

Integrated Modeling and Analysis of Physical Oceanographic and Acoustic Processes

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LONG TERM GOALS

The long term goal is to improve ocean physical state and acoustic state predictive capabilities. The goal fitting the scope of this project is the creation of physics-based, broadly applicable and portable acoustic prediction capabilities that include the effects of internal waves, surface waves, and larger scale features, with an emphasis on continental shelf and slope regions.

OBJECTIVES

Specific physics objectives are completion of targeted studies of oceanographic processes that are relevant to underwater acoustic conditions, yet not completely understood, in a few environmental regimes, accompanied by studies of the acoustic propagation and scattering processes in those regimes. Internal gravity waves and other submesoscale features are of specific interest. There are many open questions regarding the processes of internal-wave formation and propagation in the presence of low-frequency and large-scale ocean features, to be pursued by the basic research efforts of this project.

An additional objective is development of improved computational tools for acoustics and for the physical processes identified to be important via the targeted ocean dynamics studies, including work on data-driven ocean flow models. Time-stepped three-dimensional (3.5D) and true four-dimensional (4D) computational acoustic models are to be improved, as well as the methods used to couple them with ocean flow models. Fully numerical ocean flow modeling will also be improved by coupling models having nonhydrostatic pressure (NHP) physics, data-driven regional models having hydrostatic pressure (HP) physics, and surface wave models. Stochastic acoustic prediction models will be developed. The entire suite of models is to be tested for acoustic prediction effectiveness using existing data sets.

APPROACH

The approach toward advancing the state of the art is to identify acoustically relevant ocean processes, to improve computational models that include these processes, and to eventually model the acoustic effects in detail, for comparison with existing data. The acoustical relevance of each aspect of the features generated by these processes is best obtained with state-of-the-art acoustics research using the latest tools, with some tools in need of refinement or development, which forms part of this project. The acoustical relevancy of the processes is intended to prioritize research into each process into a ranking that may differ from that obtained from ocean ecosystem, climate, or ocean biogeochemical cycle research. The research plan anticipates feedback: as the acoustics research evolves, the goals and priorities in the process research plan may change.

This approach has been broken into seven tasks directed toward two goals, equivalent to the objectives listed above: (Goal #1) Development of integrated tools for joint oceanography/acoustic study and prediction, i.e. a modeling system; and (Goal #2) Development of an understanding of the physics of coastal linear and nonlinear internal wave generation and transformation, as observed in the model, lab and field-measured features, coupled with study of acoustical propagation in these features.

Achieving Goal #2 is intimately linked with the creation of modeling tools (satisfying Goal #1) because of the need to address the open questions on coastal internal wave physics, on coastal flow features, on details of NHP fluid computational modeling, and on coastal acoustic propagation effects. Each of the seven tasks is handled by a subset of the 15 total PI's. (The subsets intersect.) The name "Integrated Ocean Dynamics and Acoustics" (IODA) was given to the project at the initial PI meeting.

Goal 1 tasks: Modeling tools

1. *NHP ocean model nested within data-driven HP model, tied to 4D acoustic models.* Under this task a nonhydrostatic physics computational flow model will be nested numerically within a spatial subdomain of data-assimilative hydrostatic physics regional flow model.
2. *Hybrid model: Hierarchical internal-wave models nested within HP model, tied to 3.5D acoustic models. (Also called composite physics model.)* Under this task, reduced-physics flow models capable of rapidly and efficiently modeling NHP nonlinear internal waves will be integrated with (nested within) regional flow models. These nested models can more rapidly compute internal wave positions and sizes than the full NHP model (Task 1), but may sacrifice detail and accuracy. Candidate models include those based on Korteweg-deVries type wave evolution equations and 2D NHP numerical models.
3. *Improved 4D deterministic and stochastic acoustic modeling.* Improvements to time-stepped 3D (3.5D) and 4D acoustic models will be made to increase speed and accuracy. A transport equation model for mean fields and higher order statistics will also be improved. The models will be coupled with the Task 1 and 2 models (i.e. will use output of those models as environmental condition input.)
4. *Unified waveguide model.* A prototype acoustic waveguide model solving for acoustic conditions in areas of rough seabed, internal fluid motions (including the nonlinear waves that form our main emphasis), and atmospherically forced rough surface waves will be developed. This coordinated effort is an attempt towards fully integrated physics, studying and solving for the dynamics of the ocean, surface and internal waves, and seabed and acoustics processes with atmospheric forcing, all in a fully synoptic and evolving fashion.
5. *Integration work necessary for the above.* Protocols for data output resolution, data format, metadata, and so on, will be developed. Protocols for physically correct nesting of the models with differing dynamics will be developed. Protocols for sufficient interpolation within the widely separated resolutions of the flow models and acoustic models will be established.

Goal 2 tasks: Physics studies and model verification

6. *Spatially 3D internal-wave physics studies.* Basic research aimed at improving our knowledge of internal tide and nonlinear wave dynamics in 2D and 3D systems will be conducted using laboratory, theoretical and numerical models as appropriate, on a parallel track with the modeling tool development. These results will guide model development, testing, and use.
7. *Comparison of outputs and predictions with ground-truth field and lab data.* To gain understanding and confidence in the model fields, the generation, propagation, and dissipation of internal waves will be studied using laboratory experiments and direct numerical simulations of fluid flow (DNS). Additionally, for both the NHP and hybrid models, 3.5D acoustic effects will be modeled, analyzed and compared to experiment and field observations.

WORK COMPLETED

The end of FY 2013 corresponds to 2 years, 4 months completed in this project. Progress in many of the tasks has been substantial. The work completed thus far is reported here grouped according to the efforts of individual PI's or PI groups, with some work directed at multiple project goals.

This year the third project meeting was held, the IODA Physical Oceanography Meeting, 13-14 March 2013, at Woods Hole Oceanographic Institution (WHOI). Twenty project personnel attended (10 PI's). Work completed and meaningful results were reported by many attendees. These and other accomplishments are presented in the remainder of this report.

Acoustic modeling (Tasks 3 and 5; Lin, Duda, Collis): Five new three-dimensional acoustic propagation codes in the MATLAB® programming environment have been completed thus far. Three of these are split-step Fourier (SSF) wide-angle codes at the typically used expansion order that is explained in papers by Thomson and Chapman [1] and Feit and Fleck [2]. The other codes are each unique, varying in expansion order and in method from the three standard-order SSF codes.

