# Coastal Engineering Technical Note

## SPECTRAL ANALYSIS OF DIGITAL WAVE DATA
## COMPUTER PROGRAM: SPECTRUM

Purpose: This CETN introduces an interactive program for spectral analysis of water elevation data capable of fast execution on microcomputers.

Capabilities: Computer program SPECTRUM takes a time series of evenly spaced water elevation data and breaks it down into its surface variance spectral density. Under linear wave theory, the surface variance spectral density can be converted to spectral energy density upon multiplication by $\rho g$ - where $\rho$ is the density of water and g is acceleration due to gravity. SPECTRUM is written in FORTRAN 77 and can handle a time series of up to 4096 points. SPECTRUM is very efficient in terms of memory usage and thus is attractive for use on microcomputers.

SPECTRUM will test for a trend in the time series and optionally delete it upon consent of the user. SPECTRUM also produces 90 percent confidence intervals for the actual surface variance spectral density values. SPECTRUM calculates the zeroth through fourth moments of the spectrum and several useful wave and spectral parameters including significant wave height and peak period. SPECTRUM also allows the user a choice of data windows or tapers. The interactive input has optional default settings for most of the input data.

Methods: The data analysis routine is performed using the Welch method (Welch, 1967) of spectral estimation which may be briefly summarized as follows:

(1) Segment the N point time series $(Y_n)$ $n = 0,1,\ldots\ldots,N-1$ into K half-overlapping segments of L points each. This implies that

$K = (N - L/2) / (L/2)$ and we define the jth segment as

$Z_{jn} = Y_{n+(j-1)\,L/2}$ where

$j = 1,2\ \ldots\ldots,K$

$n = 0,1,\ \ldots\ldots,L-1$

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13. SUPPLEMENTARY NOTES

14. ABSTRACT

**Computer program SPECTRUM takes a time series of evenly spaced water elevation data and breaks it down into its surface variance spectral density. Under linear wave theory, the surface variance spectral density can be converted to spectral energy density upon multiplication by pg - where p is the density of water and g is acceleration due to gravity. SPECTRUM is written in FORTRAN 77 and can handle a time series of up to 4096 points. SPECTRUM is very efficient in terms of memory usage and thus is attractive for use on microcomputers.**

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(2) Subtract out the mean of each segment

$X_{jn} = Z_{jn} - \overline{Z}_j$ where

$j = 1,2, \ldots\ldots, K$

$n = 0,1, \ldots\ldots, L-1$

$$\overline{Z}_j = \sum_{n=0}^{L-1} Z_{jn}/L$$

(3) Compute $G_j(m\Delta f)$, the discrete Fourier transform of the jth segment at frequency $m\Delta f$

$$G_j(m\Delta f) = \Delta t \sum_{n=0}^{L-1} X_{jn} W_n e^{-2\pi imn/L}$$

$j = 1,2 \ldots\ldots, K$

$m = 0,1, \ldots\ldots, L/2-1$

$i^2 = -1$

$\Delta f = 1/(N\Delta t)$

$\Delta t$ = the recording time between consecutive data points

$W_n$ = the data window for the nth point of the segment

(4) Compute the estimated surface variance spectral density from the jth segment - $\hat{S}_j(m\Delta f)$

$$\hat{S}_j(m\Delta f) = 2C_j \left| G_j(m\Delta f) \right|^2 / L\Delta t \qquad \text{where}$$

$j = 1,2, \ldots\ldots, K$

$m = 0,1, \ldots\ldots, L/2-1$

$\left| G_j(m\Delta f) \right|$ = the complex modulus (length) of $G_j(m\Delta f)$

$$C_j = L/\sum_{n=0}^{L-1} W_n^2$$

(5) Average the surface variance spectral density from each segment to obtain a final estimate which we shall denote by $\hat{P}(m\Delta f)$.

$$\hat{P}(m\Delta f) = \sum_{j=1}^{K} \hat{S}_j(m\Delta f)/K$$

The principle advantage of the Welch method in comparison to other non-overlapping segmenting techniques is that it reduces the width of the confidence interval for the actual surface variance spectral density values.

Additional details pertaining to the mathematics of spectrum and time series analysis may be found in Bendat and Piersol (1971), Borgman (1972), Borgman (1973), Lund (1986), and Welch (1967).

