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Kinetic Energy-Preserving Discretization Schemes for High Reynolds Number Propulsive Applications

Ayaboe Edoh, Ann Karagozian, Charles Merkle
and Venke Sankaran

66th Annual APS Meeting
Fluid Dynamics Division

Pittsburgh, PA

Nov 24-26, 2013

Objectives

Investigate dispersion and dissipation of numerical schemes with ultimate application to high-Re reacting LES

Schemes

Standard Collocated Grid Schemes
Standard Staggered Grid Schemes
Kinetic Energy Preserving Schemes

Analysis

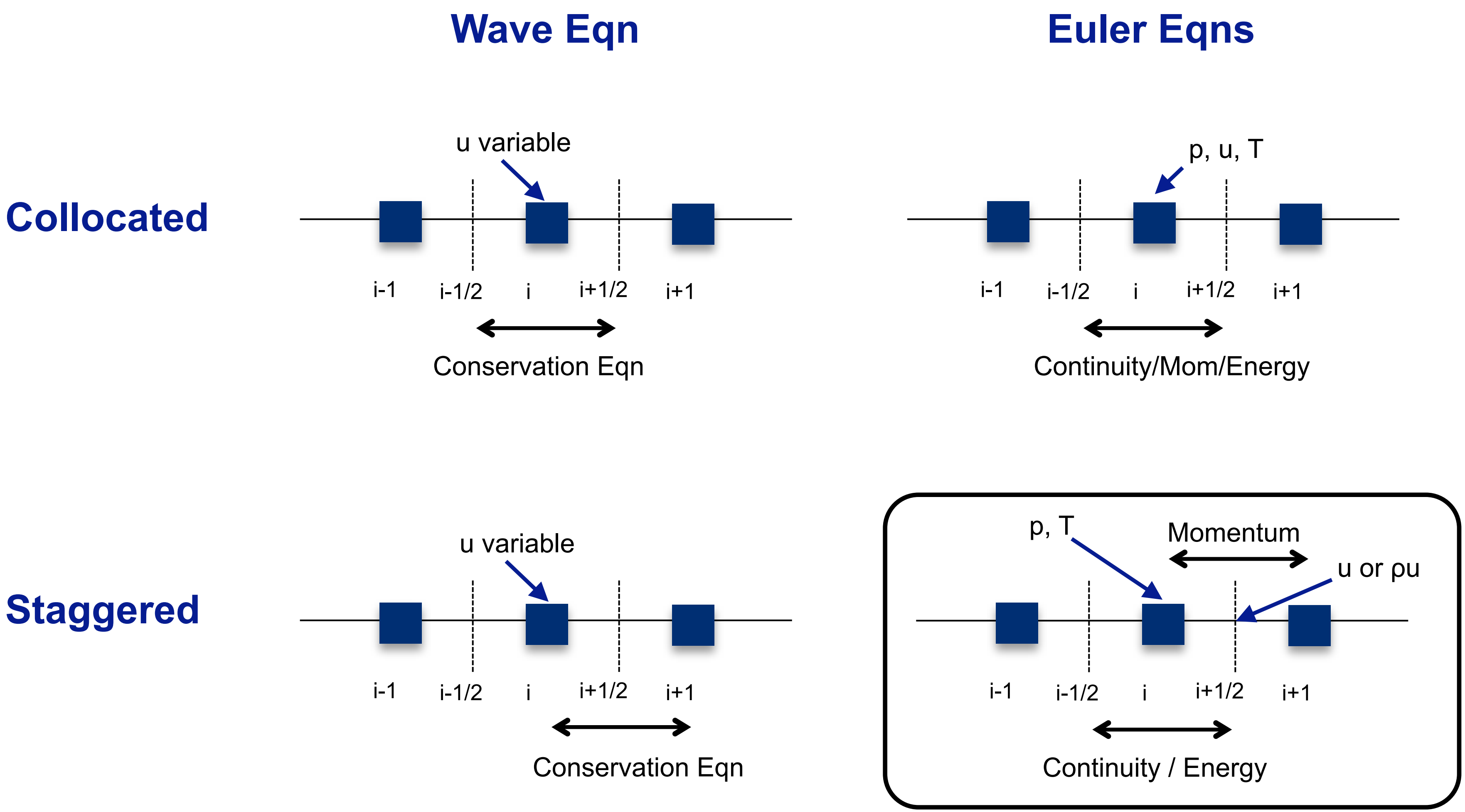
Von Neumann Stability Analysis
1D Periodic Test Problem

