NEW COMPUTATIONAL FRAMEWORK FOR THE TREATMENT OF JOINT CONSTRAINTS AND CONNECTIVITY CONDITIONS IN FINITE ELEMENT/MULTIBODY SYSTEM ALGORITMS

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14. ABSTRACT
1.Integration of CG, FE, and MBS algorithms is necessary in order to have CAD and analysis models that are consistent. 2.This integration was difficult to achieve because of the fundamental differences between CG and most FE representations. 3.A simple interface between CG-based CAD systems and FE codes can be established (I-CAD-A). 4.The new MBS software technology Sigma/Sams is being developed (www.computational-dynamics.com). 5.Sigma/Sams is being designed to eliminate the need for using other commercial FE and CAD codes for flexible MBS simulations.

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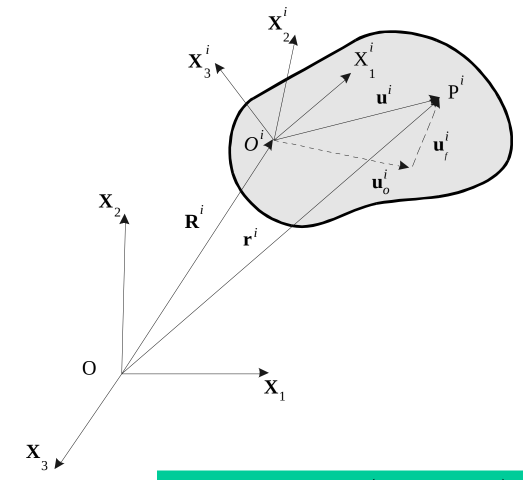
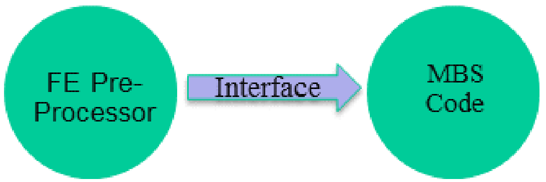
Contents

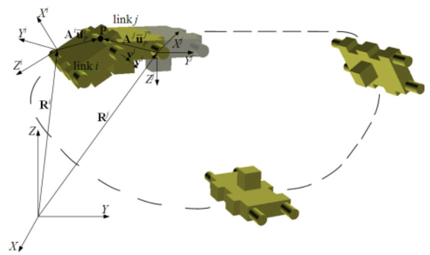
Multibody System (MBS) Approaches

1. MBS algorithms are designed to solve the differential and algebraic equations (DAE's) of complex systems.
2. MBS systems may consist of rigid, flexible or rigid and flexible components.
3. For flexible MBS dynamics, the floating frame of reference (FFR) formulations is used for small deformation analysis.
4. Two separate computer codes are required; a finite element (FE) code and a MBS code.
5. In the case of rigid body dynamics or small deformation problem, the joint constraint equations are highly nonlinear.
6. The ideal joint formulations do not capture important deformation modes.



$$\mathbf{r}^i = \mathbf{R}^i + \mathbf{A}^i\left(\bar{\mathbf{u}}_o^i + \bar{\mathbf{u}}_f^i\right)$$



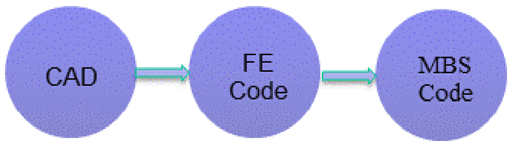


$$\begin{bmatrix} \mathbf{r}_P^{ij} & \mathbf{v}_1^{iT}\mathbf{v}^j & \mathbf{v}_2^{iT}\mathbf{v}^j \end{bmatrix}^T = \mathbf{0}$$

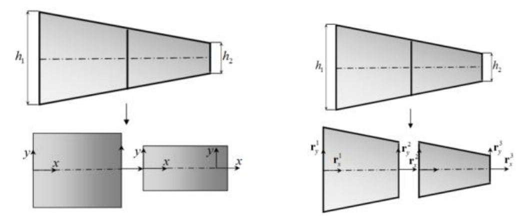
$$\begin{aligned} \mathbf{r}_P^{ij} &= \mathbf{r}_P^i - \mathbf{r}_P^j \\ &= \mathbf{R}^i + \mathbf{A}^i\bar{\mathbf{u}}_P^i - \mathbf{R}^j - \mathbf{A}^j\bar{\mathbf{u}}_P^j \end{aligned}$$

