REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
September 2013

2. REPORT TYPE
Viewgraph

3. DATES COVERED *(From - To)*
September 2013- October 2013

4. TITLE AND SUBTITLE
Multidimensional Effects on Conservative Particle Merging

5a. CONTRACT NUMBER
In-House

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Robert Martin, J. Koo, C. Lederman, J.-L. Cambier

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER
Q0AE

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory (AFMC)
AFRL/RQRS
1 Ara Drive
Edwards AFB CA 93524-7013

8. PERFORMING ORGANIZATION REPORT NO.

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Research Laboratory (AFMC)
AFRL/RQR
5 Pollux Drive
Edwards AFB CA 93524-7048

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
AFRL-RQ-ED-VG-2013-241

12. DISTRIBUTION / AVAILABILITY STATEMENT
Distribution A: Approved for Public Release; Distribution Unlimited. PA#13501

13. SUPPLEMENTARY NOTES
Viewgraph for the DSMC13, Santa Fe, NM, 21 October 2013

14. ABSTRACT
The number of particles simulated within a kinetic simulation has a direct impact on the accuracy of the results. In the case of chain-branching reactions such as those found in ionization and combustion events, the exponential growth of computational particle populations may also result in computationally intractable problems. Adaptive control of the number of computational particles is therefore an important topic for improving these types of simulations. Particle merging and its inverse splitting procedures can potentially enable this type of control, but only if they do not result in additional accumulated error. Merging multiple particles down to a single particle can be shown to either violate conservation of momentum or kinetic energy because a single particle consists of too few degrees of freedom to fully represent the original two. This has resulted in a proliferation of merging strategies relying on nearby particle pairs in velocity space or merging moments to computational grids as shown for example in Refs. [1-3]. If instead multiple particles are merged down to two rather than one, it can be shown that mass, momentum, and kinetic energy as well as center of mass and mean square deviation of position can be conserved simultaneously[4]. However, when previously attempted in this reference for electromagnetic particle-in-cell (PIC), the approach was found to result in excessive thermalization, incorrect collisionless shock wave-speeds, and was not obviously amenable to near-neighbor particle selection. To mitigate the thermalization effects, the ternary merge has been coupled with octree velocity space binning[5]. This method has been shown to match direct unmerged solutions well for several 3D3V simulations with predominately one-dimensional variations[5,6] aligned to the original coordinate system. Though these results were encouraging, the preferential selection of original spatial coordinate system for the moment decomposition suggested an orientation bias within the merge procedure. In this talk, we explore the impact of this orientation bias and several potential strategies to mitigate or eliminate it. In the process, we examine the impact of mixed second-order moments which leads to development of a merge strategy that conserves all six independent second moments using four particles. The addition of full 2nd-order spatial moment conservation can also conserve electrostatic energy for electrostatic PIC simulations to higher order in cell size for smooth electric fields. It is also shown that conserving the 0th-, 1st-, and full 2nd-moments simultaneously is in general impossible with only three particles as previously postulated in Ref. [5] except in degenerate cases. We also briefly consider the impact of dispersion between kinetic and potential energy for a sensitive 3D example case.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					Jean-Luc Cambier
a. REPORT	b. ABSTRACT	c. THIS PAGE	SAR	74	19b. TELEPHONE NO (include area code)
Unclassified	Unclassified	Unclassified			661-525-5649

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. 239.18

Multidimensional Effects on Conservative Particle Merging

Robert Martin[1], J. Koo[2], C. Lederman[1], J.-L. Cambier[2]

ERC Inc.[1], In-Space Propulsion Branch[2],
Air Force Research Laboratory
Edwards Air Force Base, CA USA

DSMC13,
Santa Fe, NM,
October 21, 2013

Distribution A: Approved for public release; unlimited distribution
PA (Public Affairs) Clearance Number TBD

Outline

1. Introduction and Prior Work
2. 0D Results
3. 1D3V Results
4. 3D3V Results
5. Future Work

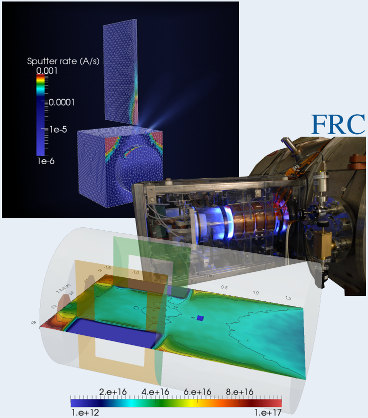
Spacecraft Plasma Modeling Challenges

Spacecraft Propulsion Relevant Plasma:

- From hall thrusters to plumes and fluxes on components
- Complex reaction physics i.e. Discharge and Breakdown in FRC
- Relevant Densities often Span 6+ Orders of Magnitude
- Spatial scales of interest span μm-100m range

Electric Propulsion Plumes

FRC



Chamber Environment

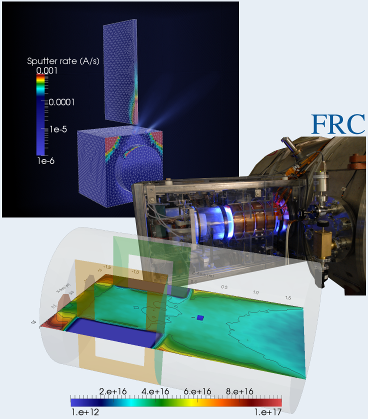
Spacecraft Propulsion Relevant Plasma:

- From hall thrusters to plumes and fluxes on components
- Complex reaction physics i.e. Discharge and Breakdown in FRC
- Relevant Densities often Span 6+ Orders of Magnitude
- Spatial scales of interest span μm-100m range

Solution?

Multi-Scale and Multi-Physics Adaptive Algorithms

Electric Propulsion Plumes



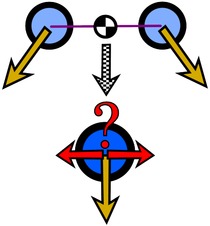
Chamber Environment

Numerous Previous Merge Methods:

Prior Merging Techniques

Numerous Previous Merge Methods:

- 2:1 - Cannot Conserve Energy
  (Lapenta & Brackbill, JCP 1994)

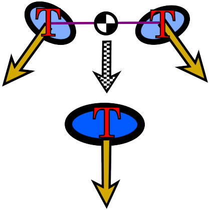


$$w_m = \sum w_i$$
$$\vec{v}_m = \sum w_i \vec{v}_i / w_m$$

$$w_m v_m^2 < \sum w_i v_i^2 \quad !!!$$

Numerous Previous Merge Methods:

- 2:1 - Cannot Conserve Energy
  (Lapenta & Brackbill, JCP 1994)
- Complex Macro-particles with Internal Energy
  (Hewett, JCP 2003)



$$w_m = \sum w_i, \quad \vec{v}_m = \sum w_i \vec{v}_i / w_m$$

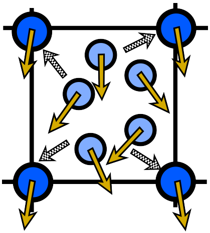
$$T_m^{(int)} = \left(\sum w_i T_i^{(int)} + \sum w_i v_i^2 - w_m v_m^2 \right) / w_m$$

$$w_m \left(v_m^2 + T_m^{(int)} \right) = \sum \left(w_i v_i^2 + w_i T_i^{(int)} \right) \surd$$

Particle Push with $T^{(int)}$?...
Split if $(T^{(int)})^{1/2} \gg v_m$?...
In What Coord-System?...

Numerous Previous Merge Methods:

- 2:1 - Cannot Conserve Energy
  (Lapenta & Brackbill, JCP 1994)
- Complex Macro-particles with Internal Energy
  (Hewett, JCP 2003)
- Merge to Grid
  (Assous et al., JCP 2003, Welch et al., JCP 2007)



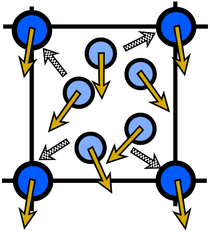
Conserve Arbitrary Moments to Grid

Not Explicitly Conserved Lost?
Entropy Generation?
Shape Functions?
Grid Dependence?