The first code is the latest version of the SSF Cartesian coordinate code. The second is our radial-grid SSF 3D cylindrical coordinate code. This code marches in radius, and uses a zero-padding method (upsampling) to maintain azimuthal resolution while marching to larger radius (r). The third code uses cylindrical coordinates and involves a grid that, while employing SSF marching, remains uniform in arc-length spacing in the azimuthal dimension. The transform variables are $k_s = k_\theta/r$ and k_z . This code retains spatial resolution while marching. At very long range this code asymptotically approaches the behavior of the Cartesian code. A paper published this year (Lin, Duda and Newhall, 2013) compares the three codes; the agreement of results from the three codes implies that they are accurate, because otherwise the details of their individual errors would differ.

The fourth new code solves a higher-order parabolic equation, but is similar to the three codes described above because the SSF algorithm is employed (Lin and Duda, 2012). This code can accurately model sound propagating at higher angle relative to the marching direction than can those three codes. The operators for the higher-order terms that are added to the first-order equation can be written so as to employ the numerical operators of the first-order equation, simplifying the coding. The higher-order terms must be computed to a desired convergence tolerance using an iterative scheme. The current version of this code runs at roughly one-tenth the speed of the standard-order code. We foresee the use for this code to be verification of the accuracy of “production” research modeling runs using the faster standard-order code (i.e. quantification of production model errors).

The fifth new code uses the Padé expansion and alternating direction implicit (ADI) methods, and 2D Galerkin discretization (Lin, Collis and Duda, 2012). The SSF method requires a smoothing of the density discontinuity at the water-seafloor interface, which compromises SSF accuracy at frequencies below 75 Hz. The Padé code can provide better results at frequencies below 75 Hz. The ADI method is implemented to reduce compute time compared to using a direct one-step Padé code, but the ADI solution of the first-order parabolic equation contains significant phase errors. The key to the good accuracy of the new code is the use of the new higher-order parabolic equation. Figure 1 shows how the code can produce fields closer to a benchmark 3D solution than other codes.

Finally, trial protocols and codes have been written to interpolate ocean sound-speed fields onto the acoustic model grids. The system has been used for three different regional ocean model domains. The

ocean sound-speed is not interpolated at the acoustic model grid increment in the stepping direction (order one wavelength); instead, the resolution is three to six wavelengths. The required interpolation resolution, for a given accuracy, remains to be theoretically justified.

Ocean internal wave physics (Tasks 2 and 6; Helfrich): The work to date has investigated the joint effects of rotation and topography. Rotation causes the slow decay of an initial solitary wave due to inertia-gravity (IG) wave radiation. That radiated IG wave subsequently shoals behind the lead wave and forms a trailing packet of solitary-like waves. The initial wave also undergoes the usual fissioning process. A paper has been submitted supported partially by this grant (Grimshaw et al., 2013). The paper includes weakly nonlinear theory based on the variable-coefficient Ostrovsky equation, numerical solutions of that equation, and MIT gcm calculations. All the calculations are for a South China Sea setting. Future work is planned to compare effects in South China Sea and New Jersey Shelf conditions. The outer New Jersey Shelf was the location of the ONR Shallow-Water 2006 (SW06) study, linked to the Littoral Environmental Acoustics Research (LEAR) and Nonlinear Internal Wave Initiative (NLIWI) programs. Measurements from these field efforts form the primary data set that we are using to guide our dynamics research and model design, and to test our models.

Ocean internal tide physics (Task 6; Swinney): In this effort, laboratory experiments and numerical simulations of internal wave generation and propagation are conducted for tidal flow over topography, as a function of tidal frequency ω and the vertical profiles of the buoyancy frequency, $N(z)$. Numerical simulations of the non-hydrostatic Navier-Stokes equations are conducted using a parallelized finite-volume based solver and run on the University of Texas Ranger supercomputer, which makes it possible to achieve the large spatial dynamic range needed to span scales from short-wavelength nonlinear internal waves to the long wavelength internal tides. Laboratory experiments and simulations are being conducted for different topographies including knife-edge, Gaussian, and tents of different heights and slopes.

During the current grant period the relationship between boundary currents generated by tidal flow over topography and the radiated internal wave power has been examined in experiments and in simulations of tidal flow of a uniformly stratified fluid over various topographies (knife-edge, tent, Gaussian, sinusoidal). Previous work used the hydrostatic approximation (where $N \gg \omega$), while the present work examines situations where the hydrostatic approximation fails, as in the abyssal ocean, where N is small and comparable to, or even smaller than, ω (the M2 tidal frequency). Our analysis of oceanic data last year revealed that there are many locations in the oceans where $N < \omega$ (King et al., 2012). In the current year we have examined the conversion of tidal energy into radiated internal wave power particularly for cases where there are intense resonant boundary currents, which occur when a topographic slope matches the internal wave beam slope. We find that this resonance phenomenon does not extend to the far field power radiated by internal waves. Kinetic energy density in boundary currents has often been used as a proxy to characterize the conversion of tidal energy to radiated internal wave power, but the present work shows that high boundary current energy density is a poor indicator of radiated power. A manuscript entitled “Internal tide and boundary current generation by tidal flow over topography” by Dettner, Paoletti, and Swinney has been submitted to *Physics of Fluids* and is now under review. The next step in this research will be to examine the effect of internal waves on acoustic propagation. An instrumented laboratory tank system is now being constructed at the University of Texas for these studies.

Ocean internal tide generation physics (Tasks 2 and 6; Zhang and Duda): The goal of effectively modeling internal waves must include reliable modeling of tidally-forced internal waves that appear at

the tidal frequencies (internal tides). A study of internal tide generation made with ROMS [3] for supercritical continental slope geometry has been performed. Here, barotropic tidal flows force internal tides at the continental shelf edge, with the slope where the internal waves are generated being steep with respect to internal wave characteristics (supercritical slope) and the barotropic currents being slow with respect to internal wave speed (subcritical Froude number). The work is described in a conference paper (Duda, Zhang and Lin, 2012), and a recently accepted journal article (Zhang and Duda, 2013). The modeling is fully nonlinear, using the Boussinesq-approximation based primitive equations, although the pressure is approximated as hydrostatic (NHP physics not allowed). The fields show signatures of significant nonlinear effects, some of which have undergone limited prior study, e.g. [4]. The nonlinear effects are being studied by modifying nonlinear terms in the equation set (equation of state, momentum equation, turbulence closure). The internal tide energy is higher if nonlinear advection is included in the model, approximately double that seen with this nonlinear effect removed. The model treatment of (nonlinear and irreversible) subgrid-scale dissipation also has a large effect on the internal tide generation process. The simulations show a rich field of harmonics, and the adjustments of nonlinear physics strongly influence the radiated waves. This work ties in well with the work of the U. Texas wave physics group [5], showing that incorrect handling of the nonlinearity is likely to compromise model comparisons with nature. Recently, the ROMS study was expanded to show the internal tide field radiated from a canyon cut into the slope. The internal tide radiates in horizontal beams. An explanation for variable generation efficiency in the canyon in terms of something that resembles the distorted-wave Born approximation, and an explanation for the radiation pattern, have been put forward in a submitted journal article (Zhang et al., 2013).

