REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

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1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
19-12-2013	Final Report	5-Sep-2009 - 4-Sep-2013

4. TITLE AND SUBTITLE
Final Report - Engineered Laser Filaments in air for defense stand-off sensing and interaction applications

5a. CONTRACT NUMBER
W911NF-09-1-0500

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
611102

6. AUTHORS
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5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES
University of Central Florida
Off of Res & Commercialization
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Orlando, FL 32826 -3246

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSOR/MONITOR'S ACRONYM(S)
ARO

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
56712-PH.17

12. DISTRIBUTION AVAILIBILITY STATEMENT
Approved for Public Release; Distribution Unlimited

13. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT
Previous research into the generation of non-diffracting beams was extended into the filamentation regime. Helical beams, previously developed as a proof of concept demonstrating the use of Bessel beam superposition to create complex beam arrays, were used in the formation of filaments. The resulting filaments mimicked the double helix structure of the helical beams used in their formation, demonstrated that non-diffracting beams can be used to control the formation of complex filament arrays. Research into non-diffracting beams has also been extended into other geometries, and the conditions for the formation of arbitrarily large rectangular arrays of non-diffracting

15. SUBJECT TERMS
Laser filaments, stand-off sensing

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Martin Richardson
UU	UU	UU	UU		19b. TELEPHONE NUMBER 407-823-6819

Report Title

Final Report - Engineered Laser Filaments in air for defense stand-off sensing and interaction applications

ABSTRACT

Previous research into the generation of non-diffracting beams was extended into the filamentation regime. Helical beams, previously developed as a proof of concept demonstrating the use of Bessel beam superposition to create complex beam arrays, were used in the formation of filaments. The resulting filaments mimicked the double helix structure of the helical beams used in their formation, demonstrated that non-diffracting beams can be used to control the formation of complex filament arrays. Research into non-diffracting beams has also been extended into other geometries, and the conditions for the formation of arbitrarily large rectangular arrays of non-diffracting beams have been established and tested using numerical simulation. Future research will involve generating these rectangular arrays in the laboratory, first as non-diffracting beams and then as filaments, as well as increased the quality and propagation range of helical filaments.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
12/11/2013	2.00	Matthew Weidman, Khan Lim, Mark Ramme, Magali Durand, Matthieu Baudelet, Martin Richardson. Stand-off filament-induced ablation of gallium arsenide, Applied Physics Letters, (07 2012): 0. doi: 10.1063/1.4734497
12/11/2013	3.00	Magali Durand, Khan Lim, Vytautas Jukna, Erik McKee, Matthieu Baudelet, Aurélien Houard, Martin Richardson, André Mysyrowicz, Arnaud Couairon. Blueshifted continuum peaks from filamentation in the anomalous dispersion regime, Physical Review A, (04 2013): 438201. doi: 10.1103/PhysRevA.87.043820
TOTAL:		**2**

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received		Paper
12/11/2013	1.00	Nicholas Barbieri, Matthew Weidman, Gregory Katona, Matthieu Baudelet, Zachary Roth, Eric Johnson, Georgios Siviloglou, Demetrios Christodoulides, Martin Richardson. Double Helical Laser Beams Based on Interfering First-Order Bessel Beams, Journal of the Optical Society of America A, (06 2011): 1462. doi:
TOTAL:		**1**

Number of Papers published in non peer-reviewed journals:

(c) Presentations

[1] Nicholas Barbieri, Matthew Weidman, Khan Lim, Matthew Fisher, Georgios Siviiglou, Zahra Hosseinimakerem, Zachary Roth, Eric Johnson, Demetrious Christodoulides, Matthieu Baudelet, and Martin Richardson, "Helical and Ionizing Helical Beams ", ARO MURI "Light Filamentation Science" annual workshop (10/05/2012).
[2] N. Barbieri, M. Weidman, M. Baudelet, Z. Roth, E. Johnson, G. Siviloglou, D. Christodoulides, and M. Richardson, "Helical plasma filaments", COFIL 2012, Tucson, AZ, USA, October 8, 2012. Poster presentation.
[3] N. Barbieri, M. Weidman, K. Lim, M. Baudelet, R. Bernath and M. Richardson, “RF emissions from filament-matter interaction”, COFIL 2012, Tucson, AZ, USA, 2012.
[4] M. Durand, A. Jarnac, S. Grabielle, A. Houard, A. Durecu, O. Vasseur, N. Forget, A. Couairon, and A. Mysyrowicz. “Filaments in the anomalous dispersion regime of fused silica”, COFIL 2012, Tucson, AZ USA, October 7-12, 2012.
[5] M. Durand, A. Jarnac, A. Houard, Y. Liu, B. Prade, M. Richardson, A. Mysyrowicz, "Optical phase conjugation with filaments", ARO MURI "Light Filamentation Science" annual workshop (10/05/2012).
[6] K. Lim, B. Bousquet, M. Weidman, M. Baudelet, and M. C. Richardson, "Broadband terahertz detection with laser-induced air plasma in counter-propagating scheme", COFIL 2012, Tucson, AZ, USA. Poster presentation, October 11, 2012.
[7] Khan Lim, Magali Durand, Matthew Weidman, Matthieu Baudelet, Martin Richardson, Bruno Bousquet, Xuan Sun, Fabrizio Buccheri, Xi-Cheng Zhang, "Broadband THz-REEF detection in counter-propagating scheme", ARO MURI "Light Filamentation Science" annual workshop (10/05/2012).
[8] Khan Lim, Erik McKee, Magali Durand, Matthieu Baudelet, Martin Richardson, Arnaud Couairon, "Supercontinuum from normal to anomalous dispersion", ARO MURI "Light Filamentation Science" annual workshop (10/05/2012).
[9] K. Lim, M. Durand, R. S. Shankar, M. Baudelet, T. Seideman, M. Richardson, “Molecular Studies of filamentation in carbon dioxide”, COFIL 2012, Tuscon, AZ, USA, 2012.
[10] M. Richardson, M. Durand, M. Baudelet, M. Weidman, N. Barbieri, R. Bernath, K. Lim, “Long range projection of high energy densities with laser filamentation and selected interaction studies”, DEPS: 2012, Albuquerque, NM, USA, November 27-29, 2012.
[11] M. Richardson, et al. “Recent studies of air filamentation’, Fundamentals and Applications of Laser Filaments, Institute for Molecular Science, Okazaki, Japan, April 4-6, 2013.
[12] Martin Richardson, Robert Bernath, Matthew Weidmann, Nicholas Barbieri, Khan Lim, Magali Durand and Matthieu Baudelet, "Remote Plasmas produced by Laser Filaments", ICON/LAT 2013, Section on Ultrafast diagnostics in laser research, Moscow, Russia, June 16-22, 2013.
[13] M. Richardson, J. Diels, A. Aceves, L. Arissian, M. Baudelet, E. Johnson, Z. Chang, N. Litchinitser, T. Seideman, X. Zhang, and R. Hammond, "The ARO MURI Program on Air Filamentation Science", COFIL 2012, Tucson, AZ, USA. Invited presentation (08 October 2012) INVITED.
[14] Martin Richardson, Jean-Claude Diels, Alejandro Aceves, Ladan Arissian, Matthieu Baudelet, Eric Johnson, Zenghu Chang, Natalia Litchinitser, Tamar Seideman, Xie-Cheng Zhang, Richard Hammond, "The ARO MURI Program on Air Filamentation Science After One Year", 10th annual ultrashort pulse laser workshop, Directed Energy Professional Society; Broomfield, CO, USA; 06/12, 2012 INVITED.
[15] Matthew Weidman, Khan Lim, Nicholas Barbieri, Erik McKee, Magali Durand, Matthieu Baudelet, Martin Richardson, "Quantitative studies of filament interaction with matter for spectroscopic applications", Seminar at GAP Biophotonics group, University of Geneva; Geneva, Switzerland; 11/30, 2012.

Number of Presentations: 15.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper	
12/16/2013	11.00	Magali Durand, Khan Lim, Vytautas Jukna, Erik McKee, Matthieu Baudelet, Aur´elien Houard, Martin Richardson, Andr´e Mysyrowicz, Arnaud Couairon. Influence of the anomalous dispersion on the supercontinuum generation by femtosecond laser filamentation, CLEO:2013. 30-JUN-13, . : ,