The discrete Fourier transform is performed using the fast Fourier transform algorithm which is an extremely efficient computational scheme for $G_i(m\Delta f)$. The fast Fourier transform routine in the program requires that the time series length N be a power of 2 - if not, the time series length is truncated to the largest power of 2 less than N. A 90 percent confidence interval for the actual surface variance spectral density value $P(m\Delta f)$ is calculated using the value of $\hat{P}(m\Delta f)$ and the chi-square probability law. The user may easily adjust SPECTRUM for percent values other than 90 percent by changing the upper tail $Z_{\alpha/2}$ value from the standard normal distribution at runtime. The zeroth through the fourth moments of the spectrum are calculated by the program. The mth moment, $U_m$, is defined as follows:

$$U_m = \int_0^\infty f^m P(f)\, df \qquad m = 0,1,2, \ldots .$$

The spectral parameters are numbers which quantify the shape of the wave spectrum. They are usually functions of the moments. The spectral parameters calculated in SPECTRUM are listed and defined as follows:

The spectral width parameter $= \varepsilon = \sqrt{1 - (U_2^2/U_0U_4)}$

The spectral narrowness parameter $= \nu = \sqrt{(U_2U_0/U_1^2) - 1}$

The spectral peakedness parameter $= Qp = (2/U_0^2) \int_0^\infty f\, P(f)^2\, df$

The significant wave height $= H_s = 4\sqrt{U_0}$

These parameters are discussed in Rye (1979). Note that $H_s$ by this definition is not necessarily equivalent to the average height of the one-third highest waves, particularly in shallow water. The integrals in the moments and equations defining the spectral parameters are calculated using the trapezoidal rule as the numerical integration scheme and $\hat{P}(m\Delta f)$ as an approximation for $P(m\Delta f)$.

The peak frequency is calculated by two different methods and is defined as the frequency at which the maximum surface variance spectral density occurs. The first method searches the array of variance density estimates for its maximum value; the second method averages all frequencies which have variance densities that exceed 0.6 times the peak frequency calculated by the first method. The peak period is computed as the reciprocal of the peak frequency computed in the first method.

The data windows are used in the reduction of side lobe leakage. This helps keep variance from "leaking" into frequencies where it does not belong. SPECTRUM gives the user a choice of three data windows:

1) The rectangular boxcar window

$$W_n = 1 \qquad\qquad 0 \leq n \leq L-1$$

2) The 10 percent cosine bell window

$$W_n = \begin{cases} 1/2[1 - \cos(10\pi n/L)] & 0 \leq n \leq L10 \\ 1 & L10+1 \leq n \leq L-L10-2 \\ 1/2[1 - \cos(10\pi(L-n-1)/L)] & L-L10-1 \leq n \leq L-1 \end{cases}$$

where L10 = [L/10] = the greatest integer less than or equal to L divided by 10.

3) The Hamming window

$$W_n = .54 - .46 \cos(2\pi n/L) \qquad\qquad 0 \leq n \leq L-1$$

Input: A title for each run is entered followed by the number of data points N, the segment length L, the data window choice, and the sampling frequency fs ($\Delta t = 1/fs$). Default values for the segment length and data window type are provided by the program. The default data window is the 10 percent cosine bell; while the default segment length is chosen such that the number of segments is seven. These values can be easily changed by the user at runtime. The sampling frequency can be interpreted as the number of data points recorded per second.

Next, SPECTRUM asks for lower and upper cutoffs for the computation of spectral moments and parameters (Rye, 1979). Cutoffs should be in terms of factors times the peak frequency. Default cutoffs are 0.3 and 3.0 times the peak frequency. Finally, the units of water elevation data and the name of the data file containing the water elevation data are input. The user should make sure that the format statement in the program reads the input time series in the correct format. This should be done by the user at runtime. The default format statement reads the data in a 6X,25F3.0 formatted data file. This means the data file will contain six blank spaces at the beginning of each row of data and will be followed by 25 three digit floating point numbers with no blanks between. If the data file is not in this format, the user will be allowed to enter the format appropriate for the specific data file. If a trend in the data is detected, the user is informed and has the option of deleting it or leaving it stand as is; if no trend is found, the program continues execution without interuption. All input time series should be reviewed and edited for quality before use with this program. Input data must be water level measurements. Erroneous results may be produced if this program is used to process pressure or current time series.

