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TARGET ERRORS DUE TO ERRORS IN ALTITUDE AND AIR SPEED

by

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TARGET ERRORS DUE TO ERRORS IN ALTITUDE AND AIR SPEED

Abstract

Expressions are obtained for the errors in range and deflection due to errors in altitude and air speed. An illustrative example is given.

The condition for hitting of a bomb in range with a stationary target and horizontal flight (see the chapter on Bombing from Airplanes in Hayes' 'Elements of Ordnance' page 501) is

$$\tan \beta_0 = \tan \beta' - \tan \tau \cos \varphi \quad (1)$$

where β_0 is the value of the position angle β at the instant of release, β' is the travel angle, τ is the trail angle and φ is the azimuth of the trail.

If the correct value of β_0 is computed and the bomb dropped when $\beta = \beta_0$, the range component of the target error* will be zero.

Suppose that there is an error $\Delta \tan \beta'$ in the estimate of $\tan \beta'$ and an error $\Delta \tan \tau$ in the estimate of $\tan \tau$, then if the error in $\tan \beta_0$ is $\Delta \tan \beta_0$, we have

$$\tan \beta_0 + \Delta \tan \beta_0 = \tan \beta' + \Delta \tan \beta' - (\tan \tau + \Delta \tan \tau) \cos \varphi. \quad (2)$$

* The target error is the distance between the point of fall and the target.

From (1) and (2) it follows that

$$\Delta \tan \beta_0 = \Delta \tan \beta - \Delta \tan \tau \cos \varphi. \quad (3)$$

The value of $\tan \beta'$ is obtained by multiplying

$$\frac{d(\tan \beta)}{dt} \text{ (measured by the sight) by } t_w, \text{ or}$$

$$\tan \beta' = \frac{d(\tan \beta)}{dt} t_w.$$

If it is assumed that $\frac{d(\tan \beta)}{dt}$ is correctly measured, then

the error, $\Delta \tan \beta'$, in $\tan \beta'$ is given by

$$\tan \beta' + \Delta \tan \beta' = \frac{d(\tan \beta)}{dt} (t_w + \Delta t_w)$$

where Δt_w is the error in the time of flight. From this

$$\Delta \tan \beta' = \frac{d(\tan \beta)}{dt} \Delta t_w.$$

Inserting this value of $\Delta \tan \beta'$ in (3) we get

$$\Delta \tan \beta_0 = \frac{d(\tan \beta)}{dt} \Delta t_w - \cos \varphi \Delta \tan \tau.$$

The range component of the target error corresponding to $\Delta \tan \beta_0$ is $y_0 \Delta \tan \beta_0$. Hence the target error in range of the bomb will be

$$y_0 \Delta \tan \beta_0 = y_0 \frac{d(\tan \beta)}{dt} \Delta t_w - y_0 \cos \varphi \Delta \tan \tau.$$

$y_0 \frac{d(\tan \beta)}{dt}$ is the ground speed, v_g . Hence the target error in range is

$$v_g \Delta t_w - y_0 \cos \varphi \Delta \tan \tau.$$

If the errors in t_w and $\tan \tau$ are due solely to an error Δy_0 in y_0 ,

$$\Delta t_w \equiv \frac{\partial t_w}{\partial y_0} \Delta y_0,$$

$$\Delta \tan \tau = \frac{\partial (\tan \tau)}{\partial y_0} \Delta y_0.$$

Thus the target error in range as depending upon the error in altitude Δy_0 is

$$v_g \frac{\partial t_w}{\partial y_0} \Delta y_0 + y_0 \cos \varphi \frac{\partial (\tan \tau)}{\partial y_0} \Delta y_0. \quad (4)$$

If there is also an error, Δu_h , in the horizontal air speed, u_h , it may readily be shown that the resultant target error in range due to an error Δy_0 in y_0 and an error Δu_h in u_h is

$$v_g \left(\frac{\partial t_w}{\partial y_0} \Delta y_0 + \frac{\partial t_w}{\partial u_h} \Delta u_h \right) - y_0 \cos \varphi \left(\frac{\partial \tan \tau}{\partial y_0} \Delta y_0 + \frac{\partial \tan \tau}{\partial u_h} \Delta u_h \right) =$$

$$v_g \left(\frac{\partial t_w}{\partial y_0} \Delta y_0 + \frac{\partial t_w}{\partial u_h} \Delta u_h \right) - y_0 \cos \varphi \sec^2 \tau \left(\frac{\partial \tau}{\partial y_0} \Delta y_0 + \frac{\partial \tau}{\partial u_h} \Delta u_h \right). * \quad (5)$$

The above expression give the amount by which the bomb will fall short of the target in range. In using (5), due attention should be paid to signs of Δy_0 and Δu_h . If the estimated altitude is greater than the true altitude, Δy_0 is positive; if less, Δy_0 is negative. If (5) is negative, the bomb will overshoot the target. For a stationary target and horizontal flight, the target error in deflection may be computed from the condition for hitting in deflection,

* It is assumed that the trail is measured in radians. A mil is equal to .001 radians.

