



Design for Lifecycle Cost using Time-Dependent Reliability Analysis

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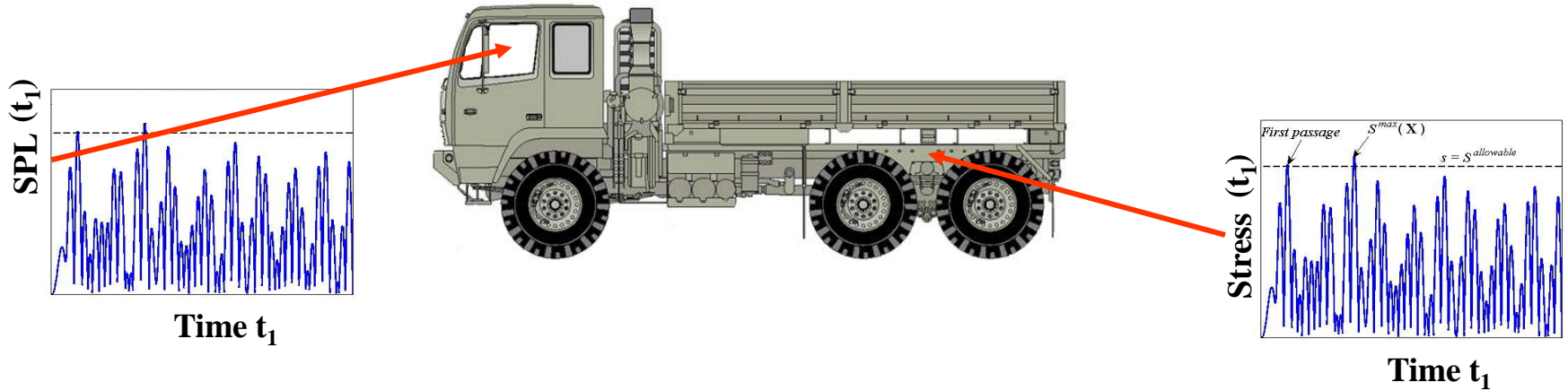
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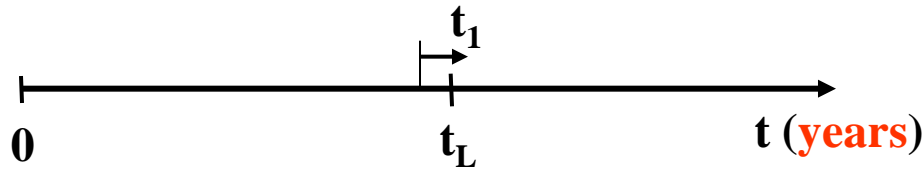
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Problem Definition



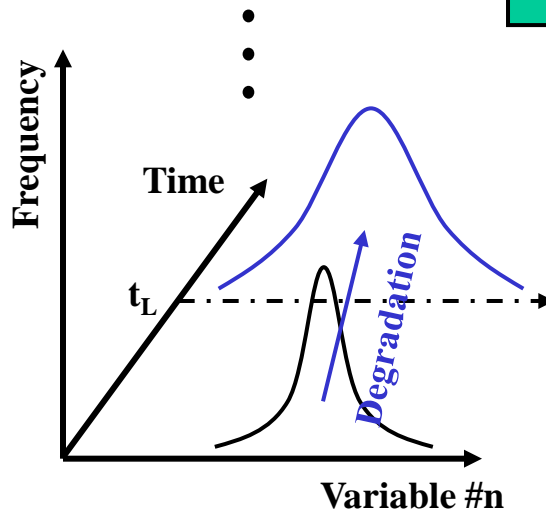
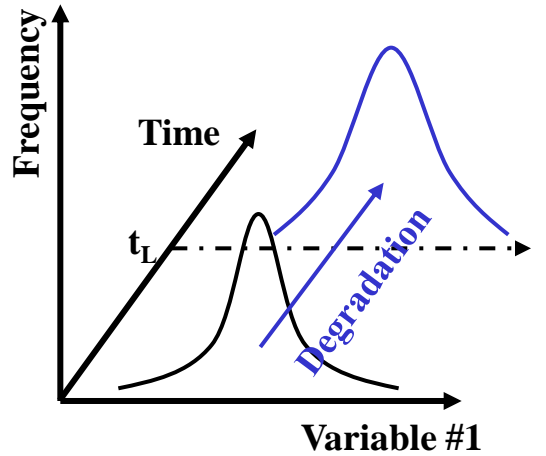
$$\text{Response}(t) = f [E(t), \text{Degradation/Wear}(t), \text{Load}(t_1)]$$



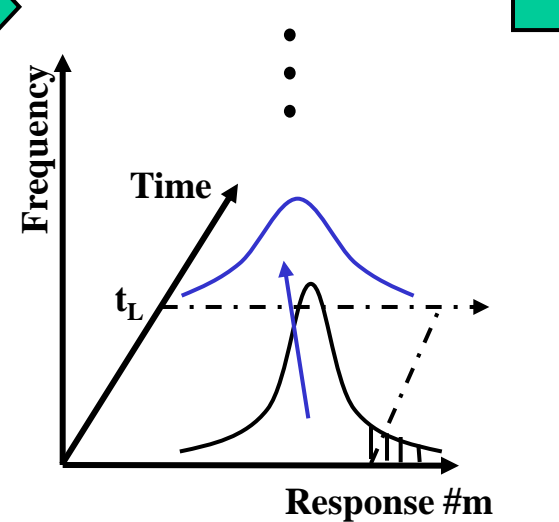
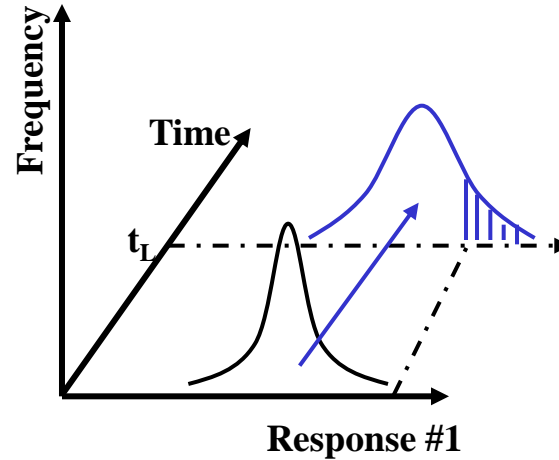
Random Process $\rightarrow E(t) = E_0(1 - D * t)$ $\rightarrow E(t_L)$ is a R. V.
Degradation Rate (Random Variable)

Two time scales; t and t_1

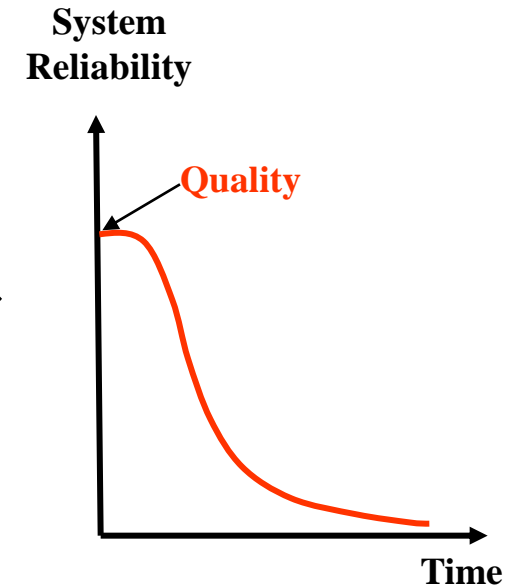
Input variables



System Responses



System Reliability



Quality = Reliability (t = 0)