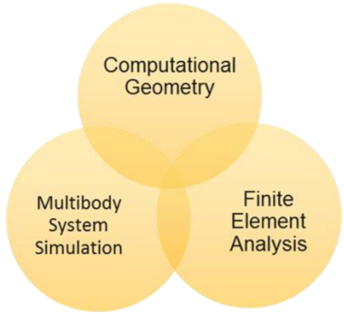
Challenges and Objectives

1. Existing flexible MBS software technology requires the use of several computer codes.
2. In addition to the compatibility issue, existing technology suffers from serious limitations.
3. Conversion of CAD models to FE/MBS meshes is costly and time consuming (more than $600 m/year for U.S. automotive industry alone, SIAM News, 2011).
4. Furthermore, existing technology is not designed to handle some important applications.
5. In order to solve this problem, new FE and MBS approaches and algorithms are required.
6. The objective is to develop a new MBS computational framework.

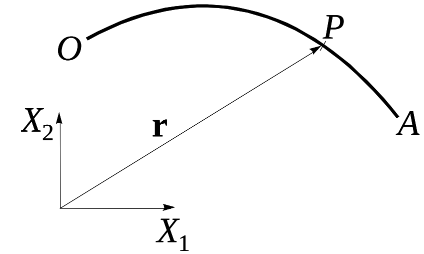


Three different codes are required. There are serious compatibility issues





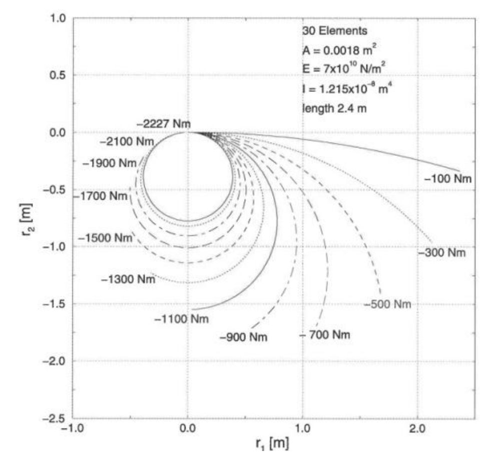
Absolute Nodal Coordinate Formulation (ANCF)

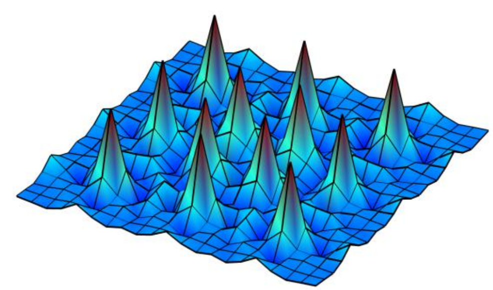


$$\mathbf{r}^j\left(\mathbf{x}^j, t\right) = \mathbf{S}^j\mathbf{e}^j$$

$$\mathbf{M}\ddot{\mathbf{e}} + \mathbf{Q}_s - \mathbf{Q}_e = \mathbf{0}$$

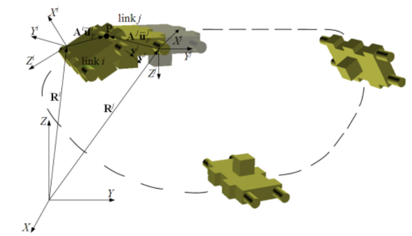
1. ANCF is a simple polynomial-based representation that defines a unique displacement and rotation field.
2. Consequently, one can always find appropriate displacement field for a particular problem (Element technology).
3. ANCF has been used in modeling a large number of statics and dynamics problems.
4. ANCF allows the use of different elastic force formulations.
5. Very complex geometry can be captured using ANCF finite elements (fully parameterized plate 1452 dof model runs 300 times faster than real time in MATLAB).





