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## **Quantum Simulation in Quantum Dot Arrays**

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**13. SUPPLEMENTARY NOTES**

**14. ABSTRACT**  
The authors use electrostatically defined quantum dot arrays to study quantum simulation, examining both large arrays to explore the ground state and excitations of the Fermi-Hubbard model, and to examine very small arrays of 3-4 quantum dots as the basis for simulating superexchange, frustration, magnetism, etc.

The large arrays were created by defining a grid-shaped gate on a semiconductor, forcing the electrons to see a periodic potential and be confined in a lattice of sites, where the low temperature reachable by the system in a dilution refrigerator allows examining the phase diagram of the model in a regime where spin correlations have an important influence. Initial plans to perform transport measurement were unsuccessful as the cleanroom realization of contacts to the two layers of the bilayered heterostructure had very low yield, so the authors performed capacitance spectroscopy measurement instead. This provides direct measurement of density of states, and the lithography process involved allowed grids with 150-200nm periods to be successfully written.

The small array examination analyzed how a system of four quantum dots on a square and filled with three electrons allowed the authors to study magnetism of few-body systems, examining whether the Nagaoka theorem could be realizable with quantum dots (conclusion: the ferromagnetic->paramagnetic state is in reach experimentally) and examining the experimental techniques to test the spin-structure of the ground state and induce the transition (finding: application of magnetic field can break the ferromagnetic ground state, in contrast to the usual). The authors also studied a linear array of three dots, resulting in publications in Nature Nanotechnology, Applied Physics Letters, and Annalen Der Physik

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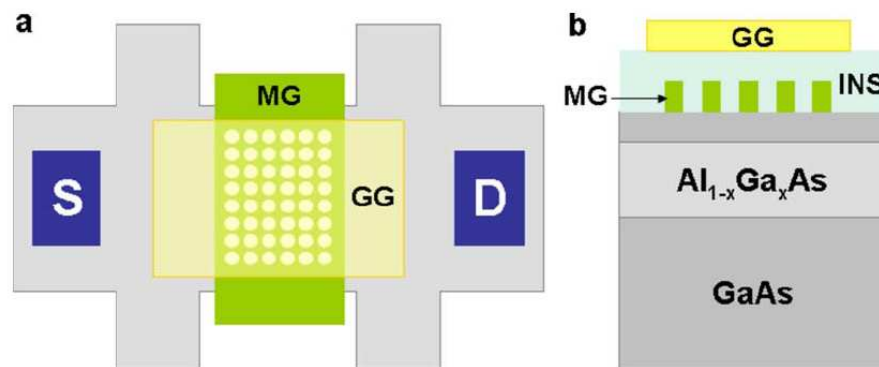
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*We use electrostatically defined quantum dot arrays to study quantum simulation. On the one hand, we have developed large two-dimensional quantum dot arrays as a means for exploring the ground state and excitations of the Fermi-Hubbard model. For this purpose, a new heterostructure design was created and a new capacitance spectroscopy set-up was built. Both are necessary to minimize the effects of disorder, so that we can extract useful information from the measurements. On the other hand, we have studied small arrays of three and four quantum dots as the basis for simulating superexchange, frustration, magnetism, and so forth.*