Definitions / Observations

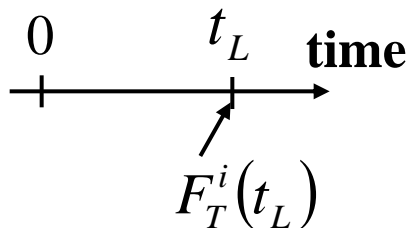
Reliability: Ability of a system to carry out a function in a time period $[0, t_L]$

$$p_f^c = P(t \leq t_L) = F_T^c(t_L) \quad \text{Prob. of Time to Failure}$$

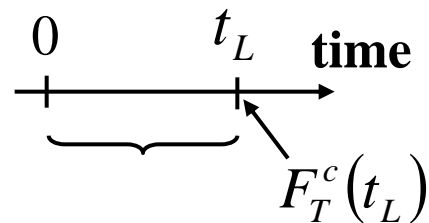
$$F_T^c(t_L) = P(\exists t \in [0, t_L], \text{ such that } g(\mathbf{X}(t), t) \leq 0) \quad \text{Cumulative Prob. of Failure}$$

$$F_T^i(t_L) = P(g(\mathbf{X}(t_L), t_L) \leq 0) \quad \text{Instantaneous Prob. of Failure}$$

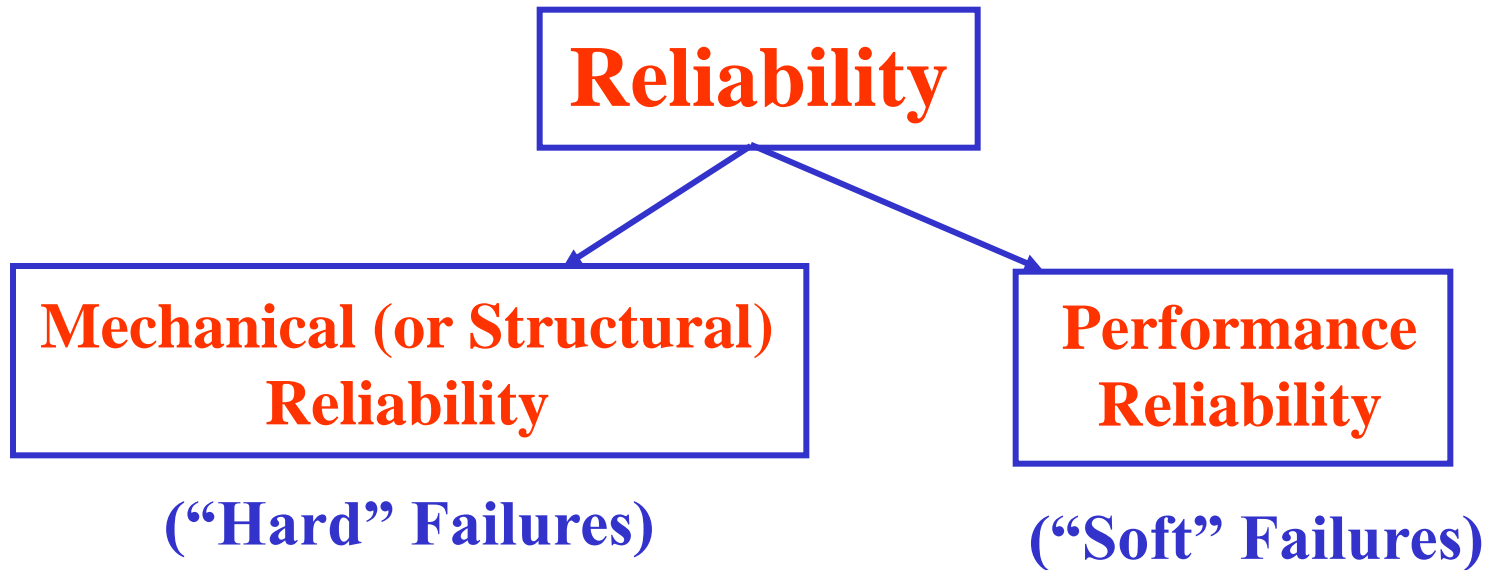
Time-Invariant Reliability



Time-Variant Reliability



Definitions / Observations



Quality: Performance Reliability at $t = 0$

OR

Conformance to Specifications at $t = 0$

Performance Reliability = Quality over Time

Design for Lifecycle Cost

$$C_L(\mathbf{d}, \mathbf{X}, t_f, r) = C_P(\mathbf{d}, \mathbf{X}) + C_I(\mathbf{d}, \mathbf{X}, t_0) + C_V^E(\mathbf{d}, \mathbf{X}, t_f, r)$$

Lifecycle Cost
Production Cost
Inspection Cost
Expected Variable Cost

$$C_V^E(\mathbf{d}, \mathbf{X}, t_f, r) = \int_0^{t_f} c_F(t) e^{-rt} f_T^c(t) dt$$

Final time
Interest rate

Cost of failure at time t
PDF of time to failure

$$F_T^c(t_L) = P(\exists t \in [0, t_L], \text{ such that } g(\mathbf{X}(t), t) \leq 0)$$

Design for Lifecycle Cost

$$\min_{\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \boldsymbol{\sigma}_{\mathbf{X}}} C_L(\mathbf{d}, \mathbf{X}, t_f, r)$$

of time-dependent
limit states



s. t. $F_{T_i}^c(\mathbf{d}, \mathbf{X}, t) \leq p_{f_i}^t$ for $i = 1, 2, \dots, n$

$$\mathbf{d}_L \leq \mathbf{d} \leq \mathbf{d}_U$$

e.g. $p_f^t = 0.05$ after $t = 5$ years

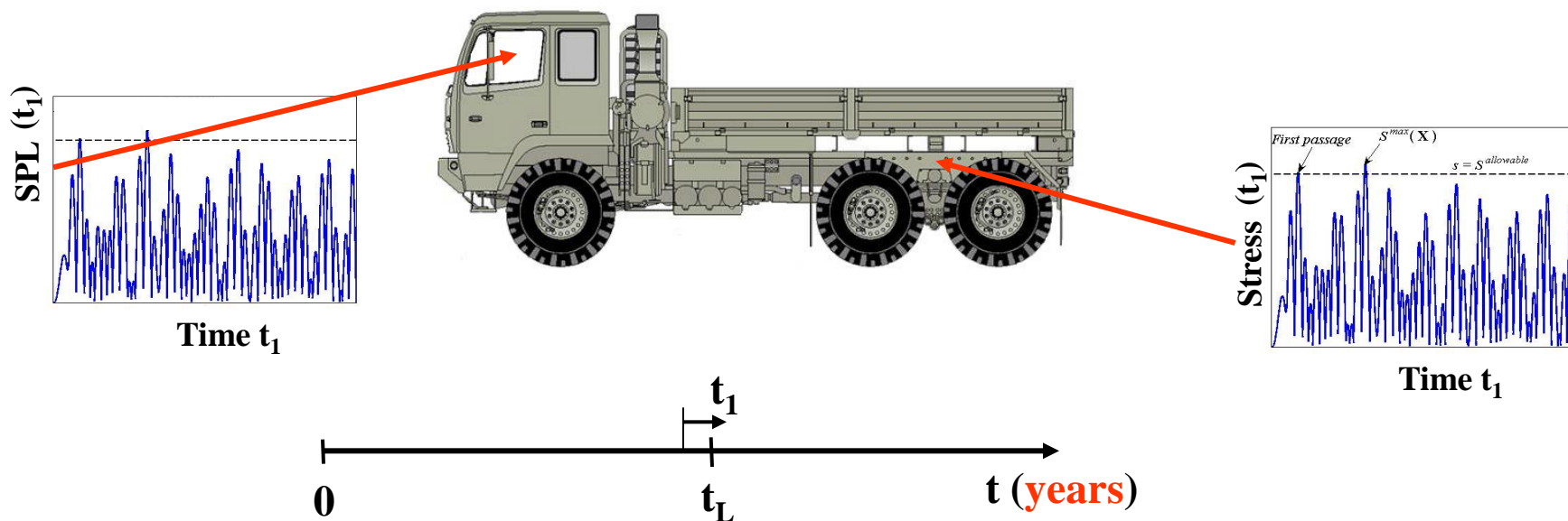


$$\boldsymbol{\mu}_{\mathbf{X}_L} \leq \boldsymbol{\mu}_{\mathbf{X}} \leq \boldsymbol{\mu}_{\mathbf{X}_U}$$

$$\boldsymbol{\sigma}_{\mathbf{X}_L} \leq \boldsymbol{\sigma}_{\mathbf{X}} \leq \boldsymbol{\sigma}_{\mathbf{X}_U}$$