Phase-resolved ocean wave computational studies (Task 4; Yue): This group has developed and improved the phase-resolved simulation capability for modeling and analysis of large-scale surface and internal wave-field evolution by including nonlinear broadband wave-wave and wave-current interactions. They developed and implemented a highly efficient smooth-particle-hydrodynamics (SPH) method for effectively simulating large-scale surface/internal breaking waves and their interactions with complex boundaries such as bottom topography (Kiara, Hendrickson and Yue, 2013). Routine large-scales phase-resolved simulations of nonlinear surface wave-field evolution are performed and used to characterize and quantify rogue wave occurrence probability, generation mechanisms, and structures and dynamics, as functions of wave-field statistical parameters (Xiao et al., 2013).

Improved analysis of field data (Task 7; Badiey): During SW06, detailed measurements of the time-varying ocean environment were made while acoustic signals were concurrently transmitted between various source and receiver pairs. The time-varying environment induced by internal waves was recorded by an array of moored thermistor chains, as well as by the attending research vessels. Using a mapping technique described in a journal paper (Badiey et al., 2013), the three-dimensional (3D) temperature field for over one month of internal wave events was reconstructed. The results of this mapping are used for the statistical analysis of the internal wave parameters, such as the wave propagation speed, direction, amplitude, coherence length, etc. This paper provides a summary of these results and also examines the implications of the detailed statistics as regards to the acoustic field. The results in this paper could be used as a database for studying internal wave generation and propagation, and internal wave impact on 3D acoustic propagation in waveguides.

In that successful work, the time-varying internal wave front for an observed SW06 event was reconstructed. Using thermistor data recorded at various clusters in the source-receiver path, the possibility of interpolating the environmental parameters was shown. The technique was shown only

for a small region of the thermistor farm area ($\sim 4 \times 4$ km). Moving forward, the interpolation technique has been expanded to include other measurements along the source/receiver track, and a statistical analysis of several internal wave events has been conducted. It is now possible to construct the 3D input parameters for an internal-wave event that has passed the source-receiver path. Using these data processing tools, it is now possible to interpolate the IW field in the entire experiment area and obtain detailed input parameters for use by the 3D propagation models. Examples of this process are shown in Figures 2 and 3 for two different periods when the wave was in the vicinity of the acoustic path. These results will be presented during the Fall 2013 Acoustical Society meeting in San Francisco.

Acoustic regimes and feature models (Tasks 2 and 3; Lynch and Colosi): As part of the IODA project, work has been done on developing simple analytical expressions for how various important acoustic quantities (horizontal array coherence length, transmission loss, scintillation index) depend on the parameters of the internal waves (and other coastal processes, such as fronts, eddies, spice.) With such “ocean feature model” derived expressions, one can easily calculate how changes in the ocean model parameters produce changes in the interesting acoustic quantities. Given “acceptability bounds” for error in the acoustic quantities, the models, in turn inform one as to how much error can be tolerated in the feature model parameters. This gives one some insight as to how much error one can tolerate in the output of a larger oceanographic model of ocean features, such as the ones that are being used for IODA. The analytic model results also give easier insight into what the physics of the more complicated models is, and also a simple way to predict acoustically useful things like frequency dependence, and source to receiver geometry dependence.

Unified waveguide model acoustics (Task 4; Makris): An analytical model derived from normal mode theory for the accumulated effects of range-dependent multiple forward scattering [6]-[8] is applied to estimate the temporal coherence of the acoustic field forward-propagated through a continental-shelf waveguide containing random three-dimensional (3D) internal waves. The inhomogeneous medium’s scatter function density is modeled using Rayleigh-Born approximation to Green’s theorem to account for random fluctuations in both density and compressibility caused by internal waves [7]. The generalized waveguide extinction theorem [9] is applied to determine the attenuation due to scattering from internal wave inhomogeneities. The modeled coherence time scale of narrow-band low-frequency acoustic field fluctuations after propagating through a continental-shelf waveguide is shown to decay with a power-law of range to the $-1/2$ beyond roughly 1 km, to decrease with increasing internal wave energy, and to be consistent with measured acoustic coherence time scales. The model should provide a useful prediction of the acoustic coherence time scale as a function of internal wave energy in continental-shelf environments (Gong et al., 2013). The acoustic coherence time scale is an important parameter in remote sensing applications because it determines (i) the time window within which standard coherent processing such as matched filtering may be conducted, and (ii) the number of statistically independent fluctuations in a given measurement period that determines the variance reduction possible by stationary averaging.

An analytic model to calculate the moments of forward scattered field that accounts for the accumulated effects of attenuation, dispersion, and Doppler-shift [10] induced by random 3D internal and surface gravity waves in a fluctuating ocean waveguide is being developed. Analytical expressions of the coherent forward field in free space with deterministic velocity and scatter function density have been derived. Numerical simulation shows that the upper bound of Doppler shift is negligibly small compared to the original frequency for typical long-range remote sensing and underwater communication scenarios involving moving surface gravity waves in free space. The analytical expressions of the moments of forward scattered field are now extended to a fluctuating ocean

waveguide. With the new analytic model, the coupled effect of accumulated Doppler-shift, attenuation and dispersion induced by the time-varying surface and internal gravity waves in the moments of forward scattered field will be investigated.

