14IDM-0085 – 2014-01-0717

A New Metamodeling Approach for Time-dependent Reliability of Dynamic Systems with Random Parameters Excited by Input Random Processes

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Abstract

We propose a new metamodeling method to characterize the output (response) random process of a dynamic system with random parameters, excited by input random processes. The metamodel can be then used to efficiently estimate the time-dependent reliability of a dynamic system using analytical or simulation-based methods. The metamodel is constructed by decomposing the input random processes using principal components or wavelets and then using a few simulations to estimate the distributions of the decomposition coefficients. A similar decomposition is also performed on the output random process. A kriging model is then established between the input and output decomposition coefficients and subsequently used to quantify the output random process corresponding to a realization of the input random parameters and random processes. What distinguishes our approach from others in metamodeling is that the system input is not deterministic but random. The quantified output random process is finally used to estimate the time-dependent reliability or probability of failure of the dynamic system using the total probability theorem. The proposed method is illustrated with a numerical example.

1. Introduction

The response of dynamic systems under uncertainty is described by a random process. The input commonly consists of a combination of random variables and random processes. A time-dependent reliability analysis is therefore, needed to calculate the probability that the system will perform its intended function successfully for a specified time. It is therefore, related to product functionality over time.

Reliability is an important engineering requirement for consistently delivering acceptable product performance through time. As time progresses, the product may fail due to time-dependent operating conditions and material properties, component degradation, etc. In this research, we use time-dependent reliability concepts associated with the first-passage of non-repairable systems.

The time-dependent probability of failure, or cumulative probability of failure [1, 2], is defined as

$$P_f\left(0,T\right)=P\left\{\exists t\in\left[0,T\right]:\, g\left(\boldsymbol{\beta},\mathbf{Z}\left(t\right),t\right)\geq S_{\mathrm{t}}\right\} \tag{1}$$

where g is a random process that maps the input random variables $\boldsymbol{\beta}$ and random processes $\mathbf{Z}\left(t\right)$ to a response of interest and S_{t} is a given threshold value. The random process $G\left(t\right)=g\left(\boldsymbol{\beta},\mathbf{Z}\left(t\right),t\right)-S_{\mathrm{t}}$ can be viewed as a collection of random variables at different time instances *t*. Since we consider a first excursion failure problem in Equation (1), the failure domain is defined as $F=\left\{\max_{t\in[0,T]} g\left(\boldsymbol{\beta},\mathbf{Z}\left(t\right),t\right)\geq S_{\mathrm{t}}\right\}$.

The time-dependent probability of failure of Equation (1) can be calculated exactly as

$$P_{\mathrm{f}}\left(0,T\right)=1-\left(1-P_{\mathrm{f}}^{\mathrm{i}}\left(0\right)\right)\exp\left\{-\int_0^T\lambda\left(t\right)\mathrm{d}t\right\} \tag{2}$$

where $P_{\mathrm{f}}^{\mathrm{i}}\left(0\right)$ is the instantaneous probability of failure at the initial time and $\lambda(t)=\lim_{\mathrm{d}t\to 0}\dfrac{P(t<T_{\mathrm{f}}\leq t+\mathrm{d}t\mid T_{\mathrm{f}}>t)}{\mathrm{d}t}$ is the failure rate with T_{f} denoting the time to failure. In the commonly used out-crossing rate approach, the failure rate is approximated by the up-crossing rate

$$\nu^{+}\left(t\right)=\lim_{\substack{\Delta\tau\to 0,\\ \Delta\tau>0}}\frac{P\left[g\left(\boldsymbol{\beta},\mathbf{Z}(t),t\right)-S_{\mathrm{t}}<0\cap g\left(\boldsymbol{\beta},\mathbf{Z}(t+\Delta\tau),t+\Delta\tau\right)-S_{\mathrm{t}}\geq 0\right]}{\Delta\tau} \tag{3}$$

under the assumptions that the probability of having two or more out-crossings in $\left[t,t+\Delta t\right]$ is negligible, and the out-crossings in $\left[t,t+\Delta t\right]$ are statistically independent of the previous out-crossings in $\left[0,t\right]$.