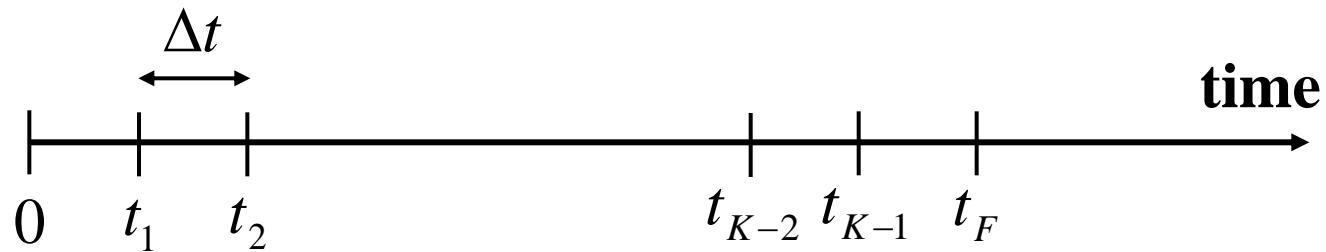
Calculation of Cumulative Probability of Failure

- Series system approach for $[0, t_f]$
- Reliability estimation at time t_L ($0 < t_L < t_f$); **multiple MPP case**



Calculation of Cumulative Probability of Failure

$$F_T^c(t_F) = P(\exists t \in [0, t_F], \text{ such that } g(\mathbf{X}(t), t) \leq 0)$$



$$t_F = K\Delta t$$

$$F_T^c(t_F) = P\left(\bigcup_{k=0}^K g(\mathbf{X}(t_k), t_k)\right)$$

Series System
Prob. of Failure

Cumulative Probability of Failure (PHI2 Method)

$$F_T^c(t_F) \approx F_T^i(0) + E[N^+(0, t_F)]$$

up-crossing rate

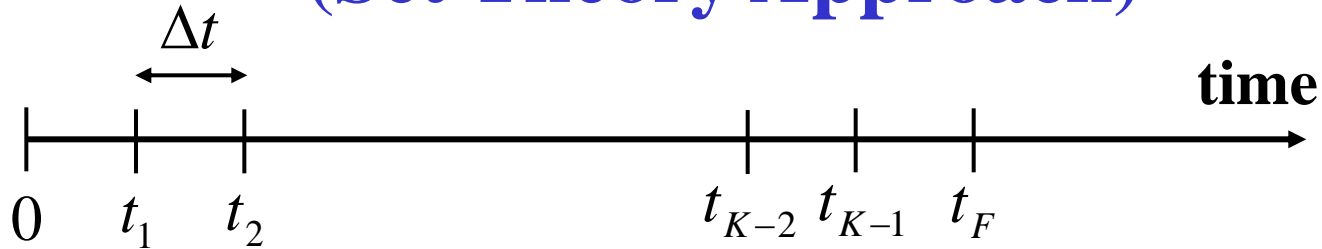
$$E[N^+(0, t_F)] = \int_0^{t_F} \nu^+(t) dt$$

of up-crossings in $[0, t_F]$

$$\nu^+(t) = \lim_{\Delta t \rightarrow 0} \frac{P[(g(\mathbf{X}(t), t) > 0) \cap (g(\mathbf{X}(t + \Delta t), t + \Delta t) \leq 0)]}{\Delta t}$$

$$\nu_{PHI2}^+(t) = \frac{\Phi_2[\beta(t), -\beta(t + \Delta t), \rho_{gg}(t, t + \Delta t)]}{\Delta t}$$

Cumulative Probability of Failure (Set Theory Approach)



$$t_F = K\Delta t$$

$$F_T^c(t_F) = P\left(\bigcup_{k=0}^K g(\mathbf{X}(t_k), t_k)\right)$$

Define: $IF_k = \{g(\mathbf{X}(t_k), t_k) \leq 0\}$ **(Instantaneous Failure Event)**

then: $CF_K = \bigcup_{k=0}^K IF_k$ **(Cumulative Failure)**

$\Delta F_k = \overline{CF_k} \cap CF_{k+1}$ **(Incremental Failure)**

$$P(\Delta F_k) \approx P(CF_k \cup CF_{k+1}) - P(CF_k)$$

Cumulative Probability of Failure (Set Theory Approach)

$$F_T^c(t_F) = P\left(\bigcup_{k=0}^K g(\mathbf{X}(t_k), t_k)\right) \approx P(IF_0) + \sum_{k=0}^{K-1} P(\Delta F_k)$$

$$P(\Delta F_k) \approx P(\mathbf{CF}_k \cup \mathbf{CF}_{k+1}) - P(\mathbf{CF}_k)$$

Probabilistic Re-Analysis MC Method

Cumulative Probability of Failure (PHI2 Method)

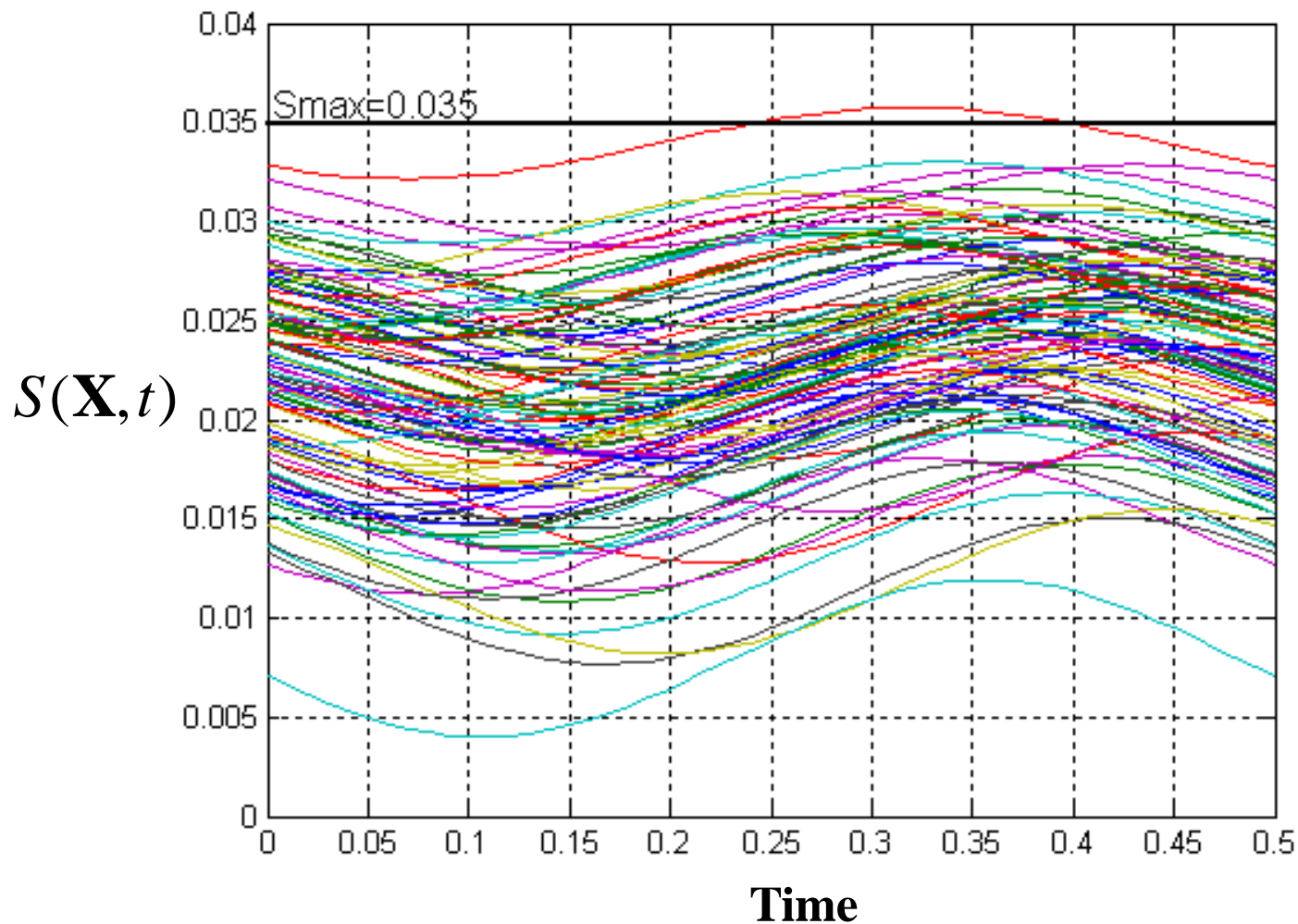
$$S(\mathbf{X}, t) = \frac{1}{18} - [12,000 - 1,000 \sin(4\pi t + X_3)] * \frac{10^4}{X_1 X_2}$$