### 1) Large arrays

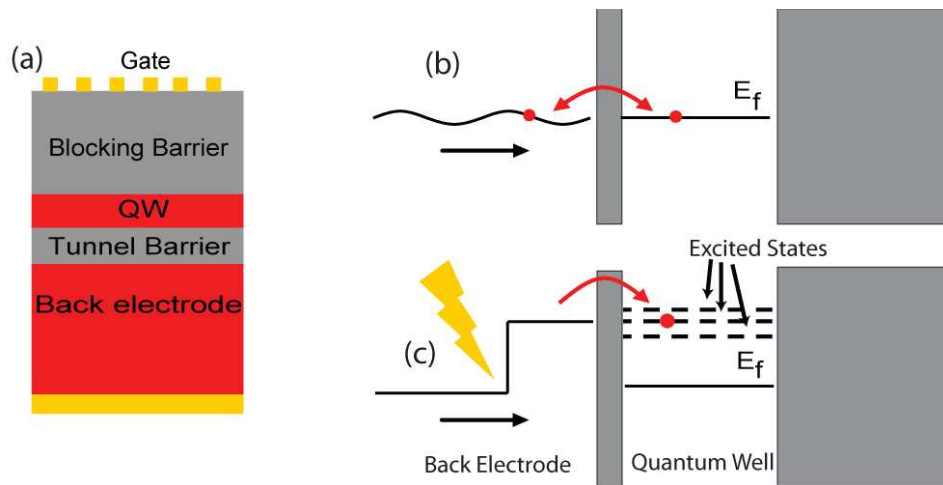
We aim to realize an array of laterally-defined semiconductor quantum dots, and use it to simulate the Fermi-Hubbard model. To do so, we lithographically define a grid-shaped gate on the surface of a semiconductor hosting a two-dimensional electron gas 50 to 100nm below the surface. The electrons in such a device see a periodic potential and can therefore be confined in a periodic lattice of sites, simulating the physics of the Hubbard model. The main advantage of using electrons in semiconducting system, compared to the highly successful BECs in optical lattices, originates from the low temperature that such a system can reach in dilution refrigerator, which allows probing the phase diagram of the Hubbard model in a regime where spin correlations are expected to have an important influence. To perform a reliable simulation of the Hubbard model, two challenges have to be addressed: first, the disorder has to be reduced: in commonly used 2DEGs, the presence of dopants and other impurities strongly perturbs the periodic potential, leading to localization of the electrons; second, the type of measurements chosen has to give access to meaningful quantities to characterize the state of the system.



*Fig. 1: Schematic of a quantum dot array device. A Schottky gate shaped like a mesh (MG) imposes a potential landscape much like an egg board carton on a 2D electron gas (2DEG) at a AlGaAs/GaAs heterointerface. A second, uniform gate is positioned on top of the MG, separated by a dielectric. The voltages on the two gates are tunable in-situ and allow us to walk through the phase diagram. Additional parameters that affect the Hamiltonian are the spacing between sites (period of the lithographically defined mesh), and the depth of the 2DEG. Typical dimensions we have in mind are  $\sim 100$  nm spacing between sites, and arrays of few micron on a side, giving us  $> 100$  sites. (Reproduced from Byrnes et al, PRB 2008)*

Ideally, we will have one electron per site. We aim at a lattice period of 100nm, which means the density will be  $10^{10} \text{ cm}^{-2}$ , about 20-30 times lower than is commonly used for quantum dot work. Clean heterostructures have been shown to conduct down to densities of  $\sim 5 \times 10^9 \text{ cm}^{-2}$ , which is encouraging. We have realized that this is not necessarily sufficient for our purposes, since electrons confined to lattice cells cannot screen disorder as efficiently as unconfined electrons. Therefore, we were concerned that our system may be Anderson localized (rather than Mott localized) already at  $10^{10} \text{ cm}^{-2}$ , and have studied ways to further reduce disorder.

The initial plans were to use an undoped bilayer heterostructure, where the deepest layer is used both as a screening layer to reduce the disorder and as a backgate to vary the electron density in the shallower 2DEG. We aimed to perform transport measurement, where the conductance of the system is probed to explore the quantum phases of the electron system. However, the clean-room realization of independent contacts to the two layers is very difficult and has a low yield.



*Fig. 2: Capacitance Spectroscopy Method. (a) Heterostructure scheme: the quantum dot is separated by a thin ( $<50\text{nm}$ ) tunnel barrier from a back electrode (degenerately doped GaAs region). (b) A low frequency signal on the back electrode induces tunnelling of electrons at the Fermi energy  $E_f$ , which modifies the sample capacitance. The capacitance measurement hence reveals the number of tunneling electrons, which is proportional to the density of states at  $E_f$ . (c) Time dependant measurements of tunnelling allows studying tunnelling to excited states and hence reveals the single-particle excitations of the system.*

We have therefore chosen to perform another type of measurements: capacitance spectroscopy (see figure 2). In this type of measurement, a single 2DEG is placed between a backgate (degenerately doped region of the semiconductor heterostructure) and lithographically defined top gates. We have simulated the disorder profile for various types of doped and undoped heterostructures. We concluded that an undoped structure with a screening layer close to the 2DEG ( $<50 \text{ nm}$  away) has the best chance of success. In collaboration with Werner Wegscheider at ETH Zurich, one of the best growers of GaAs/AlGaAs heterostructures worldwide, we have obtained specially designed and grown heterostructures for our work.