$$\tan \gamma_0 = \frac{\tan \tau \sin \varphi}{\left[1 + (\tan \tau \cos \varphi - \tan \beta')^2\right]^{\frac{1}{2}}} = \frac{\tan \tau \sin \varphi}{(1 + \tan^2 \beta_0)^{\frac{1}{2}}} = \frac{\tan \tau \sin \varphi}{\sec \beta_0},$$

where γ_0 is the position angle of the target for deflection at the instant of release.

$$\frac{y_0 \sin \varphi \sec^2 \tau}{\sec^3 \beta} (\sec^2 \beta_0 + \cos \varphi \tan \tau \tan \beta_0) \left(\frac{\partial \tau}{\partial y_0} \Delta y_0 + \frac{\partial \tau}{\partial u_h} \Delta u_h \right) \quad (6)$$

$$- \frac{\tan \tau \sin \varphi \tan \beta_0 v_g}{\sec^3 \beta_0} \left(\frac{\partial t_w}{\partial y_0} \Delta y_0 + \frac{\partial t_w}{\partial u_h} \Delta u_h \right)$$

If the above expression is positive, the bomb will fall short of the target in deflection, which is measured to the leeward.

ILLUSTRATIVE EXAMPLE

Suppose a bomb is dropped from an airplane having an air speed of 150 miles/hour at an altitude of 10,000 ft. with the azimuth of trail, φ , equal to 10° . If the ballistic coefficient is 2, what are the target errors in range and deflection due to an error of 10 miles/hour in air speed and of 100 ft. in altitude?

To obtain the error in range, expression (5) is used. First the derivatives,

$$\frac{\partial t_w}{\partial y_0}, \frac{\partial t_w}{\partial u_h}, \frac{\partial \tau}{\partial y_0} \text{ and } \frac{\partial \tau}{\partial u_h}$$

are computed approximately by the aid of figures 9 and 10 of 'Bombing from Airplanes'.

From figure 10, it is seen that, at an air speed of 100 miles/hour, a change of altitude from 7500 to 12,500 changes t_w from 22 sec. to 29 sec. The change is 7 sec. while at 200 miles/hour, t_w changes by almost the same amount.

Hence approximately, for an airspeed of 150 miles/hour and an altitude of 10,000 ft.,

$$\frac{\partial t_w}{\partial y_0} = \frac{7}{5000} = .0014.$$

From the two curves of Fig. 10 the value of $\frac{\partial t_w}{\partial u_h}$ is estimated to be approximately $\frac{.1}{100} = .001$.*

Similarly from plot 9, it is found that $\frac{\partial \tau}{\partial y_0}$ is negligible and that $\frac{\partial \tau}{\partial u_h}$

is about $\frac{47.5 - 39.5}{20} = \frac{8}{20} = .4$ mils/miles/hr. = .0004 radians/mile/hr.

For the given conditions $\tau \approx 43$ mils = .043 radians, whence $\sec^2 \tau = 1.00$.

If then we take $y_0 = 10,000$, $\Delta y_0 = 100$ and $\Delta u_h = 10$, we find from (5) that the target error in range is

$$v_g(.0014 \times 100 + .001 \times 10) - 10,000(.00040 \times 10) \cos \varphi = v_g \times (.15) - 40 \cos \varphi.$$

If φ is 10° , then $\cos \varphi = .984$; hence if v_g is 250 ft/sec. the target error is

$$37 - 39 = -2 \text{ ft.}$$

The bomb will overshoot the target by about two feet.

* The value of .001 was really obtained from the Exterior Ballistic Tables; the curves would indicate a value of about .002. The discrepancy is due to the facts that (1), the linear interpolation is inaccurate and (2), the scale of Fig. 10 is small.

If $\Delta u_h = 0$, and $\Delta y_o = 100$, the bomb will fall short by 35 ft.
 If $\Delta y_o = 0$ and $\Delta u_h = 10$ mi/hr. the bomb will overshoot by 37 ft.

To compute the error in deflection we need to know $\sin \varphi$, $\tan \tau$, $\tan \beta_o$, and $\sec \beta_o$.

For these, the following values are obtained,

$$\sin \varphi = .174$$

$$\tan \tau = .043$$

$$\tan \beta_o = \frac{v}{y_o} t_w - \tan \tau \cos \varphi = \frac{250 \times 26}{10,000} - .043 \times .984 = .61$$

$$\sec \beta_o = \sqrt{1 + .61^2} = 1.17.$$

If these values and the values of $\frac{\partial t_w}{\partial y_o}$, etc. previously obtained are inserted in (6), the result is

$$\frac{10,000 \times .174 \times 1}{1.17^3} \times (1.17^2 + .984 \times .043 \times .61) \times (.0040)$$

$$- \frac{.043 \times .174 \times .61 \times 250}{1.17^3} (.15) =$$

$$1,085 \times (1.39) \times .0040 - .71 \times .15 =$$

$$6.0 - 0.1 = 5.9 \text{ ft.}$$

Thus the bomb will fall about 6 ft. to the right in deflection.

If the calculations are repeated with $\Delta y = 0$ and $\Delta u_h = 10$ mi/hr. almost the same result will be obtained, indicating that an error in altitude will produce only a negligibly small error in deflection.

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