$$X_1 \sim N(2.29 * 10^{-3}, 0.229 * 10^{-3})$$

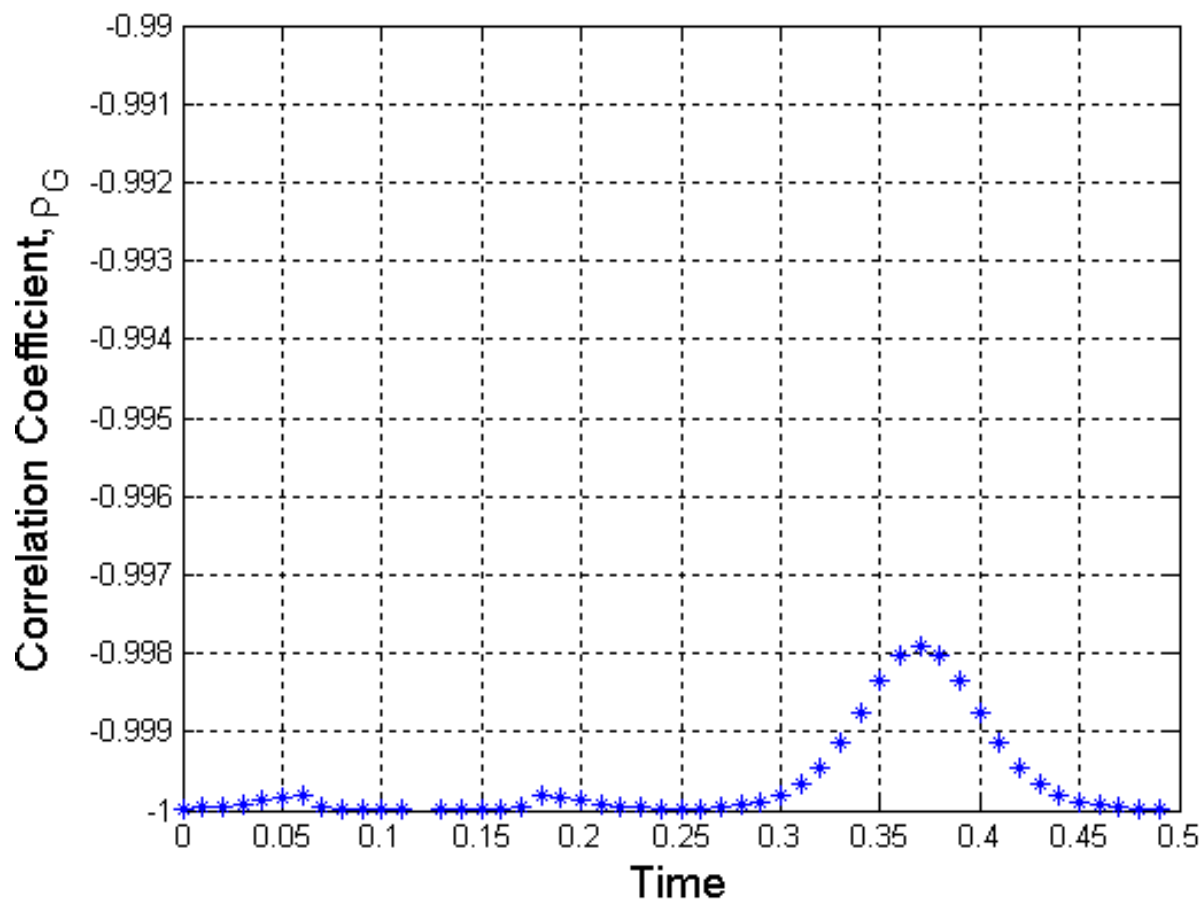
$$X_2 \sim N(2 * 10^{11}, 0.2 * 10^{11})$$

$$X_3 \sim N(0, 0.7)$$

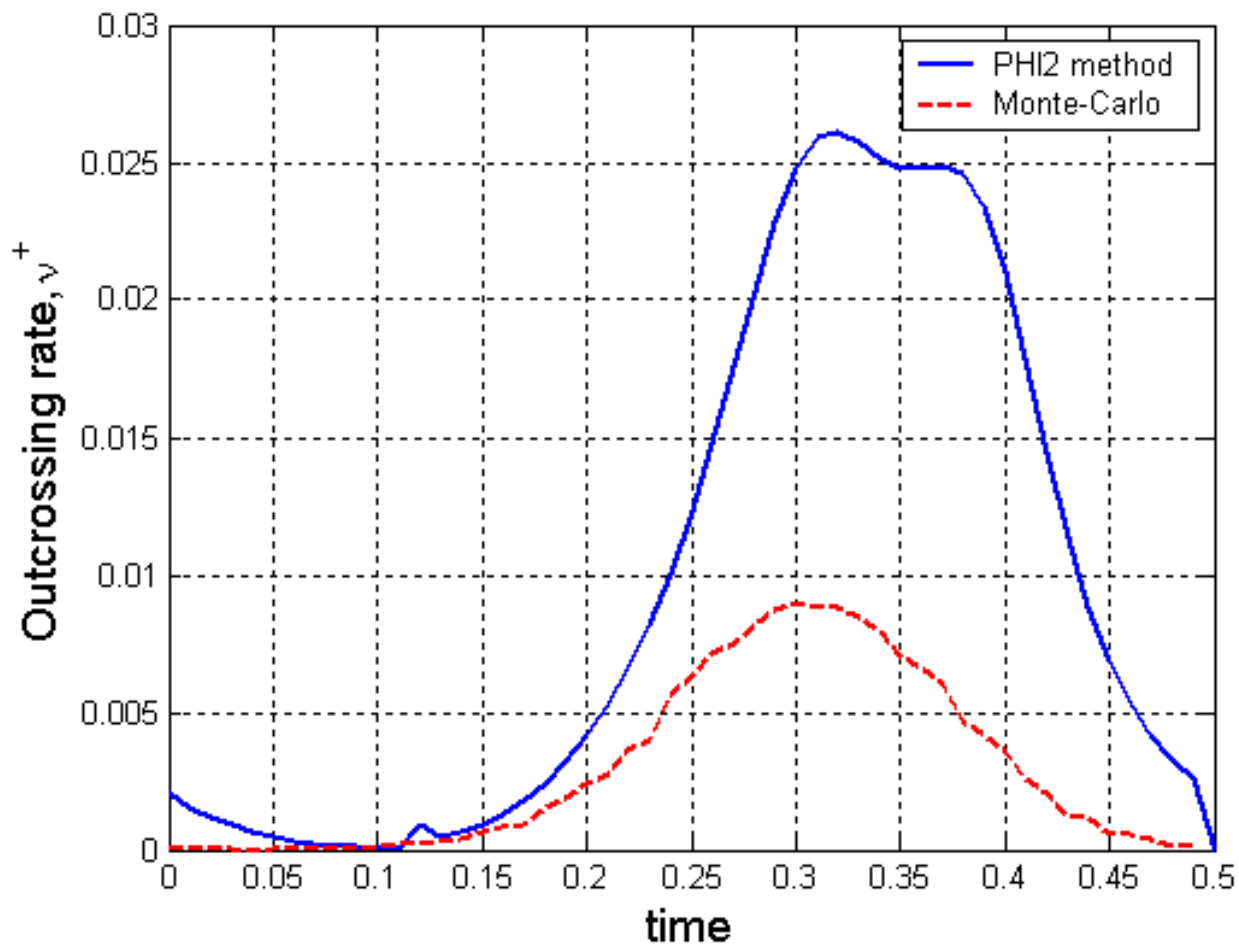
Cumulative Probability of Failure (PHI2 Method)