TOTAL: **1**

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received	Paper
12/16/2013 6.00	Khan Lim, Magali Durand, Xuan Sun, Fabrizio Buccheri, Matthew Weidman, Bruno Bousquet, Matthieu Baudelet, Xi-Cheng Zhang, Martin Richardson. Broadband THz detection in the counter-propagating configuration using THz-enhanced plasma fluorescence, Physical Review Letters (submitted) (03 2013)
12/19/2013 15.00	Nicholas Barbieri, Zahra Hosseinimakarem, Matthew Weidman, Khan Lim, Magali Durand, Matthieu Baudelet, Eric Johnson, Martin Richardson. Helical Filaments, Physical Review Letters (06 2013)
12/19/2013 16.00	Magali Durand, Aurélien Houard, Khan Lim, Anne Durécu, Olivier Vasseur, Martin Richardson. Study of filamentation threshold in Zinc Selenide, Optics Letters (12 2013)
12/19/2013 14.00	A. Jarnac, M. Durand, A. Houard, Y. Liu, B. Prade, M. Richardson, A. Mysyrowicz. Spatiotemporal cleaning of a femtosecond laser pulse through interaction with contrapropagating filaments in air, Physical Review Letters (06 2013)
12/19/2013 13.00	Khan Lim, Magali Durand, Xuan Sun, Fabrizio Buccheri, Matthew Weidman, Bruno Bousquet, Matthieu Baudelet, Xi-Cheng Zhang, Martin Richardson. Broadband THz detection in the counter-propagating configuration using THz-enhanced plasma fluorescence, Physical Review Letters (03 2013)
TOTAL:	**5**

Number of Manuscripts:

Books

Received Paper

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Andrew Housman	0.77	
Cheonha Jeon	0.94	
Yuan Liu	0.73	
Benjamin Webb	0.96	
Matthew Weidman	0.42	
FTE Equivalent:	**3.82**	
Total Number:	**5**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
Nicholas Barbieri	0.12
Matthew Weidman	0.24
FTE Equivalent:	**0.36**
Total Number:	**2**

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Martin Richardson	0.02	
FTE Equivalent:	**0.02**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period:	0.00
The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:.....	0.00
The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:.....	0.00
Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):.....	0.00
Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:.....	0.00
The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense	0.00
The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:	0.00

Names of Personnel receiving masters degrees

NAME	
Andrew Housman	
Total Number:	**1**

Names of personnel receiving PHDs

NAME	
Yuan Liu	
Matthew Weidman	
Nicholas Barbieri	
Total Number:	**3**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

1 a. Clemson University Research Foundation

1 b. Office of Sponsored Programs
Clemson University Research Founc
Clemson SC 296310946

Sub Contractor Numbers (c): 238975
Patent Clause Number (d-1):
Patent Date (d-2):
Work Description (e):
Sub Contract Award Date (f-1): 8/15/11 12:00AM
Sub Contract Est Completion Date(f-2): 9/4/13 12:00AM

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1 a. Clemson University Research Foundation

1 b. 91 TECHNOLOGY DR

ANDERSON SC 296310946

Sub Contractor Numbers (c): 238975
Patent Clause Number (d-1):
Patent Date (d-2):
Work Description (e):
Sub Contract Award Date (f-1): 8/15/11 12:00AM
Sub Contract Est Completion Date(f-2): 9/4/13 12:00AM

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1 a. Cornell University

1 b. Office of Sponsored Programs
373 Pine Tree Road
Ithaca NY 148502820

Sub Contractor Numbers (c): 186695
Patent Clause Number (d-1):
Patent Date (d-2):
Work Description (e):
Sub Contract Award Date (f-1): 5/5/09 12:00AM
Sub Contract Est Completion Date(f-2): 3/4/12 12:00AM

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1 a. Cornell University

1 b. Office of Sponsored Programs
373 Pine Tree Road
Ithaca NY 148502820

Sub Contractor Numbers (c): 186695
Patent Clause Number (d-1):
Patent Date (d-2):
Work Description (e):
Sub Contract Award Date (f-1): 5/5/09 12:00AM
Sub Contract Est Completion Date(f-2): 3/4/12 12:00AM

1 a. University of North Carolina - Charlotte

1 b. Engineering Technology
9201 University City Boulevard
Charlotte NC 282230001

Sub Contractor Numbers (c): 186582
Patent Clause Number (d-1):
Patent Date (d-2):
Work Description (e):
Sub Contract Award Date (f-1): 9/5/09 12:00AM
Sub Contract Est Completion Date(f-2): 8/14/11 12:00AM

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1 a. University of North Carolina - Charlotte

