wave resonances in the cylindrical part of the target. Analytical methods can be used to estimate the location of the Lamb wave resonances and the corresponding phase speed of the resonant mode for an ideally infinite cylindrical shell, and for a spherical shell. Figure 6 shows the analytically estimated Lamb wave resonance frequencies and associated Lamb wave wavelengths for an infinite steel shell and for a spherical steel shell having the same thickness and the same radius as the cylinder considered here. While it can be expected that the estimates of the Lamb wave wavelengths obtained from Fig. 6 are applicable to the finite cylinder with hemispherical endcaps used in the FE computations, the resonance frequency locations cannot be expected to coincide with the ones computed for the ideally infinite cylinder and for the sphere. Thus, from Fig. 6 it can be seen that the wavelength of the Lamb wave in the cylindrical body, ($\lambda_{\text{A0}-} = 0.05\,\text{m}$ at $3.5\,\text{kHz}$) is approximately 8.5 times shorter than λ_{water}. The phase speed corresponding to the shortest subsonic Lamb wave is approximately 175 m/s, while the soundspeed in water is 1500 m/s.

It should be noted, that for complex structures one cannot determine a priori whether a given Lamb wave mode resonance decays rapidly with distance from the target in the near field, or whether it propagates out into the far field. Experimental and modeling results showing the propagation of Lamb waves in the far field are presented for example in [10]. The results of reference [10] imply that the Lamb waves in the elastic structure cannot be neglected, even though one may be primarily interested in computing only far field quantities.

Since the Lamb waves travel along the thin elastic structure, it is necessary to discretize the directions inplane with the target surface using a sufficient number of degrees of freedom with respect to $\lambda_{\text{A0}-} \ll \lambda_{\text{water}}$. As observed above, the FE mesh used here has approximately a density of 18 elements per wavelength on the target surface with respect to λ_{water}. This implies that the Lamb waves are discretized with only 2 elements per wavelength. The meshing guidelines mentioned above suggest that this discretization, in conjunction with the use of quadratic (p=2) polynomial shape functions, is insufficient to represent the Lamb waves accurately. In the case analyzed here, the problem can be overcome easily by raising the p-level in the mesh from quadratic to cubic.

Results on the "v-circle" and on the "h-circle" for the three frequencies of interest, obtained with the cubic polynomial meshes using both FEMlab and the reference tool, are shown in Figs. 7–9. In all three cases, the excellent agreement between the two software tools can be seen. It has been shown in previous work [4] that the solution presented in Figs. 7–9 is in agreement with converged solutions obtained using other alternative codes for verification. The most remarkable feature of the presented solutions is the improvement obtained at the resonance frequency, as shown in Fig. 4 and in Fig. 8.

For the solution of the FE linear system in FEMlab, the SPOOLES stationary linear direct solver with symmetric option was used. The chosen preordering algorithm was "best of nested dissection and multisection". The FE linear system consisted of 17986 complex degrees of freedom (DOF) for the quadratic shape function case (p=2), and of 57352 complex DOF for the cubic shape function case (p=3). The RAM required by the solver was approximately 500MB for the first case, and 1.3GB for the second case.

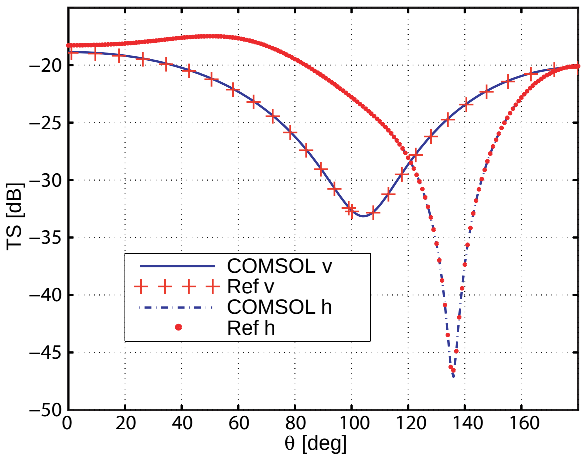

Fig. 7 Results on the v-circle and on the h-circle at $f = 2.5$kHz.

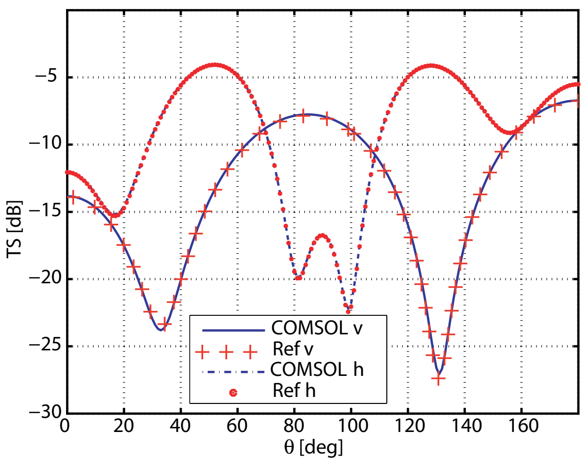

Fig. 8 Results on the v-circle and on the h-circle at the Lamb wave resonance frequency $f = 2.895$kHz.