Scope

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$ Wave Eqn

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} = 0$$ Euler Eqns

Formulation



Variables also staggered in time for fully ke preserving schemes

Von Neumann Analysis

Eigenvalues of the amplification matrix specify growth factor and phase errors.

$$Q^{n+1} = GQ^n$$

Staggered Grid Scheme

$$\Gamma_{ce}\left(\frac{\partial Q_{pT}}{\partial t}+\frac{\partial Q_u}{\partial t}\right)_i+\Gamma_m\left(\frac{\partial Q_{pT}}{\partial t}+\frac{\partial Q_u}{\partial t}\right)_{i+1/2}+A_{ce}\left(\frac{\partial Q_{pT}}{\partial x}+\frac{\partial Q_u}{\partial x}\right)_i+A_m\left(\frac{\partial Q_{pT}}{\partial x}+\frac{\partial Q_u}{\partial x}\right)_{i+1/2}=0$$

$$Q_{pT} = \begin{pmatrix} p \\ 0 \\ T \end{pmatrix}$$

Continuity/Energy

Momentum

$$Q_u = \begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix}$$

Growth Factor

$$||g_i||$$

Phase Error

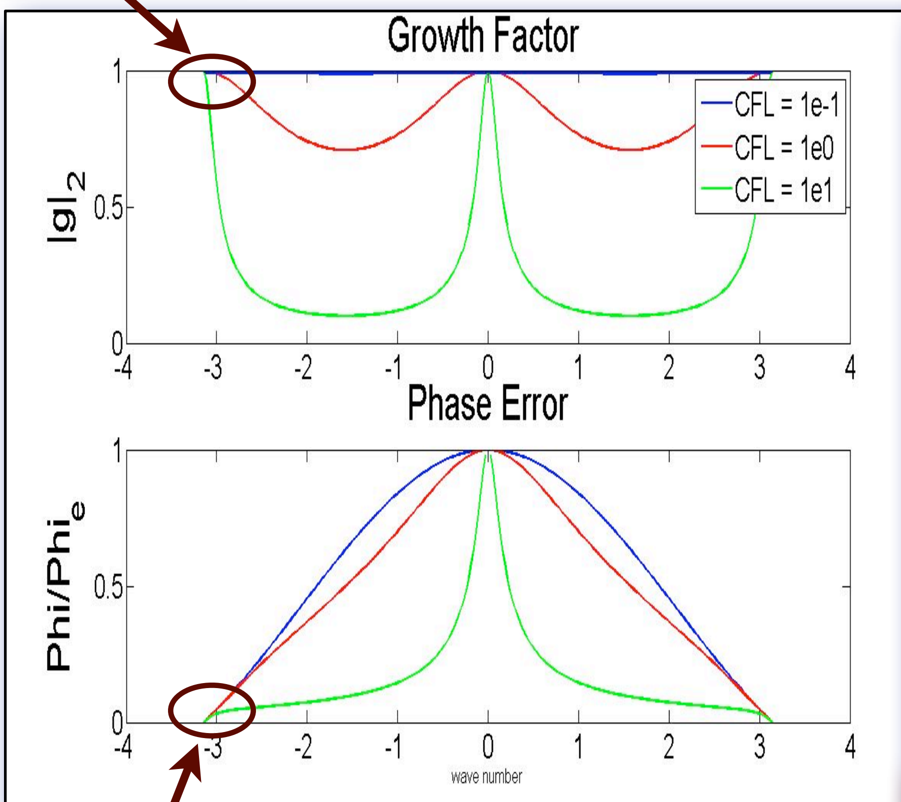
$$\frac{\phi}{\phi_{exact}} = \frac{-\tan^{-1}\{Imag(g_i)/Re(g_i)\}}{CFL \times \beta}$$