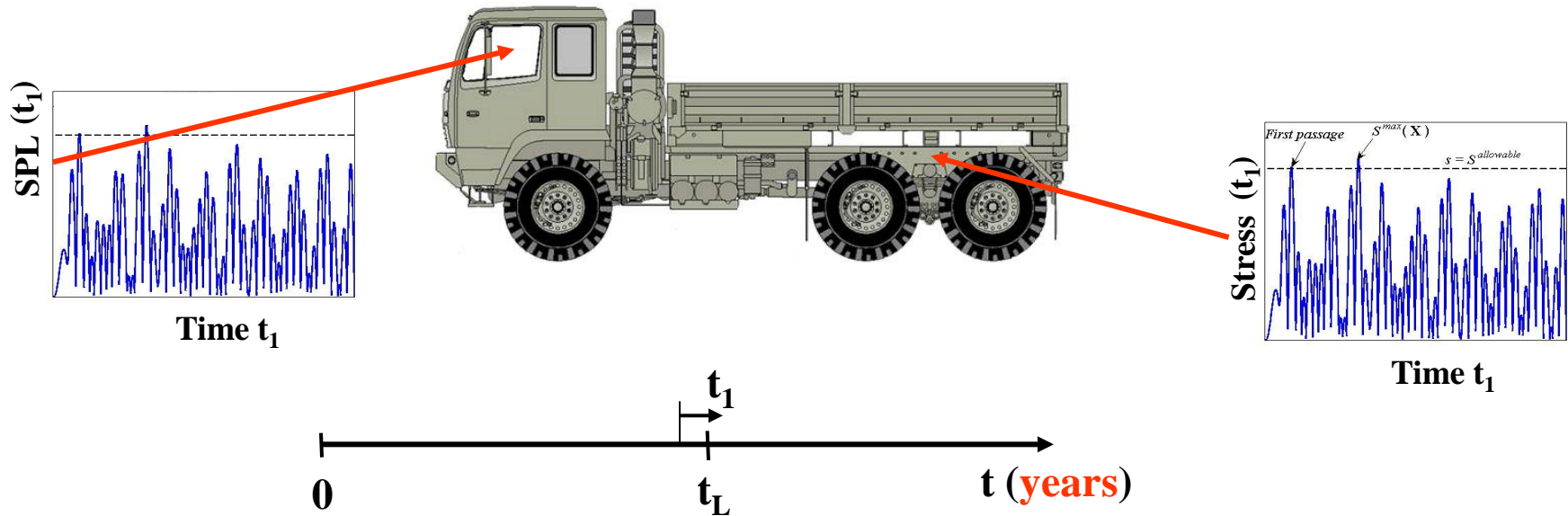
Cumulative Probability of Failure (PHI2 Method)



Cumulative Probability of Failure (PHI2 Method)



Reliability Estimation at Time t_L

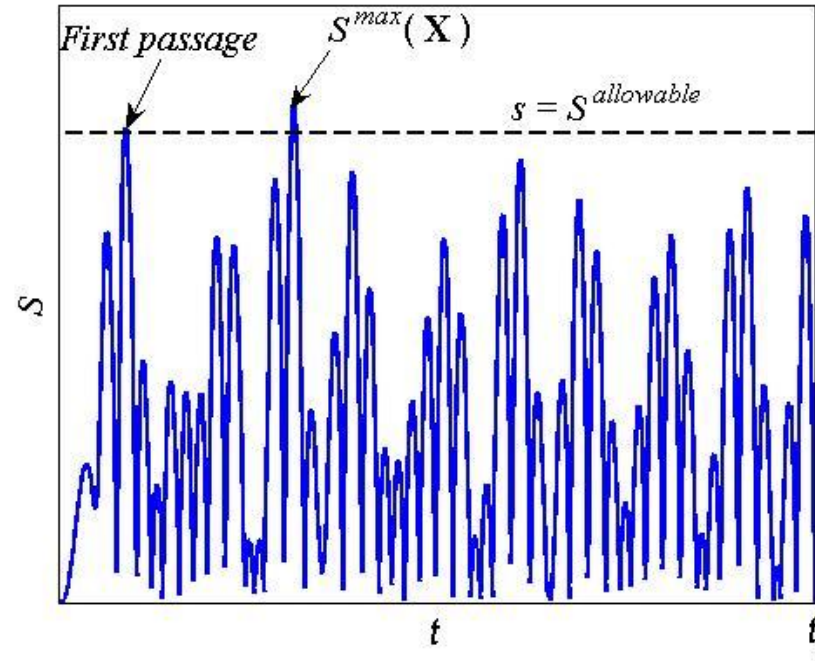


- **Time-dependent reliability**
- **Multiple MPPs**
- **Niching genetic algorithm**

Main Points

- **Time-dependent** reliability problem is **transformed** into a **time independent** reliability problem for solving a level-crossing problem
- **Multiple disjoint failure domains** may exist

Time-dependent Reliability Analysis of a Random Process



- **Transfer a time-dependent process to a time independent response:**

$$S = S(\mathbf{X}, t), \quad t_{\min} \leq t \leq t_{\max} \Rightarrow S^{\max}(\mathbf{X}) = \max_{t_{\min} \leq t \leq t_{\max}} S(\mathbf{X}, t)$$

- **Time-independent limit state:**

$$g(\mathbf{X}) = S^{\text{allowable}} - S^{\max}(\mathbf{X}) = 0$$

Time-dependent Reliability Analysis of a Random Process

➤ Double loop approach:

Outerloop: *Find MPP' s*

$$\beta_j = \min_{\mathbf{U} \in R^n} \|\mathbf{U}\|_2$$

$$s.t. \quad g(\mathbf{X}) = S^{allowable}(\mathbf{X}) - S^{\max}(\mathbf{X}) = 0$$

$$\mathbf{X} = \boldsymbol{\mu}_X + \boldsymbol{\sigma}_X \mathbf{U}$$

Inner loop:

*Find maximum response
in time domain*

$$S_j^{\max}(\mathbf{X}) = \max_{t_{\min} \leq t \leq t_{\max}} S_j(\mathbf{X}, t)$$

