Report: Low Frequency Predictive Skill Despite Structural Instability and Model Error

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I. Findings: Scientific Results

The PI, Andrew Majda, the co-PI, Rafail Abramov, and the postdoctoral associates, Dimitris Giannakis, and Michal Branicki (funded by the DRI) have submitted the following papers all with the PI as author (and other collaborators listed)

A) Mathematical Techniques for Quantifying Uncertainty in Complex Systems with Model Error with Prototype Applications

Development of new uses of information theory to quantify uncertainty, irreducible impression, sensitivity, and long range forecasting skill (7,8,9,12, 4,6,1). This work includes explicit simple examples of irreducible imprecision where imperfect models can be tuned to match the climate mean and variance of the perfect model; nevertheless the response to external forcing has an intrinsic information barrier which cannot be improved within the class of imperfect models. The development and application of these ideas to unambiguous simple models for turbulent diffusion with complex features. The theoretical development of algorithms based on fluctuation dissipation theorems for sensitivity and long-range forecasting including intrinsic skill barriers for popular linear regression models. New non-Gaussian filtering algorithms for multi-scale filtering of turbulent signals (14,11,3,2).

B) Nonlinear Laplacian Spectral Analysis (NLSA) for Time Series: Capturing intermittency and Low Frequency Variability

Majda and Giannakis have developed novel NLSA algorithms and applied them to comprehensive climate models (13,10,5). Many processes in science and engineering develop multiscale temporal and spatial patterns, with complex underlying dynamics and time-dependent external forcings. Because of the importance in understanding and predicting these phenomena, extracting the salient modes of variability empirically from incomplete observations is a problem of wide contemporary interest. Here, we present a technique for analyzing high-dimensional, complex time series that exploits the geometrical relationships between the observed data points to recover features characteristic of

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strongly nonlinear dynamics (such as intermittency and rare events), which are not accessible to classical singular spectrum analysis. The method employs Laplacian eigenmaps, evaluated after suitable time-lagged embedding, to produce a reduced representation of the observed samples, where standard tools of matrix algebra can be used to perform truncated singular-value decomposition despite the nonlinear geometrical structure of the dataset. We illustrate the utility of the technique in capturing intermittent modes associated with the Kuroshio current in the North Pacific sector of a general circulation model and dimensional reduction of a low-order atmospheric model featuring chaotic intermittent regime transitions, where classical singular spectrum analysis is already known to fail dramatically.

We apply NLSA to study North Pacific SST monthly data from the CCSM3 and ECHAM5/MPI-OM models. Without performing spatial coarse graining (i.e., operating in ambient-space dimensions up to 1.6 _ 105 after lagged embedding), or seasonal-cycle subtraction, the method reveals families of periodic, low-frequency, and intermittent spatiotemporal modes. The intermittent modes, which describe variability in the Western and Eastern boundary currents, as well as variability in the subtropical gyre with year-to-year reemergence, are not captured by SSA, yet are likely to have high significance in a predictive context and utility in cross-model comparisons.

II. Contributions to Discipline

Development of new uses of information theory to quantify uncertainty, irreducible impression, sensitivity, and long range forecasting skill (7,8,9,12, 4,6,1). This work includes explicit simple examples of irreducible imprecision where imperfect models can be tuned to match the climate mean and variance of the perfect model; nevertheless the response to external forcing has an intrinsic information barrier which cannot be improved within the class of imperfect models. The development and application of these ideas to unambiguous simple models for turbulent diffusion with complex features. The theoretical development of algorithms based on fluctuation dissipation theorems for sensitivity and long-range forecasting including intrinsic skill barriers for popular linear regression models. New non-Gaussian filtering algorithms for multi-scale filtering of turbulent signals (14,11,3,2).

