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Construction of Theoretical Model for Antiterrorism: From Reflexive Game Theory Viewpoint

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Name of Authors: * Vladimir Lefebvre, Ph.D.
Kofi Nyamekye, Ph.D.

Point of Contact: Kofi Nyamekye, Ph.D.

Name of Organization: Integrated Activity-Based Simulation Research, Inc.

Complete Address: 1919 South Grand Boulevard, Suite 412, Saint Louis, MO 63104-1573

Telephone: 314-705-1565

E-mail Address: kofinsoyameye@iabsri.net

* Senior Research Scientist in Cognitive Science, Socio-Cultural Systems and Decision Sciences at the School of Social Sciences, University of California in Irvine, CA.

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ABSTRACT

This work is devoted to the use of Reflexive Game Theory (RGT) for modeling the processes of decision making by terrorists. In the antiterrorist operations, an expert plays an extremely important role. With the help of RGT, experts can model terrorists' interactions and predict their decisions. In the RGT framework, a group of terrorists is represented as a graph with two types of sides: one - expressing confrontation, and the other - expressing collaboration. By decomposition of this graph, an expert can find each terrorist's self-images and construct a choice function which leads to writing equation for each member of the group of terrorists. Every solution of this equation is interpreted as a terrorist's possible choice. The authors then provide four examples to demonstrate how an expert can use RGT to model antiterrorist operations.

INTRODUCTION

As terrorism has become a serious threat to the modern world, the development of formal methods of modeling the terrorist activity, terrorist group structures and decision making, plays the crucial role in helping intelligence and antiterrorist forces.

The book *Computational Methods for Counterterrorism* (Argamon and Howard, Eds., 2009) emphasizes four main directions in using mathematical methods in the fight against terrorism. First - general methods of processing a wealth of information which is important for prevention or/and response to a terrorism act including retrieval strategy in searching a large repository of scanned documents. Second - text analysis for extracting useful information using categorization technique to discover sensitive unclassified information in documents. Third - use of algebraic and graphic methods – is the most significant in this book. It includes works on modeling individuals' and group's behavior in applying to terrorist activity. Lefebvre (2009) further develops his earlier introduced model of an agent facing a choice between the positive and negative poles. Koester and Schmidt (2009) start with an introduction to the fundamentals of Formal Concept Analysis and applied lattice theory to show how they are used to uncover hidden relationships in relational data. Grice and McDaniel (2009) report their findings in using Lefebvre's Algebraic Model of Self-Reflexion for modeling human judgments, and discuss the implications of their findings. And fourth - general problems in conflict analysis, such as classification of patterns of nation-state instability, discussion of adversarial planning in network, and structured method to aid intelligence and security analysts.

Several aspects of applying mathematics to counterterrorism are described in the book *Mathematical Methods in Counterterrorism* (Memon et al., Eds., 2009). Its editors state that counterterrorism efforts must detect and prevent potential future acts and also assist in the response to events that have already occurred and that mathematics can serve as a powerful tool to combat terrorism. Brantingham et al. (2009) offer a method of modeling criminal activity in urban areas. McGough (2009) discusses the strengths of different terrorist cell structures, using the partially ordered set model. Gutfraind (2009) analyzes terrorist organizations with a dynamic model which makes it possible to predict whether counterterrorism measures would be sufficient to defeat a

particular organization. Pinker (2009) developed a terror threat information model capturing the uncertainty in timing and location of terror attacks to create a mathematical framework for analyzing counterterrorism decision making. Ozgul et al. (2009) have developed two models for detection semi-supervised terrorist groups and tested them on nine groups. Shapiro and Siegel (2009) model a hierarchical terror organization in which leaders delegate financial and logistical tasks to middlemen who do not always share their leaders' interests.

Please note also Farley's (2003) work -- "Breaking Al Qaeda Sells" -- where the author uses order theory to qualify the degree to which a terrorist network is still able to function. This tool will help law enforcement know when a battle has been won. Kramer et al. (2003) describe two algorithms for generating disinformation schemes intended to influence an adversary, who attempt to penetrate an international border, to make a predetermined decision. These algorithms are implemented in a computer program. Goldstein (2006) discusses new ways of modeling terrorists that could help intelligence analysts think like an enemy. Lefebvre and Farley (2007) analyze the threat of terrorism to Western civilization and the threat of the fight against terrorism that may significantly change the nature of the society.

The book written by one of the authors (Lefebvre, 2010) thoroughly describes the Reflexive Game Theory. Its applications include predicting and influencing choices made by individual agents belonging to groups that have their own collective goals and interests.

The organization of this paper is as follows. We will first describe the use of the model constructed on the basis of a Boolean lattice for modeling the terrorists' decision-making processes. Then three examples will be given, to demonstrate how an expert can use RGT to model antiterrorist operations. Next, we will discuss an additional example which shows the relations of the terrorist group members, which in turn will help an expert to model antiterrorist operations. Conclusion will then follow.