Joint Formulations

1. Several methods can be used to define connectivity between bodies; ideal joints, penalty method, and bushing elements.
2. The ideal joints are defined using algebraic constraint equations that eliminate degrees of freedom.
3. The penalty method enforces the same algebraic equations at the position level. No degrees of freedom are eliminated.
4. The bushing element allows for the use of six stiffness coefficients to define the connectivity between bodies.
5. The penalty and bushing methods lead to compliant joints with discrete elastic elements.
6. This paper introduces a new class of ideal compliant joints that have interesting features.

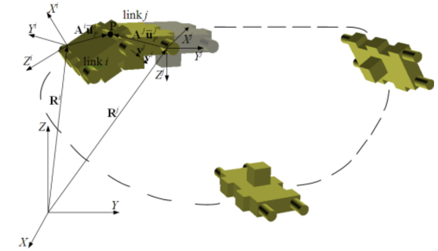


$$\begin{bmatrix} \mathbf{r}_P^{ij} & \mathbf{v}_1^{iT}\mathbf{v}^j & \mathbf{v}_2^{iT}\mathbf{v}^j \end{bmatrix}^T = \mathbf{0}$$

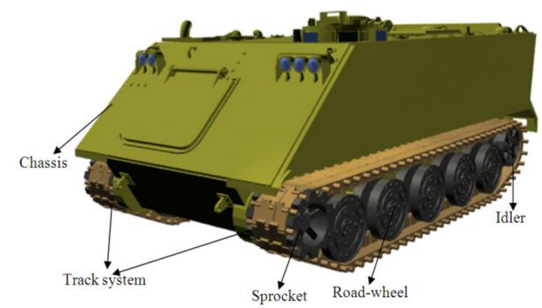
$$\mathbf{r}_P^{ij} = \mathbf{r}_P^i - \mathbf{r}_P^j = \mathbf{R}^i + \mathbf{A}^i\bar{\mathbf{u}}_P^i - \mathbf{R}^j - \mathbf{A}^j\bar{\mathbf{u}}_P^j$$

New ANCF Joint Formulation

1. ANCF finite elements allow for developing new joints using linear algebraic equations.
2. These joints have distributed elasticity and inertia and have modes of deformation that cannot be captured using other formulations.
3. ANCF joints allow for the elimination of the dependent variables at a preprocessing stage.
4. Because ANCF finite elements have constant mass matrix and linear joints, one obtains new FE meshes with desirable features.
5. The computational and storage saving can be very significant.
6. Conditions on the gradients can be imposed without imposing conditions on the positions.
7. The new ANCF meshes allow for developing models with significant details.



$$\left.\begin{aligned}\mathbf{r}^i &= \mathbf{r}^j \\ \mathbf{r}^i_\alpha &= \mathbf{r}^j_\alpha\end{aligned}\right\}$$



$n_l = 128, \quad n_j = 128$

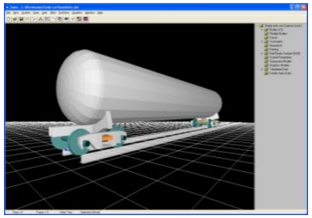
12,000 non-zero entries

The new ANCF meshes have constant mass matrix and linear connectivity.

New Generation of MBS Computer Codes

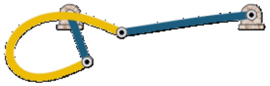
1. Sigma/Sams is based on successful integration of CG/FE/MBS.
2. It will have an FE preprocessor that have a comprehensive element library eliminating the need for the use of commercial FE codes in MBS simulations.
3. A linear transformation can be developed between CAD systems and MBS codes.
4. Sigma/Sams will allow for the integration of different formulations that employ different sets of coordinates.
5. ANCF finite elements are used to develop new linear joint formulations, leading to new ANCF meshes that have constant inertia and linear connectivity.

A new code, Sigma/Sams (**S**ystematic **I**ntegration of **G**eometric **M**odeling and **A**nalysis for the **S**imulation Articulated **M**echanical **S**ystems) is being developed.