III. Contributions to Outside Disciplines

Majda and Giannakis have developed novel NLSA algorithms and applied them to comprehensive climate models (13,10,5). Many processes in science and engineering develop multiscale temporal and spatial patterns, with complex underlying dynamics and time-dependent external forcings. Because of the importance in understanding and predicting these phenomena, extracting the salient modes of variability empirically from incomplete observations is a problem of wide contemporary interest. Here, we present a technique for analyzing high-dimensional, complex time series that exploits the geometrical relationships between the observed data points to recover features characteristic of strongly nonlinear dynamics (such as intermittency and rare events), which are not accessible to classical singular spectrum analysis. The method employs Laplacian eigenmaps, evaluated after suitable time-lagged embedding, to produce a reduced representation of the observed samples, where standard tools of matrix algebra can be used to perform truncated singular-value decomposition despite the nonlinear geometrical structure of the dataset. We illustrate the utility of the technique in capturing intermittent modes associated with the Kuroshio current in the North Pacific sector of a general circulation model and dimensional reduction of a low-order atmospheric model featuring chaotic intermittent regime transitions, where classical singular spectrum analysis is already known to fail dramatically.

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PUBLICATIONS

1. (A. Majda, M. Branicki, N. Chen) " Non-Gaussian Test Models for Prediction and State Estimation with Model Errors", submitted, *Chinese Annals Math.,* September 1, 2012
2. (A. Majda, I. Grooms) "Stochastic Superparameterization in a one-dimensional model for wave turbulence", submitted, *SIAM J. MMS,* June 15, 2012
3. (A. Majda, M. Branicki) "Dynamic Stochastic Superresolution of sparseley observed turbulent systems", submitted, *Journal of Computational Physics*, May 17, 2012
4. (A. Majda, M. Branicki) "Fundamental Limitations of Polynomial Chaos for Uncertainty Quantification in Systems with Intermittent Instabilities", accepted and in press, *Comm.Math.Sci.* , January 2012
5. (A. Majda, D. Giannakis) "Nonlinear Laplacian spectral analysis: Capturing intermittent and low-frequency spatiotemporal patterns in high-dimensional data", submitted, *American Statistical Association*, January 2012
6. (A. Majda, E. Kang, J. Harlim) "Regression Models with Memory for the Linear Response of Turbulent Dynamical Systems," accepted, *Comm. Math. Sci*, December 2011
7. (A. Majda, M. Branicki) "Lessons in Uncertainty Quantification for Turbulent Dynamical System", *Discrete and Continuous Dynamical Systems,* September 2012,doi: 10.3934/dcds.2012.32.3133
8. (A. Majda, M. Branicki) "Quantifying uncertainty for predictions with model error in non-Guassian systems with intermittency", *Nonlinearity*, August 3, 2012 doi: 10.1088/0951-7715/25/9/2543
9. (A. Majda, B. Gershgorin) "Quantifying uncertainty for climate change and long range forecasting scenarios with model errors. Part I: Gaussian models" , *J. Climate*, July 2012, Vol 25, pp. 4523-4548
10. (A. Majda, D. Giannakis) "Comparing low-frequency and intermittent variability in comprehensive climate models through nonlinear Laplacian spectral analysis", *Geophysical Research Letters,*May 30, 2012, doi: 10.1029/2012GL051575
11. (A. Majda, J. Harlim) "Test Models for Filtering and Prediction of Moisture-Coupled Tropical Waves," *QJRMS,* (early online release)May 30, 2012, doi: 10.1002/qj.1956

12. (A. Majda) "Challenges in Climate Science and Contemporary Applied Mathematics," Invited Paper for the Special Volume of Communications on Pure and Applied Mathematics for 75th Anniversary of the Courant Institute, April 12, 2012, doi: 10.1002/cpa.21401
13. (A. Majda, D. Giannakis) "Nonlinear Laplacian spectral analysis for time series with intermittency and low-frequency variability," *PNAS*, Feburary 14, 2012, doi: 10.1073/pnas.1118984109
14. (A. Majda, S. Keating, K. S. Smith) "New methods for estimating poleward eddy heat transport using satellite altimetry," *Monthly Weather Review,*(early online release), February 9, 2012, doi:10.1175/MWR-D-11-00145.1