Numerous Previous Merge Methods:

- 2:1 - Cannot Conserve Energy
  (Lapenta & Brackbill, JCP 1994)
- Complex Macro-particles with Internal Energy
  (Hewett, JCP 2003)
- Merge to Grid
  (Assous et al., JCP 2003, Welch et al., JCP 2007)

All Introduce Significant Error and/or Complexity



Conserve Arbitrary Moments to Grid

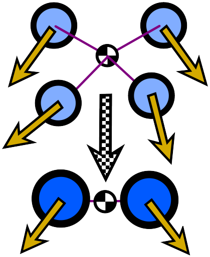
Not Explicitly Conserved Lost?
Entropy Generation?
Shape Functions?
Grid Dependence?

Conservative Merge

Merge to Pair → DOF for Conservation:

- (n+2):2 yields Exact Mass, Momentum, and Kinetic Energy Conservation
- Applied Spatially also Shown to Conserve Electrostatic Energy
- Though Energy Conserving, Still Thermalizes VDF

(AFOSR Review 2006)



$$w_m = \sum_i^{(n+2)} w_i$$

$$\bar{\vec{v}} = \left(\sum_i^{(n+2)} w_i \vec{v}_i\right) / w_m$$

$$\overline{V^2} = \left(\sum_i^{(n+2)} w_i \left(\vec{v}_i - \bar{\vec{v}}\right)^2\right) / w_m$$

$$w_{(a/b)} = w_m / 2$$

$$\vec{v}_{(a/b)} = \bar{\vec{v}} \pm \hat{\mathcal{R}} \sqrt{\overline{V^2}}$$

$$\left(\text{Similarly: } \vec{x}_{(a/b)} = \bar{\vec{x}} \pm \hat{\mathcal{R}} \sqrt{\overline{X^2}}\right)$$

Conservative Merge

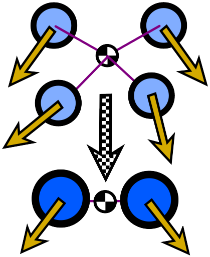
Merge to Pair → DOF for Conservation:

- (n+2):2 yields Exact Mass, Momentum, and Kinetic Energy Conservation
- Applied Spatially also Shown to Conserve Electrostatic Energy
- Though Energy Conserving, Still Thermalizes VDF

(AFOSR Review 2006)

Selection of Near Neighbors in VDF Limits Thermalization

(≈ Near Neighbor Pairs in 2:1 Merges that Limit Numerical Cooling)

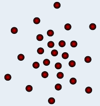


$$w_m = \sum_i^{(n+2)} w_i$$
$$\bar{\vec{v}} = \left(\sum_i^{(n+2)} w_i \vec{v}_i\right) / w_m$$
$$\overline{V^2} = \left(\sum_i^{(n+2)} w_i \left(\vec{v}_i - \bar{\vec{v}}\right)^2\right) / w_m$$

$$w_{(a/b)} = w_m / 2$$
$$\vec{v}_{(a/b)} = \bar{\vec{v}} \pm \hat{\mathcal{R}} \sqrt{\overline{V^2}}$$
$$\left(\text{Similarly: } \vec{x}_{(a/b)} = \bar{\vec{x}} \pm \hat{\mathcal{R}} \sqrt{\overline{X^2}}\right)$$

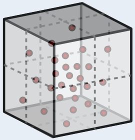
Phase-Space Decomposition

- Given a Set of Particles...



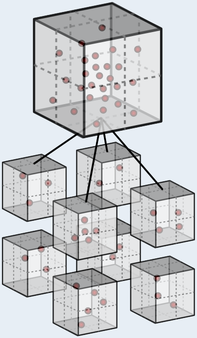
Phase-Space Decomposition

- Given a Set of Particles...
- Particles Binned in Octants



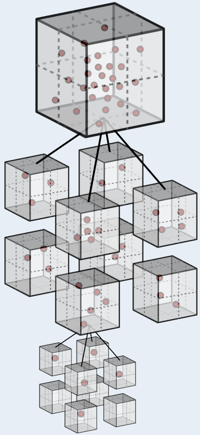
Phase-Space Decomposition

- Given a Set of Particles...
- Particles Binned in Octants
- Octants Recursively Sub-Divided



Phase-Space Decomposition

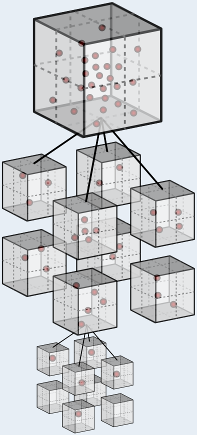
- Given a Set of Particles...
- Particles Binned in Octants
- Octants Recursively Sub-Divided
- Recursion Halted at 1-Particle/Bin or Other Criteria such as Bin-Density



Phase-Space Decomposition

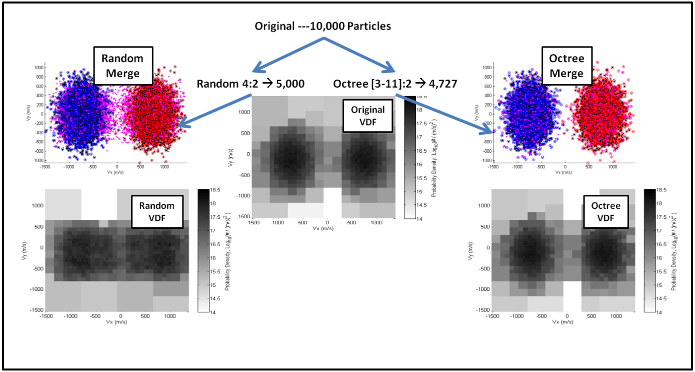
- Given a Set of Particles...
- Particles Binned in Octants
- Octants Recursively Sub-Divided
- Recursion Halted at 1-Particle/Bin or Other Criteria such as Bin-Density

Restricts Phase-Space Diffusion to Within Local Bins

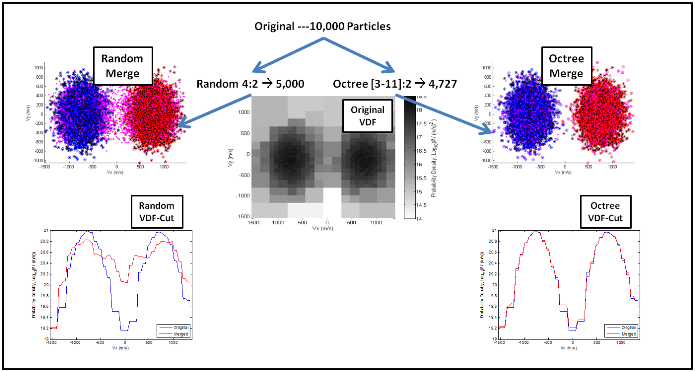


Comparison of Random vs. Octree Merge Partner Selection

(Note: Mass, Momentum, and Kinetic Energy Both Exactly Conserved)

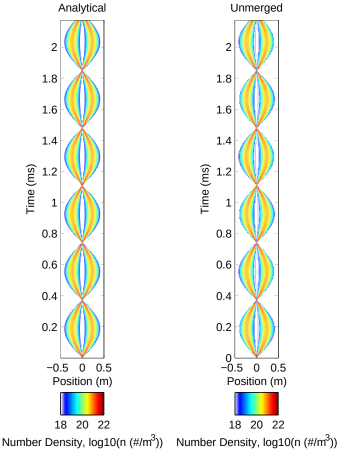


Comparison of Random vs. Octree Merge Partner Selection
(Note: Mass, Momentum, and Kinetic Energy Both Exactly Conserved)