Wave Equation

Euler Implicit Scheme

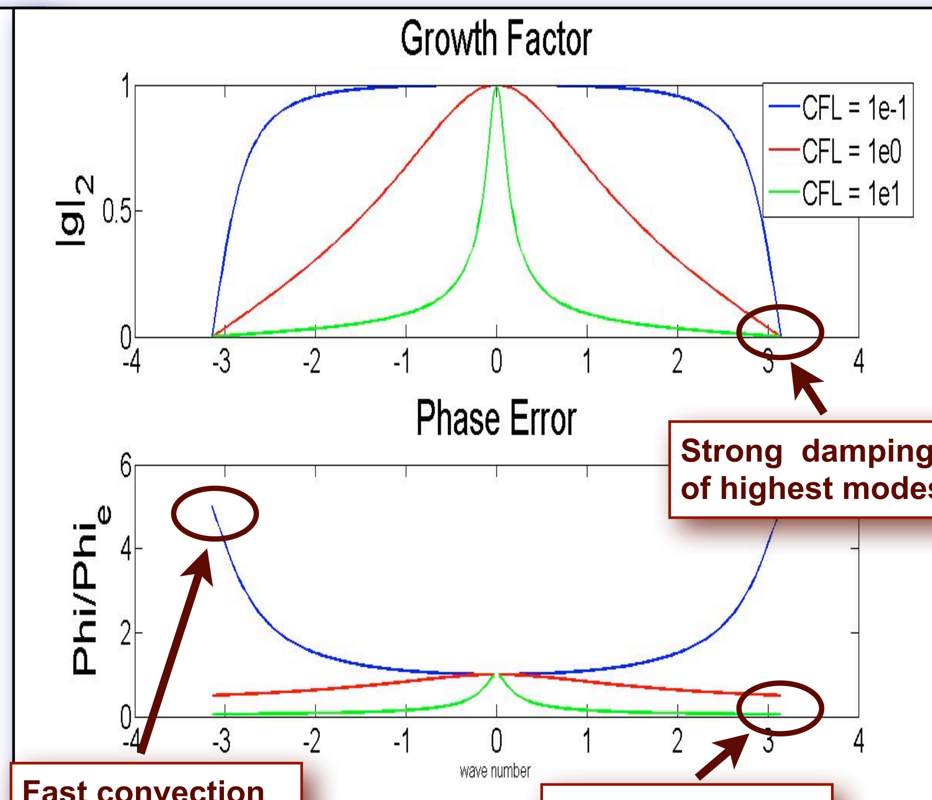
No damping of highest modes

Collocated



No convection of highest modes

Staggered



Strong damping of highest modes

Fast convection of highest modes for low CFL

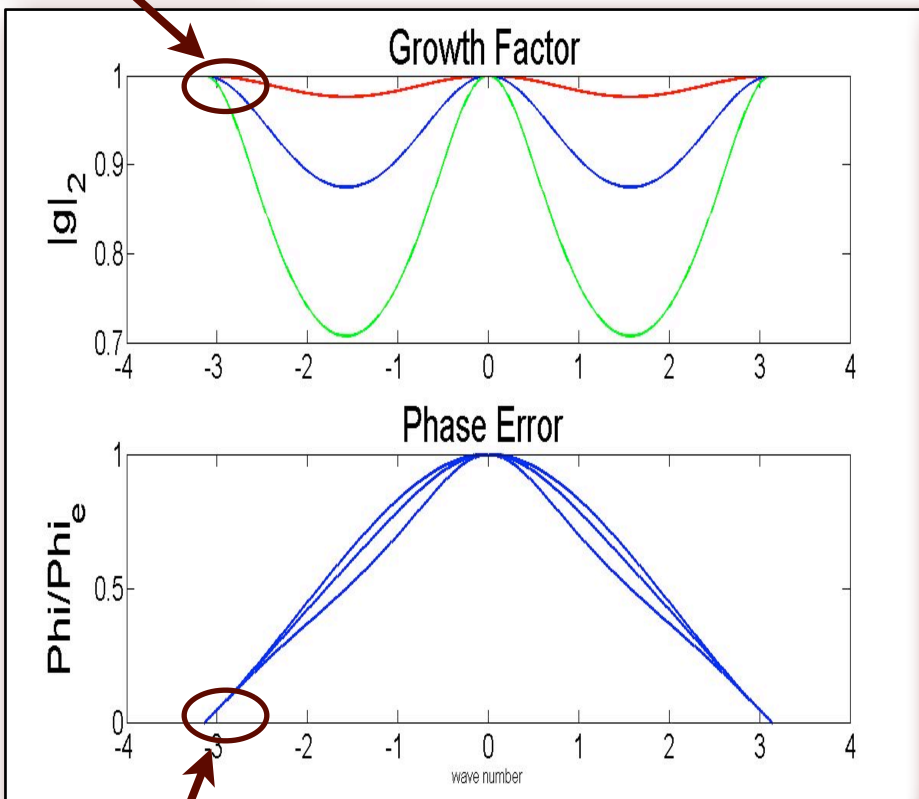
Slow convection of highest modes for high CFL