1. MODELING TERRORISTS' ACTIVITY AND ITS SPECIFICS

A specific feature in the fight against terrorism is the role of the experts' experience in preparing antiterrorist operations. Only highly experienced experts can select significant factors from information received and correctly determine a degree of real threat in a given situation by comparing it with similar instances in the past. Modeling must help an expert to work. The expert is the one to prepare the information to insert into a model. We will show in this paper how modeling can be done with the help of RGT, which allows us to include the experts' experience into a model (Lefebvre, 2010; Nyamekye, 2013).

2. THE MAIN IDEAS OF THE REFLEXIVE GAME THEORY

At the basis of the reflexive game theory is the assumption that a purposeful agent (Nyamekye, 2013) is reflexive, i.e., the agent has images of the self, which have images of the self, etc. (Lefebvre, 2010). Please see APPENDIX A for detailed discussion of images of the self. There is a formal connection between the structure of a purposeful agent's images of the self and a graph representing relations in a group. The graph nodes correspond to purposeful agents, and its sides to relations of agreement or disagreement between them. Graphs may be either decomposable or non-decomposable. The non-decomposable graphs do not allow reconstruction of the structure of self-

images. We presume that in this case the purposeful agent's cognitive system simplifies the graph to make it decomposable.

To obtain the structure of self-images, a special polynomial corresponding to the decomposable graph is constructed and, from it, a so-called diagonal form is written. This form is a depiction of the structure of a purposeful agent's images of the self. Then, by using formal rules we write an equation of choice for each purposeful agent in a group. This equation links a purposeful agent's choice with influences from other group members. Each solution corresponds to a purposeful agent's possible decision, so, we have a prediction. The equation may have no solutions. This case is interpreted as impossibility (or difficulty) to make a choice under given conditions.

The following information is needed for constructing a diagonal form corresponding to a purposeful agent in a group:

1. A list of purposeful agents in the group.
2. Pair wise relations between the purposeful agents (cooperation or confrontation).
3. A set of actions from which a particular purposeful agent can make a choice.
4. Influences on this purposeful agent by other group members.
5. The order of other purposeful agents' importance for this purposeful agent.

The order of importance is used only when the relation graph of a group is not decomposable.

Consider a situation in which every group member can either do an act α, or refrain from it. In this case the set of choices consists of two elements: $1=\{\alpha\}$, $0=\{\ \}$. The first set, 1, consists of one element - α, and the second set, 0, is the empty set.

Imagine that, using a description of the situation we construct a diagonal form $\Phi = \Phi(a, b, c, ...)$. To find the purposeful agent's choice we have to write the equation for the agent a, where the values of other purposeful agents (b, c, ...) are known, and solve it:

$$a = \Phi(a, b, c, ...). \tag{1}$$

For the cases when the set of choices consists of two elements, 1 and 0, Boolean equation (1) can be reduced to one of the four forms:

$$\begin{aligned} &\text{I.} \quad a = a \\ &\text{II.} \quad a = 1 \\ &\text{III.} \quad a = 0 \\ &\text{IV.} \quad a = \overline{a} \end{aligned}$$

Equation I have two roots, 1 and 0. In this case, a group does not impose restriction of the purposeful agent's choice. Being in this state, the agent may choose either set $\{\alpha\}$ consisting of one action α, or the empty set $\{\ \}$, i.e., a refrain from acting.

Equation II has one root, 1. In this case, the group imposes the restrictions on the purposeful agent; in this state the agent can choose only $\{\alpha\}$.

Equation III has one root, 0. This means that in this state the purposeful agent makes decision to refrain from action.

Equation IV does not have roots. This means that the group influence on the purposeful agent is such that the agent cannot make any decision at all.

3. EXAMPLES

Example 1. A group consists of four terrorists - a, b, c and d. Each of them faces a choice: to participate in terrorist action α, or not to participate. Terrorist b is in cooperation (union) with a, c and d each, but a, c and d are in confrontational relations (conflict) with each other. Based on these data we construct the following graph:
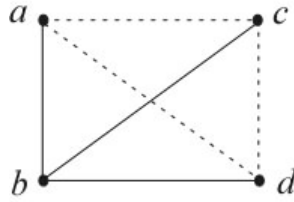

Fig. 1. Group-relations graph. Solid lines depict union, dotted lines – conflict.

This graph is not isomorphic to graph S_4, so, it is decomposable (see Lefebvre, 2010):
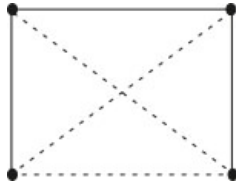

Fig. 2. Non-decomposable graph S_4.

The graph in Fig.1 can be presented as polynomial

$$b \bullet (a + c + d) \tag{2}$$

and the following diagonal form corresponds to it (see Appendix A):

$$
\begin{array}{c}
[a] + [c] + [d] \\
[b] \ \bullet \ [a + c + d] \\
[b \bullet (a + c + d)]
\end{array} \tag{3}
$$

where '$\bullet$' means union, and '$+$' means conflict.

The terrorists' influences on each other are given in Table 1. The numbers in columns show the influences of other terrorists to the one whose name is on the top of the column and the self-influence, which is the terrorist's intention to choose a set of actions. For example, terrorist a self-

influence is the value of variable a, unknown to us prior to computations. Terrorist b inclines a to participate in the act of terrorism, while c and d incline a to restrain from it.