Monte Carlo simulation (MCS) can accurately estimate the probability of failure in Equation (1) but it is computationally prohibitive for dynamic systems with a low failure probability. To

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
09 APR 2014	**Journal Article**	**08-02-2014 to 15-03-2014**

4. TITLE AND SUBTITLE
A New Metamodeling Approach for Time-dependent Reliability of Dynamic Systems with Random Parameters Excited by Input Random Processes

5a. CONTRACT NUMBER
W56HZV-04-2-0001

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Igor Baseski; Dorin Drignei; Zissimos Mourelatos; Monica Majcher

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Oakland University,2200 N. Squirrel Road,Rochester,Mi,48309

8. PERFORMING ORGANIZATION REPORT NUMBER
; #24310

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army TARDEC, 6501 East Eleven Mile Rd, Warren, Mi, 48397-5000

10. SPONSOR/MONITOR'S ACRONYM(S)
TARDEC

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
#24310

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
Submitted to SAE World Congress 2014

14. ABSTRACT

We propose a new metamodeling method to characterize the output (response) random process of a dynamic system with random parameters, excited by input random processes. The metamodel can be then used to efficiently estimate the time-dependent reliability of a dynamic system using analytical or simulation-based methods. The metamodel is constructed by decomposing the input random processes using principal components or wavelets and then using a few simulations to estimate the distributions of the decomposition coefficients. A similar decomposition is also performed on the output random process. A kriging model is then established between the input and output decomposition coefficients and subsequently used to quantify the output random process corresponding to a realization of the input random parameters and random processes. What distinguishes our approach from others in metamodeling is that the system input is not deterministic but random. The quantified output random process is finally used to estimate the time-dependent reliability or probability of failure of the dynamic system using the total probability theorem. The proposed method is illustrated with a numerical example.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**Public Release**	**7**	

address the computational issue of MCS, analytical methods have been developed based on the out-crossing rate approach which was first introduced by Rice [3] followed by extensive studies [2, 4-6] under the assumption that the out-crossings are statistically independent and Poisson distributed. The PHI2 method [2] uses two successive time-invariant analyses based on FORM, and the binomial cumulative distribution to calculate the probability of the joint event in Equation (3). A Monte-Carlo based set theory approach has been also proposed [7, 8] using a similar approach with the PHI2 method. Because, analytical studies (e.g., [9-11]) have shown that the PHI2 based approach lacks sufficient accuracy for vibratory systems, analytical approaches have been proposed to accurately estimate the time-dependent probability of failure considering non-monotonic behavior [9, 12, 13].

The limited accuracy of the out-crossing rate approach has been recently improved by considering the correlations between the limit-state function at two time instants [11]. The method estimates the up-crossing rate $\nu^+(t)$ by solving an integral equation involving $\nu^+(t)$ and $\nu^{++}(t, t_1)$, the joint probability of up-crossings in times *t* and t_1 [14].

Among the simulation-based methods, a MCS approach was proposed in [15] to estimate the time-dependent failure rate over the product lifecycle and its efficiency was improved using an importance sampling method with a decorrelation length [16] in order to reduce the high dimensionality of the problem. Subset simulation [17, 18] has been recently developed as an efficient simulation method for computing small failure probabilities for general reliability problems. Its efficiency comes from introducing appropriate intermediate failure sub-domains to express the low probability of failure as a product of larger conditional failure probabilities which are estimated with much less computational effort. Because it is very challenging to generate samples in the conditional spaces, subset simulation with Markov Chain Monte Carlo (SS/MCMC) [19] and subset simulation with splitting (SS/S) [20, 21] methods have been introduced.

In this paper, we describe a new metamodeling method to characterize the output random process of a dynamic system with random parameters, excited by input random processes which can be used to efficiently estimate the time-dependent probability of failure using either analytical or simulation-based methods. The metamodel is constructed by decomposing the input random processes using principal components or wavelets and then using a few simulations to estimate the distributions of the decomposition coefficients. A similar decomposition is also performed on the output random process. A kriging model is then established between the input and output decomposition coefficients and subsequently used to quantify the output random process corresponding to a realization of the input random parameters and random processes. The proposed approach is new because it considers the system input as random and not deterministic as is usually the case.

The paper is arranged as follows. Section 2 describes the proposed metamodeling methodology and Section 3 uses a mathematical example to demonstrate its applicability. Finally, Section 4 provides a brief summary and concludes.