End of innerloop

End of outerloop

Inner
loop

Outer
loop

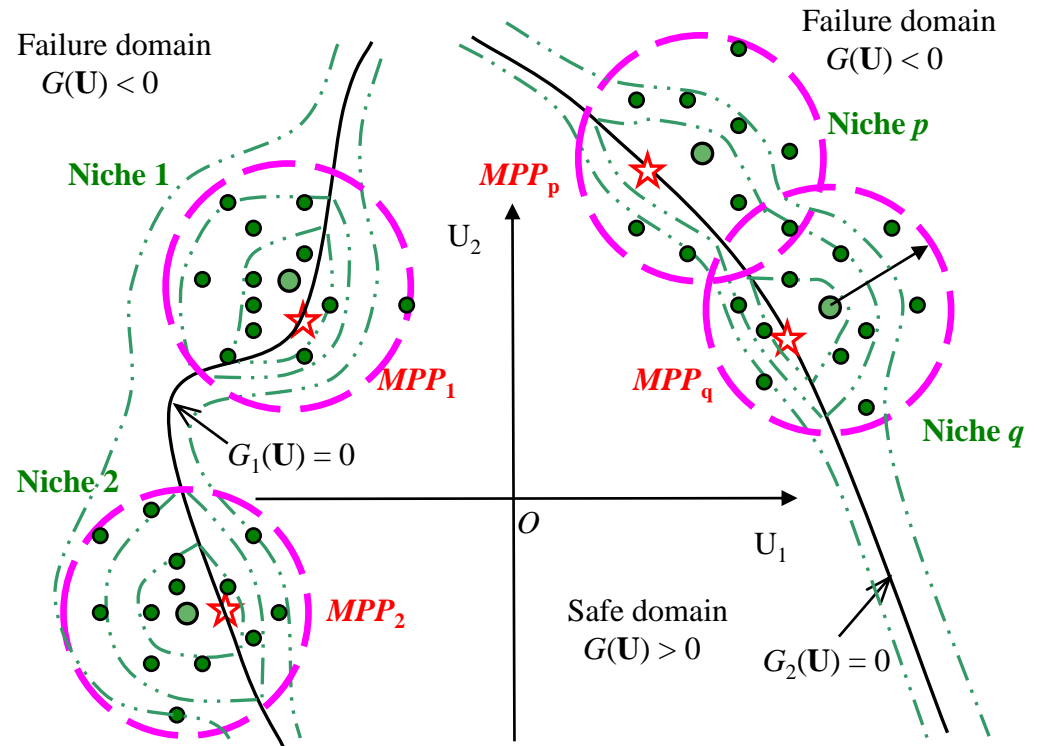
Time-dependent Reliability Analysis of a Random Process

➤ Definitions

- ✓ Actual MPPs
- ✓ Niches

➤ Observations

- ✓ Niche center is an approximate MPP
- ✓ Niching GA finds ALL approximate MPPs



Time-dependent Reliability Analysis of a Random Process

➤ Identification of Actual MPPs

- ✓ Linearized limit states

$$G_p^L(\mathbf{U}) = (\mathbf{U} - \mathbf{MPP}_p) \cdot \mathbf{MPP}_p = 0$$

or

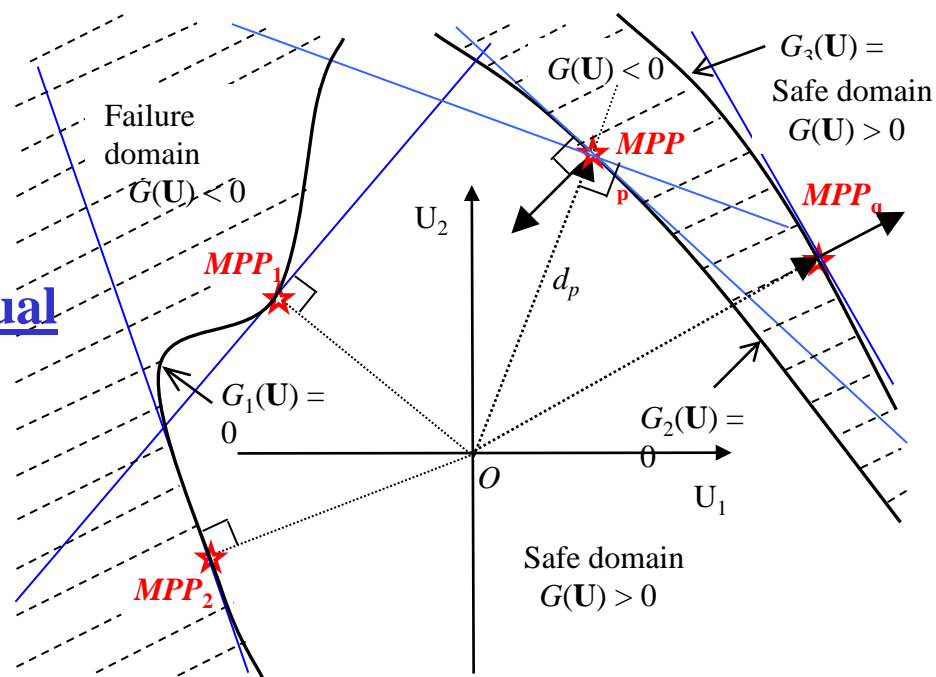
$$G_p^L(\mathbf{U}) = (\mathbf{U} - \mathbf{MPP}_p) \cdot \nabla G_p = 0$$

- ✓ Collinearity criterion for actual MPP

$$\cos \angle(\mathbf{MPP}_p, \nabla G_p) = \frac{\mathbf{MPP}_p \cdot \nabla G_p}{\|\mathbf{MPP}_p\| \cdot \|\nabla G_p\|} = \pm 1$$

- ✓ Practical criterion

$$\left| \frac{\mathbf{MPP}_p \cdot \nabla G_p}{\|\mathbf{MPP}_p\| \cdot \|\nabla G_p\|} \right| \geq 1 - \varepsilon_{\text{MPP}} \quad 0 < \varepsilon_{\text{MPP}} < 1$$

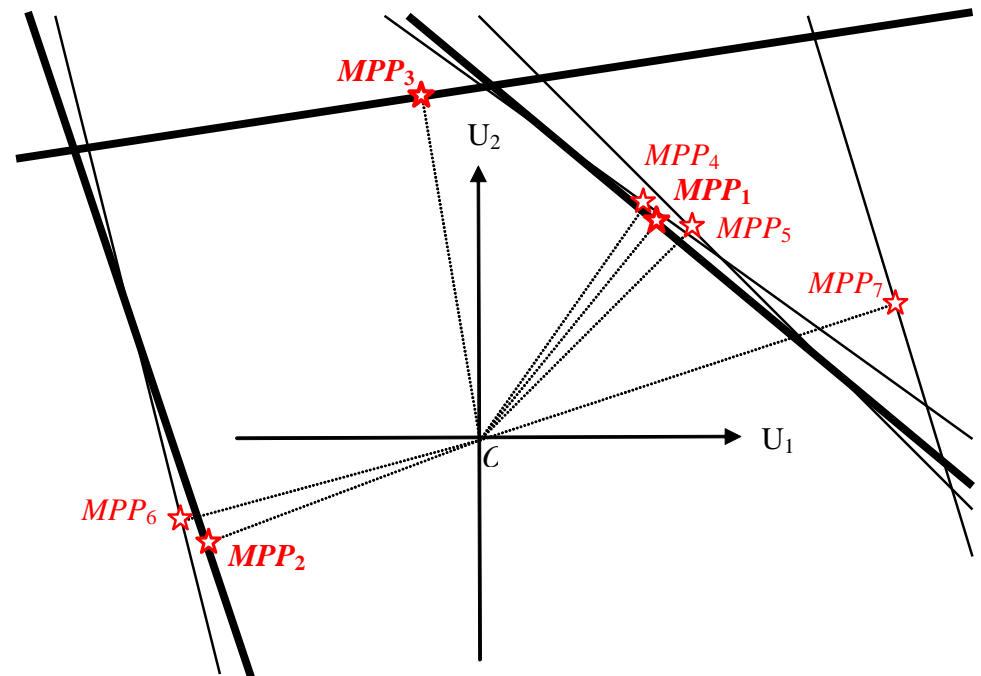


Time-dependent Reliability Analysis of a Random Process

➤ Identification of Independent and Significant MPPs

- Two MPPs are independent if the correlation coefficient is very close to **+1**
- Reliability index identifies significant MPPs

$$\rho_{pq}^L = \cos \angle(\mathbf{MPP}_p, \mathbf{MPP}_q) = \frac{\mathbf{MPP}_p}{\|\mathbf{MPP}_p\|} \cdot \frac{\mathbf{MPP}_q}{\|\mathbf{MPP}_q\|}$$