1 b. 9201 University City Boulevard
Academic Grants and Contracts
Charlotte NC 282230001

Sub Contractor Numbers (c): 186582
Patent Clause Number (d-1):
Patent Date (d-2):
Work Description (e):
Sub Contract Award Date (f-1): 9/5/09 12:00AM
Sub Contract Est Completion Date(f-2): 8/14/11 12:00AM

Inventions (DD882)

Scientific Progress

Technology Transfer

Townes Laser Institute
CREOL, the College of Optics & Photonics

Contract Title:	**Engineered Laser Filaments in air for defense stand-off sensing and interaction applications**
Contract Number:	**ARO W911NF0910500**
Reporting period:	Progress Report 8-30-2012 to 8-31-2013
Program Manager:	Richard Hammond
Program Office:	P.O. Box 12211. Research Triangle Park, **NC** 27709- 2211.
Principle Investigator:	Martin Richardson Director, Laser Plasma Laboratory and Townes Laser Institute College of Optics & Photonics, University of Central Florida 4304 Scorpius Street, Orlando 32618-2700 Tel: 407 823 6819 Email:mcr@creol.ucf.edu
Report prepared by	Dr. Nicholas Barbieri Laser Plasma Laboratory , Townes Laser Institute, University of Central Florida Phone: +1-407-823-6910, email:barbieri@creol.ucf.edu
UCF Contract Officer	Ms Arlisia Potter Tel: 407 882 2018, Email: apotter@ucf.edu
UCF Contract number	65016195
Performing Organization	University of Central Florida College of Optics & Photonics 4000 Central Florida Blvd CREOL Bldg 53 Orlando, FL 32826

Abstract – Continued Development of Non-Diffracting Beams for Filament Engineering

Previous research into the generation of non-diffracting beams was extended into the filamentation regime. Helical beams, previously developed as a proof of concept demonstrating the use of Bessel beam superposition to create complex beam arrays, were used in the formation of filaments. The resulting filaments mimicked the double helix structure of the helical beams used in their formation, demonstrated that non-diffracting beams can be used to control the formation of complex filament arrays. Research into non-diffracting beams has also been extended into other geometries, and the conditions for the formation of arbitrarily large rectangular arrays of non-diffracting beams have been established and tested using numerical simulation. Future research will involve generating these rectangular arrays in the laboratory, first as non-diffracting beams and then as filaments, as well as increased the quality and propagation range of helical filaments.

ENGINEERED LASER FILAMENTS IN AIR FOR DEFENSE STAND-OFF SENSING

Previous attempts at generating helical filaments were unsuccessful because the phase plates necessary for helical beam generation could not withstand the laser pulse energy required for filamentation. To address this issue, Dr. Johnson's group fabricated a single 10 mm coaxial vortex plate to replace the pair of 5 mm vortex plates used in previous experiments. The new phase plate element applied a unit azimuthal phase shift at 800 nm for radial values of less than 2.5 mm and the opposite azimuthal phase shift for radial values greater than 2.5 mm. The increased diameter of the phase plate increases the energy throughput of the phase plates by a factor of four, enabling helical filament experiments to be carried out at higher pulse energies.

With the new 10 mm diameter vortex plate, the helical beam experiments were repeated. A telescope composed of a diverging-converging lens pair was used to expand the MTFL laser beam to a diameter of 1 inch. The vortex plate was placed inside the telescope to enlarge the vortex phase pattern to a diameter of 1 inch. This telescope was placed in series with the double angle axicon and the conventional 179.4 angle axicon. Pulses from MTFL were used in conjunction with experimental setup shown in Figure 1 to generate both 133 uJ and 11.9 mJ helical pulses. The 133 uJ pulses were used to evaluate the linear optical behavior of the experimental setup while the 11.9 mJ pulses are intended to generate helical filaments.