Euler Equations

Euler Implicit Scheme

No damping of highest modes

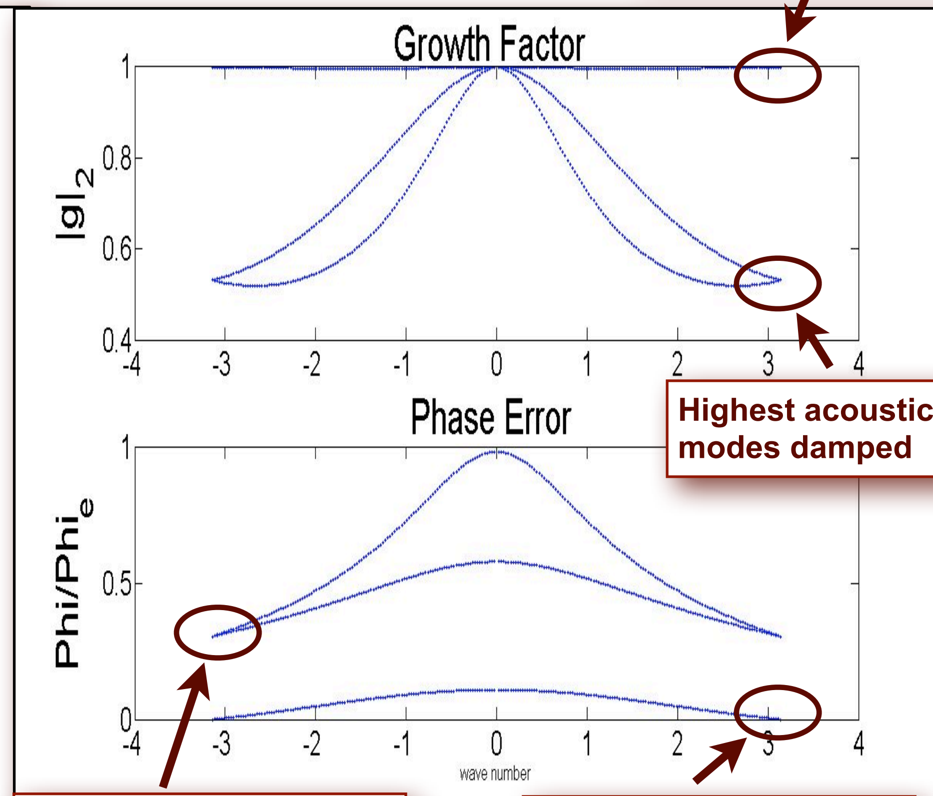
Collocated



No convection of highest modes

Staggered

Highest particle modes not damped



Highest acoustic modes damped

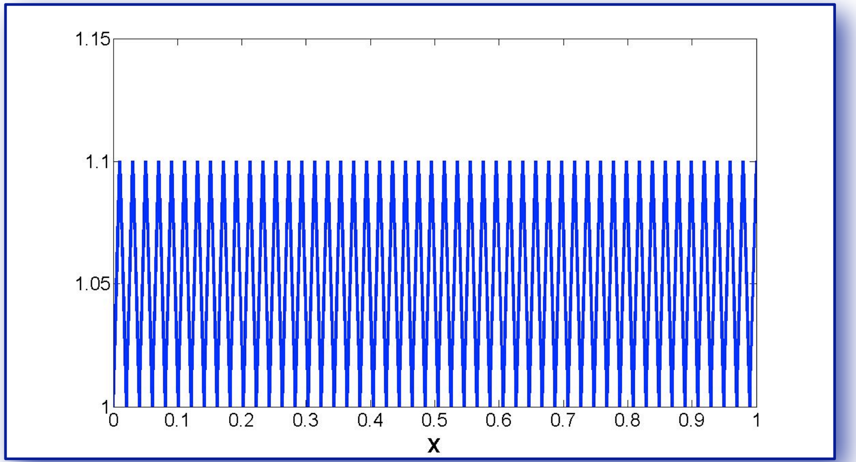
Slow convection of highest acoustic modes