Time-dependent Reliability Analysis of a Random Process

➤ Estimation of Probability of Failure

- ✓ Convex safe domain formed by linearized limit states

$$M_p : \quad G_p^L(\mathbf{U}) = \mathbf{U} \cdot \mathbf{MPP}_p - \mathbf{MPP}_p \cdot \mathbf{MPP}_p \leq 0 \quad \text{for } p = 1, 2, \dots, M \quad \beta_p^L = \|\mathbf{MPP}_p\|_2$$

- ✓ Second order (bi-modal) bounds

$$p_{f_1} + \sum_{p=2}^M \max \left(p_p - \sum_{q=1}^{p-1} P(M_p \cap M_q), 0 \right) \leq p_f \leq \sum_{p=1}^M p_{f_i} - \sum_{p=2}^M \max_{q < p} P(M_p \cap M_q)$$

- ✓ MCS on the linearized convex safe domain

$$\Omega_f^L = \bigcup_p M_p = \bigcup_p (\mathbf{U} \cdot \mathbf{MPP}_p - \mathbf{MPP}_p \cdot \mathbf{MPP}_p \leq 0)$$

A Niching GA Method for Identifying Multiple MPPs

➤ Greedy Fitness Sharing

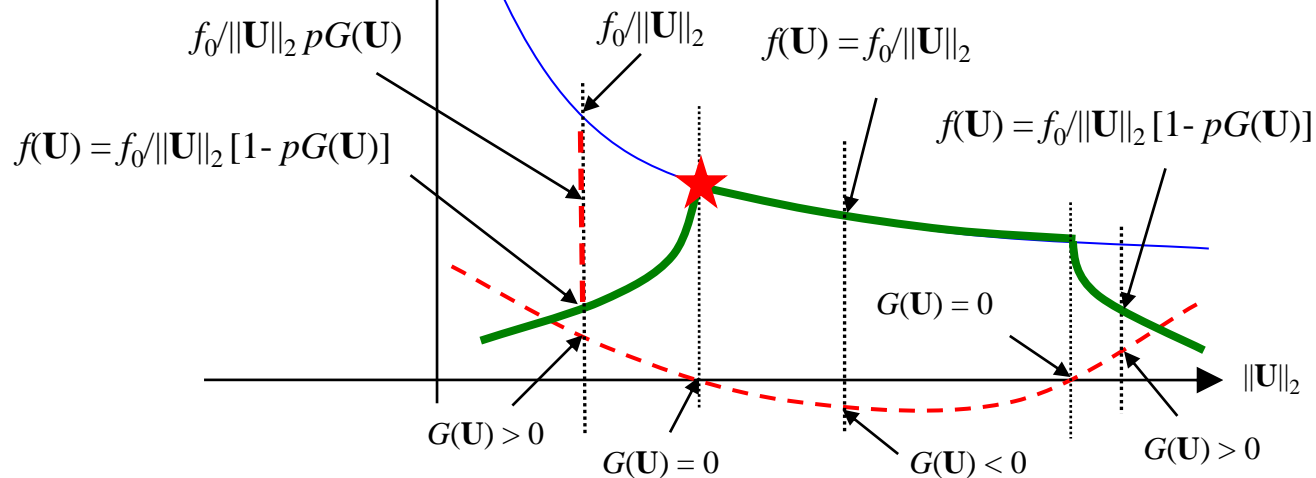
- ✓ Fitness sharing is a popular niching technique.
 - Decreases chance of mating in densely populated niches
 - Increases chance of mating in sparse niches.

A Niching GA Method for Identifying Multiple MPPs

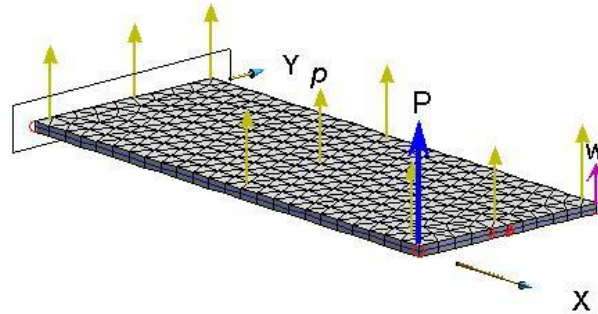
- ✓ Fitness function for identification of MPPs using RIA

$$\left. \begin{array}{l} \beta_j = \min_{\mathbf{U}} \|\mathbf{U}\|_2 \\ s.t. \quad g(\mathbf{U}) = 0 \end{array} \right\} \Rightarrow \boxed{\max f(\mathbf{U}) = \frac{f_0}{\|\mathbf{U}\|_2} \{1 - p \max[G(\mathbf{U}), 0]\}}$$

Penalty Function Approach



Numerical Example



• Model Description

- 3% critical modal damping
- Carried out for 0.1 seconds.

$$P(t) = 25 \cos(300\pi u(t - 0.004)) \text{ lbf}$$

$$p(t) = \cos(500\pi t) \text{ psi}$$

$$w(t) < \underbrace{0.3}_{|w|_{\max}} \text{ in}$$

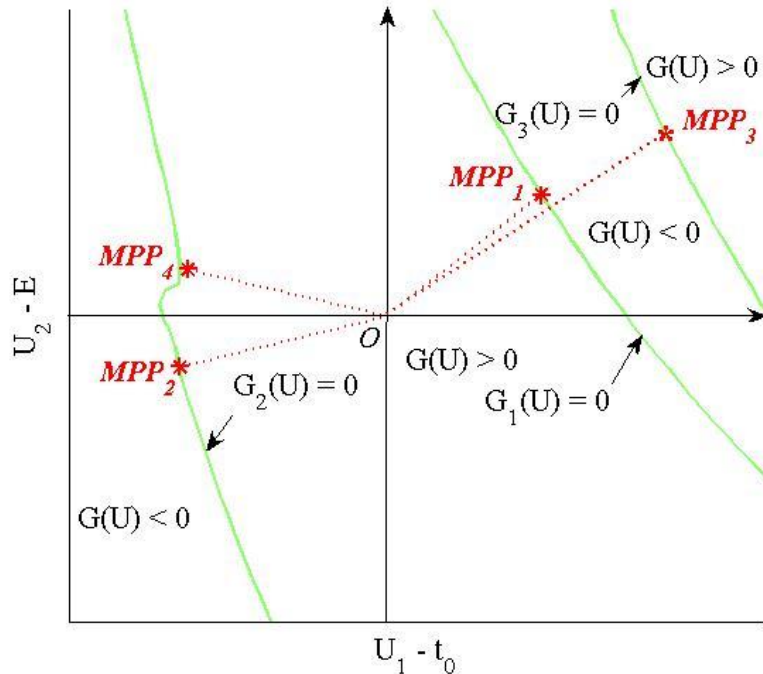
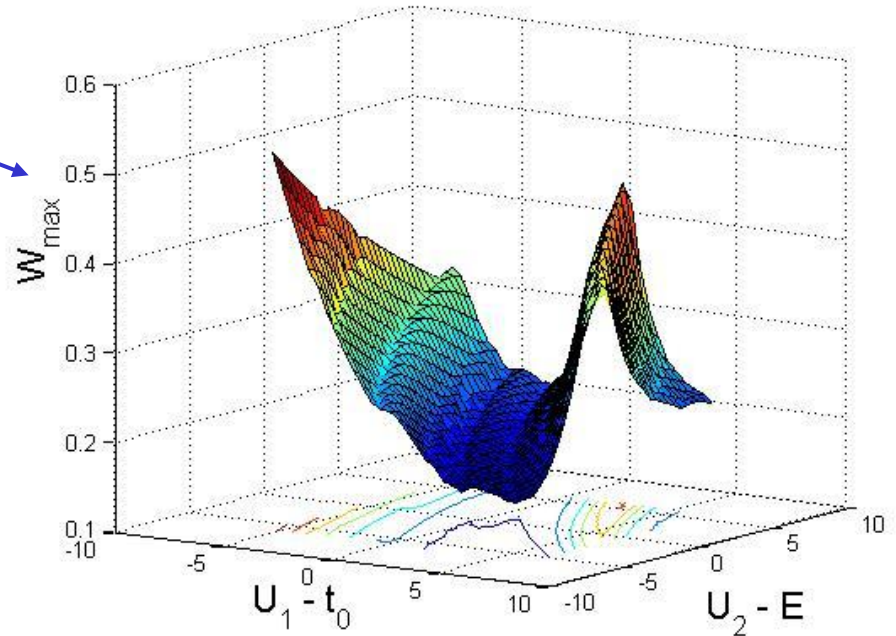
Plate properties