2. PROPOSED METAMODELING APPROACH

Assume that the simulation model has two types of inputs: random time-dependent inputs represented by the random processes $\mathbf{Z}(t)$ and random time-independent inputs represented by random variables $\boldsymbol{\beta}$. Assume that the output is also time-dependent and is represented by the random processes $\mathbf{Y}(t)$. We decompose each process $Z(t)$ using a well-defined basis of functions $\mathbf{B}(t)$ such as wavelets or principal components. For principal components, consider *D* realizations of the random process $Z(t)$ denoted by $Z_1(t), \ldots, Z_D(t)$. An eigenvalue decomposition of their covariance matrix defines the principal components. If we retain a small number *r* of dominant eigenvalues and $\mathbf{B_1}(t)$ is a vector whose rows are the corresponding eigenvectors, each $Z_j(t),\ j = 1, \ldots, D$ can be expressed as $Z_j(t) = \sum_{i=1}^{r} \alpha_i B_{1i}(t) = \boldsymbol{\alpha}_j \mathbf{B_1}(t)$ where $\boldsymbol{\alpha}_j = Z_j(t)\mathbf{B_1}(t)^T$. A similar principal decomposition can be considered for the output process $Y(t)$ so that each output realization (sample function) $Y_j(t),\ j = 1, \ldots, D$ can be expressed as $Y_j(t) = \sum_{i=1}^{s} \delta_i B_{2i}(t) = \boldsymbol{\delta}_j \mathbf{B_2}(t)$ using basis functions $\mathbf{B_2}(t)$ and $\boldsymbol{\delta}_j = Y_j(t)\mathbf{B_2}(t)^T$, if we retain a small number *s* of dominant eigenvalues of the output covariance matrix.

To simplify the description of the proposed method, we assume that we have only one input random variable β, one input random process $Z(t)$, and one output random process $Y(t)$ which is represented with only one principal component. The input vector can be therefore, expressed in terms of the vector (**α**, β) where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_r)$ and the output is expressed in terms of δ_1. We use the principal component decomposition in this paper although other bases may be chosen (e.g. wavelets).

What distinguishes our approach from others in metamodeling is that the input vector (**α**, β) is not deterministic but random. The distributions of all components of (**α**, β) are estimated using a small number of computer runs. If we assume that all α_i's are independent and normally distributed as $N(\mu_i, \sigma_i^2)$ where μ_i is the mean of α_i over all performed computer runs and σ_i is the standard error of α_i over all computer runs, and also assume that β is a normal random variable with mean μ_β and standard error σ_β, independent of the *α's,* the joint probability density function between **α** and β is given by

$$f(\boldsymbol{\alpha}, \beta) = \frac{1}{\left(\sqrt{2\pi}\right)^r \sigma_1 \ldots \sigma_r} \exp\left[-\frac{1}{2}\sum_{i=1}^{r}\frac{(\alpha_i - \mu_i)^2}{\sigma_i^2}\right] \cdot \frac{1}{\sigma_\beta\sqrt{2\pi}} \exp\left[-\frac{1}{2}\frac{(\beta - \mu_\beta)^2}{\sigma_\beta^2}\right] \qquad (4)$$

We run the mathematical model for D input vectors ($\boldsymbol{\alpha}^{(1)}$, $\beta^{(1)}$), … , ($\boldsymbol{\alpha}^{(D)}$, $\beta^{(D)}$) and obtain the corresponding time-dependent outputs whose first principal component is ($\delta^{(1)}$,…, $\delta^{(D)}$). Denote the vectors $\underline{\boldsymbol{\alpha}}$ =($\boldsymbol{\alpha}^{(1)}$,…, $\boldsymbol{\alpha}^{(D)}$), $\underline{\boldsymbol{\beta}}$=($\beta^{(1)}$,…, $\beta^{(D)}$) and $\underline{\boldsymbol{\delta}}$ = ($\delta^{(1)}$,…, $\delta^{(D)}$). Conditioned on ($\underline{\boldsymbol{\alpha}}$, $\underline{\boldsymbol{\beta}}$), the vector $\underline{\boldsymbol{\delta}}$ has a multivariate Gaussian distribution with mean $\eta\,\mathbf{1}$ and covariance matrix $\tau^2\mathbf{C}$ where $\mathbf{C}$ is a Gaussian correlation matrix whose elements depend on the distances between sampled inputs ($\boldsymbol{\alpha}$, β). The parameters of this distribution are estimated by maximum likelihood.