No convection of highest acoustic modes

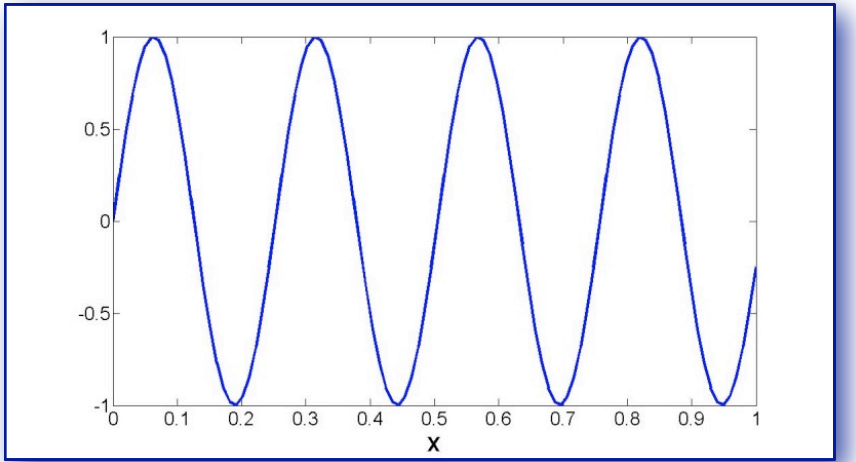
Test Cases

1D Duct
– Non-dissipative BC's $\Delta U_{IL} = \Delta U_{IL-1}$
– Periodic BC's avoid issues with reflections

Saw-tooth i.c.



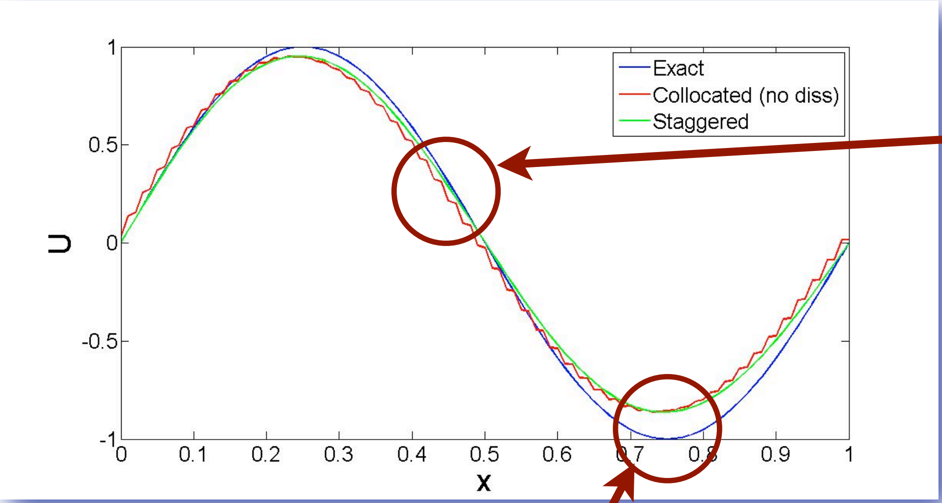
Sinusoidal i.c.