Advanced ocean modeling (Tasks 2 and 4; Lermusiaux): The MSEAS group has improved the simulation of multiscale ocean interactions (internal tides, sub-mesoscale eddies, fronts, currents and storm responses) for the SW06 experiment [11], with the product being new multiscale reanalysis fields (http://mseas.mit.edu/Research/SW06/MSEAS_reanalysis/2013_May07/). A sponging scheme with a novel, efficient, time dependent sponging target field was designed to inhibit spurious reflections from open boundaries while preserving the incoming tidal forcing and permitting realistic subtidal dynamics (e.g. permitting the advection of eddies and upwelled water in and out of the computational domain). The temporal updating of the target field was also incorporated into the open boundary conditions. The atmospheric forcing applied to the simulation was corrected to improve the merging of the WRF fields into the larger NOGAPS fields. Improved barotropic tidal forcing was obtained from Oregon State U. Analyses of the new simulation will provide several publications. A study will examine the effects of winds, shelfbreak exchanges and subsurface intrusions, the Gulf Stream and recirculation gyres as well as comparisons to data and skill metrics. Another study examines the prediction and global analysis of internal tides in the Middle Atlantic Bight region, this includes regional and event characterizations of the internal tides as well as skill evaluation of the internal-tide predictions. A third study examines the interactions of the internal tide with the shelfbreak and Gulf Stream. The results are interpreted in the context of a novel weakly-nonlinear model derived by internal-tide vertical mode decomposition of the primitive equations. A fourth study characterizes the impacts of the wind forcing on the internal tide.

Three types of detided fields were produced from the new reanalysis: (i) fields filtered with a 61-hr running average; (ii) two week simulations with no tidal forcing initialized at 10-day intervals from the tidally forced reanalysis; and (iii) an identical twin simulation of the full reanalysis run but without tidal forcing. Each type of detiding results in a different subtidal mesoscale environment due to accumulated differences in their respective tidal histories. These detided fields were provided to team members for acoustic studies, first as a baseline without IT/IW and secondly as an environment in which specific IT/IW signals could be introduced and their acoustic impacts assessed.

A new portable, parallel, high-order finite element non-hydrostatic ocean code with unstructured grids continues to be developed using a combination of Python and C/C++, with the intent of allowing high-resolution ocean-acoustic modeling. An intensive verification study is being performed on the existing Python model implementation using the method of manufactured solutions (MMS). Several time-critical routines have been ported from Python to C/C++ to facilitate high performance, parallelism, and insertion of arbitrary numbers of components and Dynamically Orthogonal (DO) modes. These C/C++ routines have been thoroughly tested for proper numerical convergence rates and parallel scalability, and they run several times faster than their Python counterparts. In the C/C++ implementation, the well-tested PETSc library is being used to reorganize and parallelize the existing data structures and linear solvers, while the community-standard netCDF library (with CF conventions) is being used for model and data I/O.

A methodology and some software have been developed for solving a generic class of stochastic non-autonomous linear and nonlinear systems, with uncertainty in initial and boundary conditions and forced by random processes of both additive and multiplicative nature. The solver is used to intercompare multiple uncertainty quantification schemes in terms of accuracy and computational

effort, with a focus on idealized ocean-acoustic modeling. To allow accurate integration of uncertainty due to external stochastic forcing, two novel polynomial chaos schemes have been developed, namely, the reduced space KLgPC scheme and the modified TDgPC scheme. We then simulated shallow water waves governed by Korteweg-de Vries (KdV) dynamics with stochastic forcing.

Idealized simulations were performed to determine the impact of a background flow (for example, a wind-forced surface current) on the formation of internal waves. The simulations were made using the 2.29 Finite Volume code. Surface currents of varying depths (from surface to $0.25h$, $0.33h$ and $0.45h$, where h is the height of the domain) and varying shapes (constant amplitude, linear change, and inverse tangent shape) were studied. The simulated topography was a Gaussian with height equal to half the height of the domain, and variance of half the height of the topography. The model ocean was uniformly stratified (constant buoyancy frequency).

Hierarchical hybrid model test bed (Task 2 and 5; Lynch, Helfrich, Lin, Duda): A part of the project that is organized by Arthur Newhall of WHOI is the interfacing of various ocean dynamics computational tools with acoustics computational tools, to form a test bed for acoustically relevant ocean dynamics research threads. A dimensionful version of the solver for the regularized version of the extended Korteweg-deVries equation with rotation (eKdVf [12]) has been developed from a trial version of a dimensionless-system code of the non-regularized version. The new version runs faster and more reliably (not going unstable) than last year's version, and it is suited to interfacing with numerical models, which provide forcing (boundary values). The eKdVf code is a wave evolution model for nonlinear and nonhydrostatic waves or other features, such as bores, that fit within an ocean waveguide normal-mode framework. Precise behaviors of the quadratic-nonlinear version of this (KdVf) are the subject of the submitted Grimshaw et al. paper mentioned above.

The background state governing the modes and the coefficients of the code, along with the initial waveform forcing at one end of the domain, are taken from data-driven ocean models such as MSEAS–Primitive Equation or ROMS, with a prototype interface in place for MSEAS. The design of the model is as follows: internal wave normal-mode speeds are taken from the data-driven model, removing internal tides with one of the optional methods mentioned in the previous section; internal wave mode rays are traced (with a few options for wave-mean flow interaction complexity and source location, wave source location being one of our research topics); coefficients for the eKdVf are evaluated along each ray; eKdVf is solved along each ray; resulting high-frequency internal waves along each ray are mapped into a 3D space using interpolation schemes. Feedback regarding ambiguities may be supplied to the ray tracing to provide additional detail, when necessary. The information is then used to build a 3D sound-speed anomaly field as input for acoustic field modeling.

Nested ocean modeling (Task 2; Zhang, Wilken): Both the fully numerical (Task 1) and hybrid (Task 2) coastal internal wave modeling and analysis efforts require broad-scale ocean fields best obtained from data-driven modeling. A new eastern US model is being developed at Rutgers, an expansion of the ESPreSSO model (<http://www.myroms.org/espresso/>). Work is underway toward two-way nested subdomains having the scope and resolution required to effectively model internal tides and internal waves.

RESULTS

Advanced ocean modeling (Lermusiaux): Figure 4 (upper panel) shows the 30m salinity overlain with the 30m velocity vectors from the 2013 reanalysis of SW06, just prior to Tropical Storm Ernesto. The

reanalysis improved the frontal properties and especially the internal tide (IT) fields. Figure 4 (lower panel) shows comparisons of the mode-1 IT from the SW06 reanalysis to the IT directly estimated from the velocity data at 4 moorings. The simulation captures both the amplitude and the phase of the coherent portion of the mode-1 IT well. It also captures the overall patterns of the noncoherent portion. Analysis of the simulation reveals that the shelfbreak ocean front decreases the topographic IT generation by roughly 10% and that the Gulf Stream produces (i) IT energy-flux divergence $O(20 \text{ W m}^{-2})$ and (ii) IT refraction/reflection. Reduced IT energy densities seaward of the Gulf Stream indicate that a significant fraction of the IT is reflected back to the shelf.