Let $(\boldsymbol{\alpha}_0, \beta_0)$ be a new input vector and δ_0 the corresponding output first principal component. A kriging metamodel is generated between the input ($\underline{\boldsymbol{\alpha}}$, $\underline{\boldsymbol{\beta}}$) and the output $\underline{\boldsymbol{\delta}}$ to obtain δ_0. Details about kriging metamodeling can be found, for example, in [22-24]. The kriging Gaussian prediction distribution $\hat{f}(\delta_0 \mid \underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})$ for δ_0, is given by

$$\hat{f}(\delta_0 \mid \underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}}) = \frac{1}{\sqrt{2\pi v^0_{\underline{\boldsymbol{\alpha}},\underline{\boldsymbol{\beta}}}}} \exp\left[-\frac{1}{2v^0_{\underline{\boldsymbol{\alpha}},\underline{\boldsymbol{\beta}}}} \left(\delta_0 - m^0_{\underline{\boldsymbol{\alpha}},\underline{\boldsymbol{\beta}}}\right)^2 \right] \quad (5)$$

where the prediction variance is $v^0_{\underline{\boldsymbol{\alpha}},\underline{\boldsymbol{\beta}}} = \tau^2\left(1 - \mathbf{C}_0^T\mathbf{C}^{-1}\mathbf{C}_0\right)$ and the prediction mean is $m^0_{\underline{\boldsymbol{\alpha}},\underline{\boldsymbol{\beta}}} = \eta + \mathbf{C}_0^T\mathbf{C}^{-1}\left(\underline{\boldsymbol{\delta}} - \eta\mathbf{1}\right)$. Here $\mathbf{C}_0$ is the vector of correlations between the new input and the sampled inputs. The joint distribution of $\delta_0, \underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}}$ is

$$\hat{f}(\delta_0, \underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}}) = \hat{f}(\delta_0 \mid \underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})\, f(\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}}) \propto \frac{1}{\sqrt{2\pi v^0_{\underline{\boldsymbol{\alpha}},\underline{\boldsymbol{\beta}}}}} \exp\left[-\frac{1}{2v^0_{\underline{\boldsymbol{\alpha}},\underline{\boldsymbol{\beta}}}} \left(\delta_0 - m^0_{\underline{\boldsymbol{\alpha}},\underline{\boldsymbol{\beta}}}\right)^2 \right] \prod_{j=1}^{D} \exp\left[-\frac{1}{2}\sum_{i=1}^{r} \frac{(\alpha_i^{(j)} - \mu_i)^2}{\sigma_i^2} - \frac{1}{2}\frac{(\beta^{(j)} - \mu_\beta)^2}{\sigma_\beta^2} \right] \quad (6)$$

The marginal distribution of δ_0 at the new input vector $(\boldsymbol{\alpha}_0, \beta_0)$ is given by the integral

$$\hat{f}(\delta_0) = \int \hat{f}(\delta_0, \underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})\, d\underline{\boldsymbol{\alpha}}\, d\underline{\boldsymbol{\beta}}\,. \quad (7)$$

Note that the typically small number of runs performed in an engineering problem is used only to estimate the distributions $f(\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})$ and $\hat{f}(\delta_0 \mid \underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})$ of Equations (4) and (5), respectively. Once these distributions are estimated, a larger number *N* of statistical simulations from the distribution $\hat{f}(\delta_0)$ of Equation (7) can be obtained. For each simulated δ_0 we create a simulated output function $\hat{Y}_0(t) = \delta_0 B_2(t)$.