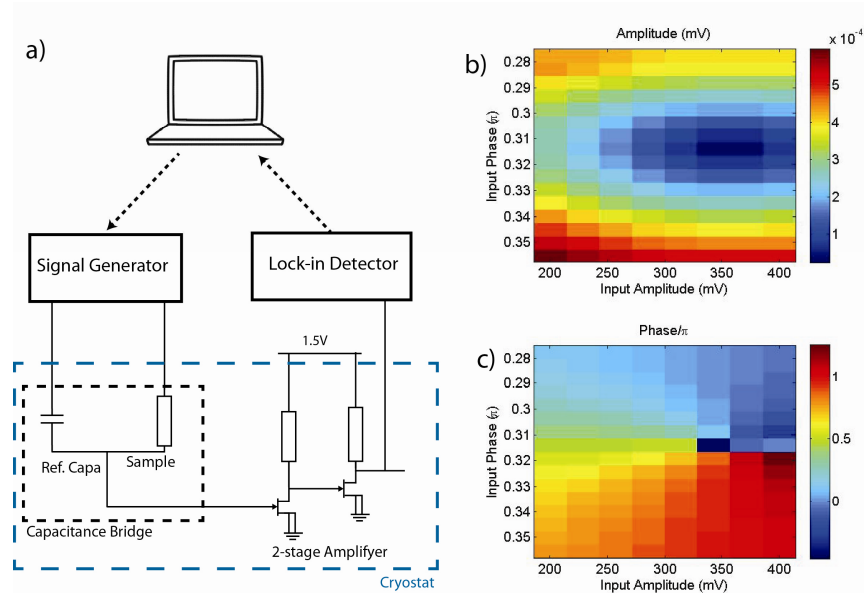


Fig. 3: *Capacitance Spectroscopy Implementation.* (a) *Measurement setup:* the sample capacitance is measured by balancing the impedance bridge in the cryostat. (b)-(c) *Amplitude and phase of the bridge output, as a function of the ref. signal amplitude and phase.* The precision on the balance condition, and hence of the capacitance measurement, is up to  $10^{-4}$ .

The measurement of the capacitance between bottom and top gate depends upon the tunnel-induced charge transfer between backgate and 2DEG and is therefore a direct measurement of the density of states, which gives important information on the phase diagram. Starting a collaboration with Ray Ashoori (MIT), we have realized a new setup to perform these capacitance spectroscopy measurements, and tested it with first samples we have realized from a heterostructure coming from Ashoori's group.

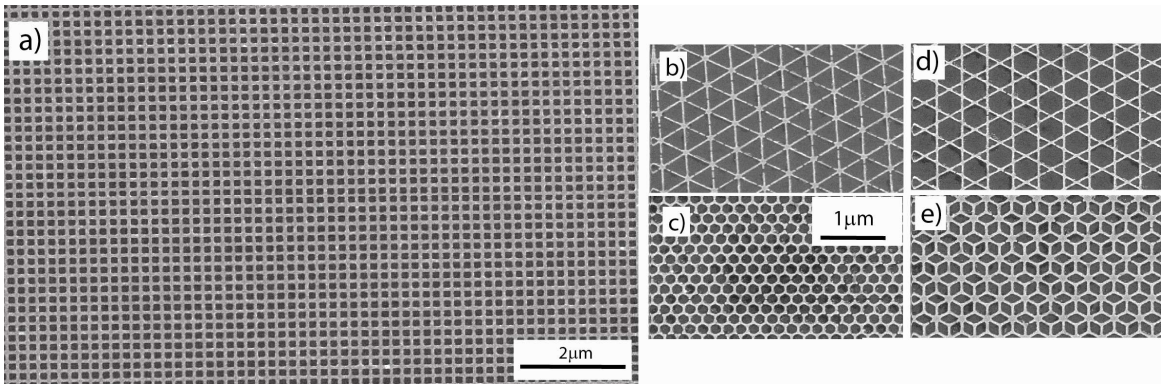


Fig. 4: *SEM images of the grid gates.* (a) *Rectangular lattice, realized with proximity effect correction and e-beam current optimized to minimize thermal drift effects.* (b) *Triangular lattice.* (c) *Hexagonal lattice.* (d) *Kagome Lattice.*