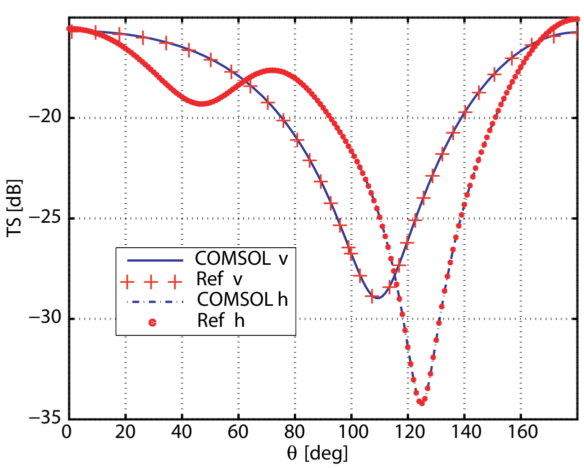

Fig. 9 Results on the v-circle and on the h-circle at $f = 3.5$kHz.

4 Conclusions and further work

The work presented here suggests that converged results for structural acoustics problems are obtained using meshes capable of resolving the short wavelength Lamb waves in the elastic structure. For complex objects, other than cylinders and spheres or combinations of those, one may attempt to estimate the Lamb wave resonances and wavelengths via analytical models, using the local curvature, material properties and shell thickness as inputs. An analytical study of this kind can guide the researcher in designing a convergent finite element mesh. The convergence behavior of the FE solution at the resonance frequency, and the fact that the resonances of fluid loaded elastic structures cannot be estimated easily a priori, imply that the convergence of a steady state FE computation must be verified not only at the highest frequency but at all the frequencies in the frequency band of interest.

As the thickness of an elastic structure increases with respect to the wavelength and with respect to the radius of curvature, the asymmetric subsonic A_0^- Lamb waves transition gradually to Rayleigh waves [11],[12]. In the elastic layer, the Rayleigh waves are confined to the close vicinity of the interface between the solid and other domains or layers. From the fluid/elastic interface, the Rayleigh waves can radiate into the acoustic far field, with the decay governed by a problem dependent constant. Also the Rayleigh wavelength can be smaller than λ_{water}, and hence it is necessary to consider such waves carefully when generating meshes for thick structures.

From the computational point of view, the restrictive meshing guidelines imposed by the elastic wave analysis put a heavy burden on the computational resources, especially if one considers more complex problems than the one presented in this paper. For this reason, it is desirable to limit the extent of the fluid domain surrounding the target in the FE part of the computation to a minimum. Far field quantities in the free field can be obtained by sampling the FE results on a surface surrounding the target, and by use of the Helmholtz-Kirchhoff integral theorem or by numerical implementations of potential theory [5]. For cases where one needs to compute far field scattering or radiation from an object located in a complex environment, it is also possible to exploit specialized propagation models. Such generally non FE-based models are efficient for predicting the acoustic propagation in a complex environment, but they lack the capability of describing the scattering from objects. For this reason, an efficient hybrid approach which can be envisioned consists of using the FE tool for the near field computations in the vicinity of the target, and passing the FE results sampled in the near field to the propagation tool of choice, which can describe the scattered echo or the radiated signal in the far field within the complex environment [5]. Such an approach makes it possible to solve problems of practical interest by exploiting optimally the resources of the FE tool.

This paper demonstrates the potential of the FEMlab multiphysics tool for modeling the complex acoustic phenomena which arise in the presence of fluid loaded vibrating or scattering structures. The wealth of different solvers offered by FEMlab is especially attractive for experimenting with different solution techniques, both iterative ones and direct ones. At this point, it should also be mentioned, that the addition of some features to future releases of FEMlab would make the tool more applicable to acoustics problems of interest to researchers and engineers working in the field. The first such feature would be the addition of efficient *local* numerical radiation conditions. Such radiation conditions can reduce drastically the size of the domain modeled with FEMlab. As a consequence, the FE problem size decreases and the range of frequencies achievable for computations with a given geometry increases. Candidate techniques for the enhancement of numerical radiation BC's could be some local form of the higher order Bayliss-Turkel radiation BC's, infinite elements (which are not boundary conditions, but rather non-conventional types of elements), or efficient perfectly matched layers (PML's). The second issue which should be addressed is the meshing of thin structures, such as the cylinder studied here. Such structures are common not only in acoustics, but in many areas of engineering and physics. A capability for generating structured 3-D meshes of thin objects would be the most desirable option, so that importing of meshes is avoided, and FEMlab's interactive refinement and p-enrichment flexibility remain available to the user.

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