Output: The title of the run is printed as a heading for the program output. This is followed by a table of frequency verses surface variance spectral density. This table also includes 90 percent confidence intervals for the actual surface variance spectral density values. The confidence intervals may be interpreted as follows - actual surface variance spectral density falls between the lower and upper bounds with 90 percent confidence. The table is followed by output of the zeroth through fourth spectral moments. Next, the degrees of freedom for the chi-square distribution and the recording interval between consecutive data points ($\Delta t$) are printed. Then the spectral parameters are output. The significant wave height, spectral peakedness parameter, spectral width parameter, and spectral narrowness parameter are all displayed by the program. Finally, peak frequency and its reciprocal, peak period, are displayed by SPECTRUM.

Sample Problem: A twenty-minute time series from a Waverider buoy in Lake Michigan (South Haven, Michigan) is analyzed in the following sample execution of SPECTRUM.

```
B>SPECTRUM
*****************************************************
*  SPECTRUM IS A PROGRAM WHICH BREAKS DOWN A TIME    *
*  SERIES OF WATER ELEVATION DATA INTO ITS SURFACE   *
*  VARIANCE DENSITY SPECTRUM.  SPECTRUM IS ABLE TO   *
*  CHECK FOR AND OPTIONALLY ELIMINATE TRENDS IN THE  *
*  DATA.  SPECTRUM USES THE WELCH METHOD OF HALF-    *
*  OVERLAPPING SEGMENTS AND WILL PRODUCE CONFIDENCE  *
*  INTERVALS FOR THE ACTUAL SPECTRAL VALUES.  THE    *
*  ZEROTH THROUGH THE FOURTH SPECTRAL MOMENTS AND    *
*  SEVERAL USEFUL SPECTRAL PARAMETERS ARE INCLUDED   *
*  IN THE PROGRAM OUTPUT.                            *
*****************************************************

INPUT THE SITE OF DATA COLLECTION OR THE TITLE OF THIS RUN.
TRIAL RUN

HOW MANY DATA POINTS DO YOU HAVE IN YOUR TIME SERIES?
4096

DO YOU WISH TO ENTER A SEGMENT LENGTH (Y/N)?
IF NOT, ONE WILL BE SELECTED FOR YOU SUCH THAT
THE NUMBER OF SEGMENTS EQUALS 7.
N

DO YOU WISH TO ENTER A WINDOW TYPE (Y/N)?  IF NOT,
THE 10% COSINE BELL WINDOW WILL BE CHOSEN FOR YOU.
Y
RECTANGULAR BOXCAR WINDOW      = 1
10 PERCENT COSINE BELL WINDOW  = 2
HAMMING WINDOW                 = 3
SELECT 1, 2, OR 3
1

HOW OFTEN ARE THE DATA POINTS SAMPLED?
USE UNITS OF RECORDINGS PER SECOND (HZ).
4

DO YOU WISH TO ENTER HIGH AND LOW FREQUENCY CUTOFFS
FOR THE COMPUTATION OF THE SPECTRAL MOMENTS AND
PARAMETERS(Y/N)?
DEFAULT VALUES ARE 0.3 AND 3.0 TIMES THE PEAK FREQUENCY
N

WHAT ARE THE UNITS OF YOUR WATER ELEVATION DATA?
FEET...........1
INCHES.........2
CENTIMETERS....3
SELECT 1, 2, OR 3.
3

WHAT IS THE NAME OF YOUR DATA FILE?
BE SURE TO INCLUDE THE EXTENSION.

C:SHAV1720.DAT
```

THE DEFAULT FORMAT IS (6X,25F3.0)
IS YOUR DATA IN THIS FORMAT (Y OR N) Y

YOUR DATA INDICATES A LINEAR TREND WITH AT LEAST 95
PERCENT CONFIDENCE. DO YOU WANT TO REMOVE IT (Y/N)?
Y

TRIAL RUN

SPECTRAL DECOMPOSITION OF THE TIME SERIES:

| FREQ(HZ) | SPECTRAL DENSITY CM^2 /HZ | 90 PERCENT CONFIDENCE INTERVALS FOR THE ACTUAL SPECTRAL VALUES | |
|---|---|---|---|
| | | LOWER | UPPER |
| .047 | 583.369 | 317.320 | 1501.989 |
| .051 | 962.267 | 523.418 | 2477.529 |
| .055 | 203.701 | 110.802 | 524.464 |
| .059 | 51.610 | 28.073 | 132.879 |
| .063 | 30.498 | 16.589 | 78.522 |
| .066 | 43.365 | 23.588 | 111.650 |
| .070 | 20.708 | 11.264 | 53.318 |
| .074 | 81.608 | 44.390 | 210.115 |
| .078 | 45.254 | 24.615 | 116.514 |
| .082 | 25.424 | 13.829 | 65.459 |
| .086 | 28.321 | 15.405 | 72.917 |
| .090 | 36.365 | 19.781 | 93.629 |
| .094 | 32.069 | 17.443 | 82.566 |
| .098 | 152.352 | 82.871 | 392.257 |
| .102 | 85.339 | 46.420 | 219.721 |
| .105 | 84.036 | 45.711 | 216.365 |
| .109 | 95.011 | 51.681 | 244.624 |
| .113 | 158.233 | 86.070 | 407.398 |
| .117 | 349.956 | 190.356 | 901.024 |
| .121 | 480.614 | 261.426 | 1237.426 |
| .125 | 1080.516 | 587.739 | 2781.982 |
| .129 | 1840.780 | 1001.280 | 4739.419 |
| .133 | 3501.738 | 1904.746 | 9015.853 |
| .137 | 4975.678 | 2706.486 | 12810.780 |
| .141 | 11350.140 | 6173.829 | 29222.960 |
| .145 | 10564.840 | 5746.672 | 27201.080 |
| .148 | 10258.390 | 5579.980 | 26412.070 |
| .152 | 12410.590 | 6750.656 | 31953.300 |
| .156 | 17176.920 | 9343.267 | 44225.060 |
| .160 | 25040.200 | 13620.440 | 64470.480 |
| .164 | 24908.960 | 13549.060 | 64132.600 |
| .168 | 10246.520 | 5573.522 | 26381.500 |
| .172 | 14238.730 | 7745.057 | 36660.160 |
| .176 | 8955.002 | 4871.011 | 23056.260 |
| .180 | 7385.563 | 4017.325 | 19015.460 |
| .184 | 5700.385 | 3100.685 | 14676.660 |
| .188 | 3712.231 | 2019.242 | 9557.804 |
| .191 | 3798.051 | 2065.923 | 9778.763 |
| .195 | 2936.927 | 1597.521 | 7561.646 |
| .199 | 3033.121 | 1649.845 | 7809.314 |
| .203 | 2535.227 | 1379.019 | 6527.396 |
| .207 | 3630.544 | 1974.809 | 9347.486 |
| .211 | 2704.000 | 1470.822 | 6961.933 |
| .215 | 3080.744 | 1675.749 | 7931.927 |
| .219 | 975.079 | 530.388 | 2510.517 |
| .223 | 2411.549 | 1311.745 | 6208.966 |
| .227 | 1527.485 | 830.865 | 3932.785 |
| .230 | 1072.366 | 583.306 | 2760.999 |
| .234 | 1679.959 | 913.802 | 4325.356 |
| .238 | 2118.298 | 1152.233 | 5453.938 |
| .242 | 1992.581 | 1083.851 | 5130.257 |
| .246 | 2086.398 | 1134.882 | 5371.806 |
| .250 | 791.320 | 430.433 | 2037.396 |
| .254 | 638.298 | 347.198 | 1643.412 |
| .258 | 2046.385 | 1113.117 | 5268.785 |
| .262 | 1157.318 | 629.515 | 2979.722 |
| .266 | 1480.093 | 805.086 | 3810.765 |
| .270 | 1685.499 | 916.815 | 4339.619 |
| .273 | 1039.320 | 565.331 | 2675.916 |
| .277 | 619.954 | 337.220 | 1596.184 |
| .281 | 1036.827 | 563.975 | 2669.496 |