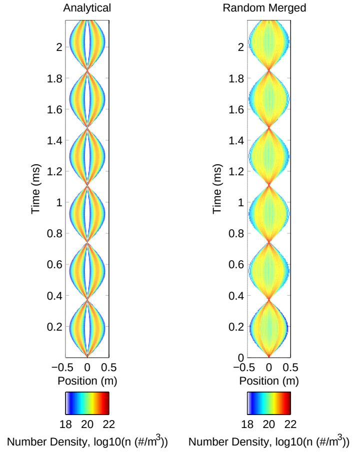
Collisionless Particles in Well

- 6000 Unmerged Particles
- Reproduces 3-4 Orders of Magnitude



Collisionless Particles in Well

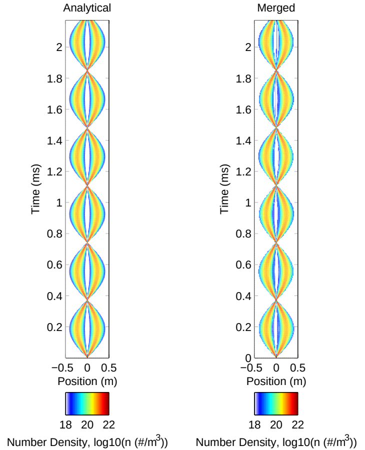
- 6000 Unmerged Particles
- Reproduces 3-4 Orders of Magnitude
- Random Merge -> Thermalization
- 3000 First Point, 1500 First Cross
- Bi-Maxwellian Specifically Difficult



Beam in Potential Well

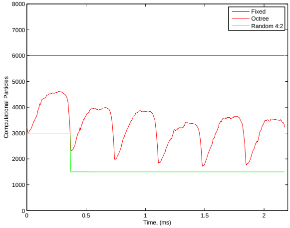
Collisionless Particles in Well

- 6000 Unmerged Particles
- Reproduces 3-4 Orders of Magnitude
- Random Merge -> Thermalization
- 3000 First Point, 1500 First Cross
- Bi-Maxwellian Specifically Difficult
- Octree Merge Significantly Better



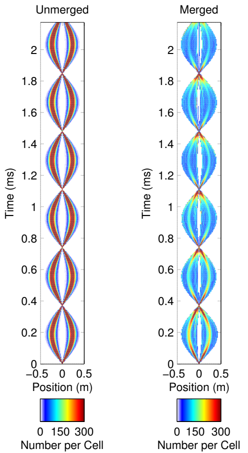
Collisionless Particles in Well

- 6000 Unmerged Particles
- Reproduces 3-4 Orders of Magnitude
- Random Merge -> Thermalization
- 3000 First Point, 1500 First Cross
- Bi-Maxwellian Specifically Difficult
- Octree Merge Significantly Better
- Merge & Split Adapts Particle Count



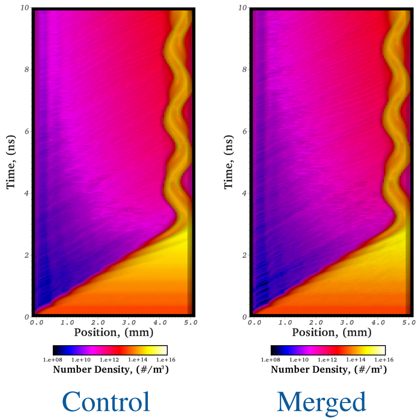
Collisionless Particles in Well

- 6000 Unmerged Particles
- Reproduces 3-4 Orders of Magnitude
- Random Merge -> Thermalization
- 3000 First Point, 1500 First Cross
- Bi-Maxwellian Specifically Difficult
- Octree Merge Significantly Better
- Merge & Split Adapts Particle Count
- Computational Particles per Cell Vastly Different



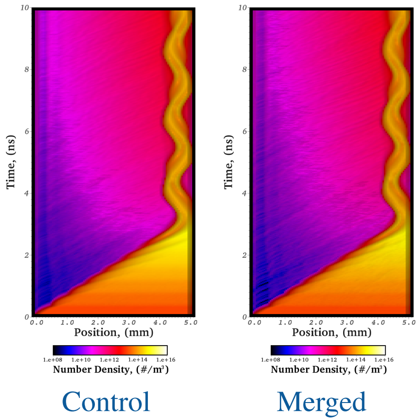
DC-Diode Test Case:

- Full 3D Electrostatic-PIC
- Averaged to 1D XT-Plot



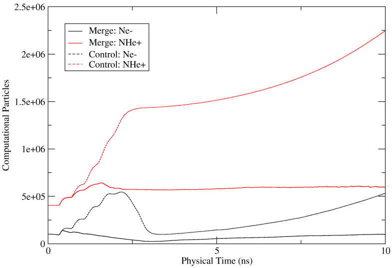
DC-Diode Test Case:

- Full 3D Electrostatic-PIC
- Averaged to 1D XT-Plot
- 250V Cathode → Anode
- MCC-Ionization Collisions
- Secondary Emission at Cathode



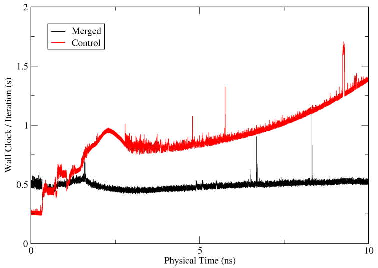
DC-Diode Test Case:

- Full 3D Electrostatic-PIC
- Averaged to 1D XT-Plot
- 250V Cathode → Anode
- MCC-Ionization Collisions
- Secondary Emission at Cathode
- Chain-Branching Needs Merge



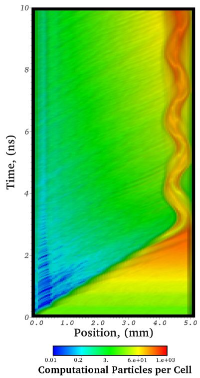
DC-Diode Test Case:

- Full 3D Electrostatic-PIC
- Averaged to 1D XT-Plot
- 250V Cathode $\rightarrow$ Anode
- MCC-Ionization Collisions
- Secondary Emission at Cathode
- Chain-Branching Needs Merge
- Merge Overhead Rapidly Negligible



DC-Diode Test Case:

- Full 3D Electrostatic-PIC
- Averaged to 1D XT-Plot
- 250V Cathode $\rightarrow$ Anode
- MCC-Ionization Collisions
- Secondary Emission at Cathode
- Chain-Branching Needs Merge
- Merge Overhead Rapidly Negligible
- Control: Parts/Cell $\propto$ Density

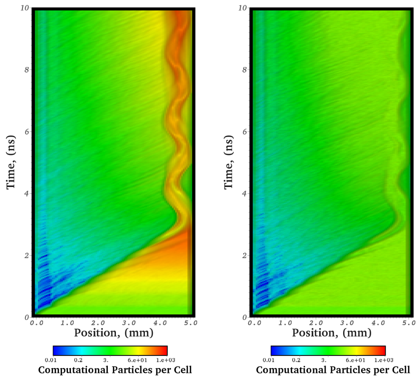


Control Merged

DC-Breakdown

DC-Diode Test Case:

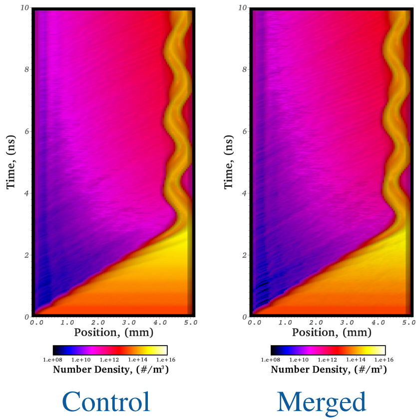
- Full 3D Electrostatic-PIC
- Averaged to 1D XT-Plot
- 250V Cathode $\rightarrow$ Anode
- MCC-Ionization Collisions
- Secondary Emission at Cathode
- Chain-Branching Needs Merge
- Merge Overhead Rapidly Negligible
- Control: Parts/Cell $\propto$ Density
- Merge: Parts/Cell Much Reduced