This group utilized numerical examples to show that the two new polynomial chaos schemes and dynamically orthogonal (DO) equations can integrate both additive and multiplicative noise over large time intervals, which is relevant for multiplicative ocean-acoustics and ocean-waves coupling. For one of the examples, they compare the Monte-Carlo and DO schemes to time-integrate shallow water surface waves governed by KdV equations with external stochastic forcing. They find that the DO scheme is computationally efficient. It is expected that it can simulate surface ocean waves with stochastic wave forcing and the coupling with internal waves and background flows, as well as the effects on ocean acoustics. This is useful for MIT-MSEAS group collaborations with Yue-Liu (Figure 5) and Duda-Lin-Lynch.

For the idealized simulations and all three variables examined (amplitude, shape, and depth of flow), the results indicated an impact on the initial formation of internal waves, but after multiple tidal cycles (3.2 periods), the internal waves show little evidence of perturbation from the presence of the background flow. The amplitude of the background flow resulted in slightly larger perturbations than did the depth of the flow. The shape of the flow resulted in the least significant impact. The project did not examine the results from varying barotropic forcing tides, or varying topographies; higher resolution runs, or longer runs, would also improve the confidence in the results.

Internal tide generation (Zhang and Duda): The idea that nonlinear physics determines the energy ultimately found in internal tides at supercritically steep slopes, and the internal tide phase, is supported by evidence in the Zhang and Duda 2013 paper. This means that linear models of internal tide generation, which are common, may be very good for some purposes, but may not be suited for detailed prediction of internal tides and the packets of nonlinear waves that they can spawn. That being said, a linearized (but multiple-scatter) internal tide model has been successfully used to explain horizontal beam patterns of internal tides generated in canyons in fully nonlinear ROMS simulations (Zhang et al., 2013). Figure 6 shows a toward-shore internal tide beam coming out of one side of a canyon in the simulation, a feature that is consistent with some poorly-understood satellite images of internal waves. Thus, linear models continue to have value, but may not be totally sufficient when wave phase (wave arrival time) is a desired parameter in a prediction system (e.g. nonlinear wave packet position, or arrival time at a specific location, for input to an acoustic model).

Internal tide generation (Swinney): The U. Texas group has demonstrated that the entire vertical relief of bathymetric features does not contribute equally to the formation of internal tides from barotropic tides. There is a shadowing effect if features are close enough to have intersecting energy-propagation paths (characteristics). An “effective height” for bathymetric internal tide generation, which has the actual height as an upper bound, can be found using water column and bathymetric parameters. In a different study, this group has also found that internal tides can be generated by bathymetric features not reaching above the lower turning depth of internal-tide rays (these don’t always reach the abyssal seafloor and tops of deep seamounts), which is consistent with full-wave internal wave dynamics.

The efficiency of internal-tide generation has also been examined using simulations, finding that for fixed slope criticality (a factor known to affect efficiency), efficiency declines below hydrostatic theoretical levels as frequency increases. Nonhydrostatic effects are found to play a role.

Stochastic acoustic modeling (Makris): A paper on the topic of acoustic field temporal coherence time scales in internal-wave filled continental-shelf waveguides has been finished (Gong et al., 2013). Coherence times for order 400-Hz sound are predicted to range from roughly 2 to 20 minutes depending upon environmental conditions.

IMPACT/APPLICATIONS

The creation of a modeling suite that includes submesoscale features as well as data assimilation is expected to be a valuable asset to apply in numerous ocean regions. Identification of acoustic propagation and noise field features that are controlled by local oceanographic processes may allow exploitation or mitigation of the effects.

The MIT analytical acoustic model enables prediction of the temporal coherence time scale and Doppler-shift of acoustic field fluctuations after propagating through a fluctuating ocean waveguide with random 3D surface and internal gravity waves.

TRANSITIONS

The MSEAS random noise ocean process models from MIT are being transitioned to NRL.

RELATED PROJECTS

There are many closely related projects among the many co-PI's. Some of the acoustics PI's have ONR grants related to shallow-water acoustics. Several of the ocean-flow modeling PI's have closely related projects on data assimilation, dynamics, and model development funded by both ONR and the National Science Foundation.

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HONORS

Recipient: Harry L. Swinney

Recipient's Institution: University of Texas at Austin

Award Name: Boltzmann Medal, awarded once every three years by the Commission on Statistical Physics of the International Union of Pure and Applied Physics (UNESCO), during the STATPHYS conference.

Date and Place of Award: 24 July 2013, Seoul, Korea

Award Citation: Harry L. Swinney – *“for his ingenious and challenging experiments which have had a large impact on many areas of statistical physics”*
http://www.statphys25.org/sub06_1.htm