For Euler Eqns:
– Use Characteristic Eqns
– However, staggered grid does not allow proper diagonalization

Wave Equation Results

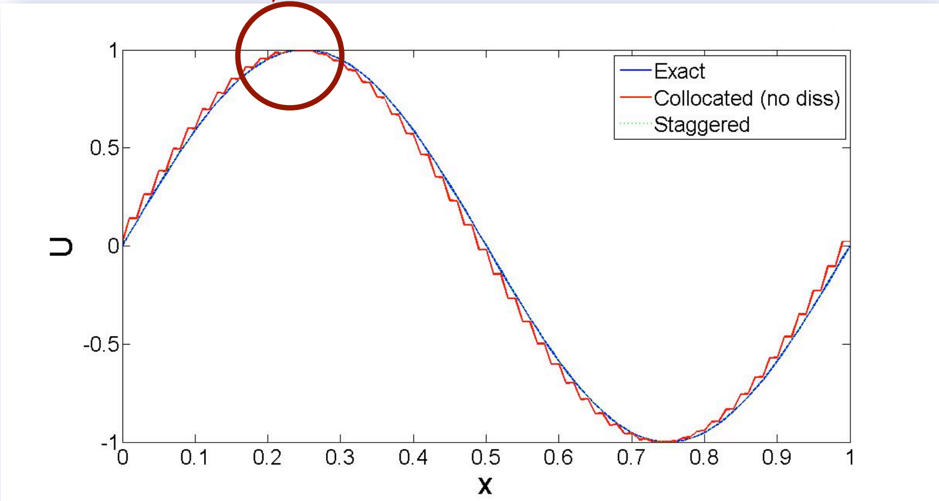
Euler Implicit



Odd-even errors in collocated grid solution; staggered solutions are smooth

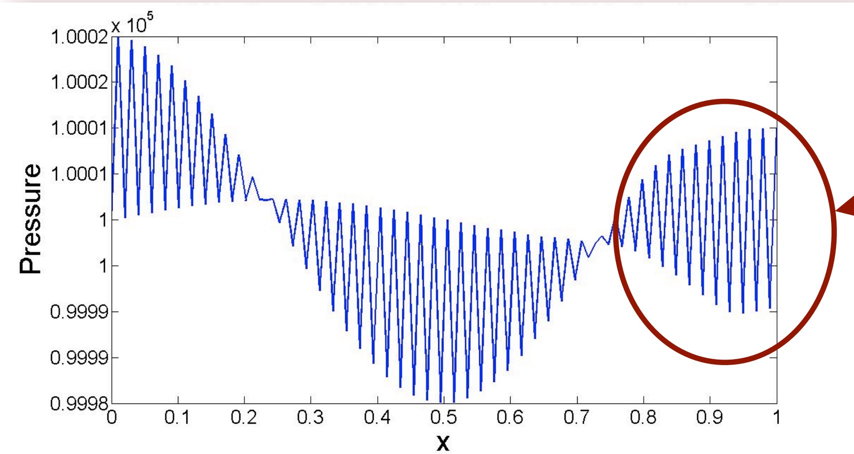
Mid-wave number damping in the Euler Implicit Scheme