Table 1

	a	b	c	d
a	a	0	0	1
b	1	b	1	0
c	0	0	c	0
d	0	0	0	d

Let us model the choice made by a. Equation (1) for a is as follows:

$$a = [b \bullet (a + c + d)]^{[b] \, \bullet \, [a + c + d]^{[a] + [c] + [d]}} . \tag{4}$$

In this case, the variables in the equation take on the values either 1, or 0; operation ' $\bullet$ ' is Boolean multiplication and operation '+' is Boolean addition. These operations are given by the following rules:

$$
\begin{array}{ll}
1 \bullet 1 = 1 & 1 + 1 = 1 \\
1 \bullet 0 = 0 & 1 + 0 = 1 \\
0 \bullet 1 = 0 & 0 + 1 = 1 \\
0 \bullet 0 = 0 & 0 + 0 = 0.
\end{array}
$$

Operation x^y is given by function

$$x^y = x + \overline{y} . \tag{5}$$

It is easy to see that

$$
\begin{array}{l}
1^1 = 1 \\
1^0 = 1 \\
0^1 = 0 \\
0^0 = 1.
\end{array}
$$

For simplicity, we will omit the multiplication sign ' $\bullet$ '. Now, we substitute the values from column a in Table 1 for variables in equation (4):

$$a = [1 \, (a + 0 + 0)]^{[1] \, [a + 0 + 0]^{[a] + [0] + [0]}} , \tag{6}$$

and, after computations obtain:

$$a = a. \tag{7}$$

The model shows that terrorist a is in the state I, in which an agent has the freedom of choice. So terrorist a can make either decision: to participate in the act of terrorism or not participate.

Consider terrorist b. The following equation corresponds to b:

$$b = [b\,(a + c + d)]^{[b]\,[a + c + d]^{[a] + [c] + [d]}} \quad . \tag{8}$$

After substituting values from column b in Table 1, we obtain

$$b = [b\,(0 + 0 + 0)]^{[b]\,[0 + 0 + 0]^{[0] + [0] + [0]}} \quad , \tag{9}$$

$$b = \overline{b} \; . \tag{10}$$

The last equation has no roots. This means that terrorist b is in the state IV, the state of frustration, in which b cannot make decision.

The equation for terrorist c is

$$c = [b\,(a + c + d)]^{[b]\,[a + c + d]^{[a] + [c] + [d]}}, \quad , \tag{11}$$

and after substitution values from column c in Table 1:

$$c = [1\,(0 + c + 0]^{[1]\,[0 + c + 0]^{[0] + [c] + [0]}} \quad , \tag{12}$$

$$c = c \; . \tag{13}$$

So, terrorist c, as terrorist a, is in the state I, in which the terrorist has the freedom of choice.

The terrorist d's equation is

$$d = [b\,(a + c + d)]^{[b]\,[a + c + d]^{[a] + [c] + [d]}} \quad . \tag{14}$$

Substituting the values from column d in Table 1, we obtain

$$d = [0\,(1 + 0 + d)]^{[0]\,[1 + 0 + d]^{[1] + [0] + [d]}} \quad , \tag{15}$$

$$d = 1 \; . \tag{16}$$

Terrorist d is in the state II and chooses to participate in the act of terrorism - α.

Example 2. Five terrorists are in the relations shown in the following graph:
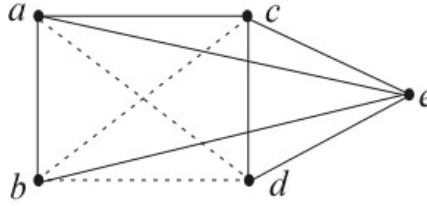

Fig. 3. Relation graph.

Terrorist *a* faces a choice: to commit a terrorist act or to restrain from it. "1" means to commit the act, "0" means to restrain from it. First, we have to find the diagonal form for *a*, but we see that the graph in Fig. 3 is not decomposable, because it contains subgraph <*a*, *b*, *c*, *d*> isomorphic to S_4 (similar to Fig. 2). This means that we need information concerning *a*'s order of importance of other terrorists to him. Let the order be like this: *c* is the most important, then *d*, then *e*, and *b* is the least important. While constructing the model we presumed that when a graph of relations is non-decomposable, the cognitive system of a purposeful agent, who is making a choice, removes the least important purposeful agents from considerations, one-by-one, until the graph becomes decomposable. In the given case, the first to be removed from the graph is node *b*. As a result, terrorist *a* corresponds to the following graph:
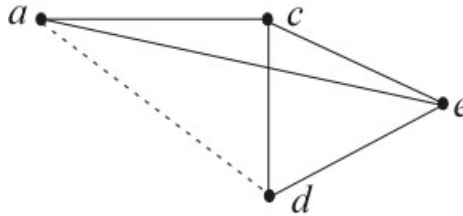

Fig. 4. Graph of relations after removal node *b*.