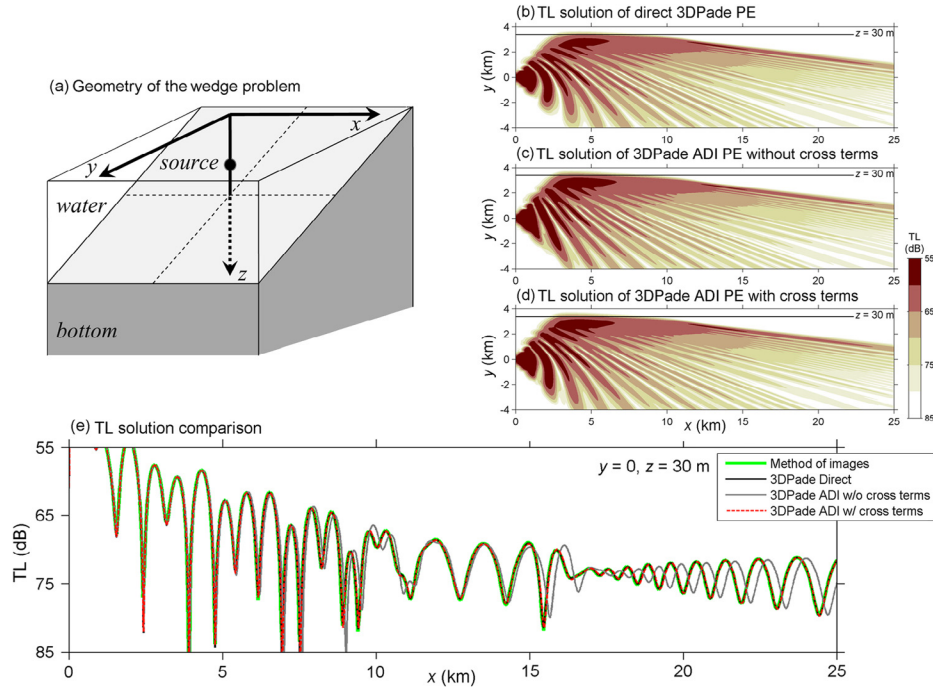


Figure 1. Modeling of 3D sound propagation in a benchmark idealized wedge with bottom density equal to one g/cm³. (a) Model geometry. The slope angle is $\pi/63$ rad (2.86°), and a 25-Hz point source is located 4 km away from the wedge apex at 100 m depth. (b)-(d) TL solutions in the x-y plane at $z = 30$ m obtained from three different Padé methods. (e) Comparison of the three Padé solutions with the image solution along the line at $y=0$ and $z=30$ m. The (higher-order) alternating direction implicit (ADI) solution agrees with the computationally intensive direct solution and the restrictive method of images solution. From Lin, Collis and Duda, 2012, copyright ASA.

[(a) x is along the wedge, y points offshore out of the wedge, and z points downward. (b-d) the direct 3D Padé and ADI 3D Padé with cross terms diagrams agree; the ADI 3D Padé without cross terms diagram differs. (e) The method of images, the direct 3D Padé and ADI 3D Padé with cross term lines agree, and the ADI 3D Padé without cross terms line diverges at distances greater than 8 km.]

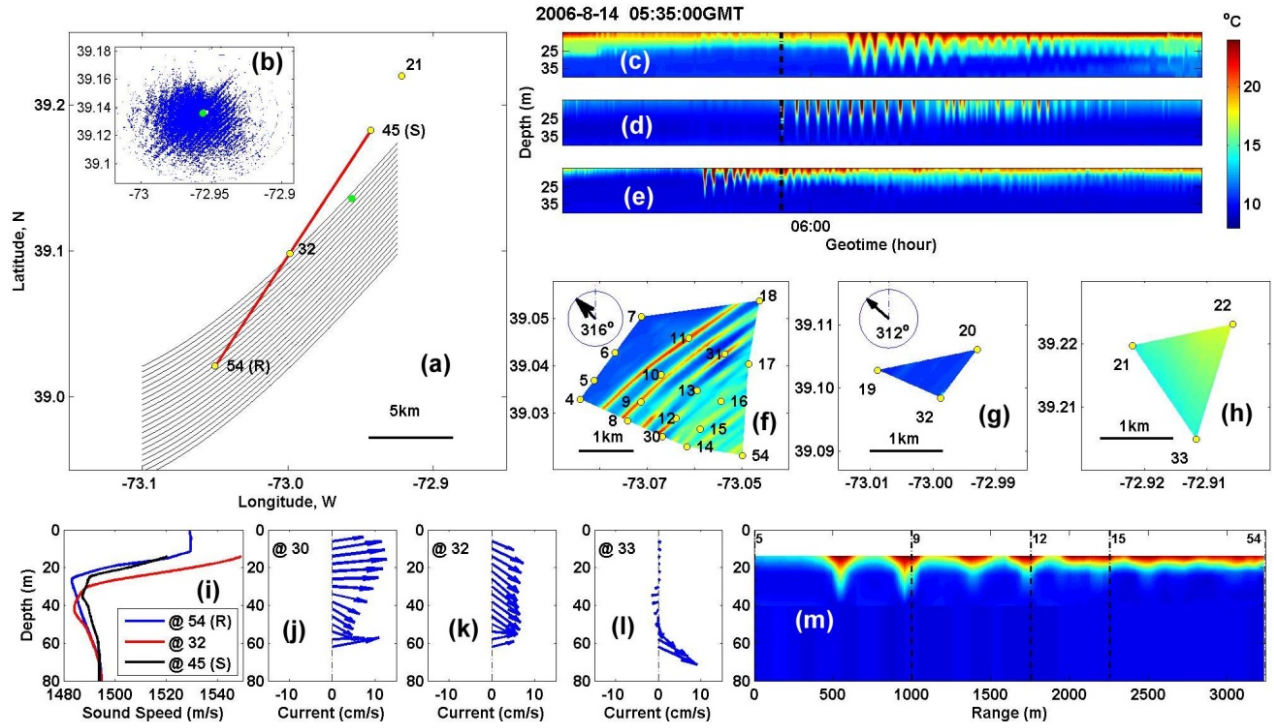


Figure 2. (a) This shows internal wave fronts (black curves) obtained by matching the internal wave (IW) propagation at three ocean patches (shown in (f-h)) within the SW06 experimental area. At time 2006-8-14 05:41:00 GMT. The numbered yellow dots are the locations of thermistor strings. The acoustic source and receivers were deployed at positions 45 and 54 respectively; the red line marks the acoustic propagation track. (b) A ship radar image (in blue dots) obtained by the R/V Sharp (position at green dot). (c-e) Measured temperature profiles at positions 45, 32 and 54, respectively. The vertical dashed dotted lines indicate the time of this figure. (f-h) These panels show the interpolated temperature data (24m below sea surface) obtained by a three-dimensional mapping technique of evolving internal waves [Badiey et al. 2013] at three patches of ocean. The arrows represent the directions of IW propagation with respect to true north; distinct IW arrive at one area, seen in (f). (m) A side view of the interpolated temperature field along the track connecting the thermistors #54, #15, #12, #9, and #5. (i) The sound speed as a function of depth at thermistor #54 (blue). #32 (red) and #45 (black). (j-l) The measured current magnitude (length of arrows) and direction as a function of depth at thermistor #30, #32, and #33, respectively.