Crank-Nicolson



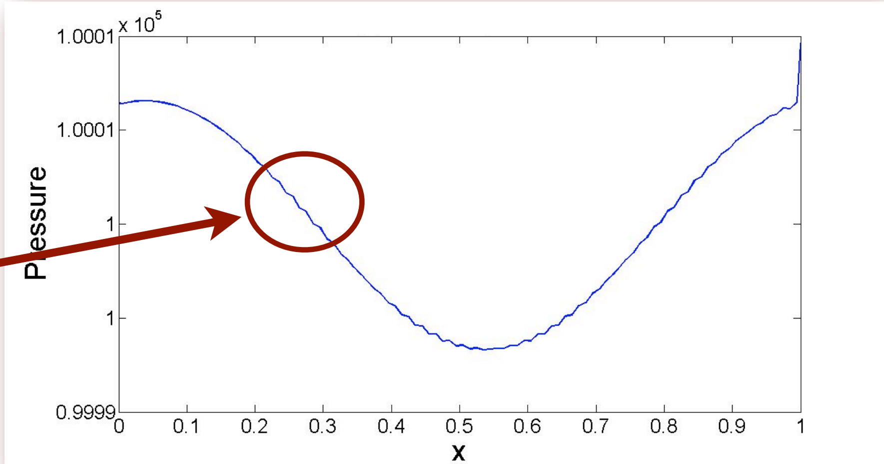
Euler Equations Results

Collocated



Collocated grid solution shows strong odd-even splitting errors

Staggered



Staggered grid solution is relatively smooth

KE Conservative Scheme

Collocated Grid

Transport Eqn

$$\frac{\left[(\rho\phi_k)^{n+1} - (\rho\phi_k)^n\right]}{\Delta t} + \Delta_x(\rho u_j)\phi_k^* = 0$$

Time-Averaging

$$\phi_k^* = \frac{(\sqrt{\rho}\phi_k)^{n+1} + (\sqrt{\rho}\phi_k)^n}{(\sqrt{\rho})^{n+1} + (\sqrt{\rho})^n}$$