In parallel, we have benchmarked clean room fabrication methods to obtain clean grid gates samples. The fabrication of these grids was challenging, not only because of the difficulty to lift-off the resist in the holes of the grid, but also because these grids require high resolution lithography (line width  $< 30\text{nm}$ ) on large surfaces ( $200 \times 200$  microns). Using proximity effect correction and optimizing the e-beam current allowed improving the large-scale homogeneity

and resulted in high-quality grids (see figure 4). Different grid designs were created, which will allow to create quantum dot arrays with different symmetries (square, triangular/hexagonal or Kagome/Diced lattices). Grids with 150-200nm periods have been successfully written. Possibilities to reduce further the grid periods are currently under investigation.

### Small arrays

We have analyzed how a system of four quantum dots arranged on a square and filled with three electrons allow us to study magnetism of itinerant few-body systems.

For a single-band Fermi-Hubbard model on a square lattice, the Nagaoka theorem states that by doping the system with either one electron or one hole away from half filling the ground state becomes ferromagnetic in the limit of infinite on-site interaction. The simplest finite square lattice consists of four sites located at the corners of a square. The Nagaoka theorem now states that if filled by three Fermions the ground state becomes ferromagnetic for sufficiently large onsite energy, which turns out to be  $U = 18.6t$  where  $t$  is nearest neighbour hopping and  $U$  is the onsite interaction.

In our work, done in collaboration with Eugene Demler of Harvard University, his postdoc Bernhard Wunsch and Mark Rudner, we studied whether this transition is realizable with quantum dots. Relevant modifications include: 1) The level spacing is smaller than charging energy so that double occupied sites can be in a triplet state. 2) The interaction is long-ranged and causes correlations that are non-diagonal in Wannier functions. 3) The experimental setup is never perfectly symmetric and the effect of disorder has to be estimated. Our analysis shows that the phase transition from the ferromagnetic to the paramagnetic state is in reach experimentally.

We then study possible experimental techniques to test the spin-structure of the ground state, and to induce the transition. One finding, which is surprising at first, is that application of magnetic field can *break* the ferromagnetic ground state, in contrast to the usual case. The origin of this feature is that the Aharonov-Bohm phase induced by the magnetic field can give destructive interference for the hole travelling around the square, thereby destroying its stabilizing effect for the ferromagnetic state. We propose to detect the nature of the ground state via an extension of the well-established Pauli spin blockade technique.

Another system of small arrays we have studied is a linear array of three dots. We were initially interested in the question whether the filling of the middle dot (zero, one or two electrons) can modify the relative alignment of the spins in the outer dots (which are occupied by one electron each). We have found that the relative alignment for the ground state alternates sign as the filling of the middle dot is increased. Unfortunately, in this case the excitation energy scales are so small that we don't think we can find an experimental window where we could realistically probe this transition. However, dynamic (time-domain) behavior of spin chains can be studied very nicely in this system.

We have realized linear arrays of three dots, with an external extra dot as charge sensor. In these devices, we have observed for the first time cotunneling of charges between the two outer dots.

In this process, electron tunnel from one outer dot to the other through the virtual occupation of the middle dot. We have observed coherent charge oscillations between the dots through Landau-Zener-Stueckelberg interferometry. The dynamics of the electrons spin in such an effect is very interesting, as it frees us from the constraints of nearest-neighbor coupling, and opens the way to long range spin-spin coupling. This allows quantum simulation of a new range of phenomena, ranging from superexchange to frustration.

These results on coherent cotunneling in a triple dot device were published in Nature Nanotechnology. Investigations of the charge stability diagram of the triple dot device were published in Applied Physics Letters. Additionally, we have published in Annalen Der Physik a review of the progress towards quantum simulations with quantum dot systems.