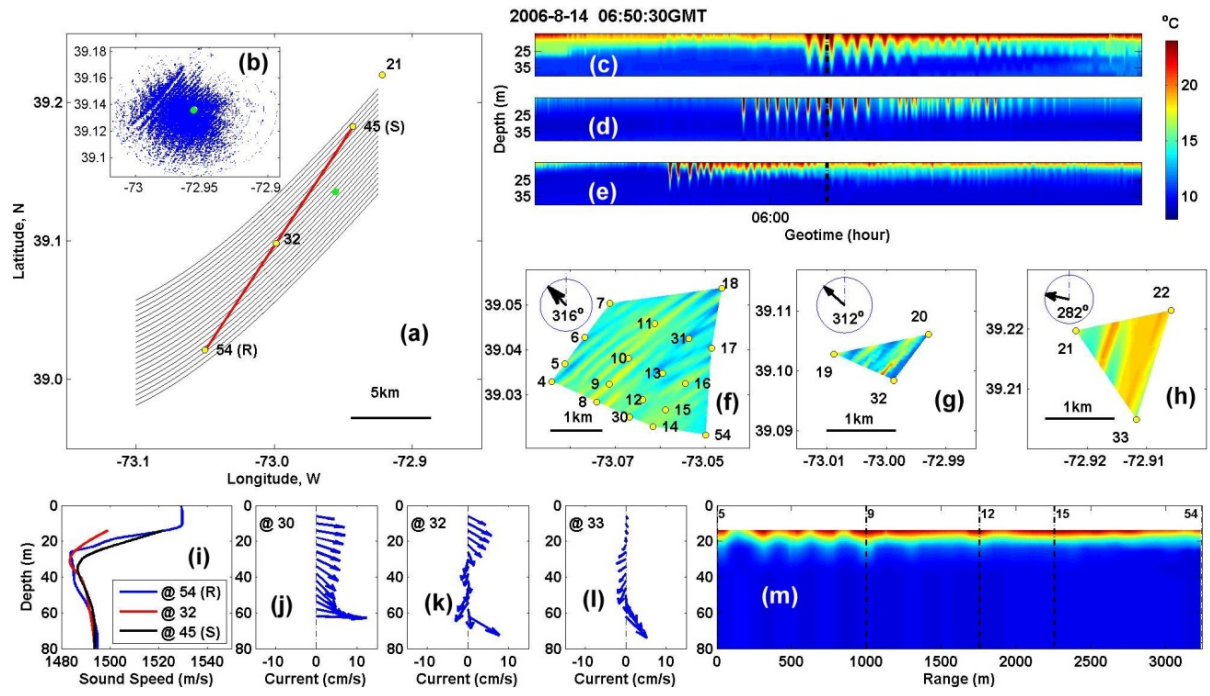


Figure 3. Similar to Figure 2 except for a different time, 2006-8-14 06:50:30 GMT, just over one hour later. At this time internal waves completely cover the acoustic path.

2013 Reanalysis of SW06: 30m Salinity

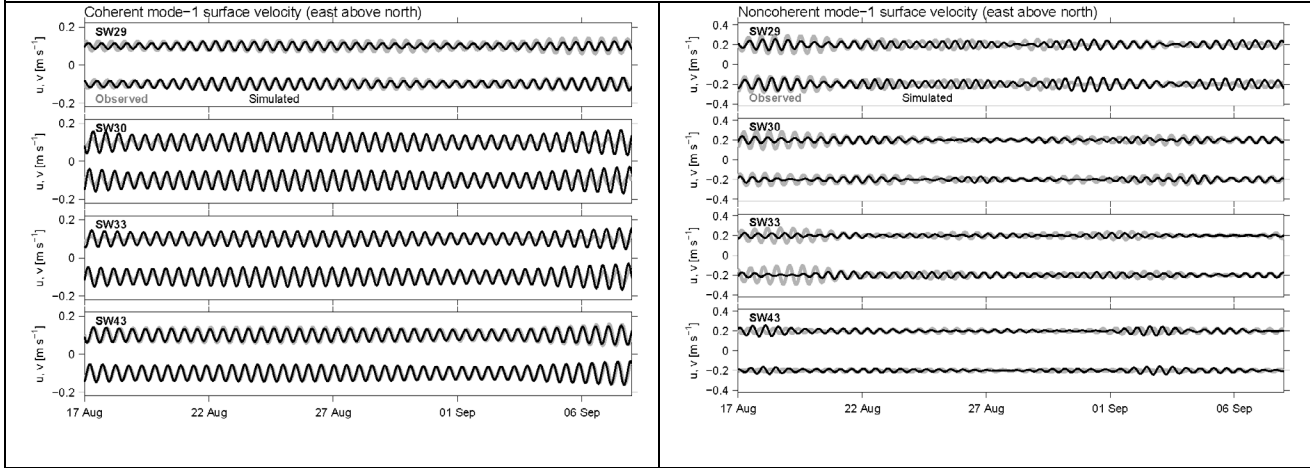
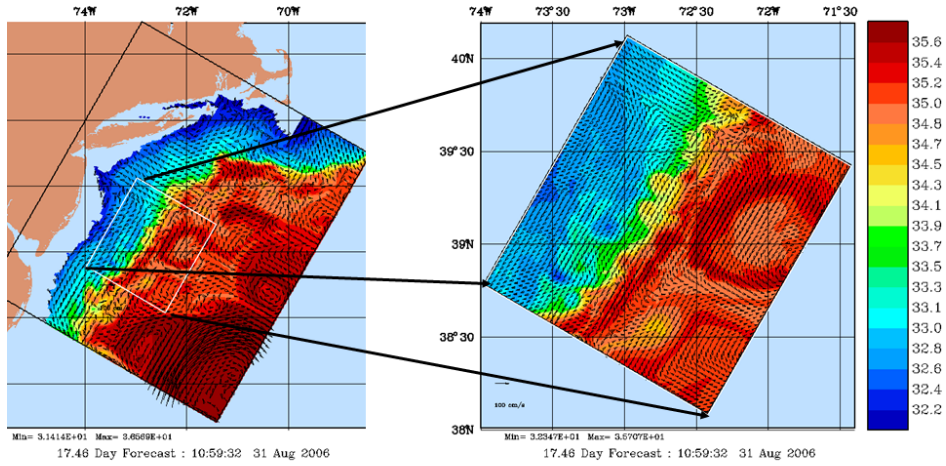


Figure 4. The top panels show salinity at 30 m depth in the reanalysis of predicted fields in the modeling domain originally configured for real-time operation and forecasting in SW06. The lower left panel shows model (dark color) and observed fixed-phase (coherent) internal tides at three mooring sites. These compare well. The lower right panel shows predictions of wandering-phase (incoherent) internal tides at the same sites. These do not completely agree, but notable observed features are reproduced in the model.

[A front in salinity at the edge of the US eastern seaboard can be seen, saltier offshore, as can eddies both onshore and offshore of the front. The modeled and measured coherent tides (rotating east and north elliptical tidal currents) agree for the 17 Aug. to 7 Sept. model interval. The modeled and measured incoherent tides differ, although measured enhancements in mid August and early September do appear in the model output, and are sometimes phased correctly, sometimes not.]