We now provide some details on the evaluation of integral in Equation (7). Consider *R* sampled sets of *D* input vectors $(\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})_1$,..., $(\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})_R$. The integral is approximated using the Monte Carlo method, up to a proportionality constant, as

$$\frac{1}{R}\sum_{k=1}^{R} \hat{f}(\delta_0, (\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})_k) = \frac{1}{R}\sum_{k=1}^{R} \hat{f}(\delta_0 \mid (\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})_k)\, f((\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})_k)\,. \quad (8)$$

While $f((\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})_k)$ can be computed easily for any number *R* of sets, $\hat{f}(\delta_0 \mid (\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})_k)$ will require running the mathematical model *R*D* times; i.e., for each set $[(\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})_k, \gamma]$ of *D* input vectors. This is computationally very demanding. Instead, we use a jackknife method (e.g. [25]), which belongs to the general class of "leave-one-out" methods for cross-validation. Specifically, we choose *R=D* and $(\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})_k := (\underline{\boldsymbol{\alpha}}^{(-k)}, \underline{\boldsymbol{\beta}}^{(-k)})$, meaning that we replace $(\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}})_k$ by the set of *D* original input vectors $\underline{\boldsymbol{\alpha}}, \underline{\boldsymbol{\beta}}$ from which we leave the k^{th} input vector out, for *k* = 1,…,*D*. The advantage of this method is that we already have data from math model runs at these $(\underline{\boldsymbol{\alpha}}^{(-k)}, \underline{\boldsymbol{\beta}}^{(-k)})$ vectors, and we do not need any additional math model runs.

The total probability theorem is used to compute the time-dependent probability of failure. We draw *M* new random inputs $(\boldsymbol{\alpha}_0, \beta_0)$ from the input distribution $f(\boldsymbol{\alpha}, \beta)$ of Equation (4) and for each of them simulate *N* output random functions $\hat{Y}_0(t)$ by sampling $\hat{f}(\delta_0)$ of Equation (7) *N* times. For each of the *M* random inputs $(\boldsymbol{\alpha}_0, \beta_0)$, a simple Monte Carlo simulation is used to calculate a probability of failure $P_f^M(0,t)$ using the *N* output functions $\hat{Y}_0(t)$. It can be easily shown using the total probability theorem that the probability of failure is the average among the $P_f^M(0,t)$ probabilities; i.e., $P_f(0,t) = \frac{1}{M}\sum_{i=1}^{M} P_f^M(0,t)$. Therefore, the time-dependent probability of failure is the percentage of the *M*N* response functions $\hat{Y}_0(t)$ that exceed a specified failure threshold T in the interval [0, *t*]. This probability considers the randomness of the inputs $Z(t)$ and β.

3. EXAMPLE

Consider the following mathematical model

$$Y(t) = 2\exp\left(Z(t)\right)\sin\left(2\pi\beta t\right) \quad (9)$$

where *Z*(*t*) is a random process generated from the function 2-*t*, to which correlated noise has been added. The correlated noise is represented by random draws from a Gaussian distribution with a zero mean vector and Gaussian correlation

for the temporal dimension of smoothness parameter 25. The random parameter β is normally distributed as $N(2, 0.05)$.

A sample of $D = 30$ computer runs is obtained (Figure 1). *Figure 1a* shows the sample of 30 random input functions $Z(t)$ for $t \in [0, 2]$ and *Figure 1b* shows their principal component reconstruction using $r = 4$ principal components which account for approximately 90% of the variance. *Figure 1c* shows the corresponding 30 random output functions $Y(t)$ and *Figure 1d* shows their principal component reconstruction using only the first principal component which accounts for about 66% of the variability. Visual inspection shows that the time-dependent output reconstruction captures the major features of actual time-dependent output, even with only the first principal component.

The random input vector is expressed with 5 components: the 4 coefficients from the principal component decomposition of the input function $Z(t)$ denoted by $\boldsymbol{\alpha}$ and the β. The time-dependent output is expressed by a scalar which is the dominant principal component coefficient δ. The four components of $\boldsymbol{\alpha}$ are assumed independent and normal, with estimated means equal to zero and standard deviations equal to 0.9062, 0.8387, 0.6854, and 0.4947, respectively. These estimated values are obtained from the sample of 30 computer runs. The estimated mean and standard deviation of β are 1.9997 and 0.0531 respectively, which are in agreement with the theoretical values of 2 and 0.05. The scatter plots of Figure 2 verify the assumption that the four components of $\boldsymbol{\alpha}$ are independent.