$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_r^T & \mathbf{q}_f^T & \mathbf{p}^T & \mathbf{s}^T \end{bmatrix}^T$$

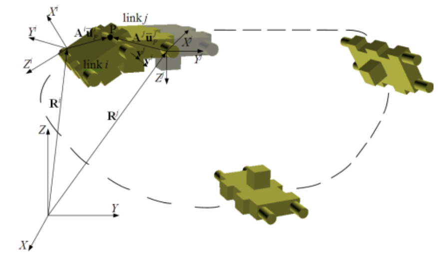
$$\begin{bmatrix} \mathbf{M}_{rr} & \mathbf{M}_{rf} & \mathbf{0} & \mathbf{0} & \mathbf{C}_{\mathbf{q}_r}^T \\ \mathbf{M}_{fr} & \mathbf{M}_{ff} & \mathbf{0} & \mathbf{0} & \mathbf{C}_{\mathbf{q}_f}^T \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{C}_{\mathbf{p}}^T \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{C}_{\mathbf{q}_s}^T \\ \mathbf{C}_{\mathbf{q}_r} & \mathbf{C}_{\mathbf{q}_f} & \mathbf{C}_{\mathbf{q}_a} & \mathbf{C}_{\mathbf{q}_s} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_r \\ \ddot{\mathbf{q}}_f \\ \ddot{\mathbf{p}} \\ \ddot{\mathbf{s}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_r \\ \mathbf{Q}_f \\ \mathbf{Q}_a \\ \mathbf{0} \\ \mathbf{Q}_d \end{bmatrix}$$

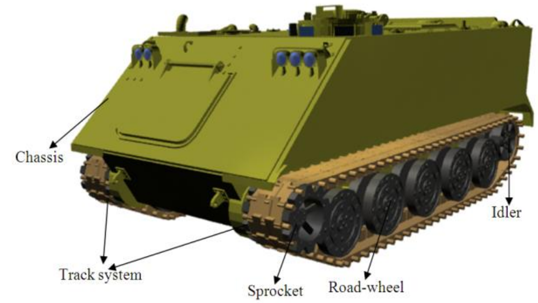


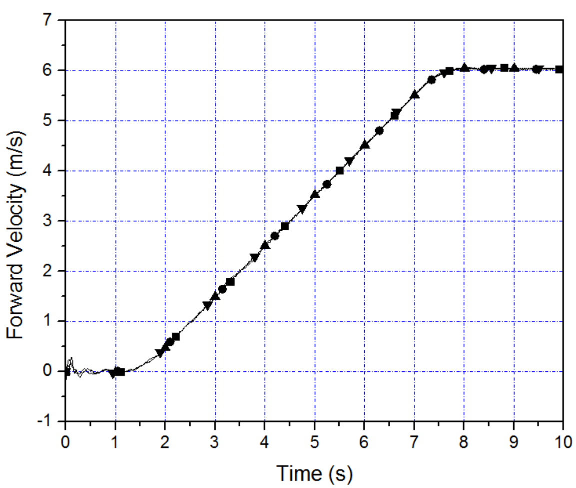
$$\left.\begin{aligned} \mathbf{r}^i &= \mathbf{r}^j \\ \mathbf{r}_\alpha^i &= \mathbf{r}_\alpha^j \end{aligned}\right\}$$

Numerical Results

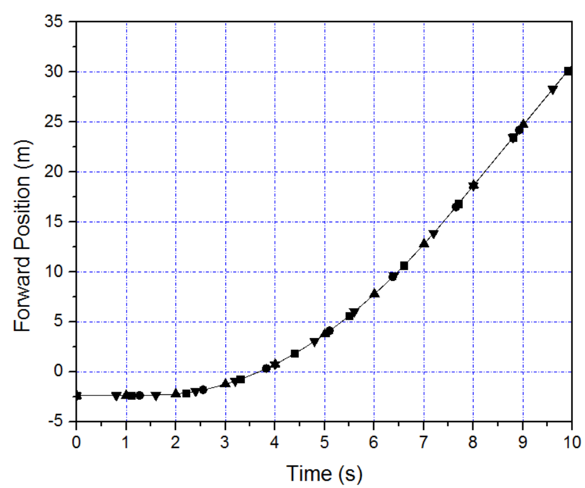
1. Closed loop ANCF meshes do not suffer from singularities encountered with rigid body chain models.
2. No significant differences in CPU time between different chain formulations.
3. Different joint models can be developed and the results can be compared.