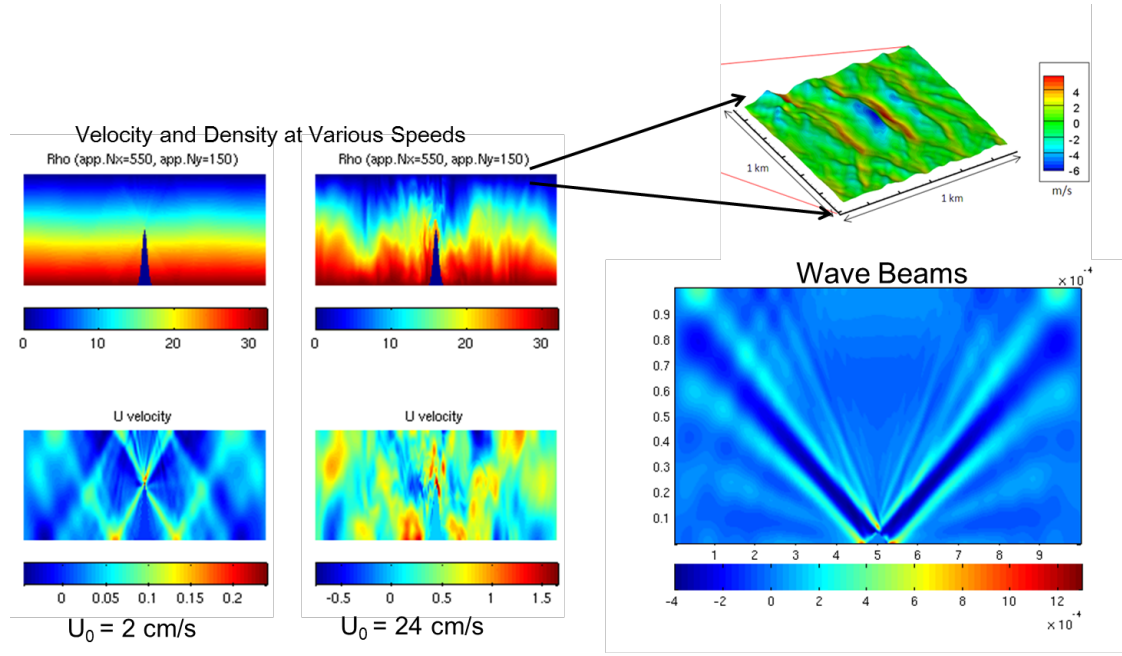


Figure 5. Results towards coupled Ocean-Waves-Acoustics simulations (MIT, MSEAS-SNOW). Idealized coupled 2D internal-tide-background-flow dynamical models (four color frames at left showing simulation fields in a $[x,z]$ plane, and bottom right plot showing internal wave beams) are coupled with surface wave dynamical models (surface shown at upper right). The whole system is stochastically forced to represent the unresolved internal-surface spectrum, and then integrated using DO equations.

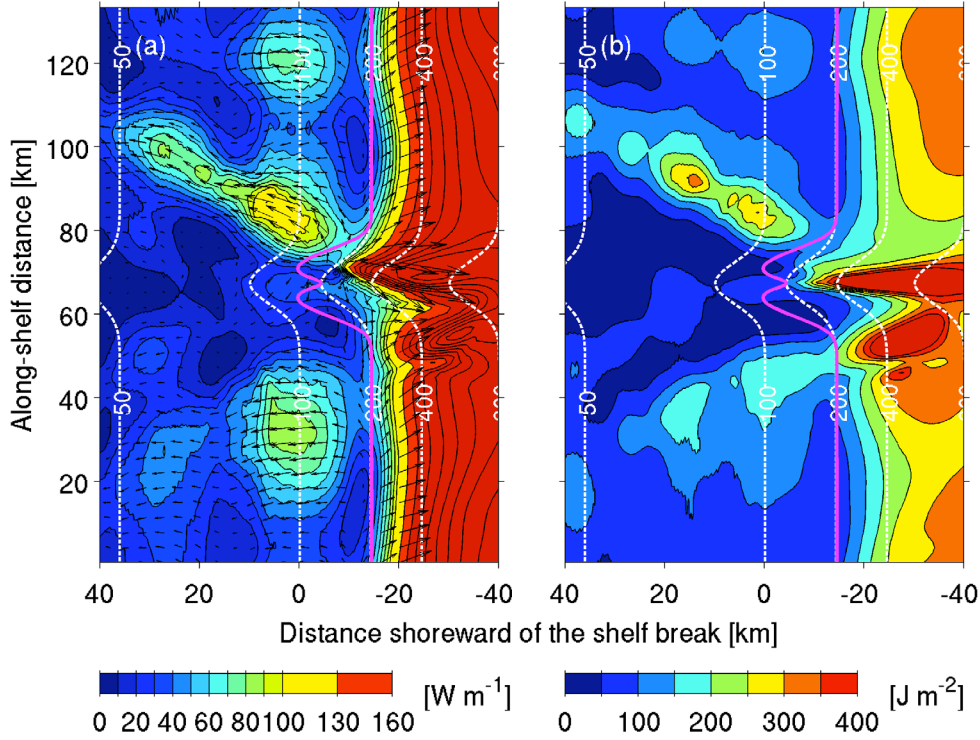


Figure 6. Vertically integrated (a) baroclinic (internal-wave) energy flux and (b) internal-wave kinetic energy density from a ROMS model run of internal tide generation in a model canyon that is studied in Zhang et al. [2013]. Arrows of energy flux greater than 160 W m^{-1} are omitted in (a) for clarity. The white dashed lines are isobath contours (meters), and the magenta lines indicate the critical slope locus along which wave sources are modeled in an explanatory modal internal tide model.

[Looking up the canyon from deep water onto the continental shelf, an internal-wave beam appears on the right, starting at the right side of the canyon and moving onto the shelf into shallow water at an angle of about 30 degrees from straight-ahead. The depth where the beam starts is about 150 m. Energy flux in the beam is about 100 watts per meter, energy is up to 300 J/m^2 . A zone of weaker energy flux (80 W/m) appears to the left of the canyon. Internal-wave flux and energy on the shelf is small outside of these areas. Outgoing flux and energy in deep water are greater than in shallow water.]