Control Merged

DC-Diode Test Case:

- Full 3D Electrostatic-PIC
- Averaged to 1D XT-Plot
- 250V Cathode → Anode
- MCC-Ionization Collisions
- Secondary Emission at Cathode
- Chain-Branching Needs Merge
- Merge Overhead Rapidly Negligible
- Control: Parts/Cell ∝ Density
- Merge: Parts/Cell Much Reduced
- Despite Identical Densities

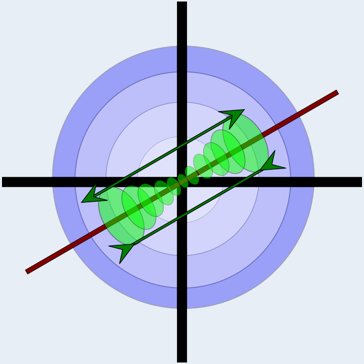


Multidimensional Concerns

- Conserving $[v_x^2, v_y^2, v_z^2] \rightarrow$ Axis Aligned Preference?
- 3D Well: $\Phi \propto (x^2 + y^2 + z^2)$

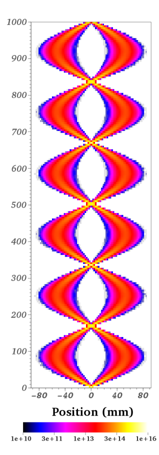
Multidimensional Concerns

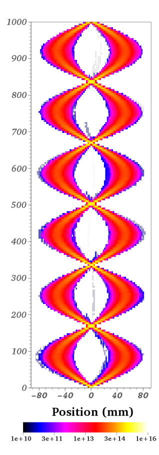
- Conserving $[v_x^2, v_y^2, v_z^2] \rightarrow$ Axis Aligned Preference?
- 3D Well: $\Phi \propto (x^2 + y^2 + z^2)$
- First: Bounce Aligned 30° to $x+$



Multidimensional Concerns

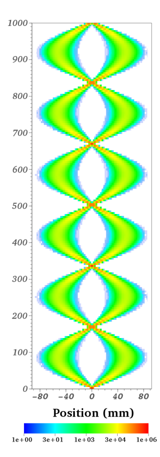
- Conserving $[v_x^2, v_y^2, v_z^2] \rightarrow$ Axis Aligned Preference?
- 3D Well: $\Phi \propto (x^2 + y^2 + z^2)$
- First: Bounce Aligned 30° to $x+$
- Slice Results like 1D Well

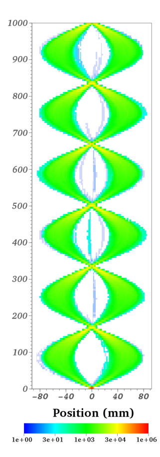




Multidimensional Concerns

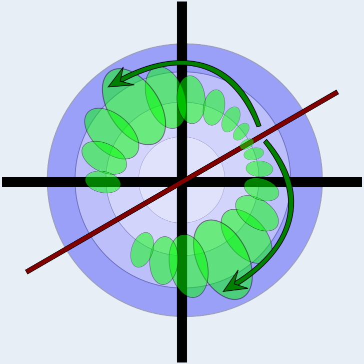
- Conserving $[v_x^2, v_y^2, v_z^2] \rightarrow$ Axis Aligned Preference?
- 3D Well: $\Phi \propto (x^2 + y^2 + z^2)$
- First: Bounce Aligned 30° to $x+$
- Slice Results like 1D Well





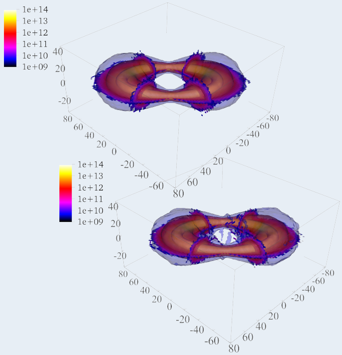
Multidimensional Concerns

- Conserving $[v_x^2, v_y^2, v_z^2] \rightarrow$ Axis Aligned Preference?
- 3D Well: $\Phi \propto (x^2 + y^2 + z^2)$
- First: Bounce Aligned 30° to $x+$
- Slice Results like 1D Well
- Next: Offset Case more Challenging



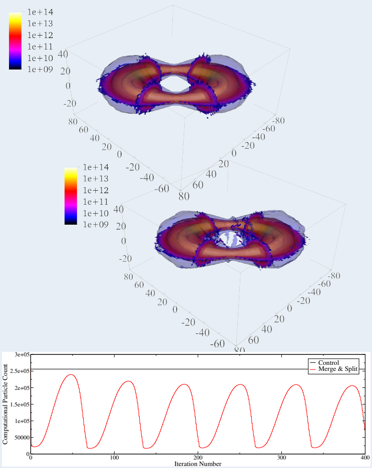
Multidimensional Concerns

- Conserving $[v_x^2, v_y^2, v_z^2] \rightarrow$ Axis Aligned Preference?
- 3D Well: $\Phi \propto (x^2 + y^2 + z^2)$
- First: Bounce Aligned 30° to $x+$
- Slice Results like 1D Well
- Next: Offset Case more Challenging
- Time Averaged Merge $\rightarrow$ Minor Scatter



Multidimensional Concerns

- Conserving $[v_x^2, v_y^2, v_z^2] \rightarrow$ Axis Aligned Preference?
- 3D Well: $\Phi \propto (x^2 + y^2 + z^2)$
- First: Bounce Aligned 30° to $x+$
- Slice Results like 1D Well
- Next: Offset Case more Challenging
- Time Averaged Merge $\rightarrow$ Minor Scatter
- Despite Multiple Merge/Split Cycles



Issues with Parabolic Case

<u>Parabolic Test is Insufficient</u>

- Conserving tr(v^2) important due to collisional invariance
- Original Scheme also Conserved tr(x^2)... Important?

Parabolic Test is Insufficient

- Conserving $\mathrm{tr}(v^2)$ important due to collisional invariance
- Original Scheme also Conserved $\mathrm{tr}(x^2)$... Important?
- Potential Energy Conservation via Taylor Expansion

$$\overline{\Phi} = \frac{\sum_p^k w^{(p)} \left[\Phi(\overline{x_i}) + \frac{\partial \Phi}{\partial x_i}\Big|_{\overline{x_i}} (x_i^{(p)} - \overline{x_i}) + \frac{1}{2} \frac{\partial^2 \Phi}{\partial x_i \partial x_j}\Big|_{\overline{x_i}} (x_i^{(p)} - \overline{x_i})(x_j^{(p)} - \overline{x_j}) \right]}{\sum_p^k w^{(p)}} + o\left(\Delta x^3 \Big|_{\max} \right)$$

$$= \Phi(\overline{x_i}) + \frac{\partial^2 \Phi}{\partial x_i \partial x_j}\Big|_{\overline{x_i}} \frac{\sum_p^k w^{(p)} \left[(x_i^{(p)} - \overline{x_i})(x_j^{(p)} - \overline{x_j}) \right]}{\sum_p^k w^{(p)}} + o\left(\Delta x^3 \Big|_{\max} \right)$$