The graph in Fig. 4 is not isomorphic to S_4, therefore, it is decomposable, and terrorist *a*'s cognitive system does not need to remove nodes anymore. The graph corresponds to the polynomial

$$ce(a + d) \tag{17}$$

and the diagonal form

$$[ce\,(a+d)]_{[c][\,e]\,[a+d]}^{[a]\,+\,[d]}. \tag{18}$$

The equation for terrorist *a* is

$$a = [ce\,(a+d)]_{[c][e]\,[a+d]}^{[a]\,+\,[d]}. \tag{19}$$

If we know only that terrorist *d* inclines terrorist *a* to commit a terrorist act and that the influence of other terrorists is unknown, we substitute $d = 1$ into (19) and obtain equation

$$a = 1 . \tag{20}$$

Therefore, according to our model, the influence of *d* to *a* is enough that *a* would commit a terrorist act.

Example 3. The leader of a terrorist group, *a*, Fig. 5, has three aims: to blow a power station - α, to blow a government building - β, to blow a historic monument - γ. The leader cannot commit all three acts at the same time or any two acts if act α is included. But acts β and γ can be committed together. In this case the set of choices consists of eight subsets of set {α, β, γ}. Each subset is an alternative.

1. {α}   If the leader chooses this alternative, the decision is to blow the power station.
2. {β}   With this choice the leader decided to blow the government building.
3. {γ}   The leader decided to blow the historic monument.
4. {α, β}   If the leader chose this alternative, this means that one of the actions, α or β, can be realized, but not both of them at the same time.
5. {α, γ}   As in the case 4, the leader can realize only one of these two actions.
6. {β, γ}   In this case, the leader can realize two actions, β and γ, at the same time, or only action β, or only action γ.
7. {α, β, γ} The three actions, α, β, γ, cannot be realized at the same time; two actions, α and β or α and γ, cannot be realized at the same time either. But actions β and γ can be realized at the same time, or any single actions - α, β or γ.
8. { }   A set of actions is empty. The leader refused to commit a terrorist act.

Let terrorists *b* and *c* support the leader's opinions (are in union with *a*), and terrorist *d* be in conflict with all others – *a*, *b,* and *c*. The following graph describes this situation:
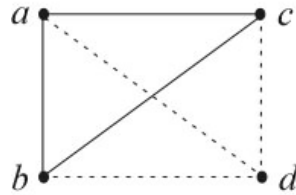

Fig. 5. Terrorist *d* is in conflict with the other three terrorists who are in union with each other.

The graph in Fig. 5 is decomposable. The equation for the leader *a* is as follows:

$$a = [d + abc] \genfrac{}{}{0pt}{}{[a][b][c]}{[d]+[abc]} . \tag{21}$$

Suppose that terrorist *d* inclines the leader to restrain from the act of terrorism:

$$d = 0, \tag{22}$$

terrorist *b* advises the leader to only blow the government building, and terrorist *c* advises the leader to only blow the monument:

$$b = \{\beta\},$$
$$c = \{\gamma\}. \tag{23}$$

By substituting these data into (21), we obtain:

$$a = [0 + a \{\beta\} \{\gamma\}] \begin{matrix} [a]\,[\{\beta\}]\,[\{\gamma\}] \\ [0] + [a \{\beta\} \{\gamma\}] \end{matrix} . \tag{24}$$

The product of sets $\{\beta\}$ and $\{\gamma\}$ is equal to their intersection, and it is empty, thus

$$\{\beta\}\{\gamma\} = 0. \tag{25}$$

And after transformation we find that

$$a = 0. \tag{26}$$

Therefore, the model predicts that with the given conditions the group leader will not perform terrorist actions.

4. DISCUSSION

We have demonstrated that under indicated assumption, RGT allows us to model decisions made by members of a group of terrorists. Such a model opens the opportunity to find effective informational influence on the terrorists (see 'Reflexive control' in Lefebvre, 1977).

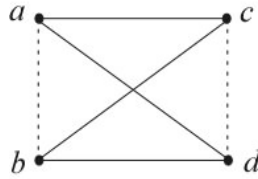Consider an example. The following graph shows relations of terrorist-group members:

Fig. 6. Pairs *a-b* and *c-d* are in conflict, other pairs are in union.

The leader *a* has to make a decision: either to commit a terrorist act or to refuse to commit a terrorist act. The three remaining members of the group influence the leader. Terrorists *b* and *c* support the idea to strike ($b=1$, $c=1$); terrorist *d* inclines the leader to refuse. Let us find the model of the leader. The graph of relations in Fig. 6 corresponds to the polynomial

$$(a + b)(c + d), \tag{27}$$

and the equation for the leader is

$$a = [(a + b)(c + d)] \begin{matrix} [a] + [b] \quad\quad [c] + [d] \\ [a + b)] \quad\quad [c + d] \end{matrix} . \tag{28}$$

With the given values of *b*, *c* and *d*, *a*=1, that is, the leader of this terrorist group will make the decision to act.

In order to make the leader to change the decision, we have to use reflexive control, that is, we have to supply the leader with specially prepared information which will serve as a basis for the decision to refrain from a strike. What kind of information should it be? Since the value of *a*=1 follows from the given values of the leader's accomplices, *b*, *c*, and *d*, the information that we have to send must persuade the leader that *c* is against the strike (*c*=0). With this value of *c*, the root of equation (28) is *a*=0 (not to strike).