We have developed a metamodel using the proposed approach using $M = 100$ new random inputs drawn from the input distribution and $N = 50$ simulations for each of these new random inputs. Therefore, we have a total of 5,000 simulations $\hat{Y}_0(t)$ shown in green in the left panel of Figure 3. We observe that the variability of these curves appears to be similar with the variability of the 30 actual math model output curves (*Figure 1c*) or their principal components reconstructions (*Figure 1d*). However, their number is 5,000 (much greater than 30), which allows for a more accurate evaluation of the probability of failure. The right panel of Figure 3 shows the time-dependent probability of exceeding the thresholds T of 4, 6, 7.5, 10 or 15. We observe as expected, that as the threshold is increased the time-dependent probability of failure reaches its maximum value of 1 at a later time. The accuracy of the calculated probabilities of failure can be easily verified using Monte Carlo simulation.

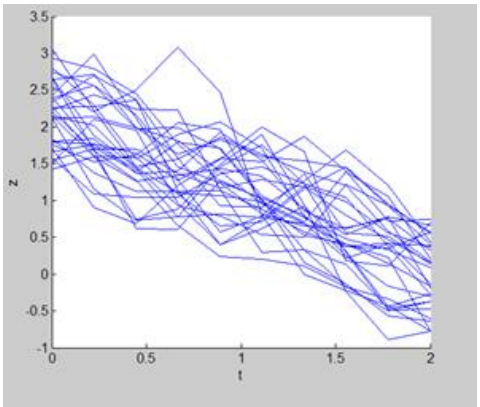