Parabolic Test is Insufficient

- Conserving tr(v^2) important due to collisional invariance
- Original Scheme also Conserved tr(x^2)... Important?
- Potential Energy Conservation via Taylor Expansion
- $O\left(\Delta x^2\big|_{\max}\right)$ Energy Conservation Requires Center of Mass, $\overline{x_i}$, Only

$$\overline{\Phi} = \frac{\sum_p^k w^{(p)}\left[\Phi(\overline{x_i}) + \frac{\partial\Phi}{\partial x_i}\Big|_{\overline{x_i}}(x_i^{(p)} - \overline{x_i}) + \frac{1}{2}\frac{\partial^2\Phi}{\partial x_i \partial x_j}\Big|_{\overline{x_i}}(x_i^{(p)} - \overline{x_i})(x_j^{(p)} - \overline{x_j})\right]}{\sum_p^k w^{(p)}} + O\left(\Delta x^3\big|_{\max}\right)$$

$$= \Phi(\overline{x_i}) + \frac{\partial^2\Phi}{\partial x_i \partial x_j}\Big|_{\overline{x_i}} \frac{\sum_p^k w^{(p)}\left[(x_i^{(p)} - \overline{x_i})(x_j^{(p)} - \overline{x_j})\right]}{\sum_p^k w^{(p)}} + O\left(\Delta x^3\big|_{\max}\right)$$

Parabolic Test is Insufficient

- Conserving $\mathrm{tr}(v^2)$ important due to collisional invariance
- Original Scheme also Conserved $\mathrm{tr}(x^2)$... Important?
- Potential Energy Conservation via Taylor Expansion
- $O\left(\left.\Delta x^2\right|_{\max}\right)$ Energy Conservation Requires Center of Mass, $\overline{x_i}$, Only
- Next Order? Requires Full $(x_i^{(p)} - \overline{x_i})(x_j^{(p)} - \overline{x_j})$ Tensor for Arbitrary Φ

$$\overline{\Phi} = \frac{\sum_p^k w^{(p)} \left[\Phi(\overline{x_i}) + \left.\frac{\partial \Phi}{\partial x_i}\right|_{\overline{x_i}} (x_i^{(p)} - \overline{x_i}) + \frac{1}{2} \left.\frac{\partial^2 \Phi}{\partial x_i \partial x_j}\right|_{\overline{x_i}} (x_i^{(p)} - \overline{x_i})(x_j^{(p)} - \overline{x_j}) \right]}{\sum_p^k w^{(p)}} + o\left(\left.\Delta x^3\right|_{\max}\right)$$

$$= \Phi(\overline{x_i}) + \left.\frac{\partial^2 \Phi}{\partial x_i \partial x_j}\right|_{\overline{x_i}} \frac{\sum_p^k w^{(p)} \left[(x_i^{(p)} - \overline{x_i})(x_j^{(p)} - \overline{x_j}) \right]}{\sum_p^k w^{(p)}} + o\left(\left.\Delta x^3\right|_{\max}\right)$$

Issues with Parabolic Case

Parabolic Test is Insufficient

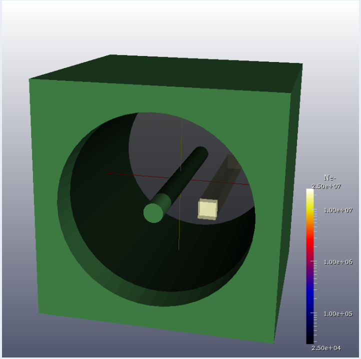
- Conserving $\mathrm{tr}(v^2)$ important due to collisional invariance
- Original Scheme also Conserved $\mathrm{tr}(x^2)$... Important?
- Potential Energy Conservation via Taylor Expansion
- $O\left(\Delta x^2\big|_{\max}\right)$ Energy Conservation Requires Center of Mass, $\overline{x_i}$, Only
- Next Order? Requires Full $(x_i^{(p)} - \overline{x_i})(x_j^{(p)} - \overline{x_j})$ Tensor for Arbitrary Φ
- But Parabolic Potential $\rightarrow$ Diagonal Hessian!
- $\therefore$ Accidental Exact Potential Conservation via Diagonal 2^{nd} Moment Cons.

$$\overline{\Phi} = \frac{\sum_p^k w^{(p)}\left[\Phi(\overline{x_i}) + \frac{\partial\Phi}{\partial x_i}\Big|_{\overline{x_i}}(x_i^{(p)} - \overline{x_i}) + \frac{1}{2}\frac{\partial^2\Phi}{\partial x_i \partial x_j}\Big|_{\overline{x_i}}(x_i^{(p)} - \overline{x_i})(x_j^{(p)} - \overline{x_j})\right]}{\sum_p^k w^{(p)}} + O\left(\Delta x^3\big|_{\max}\right)$$

$$= \Phi(\overline{x_i}) + \frac{\partial^2\Phi}{\partial x_i \partial x_j}\Big|_{\overline{x_i}} \frac{\sum_p^k w^{(p)}\left[(x_i^{(p)} - \overline{x_i})(x_j^{(p)} - \overline{x_j})\right]}{\sum_p^k w^{(p)}} + O\left(\Delta x^3\big|_{\max}\right)$$

Increased Complexity First Attempt...

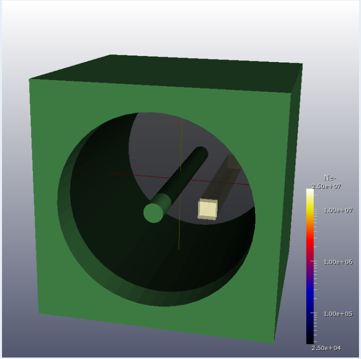
- Annular Potential:
  $\Phi \propto log(r/r_{\text{in}})/log(r_{\text{in}}/r_{\text{out}})$
- Non-Trivial Cartesian Derivative of Φ



Annular Test Case

Increased Complexity First Attempt...

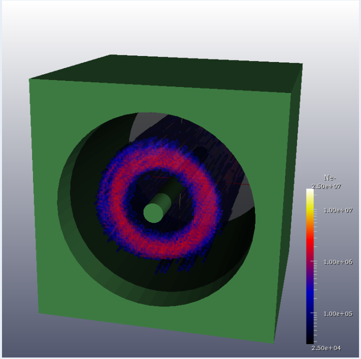
- Annular Potential:
  $\Phi \propto log(r/r_{\text{in}})/log(r_{\text{in}}/r_{\text{out}})$
- Non-Trivial Cartesian Derivative of Φ
- Stable Spiral Required C.N. Push
- Different Periods per Particle - Non-Periodic unlike Parabolic Well
- Part./Cell Drops & #-Cells Active Grows



Increased Complexity First Attempt...

- Annular Potential:
  $\Phi \propto log(r/r_{\text{in}})/log(r_{\text{in}}/r_{\text{out}})$
- Non-Trivial Cartesian Derivative of Φ
- Stable Spiral Required C.N. Push
- Different Periods per Particle - Non-Periodic unlike Parabolic Well
- Part./Cell Drops & #-Cells Active Grows

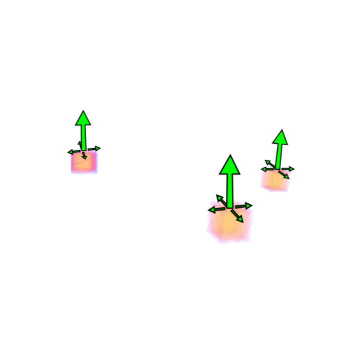
Not a Good Repeating Merge/Split Test



Increased Complexity Second Attempt...

- Spherical Potential: $\Phi \propto 1/r$

Without Merge & Split



Increased Complexity Second Attempt...

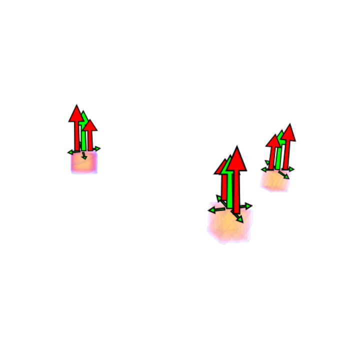
- Spherical Potential: $\Phi \propto 1/r$
- Slight $\Delta X \rightarrow$ Orbits Unsynchronized

Without Merge & Split

Increased Complexity Second Attempt...