This example demonstrates that RGT makes it possible to elaborate the reflexive control over a group of potential terrorists.

What is more important, this example also shows that even though a group of insurgents may have a common intent (an overall shared goal) to inflict harm to some innocent civilian population in a society, if some members, within the group of insurgents, believe that causing such harm could backfire the insurgents strategic vision, e.g., seeking sound political and an economic situation for the overall society for whom they are fighting, it is quite conceivable that some moderate members in the insurgents could potentially persuade other hardliners in the group to stop attacking the innocent civilian population. Thus, from this example we can assert that in any complex endeavor, "***common intent may be ideal but compatible intents or reconcilable intents, vision etc. are more realistic – managing the possible sources of friction and deconflicting actions among the entities in the complex endeavor, are the key to a successful operation!***" Reflexive control – from RGT --- can provide a powerful insight into modeling such a complex endeavor.

5. CONCLUSION

Will RGT be efficient in practice? There could be two types of efficiency. The first type - the plausibility of predictions. The second type - the better understanding of a mechanism of decision making processes in terrorist groups.

The plausibility of predictions can be found as a result of a broad empirical analysis of the descriptions of the real terrorist acts, which will allow us to compare the model predictions with the real terrorists' behavior.

Concerning the second type, our experience in analyzing international relations (Lefebvre, 2010) tells us that using RGT allows us to see many hidden features in the processes of decision making, those that are not readily available in using other methods.

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APPENDIX A

DETAILED DISCUSSION OF IMAGES OF THE SELF (SELF-IMAGES)

Let us clarify the meaning of a diagonal form. Consider the form

$$
\begin{array}{c}
[b] + [c] \\
[a]\ [b + c] \\
[a(b + c)]
\end{array}
$$

.

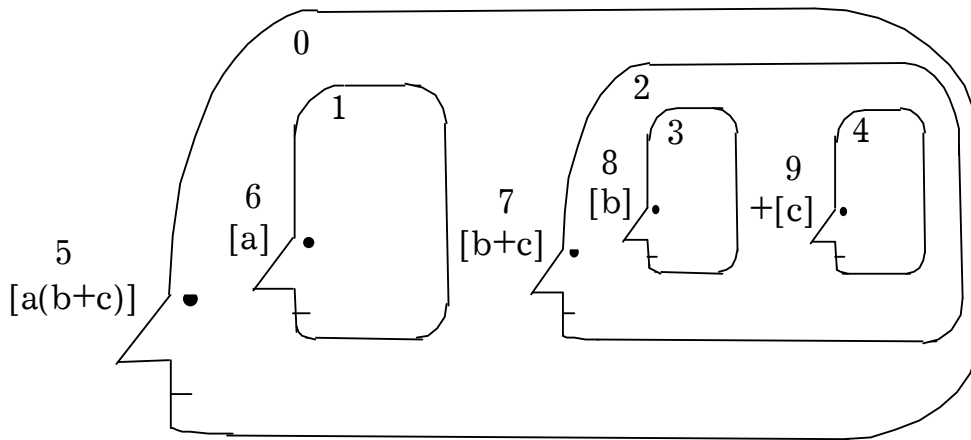It corresponds to the following cartoon:

Fig. A1. Reflexive structure of a purposeful agent.

This cartoon depicts the purposeful agent (0) with two mental images of the self (1, 2), which are in the purposeful agent head. These mental images can be considered the same way as we consider purposeful agents. In the head of image 2, there are two images of the self (3, 4).

How do we construct this picture? The purposeful agent (0) perceives the group structure; we write it as a polynomial (5). It is the lower polynomial for the purposeful agent (0). After receiving the polynomial (5), the purposeful agent's cognitive system decomposes it into two polynomials on multiplication (6, 7) and generates two images of the self (1, 2) corresponding to these polynomials. The image of the self (1) perceives polynomial (6). The image of the self (2) perceives polynomial (7), decomposes it into polynomials (8, 9) on addition and generates the own mental images (3, 4). The polynomials cannot be decomposed further, so, the construction of the picture ends. Similar explanations can be provided for constructing each diagonal form.

AUTHOR BIOGRAPHIES

DR. VLADIMIR LEFEBVRE is a senior research scientist in reflexive approach toward modeling the processes of thinking and decision making, at School of Social Sciences, University of California in Irvine, CA. Fifty years ago he introduced the concept of reflexive control and his current research work includes the reflexive control theory for influencing the insurgents' decision-making process to select particular course of actions that benefit the objectives of the Warfighters. Dr. Lefebvre has extensively published many referred journals on the notions of reflexive system, reflexive control, and reflexive game theory, all of which are now broadly used in scientific analysis of socio-cultural systems. He has received many distinguished awards in cognitive studies, mathematical psychology, reflexive approach toward modeling the processes of thinking and decision making. He holds Doctor of Philosophy degree in psychology, from Lomonosov State University, Moscow, USSR, and Master of Science degree in mathematics, from Lomonosov State University, Moscow, USSR. Email: valefebv@uci.edu