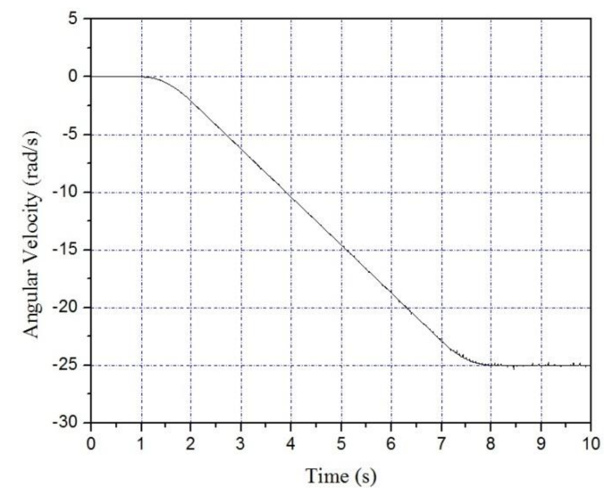




Chassis forward velocity



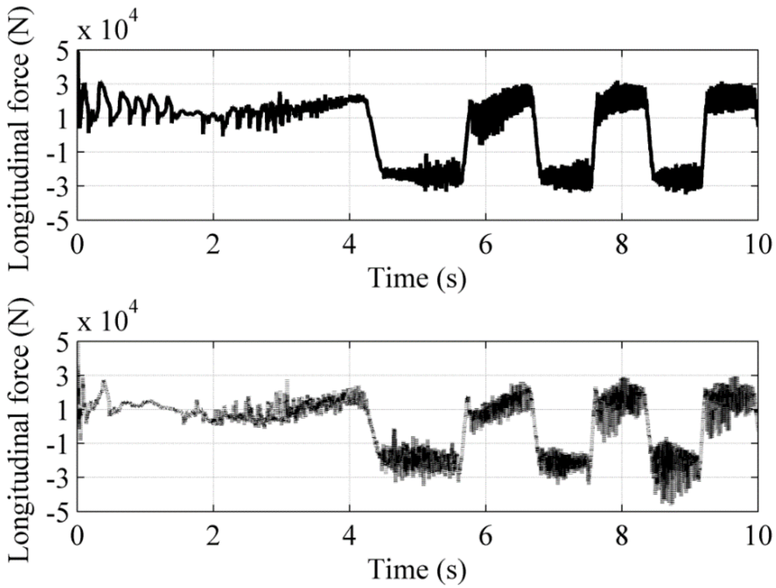
Chassis forward position



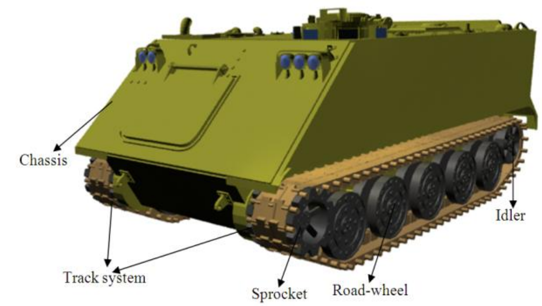
Sprocket angular velocity

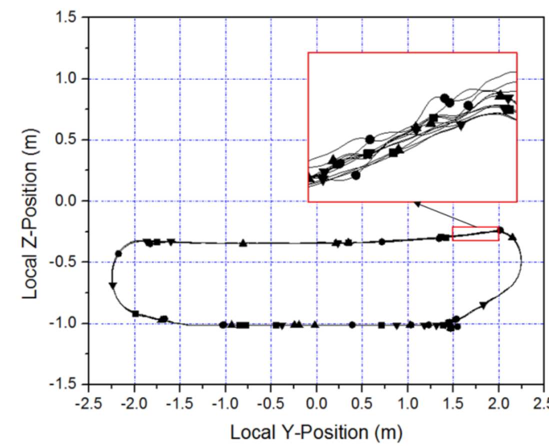
(—▲— Constrained joint model, —▼— Penalty method model,
—■— Bushing element model, —●— ANCF model)