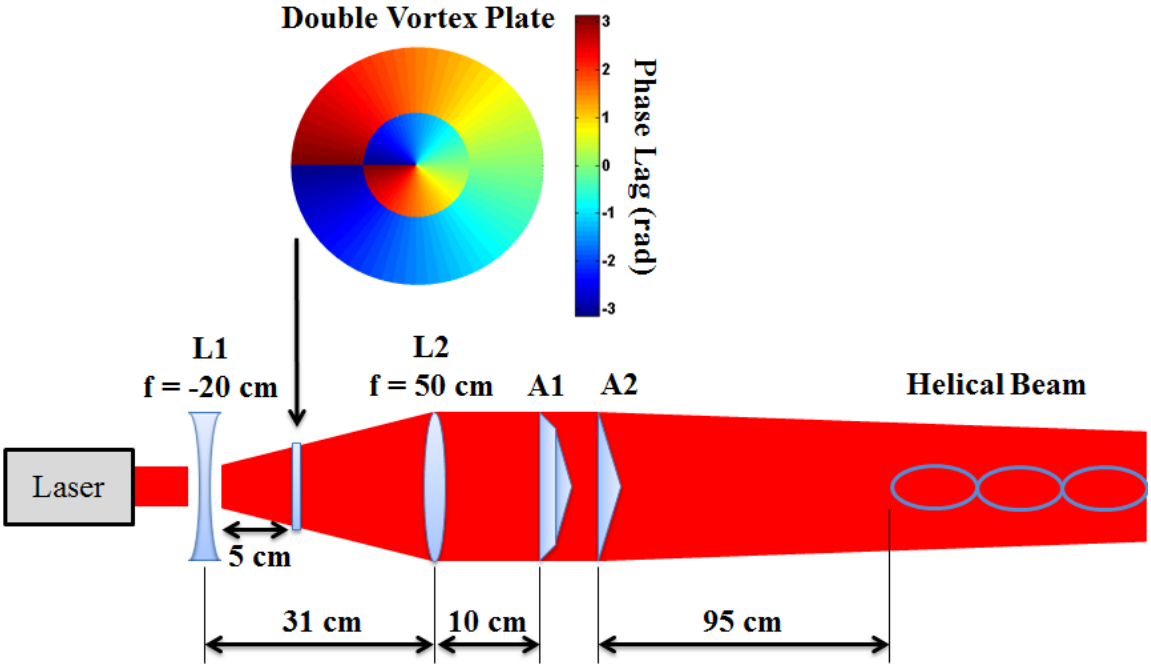

Figure 1: Experimental setup used to generate helical filaments.

Pulses were evaluated with the filament transverse imaging system along with an ionization test taken from Polykin et al [1]. The ionization test was carried out by measuring the current flowing between two 20 mm x 25 mm copper electrodes space 6 mm apart and raised to a potential of 5.6 kV while the helical pulses propagated between the electrode pair.

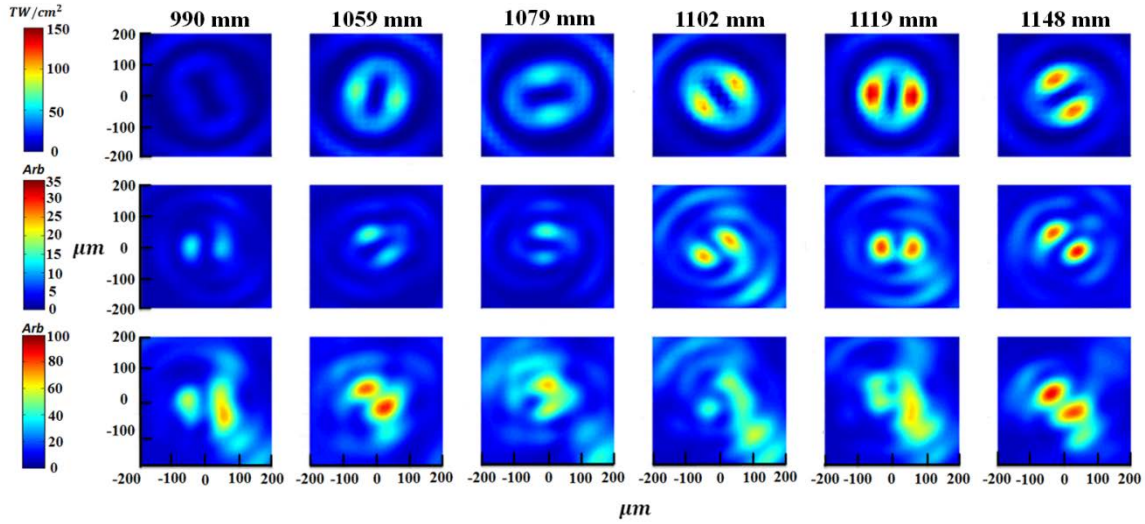

Figure 2: Transverse images obtained for the helical filament experiment. Top: MATLAB Fresnel diffraction simulation of the experimental setup. Middle: Experiment carried out using 133 *μJ pulses. Bottom: Experiment carried out using 11.9 mJ pulses.*

Helical beams were observed to extend from 883 mm to 1148 mm from the second axicon for both the 133 μJ and 11.9 mJ cases, as shown in Figure 2. As per the previous experiment, the resulting irradiance profiles were consistent with the Fresnel diffraction simulation of the experimental setup. From the corresponding ionization measurements, the beams were found to ionize the air between 1000 and 1150 mm (see Figure 3), but only for the 11.9 mJ case.