- Spherical Potential: $\Phi \propto 1/r$
- Slight $\Delta X \rightarrow$ Orbits Unsynchronized
- Periods Synched via Orbital Mech. Eqns.

Without Merge & Split



Increased Complexity Second Attempt...

- Spherical Potential: $\Phi \propto 1/r$
- Slight $\Delta X \rightarrow$ Orbits Unsynchronized
- Periods Synched via Orbital Mech. Eqns.
- Periodic with Full Nonlinear-C.N. Push

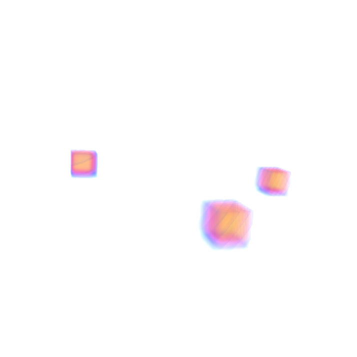
Without Merge & Split

Spherical Test Case

Increased Complexity Second Attempt...

- Spherical Potential: $\Phi \propto 1/r$
- Slight $\Delta X \rightarrow$ Orbits Unsynchronized
- Periods Synched via Orbital Mech. Eqns.
- Periodic with Full Nonlinear-C.N. Push
- Adding Merge & Split:
  Energy Conservation OK
  Accuracy: Scatter + Dispersion?

Including Merge & Split

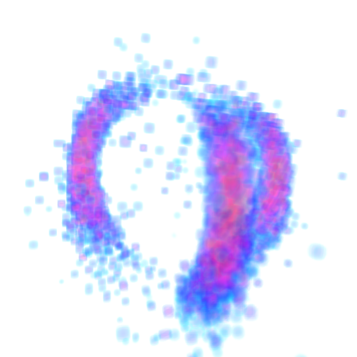


Increased Complexity Second Attempt...

- Spherical Potential: $\Phi \propto 1/r$
- Slight $\Delta X \rightarrow$ Orbits Unsynchronized
- Periods Synched via Orbital Mech. Eqns.
- Periodic with Full Nonlinear-C.N. Push
- Adding Merge & Split:
  Energy Conservation OK
  Accuracy: Scatter + Dispersion?

Source of Dispersion?

Including Merge & Split



$O(\Delta x_{\text{cell}}^3)$ Potential Conservation:

- Remap must Preserve Identical Contraction, $\partial_i \partial_j \Phi : \overline{\Delta x_i \Delta x_j}$

$$\begin{bmatrix} \Phi_{xx} & \Phi_{xy} & \Phi_{xz} \\ \Phi_{yx} & \Phi_{yy} & \Phi_{yz} \\ \Phi_{zx} & \Phi_{zy} & \Phi_{zz} \end{bmatrix} : \begin{bmatrix} \overline{\Delta x \Delta x} & \overline{\Delta x \Delta y} & \overline{\Delta x \Delta z} \\ \overline{\Delta y \Delta x} & \overline{\Delta y \Delta y} & \overline{\Delta y \Delta z} \\ \overline{\Delta z \Delta x} & \overline{\Delta z \Delta y} & \overline{\Delta z \Delta z} \end{bmatrix}$$

$O(\Delta x_{\text{cell}}^3)$ Potential Conservation:

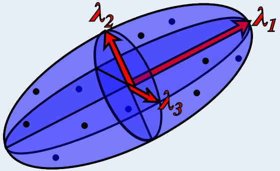
- Remap must Preserve Identical Contraction, $\partial_i \partial_j \Phi : \overline{\Delta x_i \Delta x_j}$
- 2-Particle Only Reproduces Diagonal with $x_i^{(a/b)} = \overline{x_i} \pm \epsilon_i$ if $\epsilon_i = \overline{\Delta x_i^2}^{1/2}$

$$\begin{bmatrix} \overline{\Delta x \Delta x} & \overline{\Delta x \Delta y} & \overline{\Delta x \Delta z} \\ \overline{\Delta y \Delta x} & \overline{\Delta y \Delta y} & \overline{\Delta y \Delta z} \\ \overline{\Delta z \Delta x} & \overline{\Delta z \Delta y} & \overline{\Delta z \Delta z} \end{bmatrix} \neq \begin{bmatrix} \epsilon_x \epsilon_x & \epsilon_x \epsilon_y & \epsilon_x \epsilon_z \\ \epsilon_y \epsilon_x & \epsilon_y \epsilon_y & \epsilon_y \epsilon_z \\ \epsilon_z \epsilon_x & \epsilon_z \epsilon_y & \epsilon_z \epsilon_z \end{bmatrix}$$

$O(\Delta x^3_{\text{cell}})$ Potential Conservation:

- Remap must Preserve Identical Contraction, $\partial_i \partial_j \Phi : \overline{\Delta x_i \Delta x_j}$
- 2-Particle Only Reproduces Diagonal with $x_i^{(a/b)} = \overline{x_i} \pm \epsilon_i$ if $\epsilon_i = \overline{\Delta x_i^2}^{1/2}$
- Non-Degenerate $\overline{\Delta x_i \Delta x_j} \rightarrow$ 3-Eigenvalues

$$\begin{bmatrix} \overline{\Delta x \Delta x} & \overline{\Delta x \Delta y} & \overline{\Delta x \Delta z} \\ \overline{\Delta y \Delta x} & \overline{\Delta y \Delta y} & \overline{\Delta y \Delta z} \\ \overline{\Delta z \Delta x} & \overline{\Delta z \Delta y} & \overline{\Delta z \Delta z} \end{bmatrix} =$$

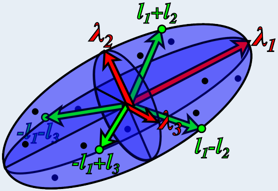


$O(\Delta x^3_{\text{cell}})$ Potential Conservation:

- Remap must Preserve Identical Contraction, $\partial_i\partial_j\Phi : \overline{\Delta x_i \Delta x_j}$
- 2-Particle Only Reproduces Diagonal with $x_i^{(a/b)} = \overline{x_i} \pm \epsilon_i$ if $\epsilon_i = \overline{\Delta x_i^2}^{1/2}$
- Non-Degenerate $\overline{\Delta x_i \Delta x_j} \rightarrow$ 3-Eigenvalues
- Non-Deg. $\rightarrow$ 4+ Non-Coplanar Particles

(3-Particles = Plane & $\lambda_3 = 0$; 2-Particles = Line & $\lambda_2, \lambda_3 = 0$)

$$\begin{bmatrix} \overline{\Delta x \Delta x} & \overline{\Delta x \Delta y} & \overline{\Delta x \Delta z} \\ \overline{\Delta y \Delta x} & \overline{\Delta y \Delta y} & \overline{\Delta y \Delta z} \\ \overline{\Delta z \Delta x} & \overline{\Delta z \Delta y} & \overline{\Delta z \Delta z} \end{bmatrix} =$$

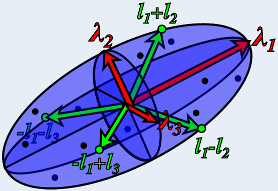


$O(\Delta x^3_{\text{cell}})$ Potential Conservation:

- Remap must Preserve Identical Contraction, $\partial_i \partial_j \Phi : \overline{\Delta x_i \Delta x_j}$
- 2-Particle Only Reproduces Diagonal with $x_i^{(a/b)} = \overline{x_i} \pm \epsilon_i$ if $\epsilon_i = \overline{\Delta x_i^2}^{1/2}$
- Non-Degenerate $\overline{\Delta x_i \Delta x_j} \to$ 3-Eigenvalues
- Non-Deg. $\to$ 4+ Non-Coplanar Particles