DR. KOFI NYAMEKYE is the president and chief executive officer of Integrated Activity-Based Simulation Research, Inc. Dr. Nyamekye has extensive prior experience as a senior research scientist in modeling and simulation of complex adaptive distributed enterprise systems for Boeing's Army Future Combat Systems (FCS). In collaboration with Dr. Vladimir Lefebvre, Dr. Nyamekye is currently using Reflexive Control theory to model Cybersecurity risks and more importantly construct a purposeful agent-based system (PABS) to mitigate Cybersecurity risks, in a Net-Centric Ecosystem (NCE). Using Experimental Laboratory for Investigating Collaboration, Information-sharing, and Trust (ELICIT) platform, he will then conduct experimental tests for information sharing and collaboration among the entities, for mitigating Cybersecurity risks, in a NCE. Dr. Nyamekye has extensively published many refereed journals on scientific design, multi-purposeful agent modeling, and simulation of integrated and adaptive C4ISR SoS. He holds a Doctor of Philosophy degree in industrial and management systems engineering from Pennsylvania State University, a Master of Science degree in mechanical engineering from Pennsylvania State University, and a Bachelor of Science degree in mechanical engineering from the University of Wisconsin-Madison. E-mail: kofinsoyameye@iabsri.net

Construction of Theoretical Model for Antiterrorism:
From Reflexive Game Theory Viewpoint

Vladimir Lefebvre, PhD

University of California, Irvine

Kofi Nyamekye, PhD

Integrated Activity-Based Simulation Research, Inc.

ABSTRACT

This presentation is devoted to the use of Reflexive Game Theory (RGT) for modeling the processes of decision making by terrorists. In the RGT framework, a group of terrorists is represented as a graph with two types of sides: one - expressing confrontation, and the other - expressing collaboration. By decomposition of this graph, an expert can find each terrorist's self-images and construct a choice function which leads to writing equation for each member of the group of terrorists. Every solution of this equation is interpreted as a terrorist's possible choice. The authors then provide examples to demonstrate how an expert can use RGT to model antiterrorist operations.

Modeling Terrorists' Activity and Its Specific

A specific feature in the fight against terrorism is the role of the experts' experience in preparing antiterrorist operations. Only highly experienced experts can select significant factors from information received and correctly determine a degree of real threat in a given situation by comparing it with similar instances in the past. Modeling will help an expert to work.

The expert is the one who prepares the information to insert into a model. We will show in this presentation how modeling can be done with the help of RGT, which allows us to include the experts' experience into a model (Lefebvre, 2010; Nyamekye, 2013).

The Main Ideas of the Reflexive Game Theory

At the basis of the reflexive game theory is the assumption that a purposeful agent (Nyamekye, 2013) is reflexive, i.e., the agent has images of the self, which have images of the self, etc. (Lefebvre, 2010).

Reflexive Structure of a Purposeful Agent
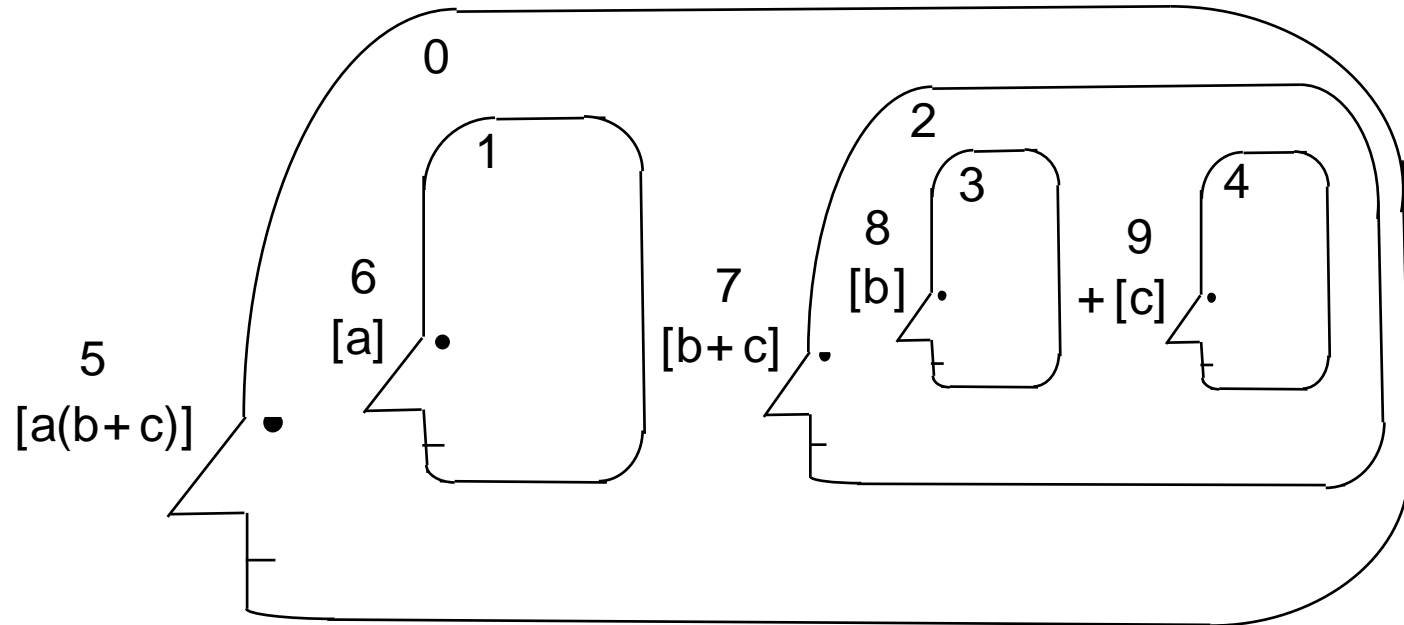
The purposeful agent (0) has two mental images of the self (1,2). In the head of image 2, there are two images of the self (3, 4). The agent (0) perceives the group structure: polynomial (5). The image of the self (1) perceives polynomial (6). The image of the self (2) perceives polynomial (7).