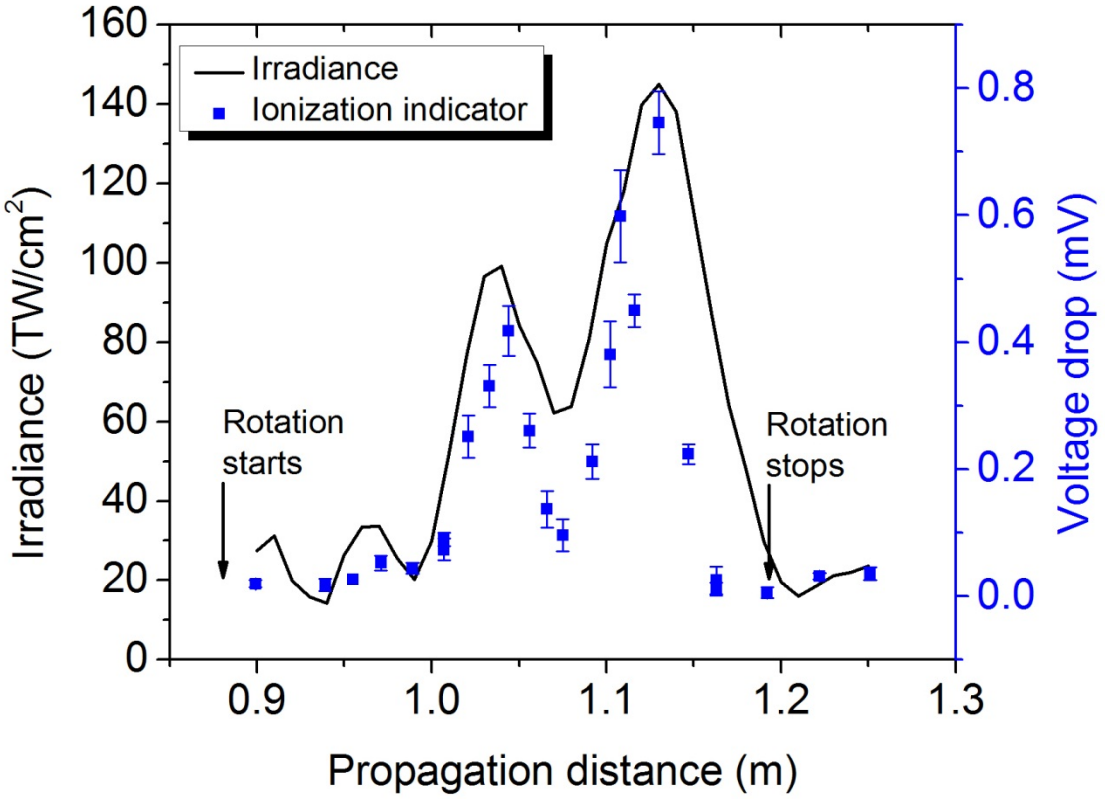

Figure 3: Ionization measurements of the helical beam along with a corresponding theoretical plot of the helical beam peak irradiance using the MATLAB Fresnel diffraction code.

The helical beam transverse profiles taken over the ionizing region are shown in Figure 2, along with profiles obtained through the MATLAB simulation. For 133 μJ pulses, a helical beam composed of two isolated, rotating, elliptical spots was obtained over a distance of 15 cm. The location, size and rotation rate of these irradiance spots are successfully reproduced by the simulation. When pulse energy was increased, a reduction in contrast between the null at the beam center and the two irradiance peaks was observed. For many of the transverse images taken of 11.9 *mJ* helical pulses, one or both of the irradiance peaks undergoes geometric distortions, losing the elliptical shape observed at 133 μJ. In some images, the formation of additional irradiance peaks within the helical beam ring structure was observed. The greatest distortions in the ionizing helical beam are observed at 1102 *mm* and 1119 *mm*, at the beginning of the second ionization peak observed in Figure 3. However, the beam continues to rotate during propagation, as observed for the 133 μJ case.