  (3-Particles = Plane & $\lambda_3 = 0$; 2-Particles = Line & $\lambda_2, \lambda_3 = 0$)
- Same $\to$ 4 Merge for 2^{nd}-Vel. Moment
- "Principal Component Analysis"

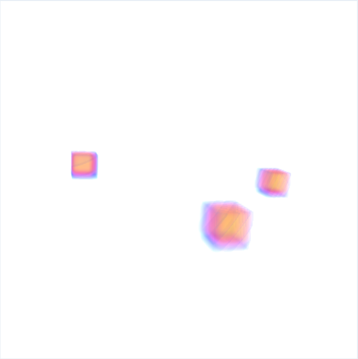
$$\begin{bmatrix} \overline{\Delta x \Delta x} & \overline{\Delta x \Delta y} & \overline{\Delta x \Delta z} \\ \overline{\Delta y \Delta x} & \overline{\Delta y \Delta y} & \overline{\Delta y \Delta z} \\ \overline{\Delta z \Delta x} & \overline{\Delta z \Delta y} & \overline{\Delta z \Delta z} \end{bmatrix} =$$



$O(\Delta x_{cell}^3)$ Potential Conservation:

- Remap must Preserve Identical Contraction, $\partial_i \partial_j \Phi : \overline{\Delta x_i \Delta x_j}$
- 2-Particle Only Reproduces Diagonal with $x_i^{(a/b)} = \overline{x_i} \pm \epsilon_i$ if $\epsilon_i = \overline{\Delta x_i^2}^{1/2}$
- Non-Degenerate $\overline{\Delta x_i \Delta x_j} \rightarrow$ 3-Eigenvalues
- Non-Deg. $\rightarrow$ 4+ Non-Coplanar Particles

  (3-Particles = Plane & $\lambda_3 = 0$; 2-Particles = Line & $\lambda_2, \lambda_3 = 0$)
- Same $\rightarrow$ 4 Merge for 2^{nd}-Vel. Moment
- "Principal Component Analysis"
- Improved but Non-Negligible Scatter and Dispersion



Kinetic Potential Energy Scatter:

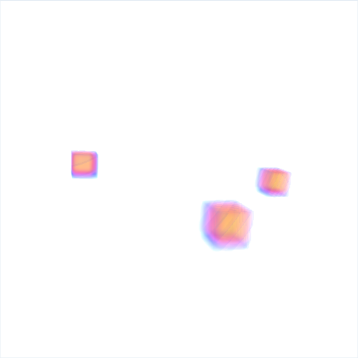
- A Closer look at Merge $\rightarrow 2$
- $\approx$ Match $Sgn\left(\epsilon_i^{(v)}\right)$ to $Sgn(\Delta v_i \Delta v_j)$ Moments
- Select $Sgn\left(\epsilon_i^{(x)}\right)$ to Match $Sgn(\Delta x_i \Delta v_i)$

$$\begin{bmatrix} - & Sgn\left(\epsilon_x^{(v)}\epsilon_y^{(v)}\right) & Sgn\left(\epsilon_x^{(v)}\epsilon_z^{(v)}\right) \\ Sgn\left(\epsilon_y^{(v)}\epsilon_x^{(v)}\right) & - & Sgn\left(\epsilon_y^{(v)}\epsilon_z^{(v)}\right) \\ Sgn\left(\epsilon_z^{(v)}\epsilon_x^{(v)}\right) & Sgn\left(\epsilon_z^{(v)}\epsilon_y^{(v)}\right) & - \end{bmatrix} \approx$$

$$\begin{bmatrix} - & Sgn\left(\overline{\Delta v_x \Delta v_y}\right) & Sgn\left(\overline{\Delta v_x \Delta v_z}\right) \\ Sgn\left(\overline{\Delta v_y \Delta v_x}\right) & - & Sgn\left(\overline{\Delta v_y \Delta v_z}\right) \\ Sgn\left(\overline{\Delta v_z \Delta v_x}\right) & Sgn\left(\overline{\Delta v_z \Delta v_y}\right) & - \end{bmatrix}$$

Kinetic Potential Energy Scatter:

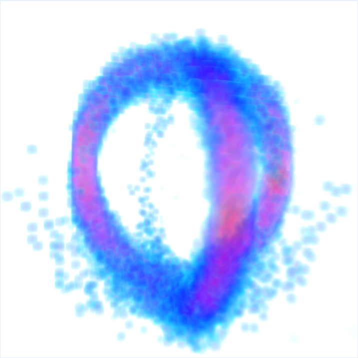
- A Closer look at Merge $\rightarrow 2$
- $\approx$ Match $Sgn\left(\epsilon_i^{(v)}\right)$ to $Sgn(\Delta v_i \Delta v_j)$ Moments
- Select $Sgn\left(\epsilon_i^{(x)}\right)$ to Match $Sgn(\Delta x_i \Delta v_i)$
- Some Improvement Scatter & Dispersion



Kinetic Potential Energy Scatter:

- A Closer look at Merge $\rightarrow$ 2
- $\approx$ Match $\mathit{Sgn}\left(\epsilon_i^{(v)}\right)$ to $\mathit{Sgn}(\Delta v_i \Delta v_j)$ Moments
- Select $\mathit{Sgn}\left(\epsilon_i^{(x)}\right)$ to Match $\mathit{Sgn}(\Delta x_i \Delta v_i)$
- Some Improvement Scatter & Dispersion
- Why Stop at just *XV* Sign?

$$\epsilon_i^{(x)} = \left(\overline{\Delta x_i \Delta v_i}\right) / \epsilon_i^{(v)}$$

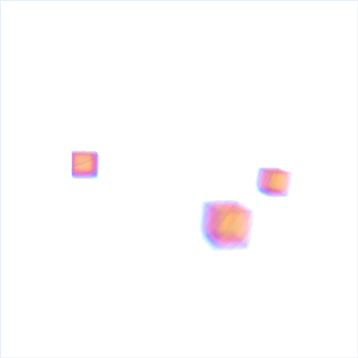


DISPERSION FROM *XV* SCATTER?

Kinetic Potential Energy Scatter:

- A Closer look at Merge $\rightarrow$ 2
- $\approx$ Match $Sgn\left(\epsilon_i^{(v)}\right)$ to $Sgn(\Delta v_i \Delta v_j)$ Moments
- Select $Sgn\left(\epsilon_i^{(x)}\right)$ to Match $Sgn(\Delta x_i \Delta v_i)$
- Some Improvement Scatter & Dispersion
- Why Stop at just *XV* Sign?

  $$\epsilon_i^{(x)} = \left(\overline{\Delta x_i \Delta v_i}\right) / \epsilon_i^{(v)}$$
- Much Improved Scatter & Dispersion

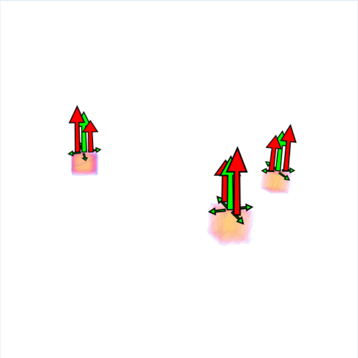


Kinetic Potential Energy Scatter:

- A Closer look at Merge $\rightarrow$ 2
- $\approx$ Match $Sgn\left(\epsilon_i^{(v)}\right)$ to $Sgn(\Delta v_i \Delta v_j)$ Moments
- Select $Sgn\left(\epsilon_i^{(x)}\right)$ to Match $Sgn(\Delta x_i \Delta v_i)$
- Some Improvement Scatter & Dispersion
- Why Stop at just *XV* Sign?