Figure 1a. A sample of 30 random input functions $Z(t)$

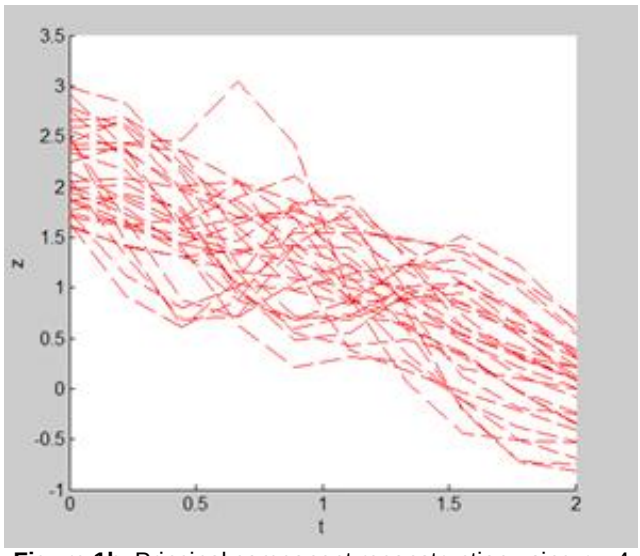

Figure 1b. Principal component reconstruction using $r = 4$

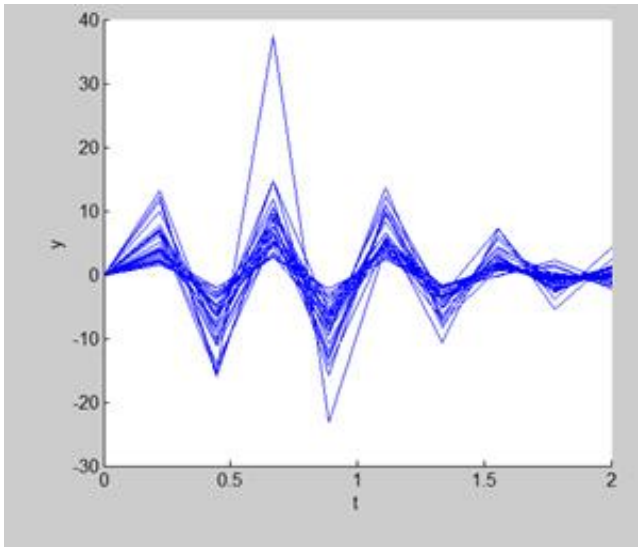

Figure 1c. Corresponding 30 random output functions $Y(t)$

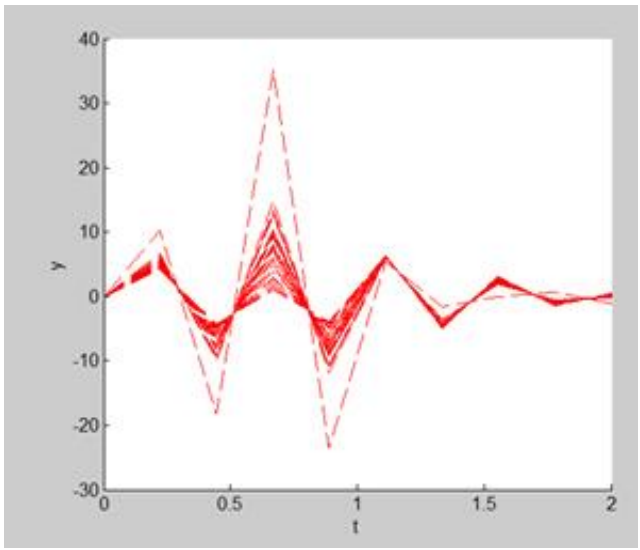

Figure 1d. Principal component reconstruction

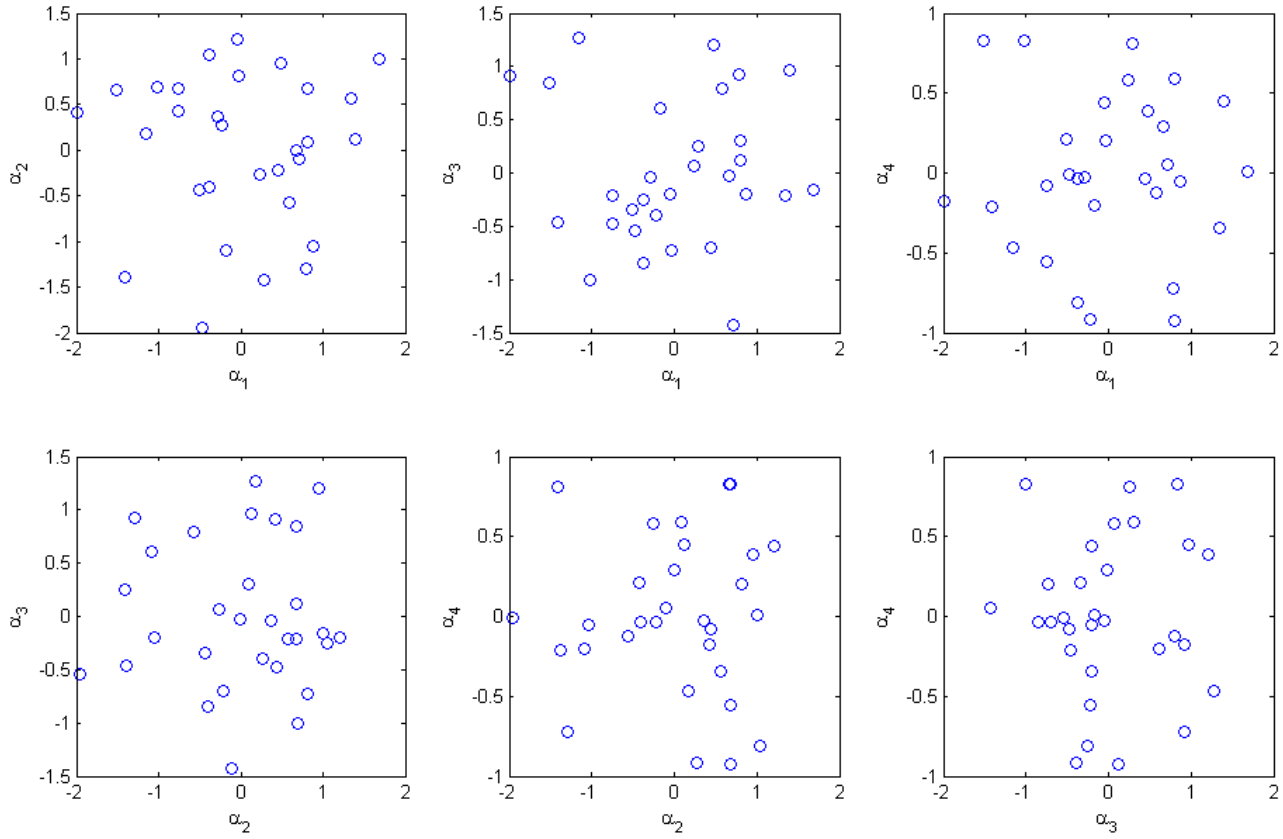

Figure 2. Scatter plots of the four components of vector $\boldsymbol{\alpha}$

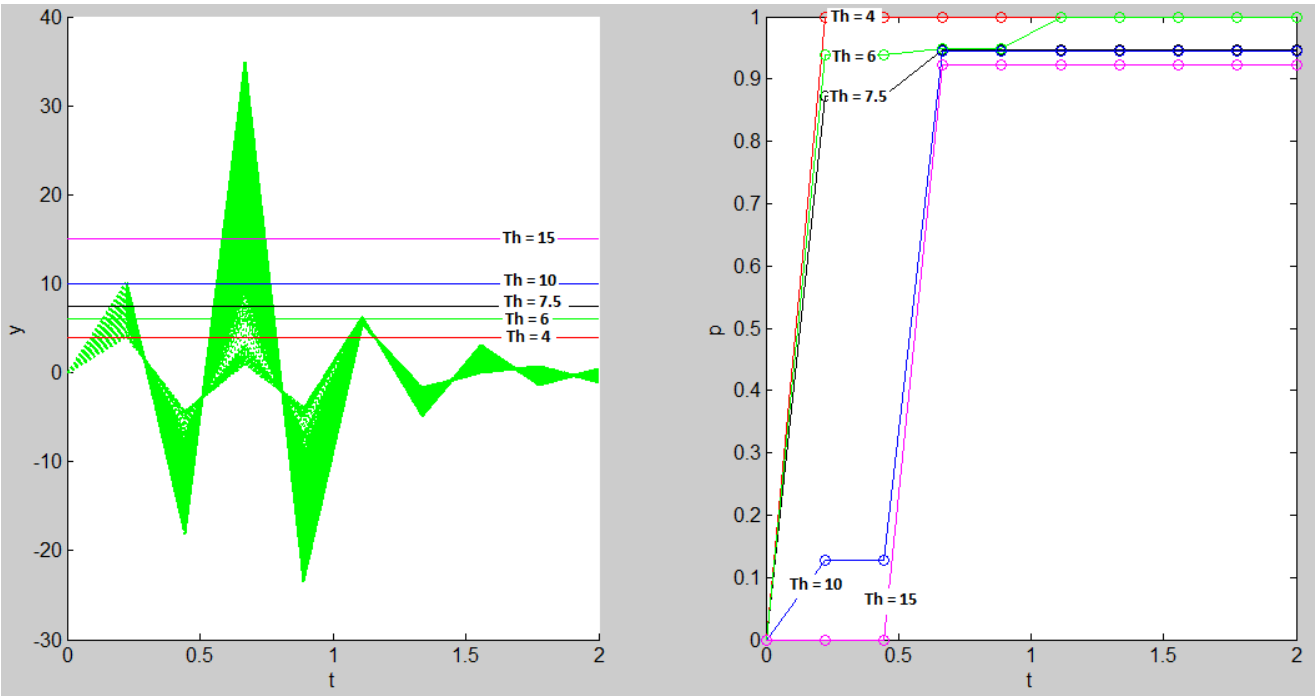

Figure 3. Metamodel simulations (left panel) and time-dependent probability for different thresholds (right panel)

4. Summary/Conclusions

We described in this paper a new metamodeling approach to characterize the output random process of a dynamic (or time-dependent in general) system with random input parameters, excited by an input random process. The methodology utilizes a decomposition of the input and output random processes using principal components or wavelets and kriging. The latter establishes a non-parametric interpolation between the input and output decomposition coefficients which is then used to quantify the output random process. The metamodel is used to efficiently estimate the time-dependent probability of failure using either analytical or simulation-based methods. This capability is very practical and desirable for large-scale, linear or non-linear, dynamic systems where the calculation of response is time intensive and/or expensive to compute or measure. The proposed approach considers the system input as random and not deterministic as is usually the case. We used a simple mathematical example to demonstrate the steps of the approach and demonstrate its practicality. Future work will generalize the method for non-Gaussian random variables and processes.

Acknowledgments

We would like to acknowledge the technical and financial support of the Automotive Research Center (ARC) in accordance with Cooperative Agreement W56HZV-04-2-0001 U.S. Army Tank Automotive Research, Development and Engineering Center (TARDEC) Warren, MI.

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