| | | | |
|---|---|---|---|
| .285 | 1399.931 | 761.482 | 3604.372 |
| .289 | 1691.631 | 920.151 | 4355.409 |
| .293 | 393.281 | 213.922 | 1012.572 |
| .297 | 268.335 | 145.959 | 690.877 |
| .301 | 723.807 | 393.710 | 1863.570 |
| .305 | 513.601 | 279.370 | 1322.357 |
| .309 | 774.767 | 421.429 | 1994.777 |
| .313 | 388.513 | 211.329 | 1000.296 |
| .316 | 541.822 | 294.720 | 1395.018 |
| .320 | 430.128 | 233.965 | 1107.441 |
| .324 | 598.140 | 325.354 | 1540.020 |
| .328 | 301.869 | 164.200 | 777.216 |
| .332 | 588.470 | 320.094 | 1515.120 |
| .336 | 636.964 | 346.472 | 1639.978 |
| .340 | 619.946 | 337.215 | 1596.162 |
| .344 | 245.769 | 133.684 | 632.775 |
| .348 | 412.251 | 224.241 | 1061.415 |
| .352 | 399.952 | 217.551 | 1029.749 |
| .355 | 340.992 | 185.480 | 877.945 |
| .359 | 219.530 | 119.412 | 565.218 |
| .363 | 402.318 | 218.838 | 1035.839 |
| .367 | 524.848 | 285.487 | 1351.314 |
| .371 | 410.288 | 223.173 | 1056.361 |
| .375 | 200.606 | 109.118 | 516.495 |
| .379 | 187.323 | 101.893 | 482.298 |
| .383 | 168.245 | 91.516 | 433.177 |
| .387 | 184.571 | 100.396 | 475.212 |
| .391 | 137.873 | 74.995 | 354.978 |
| .395 | 217.429 | 118.269 | 559.810 |
| .398 | 228.819 | 124.465 | 589.136 |
| .402 | 166.166 | 90.385 | 427.825 |
| .406 | 110.602 | 60.161 | 284.764 |
| .410 | 94.535 | 51.421 | 243.396 |
| .414 | 102.065 | 55.517 | 262.784 |
| .418 | 250.974 | 136.516 | 646.179 |
| .422 | 157.331 | 85.579 | 405.078 |
| .426 | 170.032 | 92.488 | 437.778 |
| .430 | 131.411 | 71.480 | 338.342 |
| .434 | 93.565 | 50.894 | 240.900 |
| .438 | 115.813 | 62.996 | 298.181 |
| .441 | 81.882 | 44.539 | 210.819 |
| .445 | 90.164 | 49.044 | 232.144 |
| .449 | 148.577 | 80.818 | 382.539 |
| .453 | 129.928 | 70.674 | 334.523 |
| .457 | 43.477 | 23.649 | 111.940 |
| .461 | 97.676 | 53.130 | 251.484 |
| .465 | 110.491 | 60.101 | 284.479 |
| .469 | 154.909 | 84.262 | 398.841 |
| .473 | 47.188 | 25.668 | 121.495 |
| .477 | 45.413 | 24.702 | 116.924 |
| .480 | 31.338 | 17.046 | 80.685 |

SPECTRAL MOMENTS ARE AS FOLLOW:

THE 0TH MOMENT OF THE SPECTRUM IS 934.347
THE 1TH MOMENT OF THE SPECTRUM IS 171.784
THE 2TH MOMENT OF THE SPECTRUM IS 34.466
THE 3TH MOMENT OF THE SPECTRUM IS 7.726
THE 4TH MOMENT OF THE SPECTRUM IS 1.968

THE DEGREES OF FREEDOM ARE 9.800
THE RECORDING INTERVAL IS .250 SECONDS
SIGNIFICANT WAVE HEIGHT IS 122.268 CM
SPECTRAL PEAKEDNESS PARAMETER IS 1.965
THE SPECTRAL WIDTH PARAMETER IS .595
SPECTRAL NARROWNESS PARAMETER IS .302
THE PEAK FREQUENCY IS .160 HZ
THE DELFT PEAK FREQUENCY IS .160 HZ
THE PEAK PERIOD IS 6.244 SECONDS

Stop - Program terminated.

Program Availability: SPECTRUM is available in Microsoft FORTRAN for the IBM-PC on a 5-1/4 in. diskette or as a printed program listing and may be obtained from Ms. Gloria J. Naylor at (601) 634-2581 or FTS: 542-2581, Engineering Computer Programs Library Section, U.S. Army Engineer Waterways Experiment Station, P.O. Box 631, Vicksburg, MS 39180-0631. Questions concerning applications of SPECTRUM can be directed to (601) 634-2012 or FTS: 542-2012 the Coastal Design Branch, Coastal Engineering Research Center, U.S. Army Engineer Waterways Experiment Station, P.O. Box 631, Vicksburg, MS 39180-0631.

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