Numerous phase plates are being designed and tested to future helical beam experiments. A series of 10 mm double angle Fresnel axicons (Figure 4) were designed and fabricated using the same additive microlithographic techniques used to fabricate the vortex plates used in the helical beam experiments. By using Fresnel axicons in place of conventional

axicons, fabrication precision has been increased by simultaneously reducing fabrication cost. The new set of Frensel axicons can be placed in series with the vortex plates without requiring a telescope to adapt the scales. The new Frensel axicons have also been fabricated to simulate several axicon refraction angles, enabling the investigation of helical beams with different irradiance peak spacing.

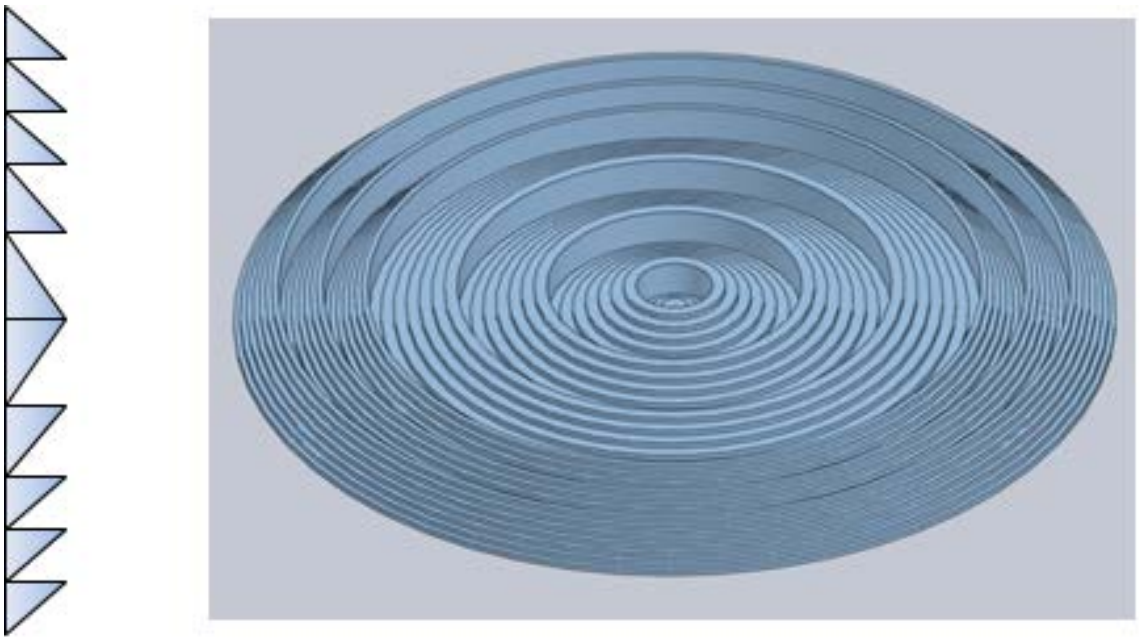

Figure 4: Layout of a Frensel axicon (left) and double angle Fresnel axicon (right).

A series of 2 inch diameter phase plates have also been designed and fabricated. Dr. Johnson's group was able to successfully fabricate both a 2 inch diameter coaxial vortex plate and a 2 inch diameter double angle Fresnel axicon. These 2 inch devices enable an energy throughput 25 times greater than that which can be achieved using 10 mm phase plates. Future experiments are planned to use the new 2 inch phase plates to generate 200 mJ helical beams 10 m in length using the 200 mJ that has become available from MTFL due to recent laser upgrades.

Knowledge obtained from the design of Bessel beams and helical beams is now being applied to the synthesis of non-diffracting rectangular beam arrays of arbitrary size, intended for the controlled and reproducible generation of rectangular arrays of filaments. Rectangular arrays of non-diffracting beams can be readily obtained from the superposition of four plane waves, as illustrated in Figure 5.