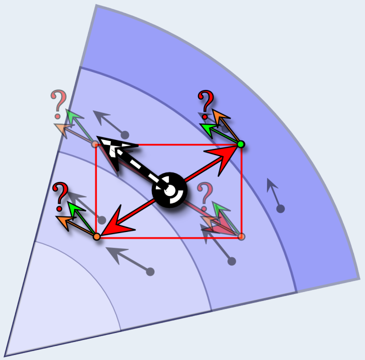
$$\epsilon_i^{(x)} = \left(\overline{\Delta x_i \Delta v_i}\right) / \epsilon_i^{(v)}$$

- Much Improved Scatter & Dispersion
- Sphere Sensitive: Kinetic <-> Potential



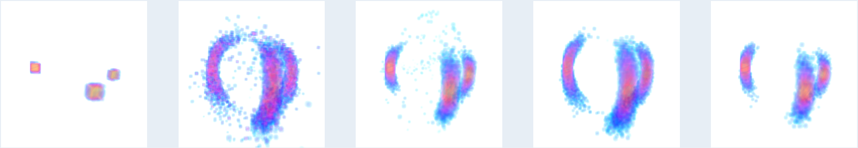
Kinetic Potential Energy Scatter:

- A Closer look at Merge $\rightarrow$ 2
- $\approx$ Match $_{Sgn}\left(\epsilon_i^{(v)}\right)$ to $_{Sgn(\Delta v_i \Delta v_j)}$ Moments
- Select $_{Sgn}\left(\epsilon_i^{(x)}\right)$ to Match $_{Sgn(\Delta x_i \Delta v_i)}$
- Some Improvement Scatter & Dispersion
- Why Stop at just *XV* Sign?
  $$\epsilon_i^{(x)} = \left(\overline{\Delta x_i \Delta v_i}\right) / \epsilon_i^{(v)}$$
- Much Improved Scatter & Dispersion
- Sphere Sensitive: Kinetic <-> Potential
- K.E. & P.E. Conserved Independently...
  XV-Moment $\rightarrow$ Better Combination



Review Sphere Test Results

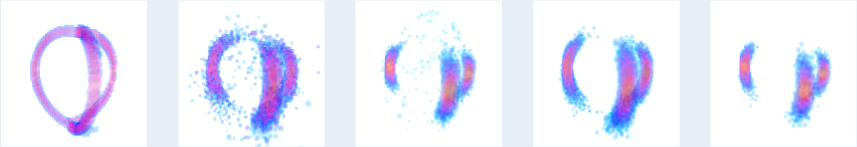
Merge Results after 1-Cycle



No Merge | Random Sign | PCA | *XV*-Sign | *XV*-Full

Note: Results Due to Extreme Case Sensitivity
C.N. Push Required for Stability
Period Sync Required for Initial Conditions

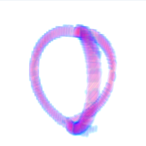
Merge Results after 1-Cycle



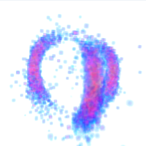
Not Synched | Random Sign | PCA | *XV*-Sign | *XV*-Full

Note: Results Due to Extreme Case Sensitivity
C.N. Push Required for Stability
Period Sync Required for Initial Conditions

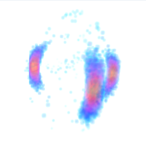
Merge Results after 1-Cycle



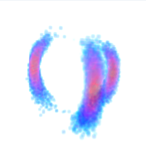
Not Synched



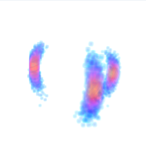
Random Sign



PCA



XV-Sign

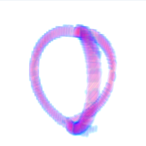


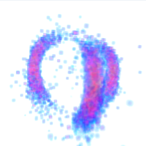
XV-Full

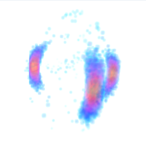
Note: Results Due to Extreme Case Sensitivity
C.N. Push Required for Stability
Period Sync Required for Initial Conditions
Result of Minor Velocity Difference Across IC Box

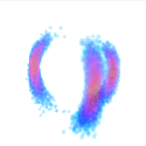
Review Sphere Test Results

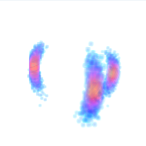
Merge Results after 1-Cycle


Not Synched


Random Sign


PCA

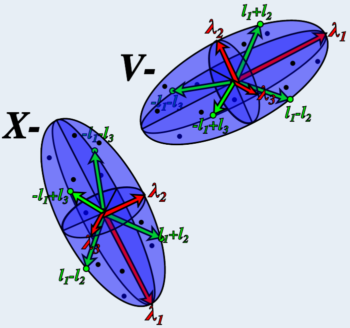

XV-Sign


XV-Full

Note: Results Due to Extreme Case Sensitivity
C.N. Push Required for Stability
Period Sync Required for Initial Conditions
Result of Minor Velocity Difference Across IC Box
Merge Scatter in Cells $<$ Synch ΔV

PCA + *XV*-Merge?

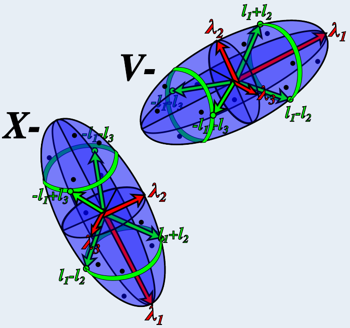
- Both Partially Inhibited Dispersion



Future Work: Combined PCA/*XV* Merge

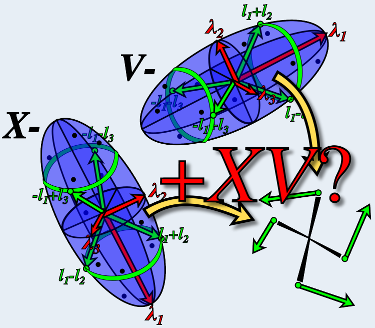
PCA + *XV*-Merge?

- Both Partially Inhibited Dispersion
- Extra Degrees of Freedom in PCA



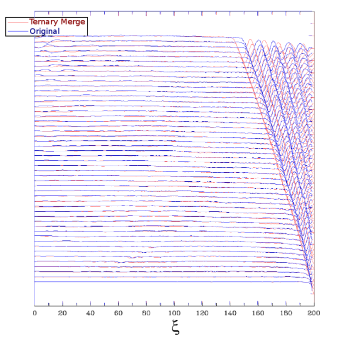
PCA + *XV*-Merge?

- Both Partially Inhibited Dispersion
- Extra Degrees of Freedom in PCA
- Use Extra DOF for XV-Match?



PCA + *XV*-Merge?

- Both Partially Inhibited Dispersion
- Extra Degrees of Freedom in PCA
- Use Extra DOF for XV-Match?
- *XV*-Dispersion Source of Error in Lapenta's MHD Slow Shock?

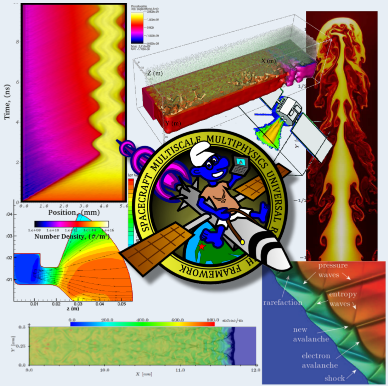


Lapenta - JCP 181, 317-337 (2002)

Integrate R&D w/ Production TODO:

- Combine PCA and *XV* Merges
  (Martin/Lederman)
- Remap <-> Vlasov <-> Fluid
  (Martin/Bilyeu/TBD)
- Implicit/Multiscale GPU-Accel. PIC
  (Lederman/Gimelsheins/Martin/TBD)
- GPU Accelerated Chemical Kinetics / CR Ar-Ne-Xe-Molecular Models
  (Le/Cole*/Kapper* PhD Research)

*Note: Former Co-op Student Work to be Integrated into Framework



Thank You

Work supported by AFOSR grant No. 11RZ12COR
(PM: Dr. J. Luginsland)

Questions?