	Mean μ_X	$Cov = \frac{\sigma_X}{\mu_X}$	Std. Variation σ_X
Length (a)	2in	0	0
Height (b)	2in	0	0
Thickness	0.0877in	0.0513	0.0045in
Weight Density	0.282 lbs/in ³	0	0
Mass/Weight Factor	$2.59 \times 10^{-3} \text{ sec}^2/\text{in}$	0	0
Young's Modulus	$30.0 \times 10^6 \text{ lbs/in}^2$	0.05	$15 \times 10^5 \text{ lbs/in}^2$
Poisson's Ratio	0.3	0	0

Numerical Example

Multimodal maximum displacement response

$$S^{\max} = W_{\max}$$



Limit State :

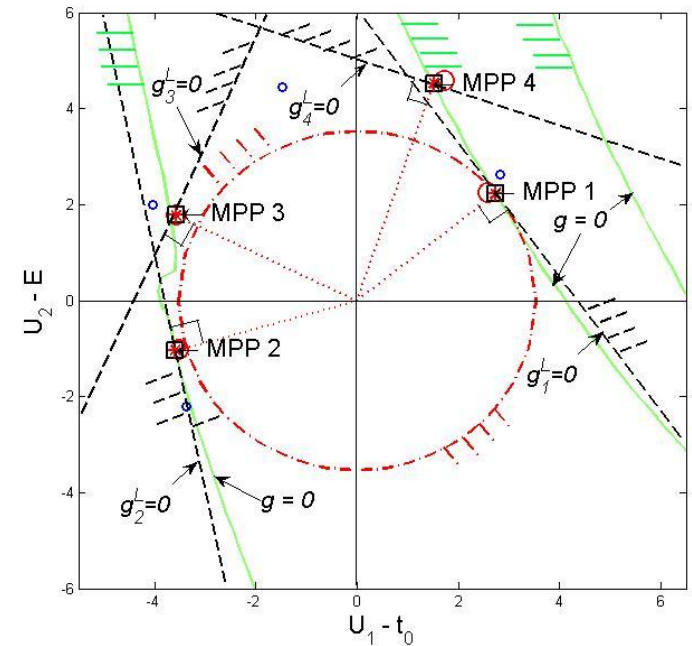
$$G(\mathbf{U}) = w_{\max} - 0.3 = 0$$

Numerical Example

- Identification of **independent** and **significant** MPPs.
 - Form a linear approximation of each limit state at each MPP.

$$\rho^L = \begin{bmatrix} \rho_{pq}^L \\ 1 & -0.9213 & -0.4146 & 0.8444 \\ -0.9213 & 1 & 0.7359 & -0.5696 \\ -0.4146 & 0.7359 & 1 & 0.1373 \\ 0.8444 & -0.5696 & 0.1373 & 1 \end{bmatrix}$$

Independent MPPs: $\rho_{ij} < +0.9$



Numerical Example

- Estimation of probability of failure using the Ditlevson's second-order bounds:

$$p_f^L = p_f^U = 3.3298 \times 10^{-4}$$

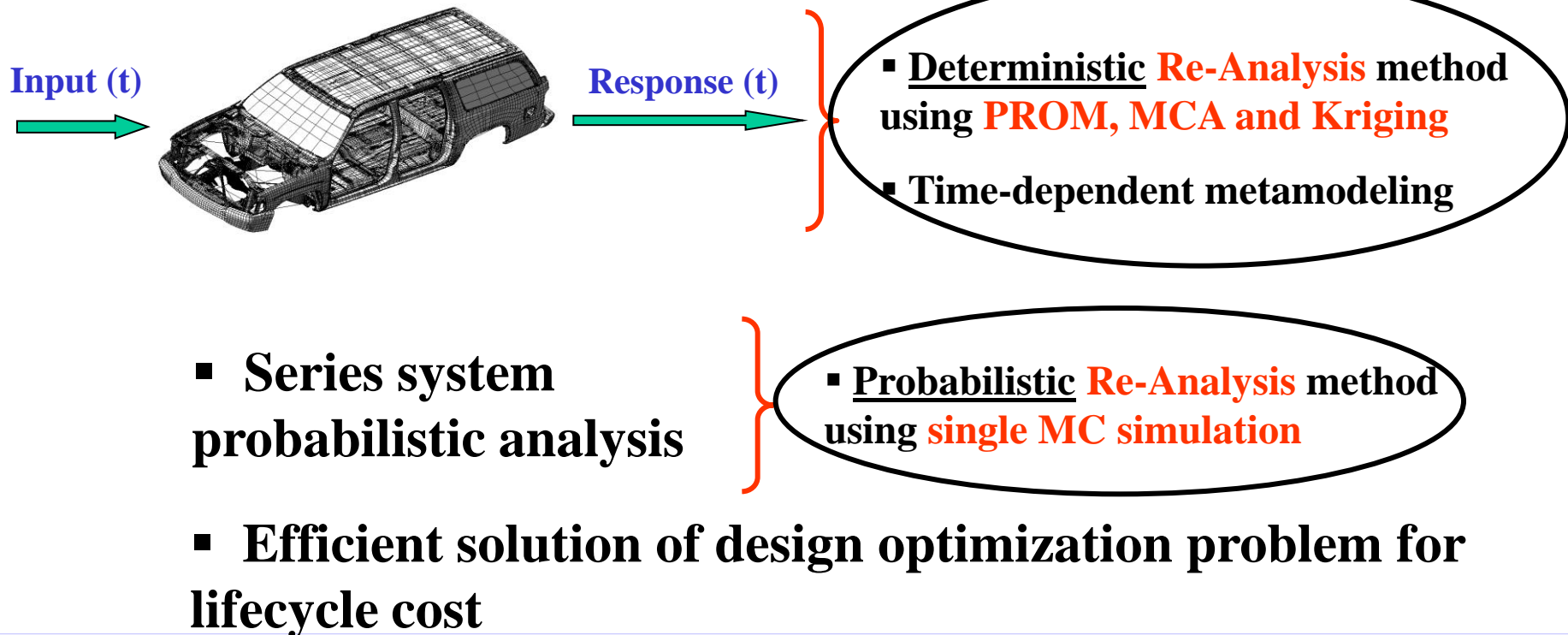
MC simulation with 1,000,000 samples using linearized safe domain yields:

$$P_{f \text{ MCS}}^L = 3.51 \times 10^{-4}$$

If MPP4 is removed: $p_f = 3.3287 \times 10^{-4}$ →

→ MPP4 is **insignificant**

- Calculation of **cumulative prob. of failure**
 - Series system approach for $[0, t_f]$
 - Reliability estimation at time t_L ($0 < t_L < t_f$); **multiple MPP case**





Q & A

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