Connection between the Structure of an Agent's Images and a Graph of Relation

There is a formal connection between the structure of a purposeful agent's images of the self and a graph representing relations in a group. The graph nodes correspond to purposeful agents, and its sides to relations of agreement or disagreement between them. Graphs may be either decomposable or non-decomposable. The non-decomposable graphs do not allow reconstruction of the structure of self-images. We presume that in this case the purposeful agent's cognitive system simplifies the graph to make it decomposable.

Construction of a Diagonal Form

To construct a diagonal form corresponding to a purposeful agent in a group the following information is needed:

1. A list of purposeful agents in the group.
2. Pair wise relations between the purposeful agents (cooperation or confrontation).
3. A set of actions from which a particular purposeful agent can make a choice.
4. Influences on this purposeful agent by other group members.
5. The order of other purposeful agents' importance for this purposeful agent.

Simple Situation

Consider a situation in which every group member can either do an act α, or refrain from it. In this case the set of choices consists of two elements: 1={α}, 0={ }. The first set, 1, consists of one element - α, and the second set, 0, is the empty set. Imagine that, using a description of the situation we construct a diagonal form

$$\Phi = \Phi(a, b, c, ...).$$

To find the purposeful agent's choice we have to write the equation for the agent a, where the values of other purposeful agents ($b, c, ...$) are known, and solve it:

$$a = \Phi(a, b, c, ...). \tag{1}$$

Choices

For the cases when the set of choices consists of two elements, 1 and 0, Boolean equation (1) is reduced to one of the four forms:

I. $a = a$. This equation has two roots, 1 and 0. The agent may choose either set $\{\alpha\}$ consisting of one action α, or the empty set $\{ \}$, i.e., refrain from acting.

II. $a = 1$. This equation has one root, 1. The purposeful agent in this state chooses only $\{\alpha\}$.

III. $a = 0$. This equation has one root, 0. This means that in this state the purposeful agent makes decision to refrain from action.

IV. $a = \bar{a}$. This equation does not have roots. This means that the group influence on the purposeful agent is such that the agent cannot make any decision at all.

Example 1

A group consists of four terrorists - *a, b, c* and *d*. Each of them faces a choice: to participate in terrorist action α, or not to participate. Terrorist *b* is in cooperation (union) with *a, c* and *d* each, but *a, c* and *d* are in confrontational relations (conflict) with each other:
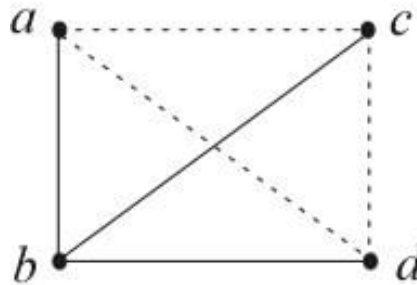
Group relations graph.
Solid lines depict union, dotted lines – conflict.

Example 1 (continue)
A polynomial and its diagonal form

The graph in slide 10 is presented as polynomial

$$b \bullet (a + c + d),$$

where $\bullet$ means union, and + means conflict.

    The following diagonal form corresponds to the above polynomial:

$$[a] + [c] + [d]$$

$$[b] \bullet [a + c + d]$$

$$[b \bullet (a + c + d)] \qquad\qquad\qquad .$$

The terrorists' influences on each other

In the table below, the numbers in columns show the influences of other terrorists to the one whose name is on the top of the column and the self-influence, which is the terrorist's intention to choose a set of actions. Terrorist b inclines a to participate in the act of terrorism (1), while c and d incline a to restrain from it (0). Terrorist's a self-influence is the value of variable a, unknown to us prior to computations

	a	b	c	d
a	a	0	0	1
b	1	b	1	0
c	0	0	c	0
d	0	0	0	d

Modeling the choice made by terrorist *a*

$$[a] + [0] + [0]$$
$$[1] \bullet [a + 0 + 0]$$
$$a = [1 \bullet (a + 0 + 0)]$$
.