Numerical Results



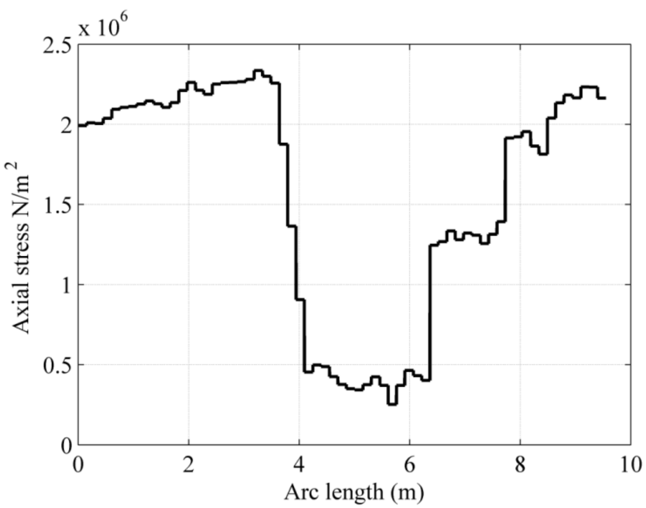
Joint longitudinal force (rigid and flexible body models)





Trajectory of a track link

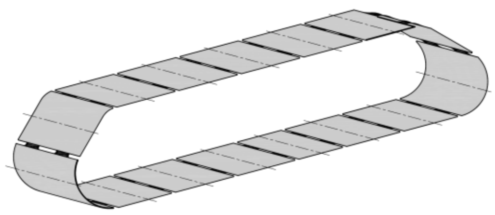
(—▲— Constrained joint model, —▼— Penalty method model, —■— Bushing element model, —●— ANCF model)



Right chain axial stresses at $t = 5$ s

Justification for Using ANCF

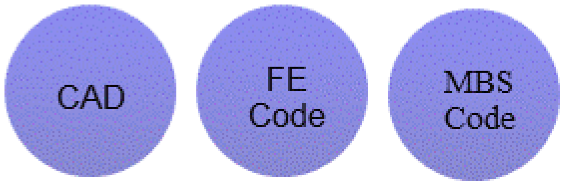
1. The FFR formulation will remain an effective method for solving small deformation problems in many applications.
2. In some small deformation problems (chains as examples) ANCF can be more efficient.
3. The joint algebraic equations can be eliminated at a preprocessing stage (640 nonlinear algebraic equations are eliminated).
4. ANCF allows for developing linear joints at arbitrary points.
5. FFR leads to highly nonlinear inertia, while ANCF leads to constant inertia.
6. FFR revolute joints do not allow for deformation at the joint definition points, while ANCF does.

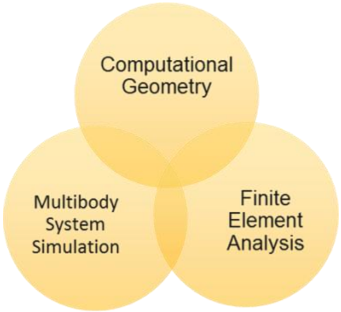


Linear joints can be defined at arbitrary points (not nodal points). The resulting algebraic equations can be eliminated at a preprocessing stage.

Summary

1. Integration of CG, FE, and MBS algorithms is necessary in order to have CAD and analysis models that are consistent.
2. This integration was difficult to achieve because of the fundamental differences between CG and most FE representations.
3. A simple interface between CG-based CAD systems and FE codes can be established (I-CAD-A).
4. The new MBS software technology Sigma/Sams is being developed (www.computational-dynamics.com).
5. Sigma/Sams is being designed to eliminate the need for using other commercial FE and CAD codes for flexible MBS simulations.





THANK YOU!