Roe-averaging in time

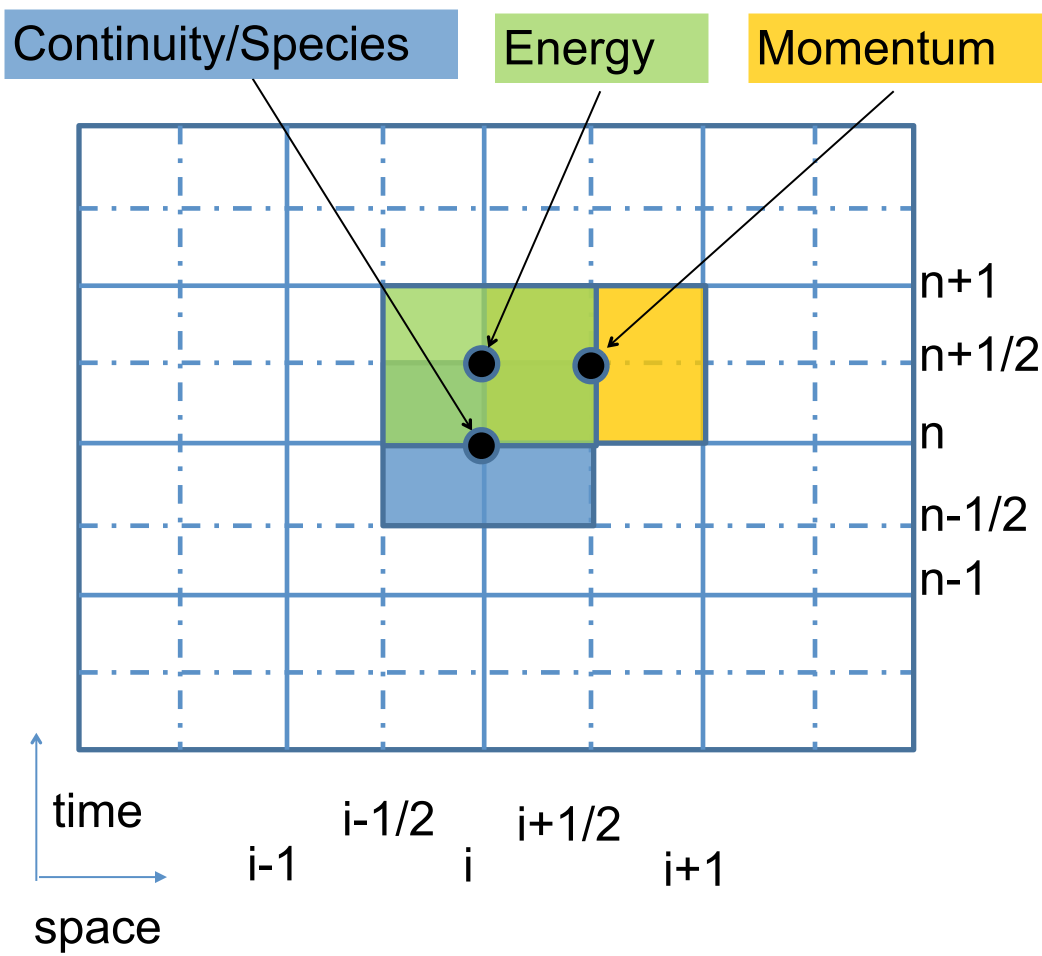
KE Transport Eqn

$$\frac{\left[(\rho\phi_k^2)^{n+1} - (\rho\phi_k^2)^n\right]}{2\Delta t} + \Delta_x(\rho u_j)\frac{\phi_k^2}{2} = 0$$

Ensures full KE preservation

KE Conservative Scheme

Staggered Grid in Space and Time



Time-Averaging

$$(u)^*_{i+1/2,j} \equiv \frac{\left(\overline{\sqrt{\overline{\rho}^{1t}}}^{1x} u_\alpha\right)^{n+1}_{i+1/2,j} + \left(\overline{\sqrt{\overline{\rho}^{1t}}}^{1x} u_\alpha\right)^{n}_{i+1/2,j}}{\left(\overline{\sqrt{\overline{\rho}^{1t}}}^{1x}\right)^{n+1}_{i+1/2,j} + \left(\overline{\sqrt{\overline{\rho}^{1t}}}^{1x}\right)^{n}_{i+1/2,j}}$$

$$(h^0)^*_{i,j} \equiv \frac{\left(\sqrt{\overline{\rho}^{1t}}\, h^0\right)^{n+1}_{i,j} + \left(\sqrt{\overline{\rho}^{1t}}\, h^0\right)^{n}_{i,j}}{\left(\sqrt{\overline{\rho}^{1t}}\right)^{n+1}_{i,j} + \left(\sqrt{\overline{\rho}^{1t}}\right)^{n}_{i,j}}$$

$$(Y_k)^*_{i,j} = \frac{\left(\sqrt{\rho} Y_k\right)^{n+1/2}_{i,j} + \left(\sqrt{\rho} Y_k\right)^{n-1/2}_{i,j}}{\sqrt{\rho}^{\,n+1/2}_{i,j} + \sqrt{\rho}^{\,n-1/2}_{i,j}}$$

Roe-averaging in time leads to full kinetic energy preservation of momentum and scalar fields.

Summary

- **Von Neumann Analysis provides dispersion & damping behavior**
  - Staggered grid schemes show natural damping even when artificial dissipation is **not** added explicitly
  - Dispersion errors are sometimes non-intuitive - faster wave speeds for small CFL's and slower wave-speeds for high CFL's
- **Periodic wave tests validate von Neumann results**
  - Staggered grid schemes provide smooth particle wave solutions with minimal dissipation
  - Acoustic wave damping is consequential for compressible LES
- **Kinetic Energy conservative schemes**
  - Formulated for both staggered and collocated grids
  - Schemes possess favorable properties for scalar energies
    - Maybe consequential for reacting-LES problems
  - Test results for improved schemes are forthcoming