Operation '$\bullet$ ' is Boolean multiplication and operation '+' is Boolean addition. Operation x^y is given by function

$$x^y = x + \overline{y}.$$

After computations we obtain: $a = a$.
This means that terrorist *a* has the freedom of choice and can make either decision: to participate in the act of terrorism or not to participate. Thus, *a*'s choice is unpredictable.
The choices of terrorists *b, c*, and *d* can be found similarly.

Example 3

The leader of a terrorist group, *a*, has three aims: to blow a power station - α, to blow a government building - β, to blow a historic monument - γ. The leader cannot commit all three acts at the same time or any two acts if act α is included. But acts β and γ can be committed together.

There are three more members in the group: terrorists *b* and *c* support the leader's opinions (are in union with *a*), and terrorist *d* is in conflict with all others – *a, b,* and *c*:
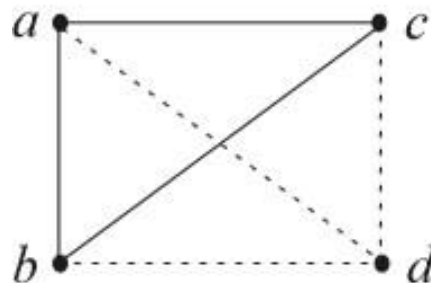
Alternatives

The set of choices consists of eight subsets of set $\{\alpha, \beta, \gamma\}$. Each subset is an alternative.

1. $\{\alpha\}$ This decision means to blow the power station.

2. $\{\beta\}$ This choice means blowing the government building.

3. $\{\gamma\}$ This decision means to blow the historic monument.

4. $\{\alpha, \beta\}$ This alternative means that one of the actions, α or β, can be realized, but not both of them at the same time.

5. $\{\alpha, \gamma\}$ Only one of these two actions can be realized.

6. $\{\beta, \gamma\}$ In this case, the leader can realize two actions, β and γ, at the same time, or only action β, or only action γ.

7. $\{\alpha, \beta, \gamma\}$ The three actions, α, β, γ, cannot be realized at the same time; two actions, α and β or α and γ, cannot be realized at the same time either. But actions β and γ can be realized at the same time, or any single action - α, β or γ.

8. $\{\ \}$ A set of actions is empty. The leader refused to commit a terrorist act.

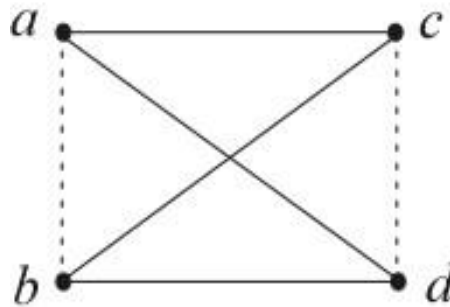# The leader's decision

The equation for the leader *a* is

$$[a] \ [b] \ [c]$$
$$[d] + [abc]$$
$$a = [d + abc] \qquad \qquad .$$

Terrorist *d* inclines the leader to restrain from the act of terrorism: *d* = 0; terrorist *b* advises the leader to blow the government building: *b* = {β}, and terrorist *c* advises the leader to blow the monument: *c* = {γ}.
   After substitutions and computations, we obtain *a* = 0. Therefore, the model predicts that with the given conditions the group leader will not perform terrorist actions.

Discussion

The terrorist group members are in the following relations:

The leader *a* has to make a decision: either to commit a terrorist act or to refuse to commit a terrorist act. The three remaining members of the group influence the leader. Terrorists *b* and *c* support the idea to strike (*b=1, c=1*); terrorist *d* inclines the leader to refuse. Let us find the model of the leader. The graph of relations corresponds to the polynomial

$$(a + b)(c + d).$$

Discussion (continue)

The equation for the leader is

$$[a] + [b] \qquad [c] + [d]$$
$$[a + b] \qquad [c + d]$$
$$a = [(a + b)(c + d)]$$

Since $b=1$, $c=1$ and $d=0$ the value of $a=1$, that is, the leader of this terrorist group will make the decision to act.

How to make the leader to change the decision?

Reflexive control

In order to make the leader to change the decision, we have to use reflexive control, that is, we have to supply the leader with specially prepared information which will serve as a basis for the decision to refrain from a strike. What kind of information should it be? Since the value of $a=1$ follows from the given values of the leader's accomplices b, c, and d, the information that we have to send must persuade the leader that c is against the strike ($c=0$). With this value of c, the root of equation is $a=0$ (not to strike).

This example demonstrates that RGT makes it possible to elaborate the reflexive control over a group of potential terrorists.

CONCLUSION

We have demonstrated that under indicated assumption, RGT allows us to model decisions made by members of a group of terrorists. How efficient it will be in practice? There could be two types of efficiency. The first type - the plausibility of predictions. The second type - the better understanding of a mechanism of decision making processes in terrorist groups.

The plausibility of predictions can be found as a result of a broad empirical analysis of the descriptions of the real terrorist acts, which will allow us to compare the model predictions with the real terrorists' behavior.

Concerning the second type, our experience in analyzing international relations (Lefebvre, 2010) tells us that using RGT allows us to see many hidden features in the processes of decision making, those that are not readily available in using other methods.