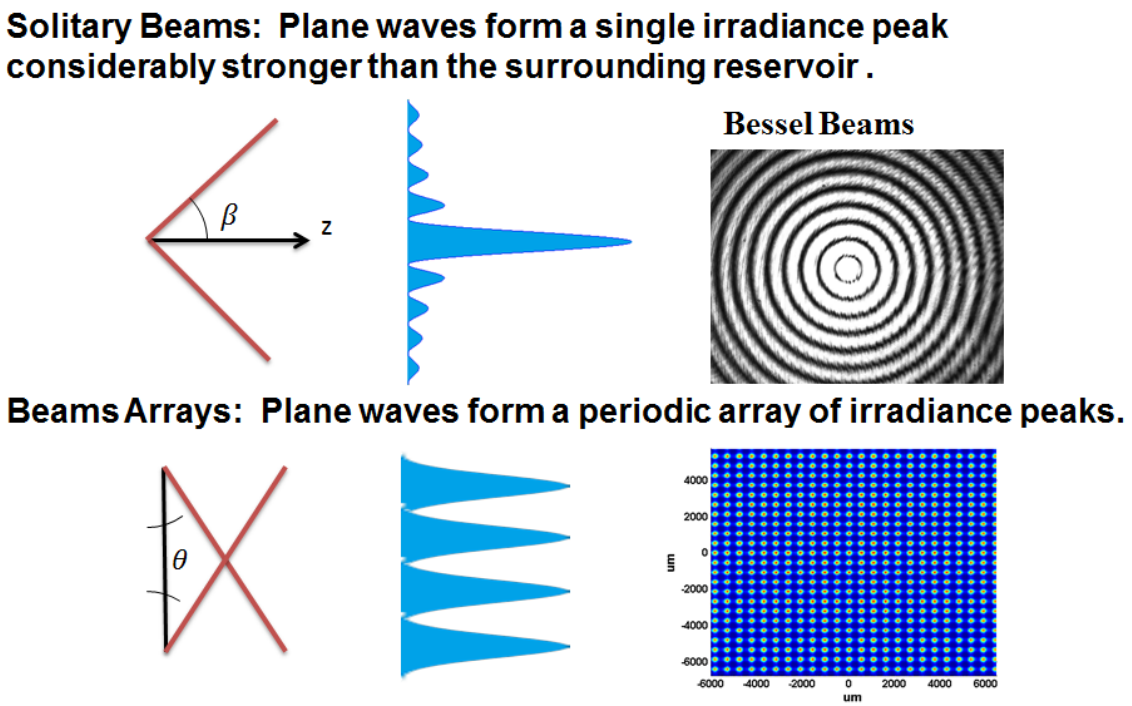

Figure 5: Conical and rectangular arrangements of plane waves giving rise to Bessel beams and a rectangular array of non-diffracting beams.

To obtain the plane wave superposition required to form the rectangular non-diffracting beam array, a diffractive grating with exactly two diffracted orders is required. Implementing this grating in two orthogonal directions will results in the desired beam array. Describing the wavefront modifications of the diffraction grating using the Fourier series

$$\phi(x) = \sum c_m \exp\left[i\frac{2\pi m}{L}x\right] \qquad 1$$

where the refracted angle associated with each Fourier series coefficient is given by

$$\theta_m = \frac{m\lambda}{L} \qquad 2$$

and the strength of each diffracted order is proportional to c_m^2.

If all of the refracted beam power is desired in the $m = \pm 1$ modes, equally distributed, the required grating coefficients are $c_1 = c_{-1} = 1$ and $c_m = 0$ otherwise. Substituting these values into equation 1 gives

$$\phi(x) = \cos\left(\frac{2\pi x}{L}\right) \qquad 3$$

This is unfortunately real, and thus requires amplitude rather than phase modulation. A direct implementation of this grating will result in a 50% transmission loss for a one dimensional grating, and a 75% transmission loss for a two dimensional grating. The effects of this grating can be approximated using a modulated neutral density filter described by the transmission function

$$T(x,y) = \sqrt[n]{\frac{1+\cos\left(\frac{2\pi}{L}x\right)}{2}}\sqrt[n]{\frac{1+\cos\left(\frac{2\pi}{L}y\right)}{2}} \qquad 4$$

The efficiency of this transmission grating is based on the value of n as follows:

n = 1 – 25%
n = 2 – 50%
n = 4 – 75%
n = 8 – 87%

While increasing the value of n increasing the transmission efficiency, it also degrades the performance of the grating, as it becomes a poorer approximation of the expression given in equation 3.

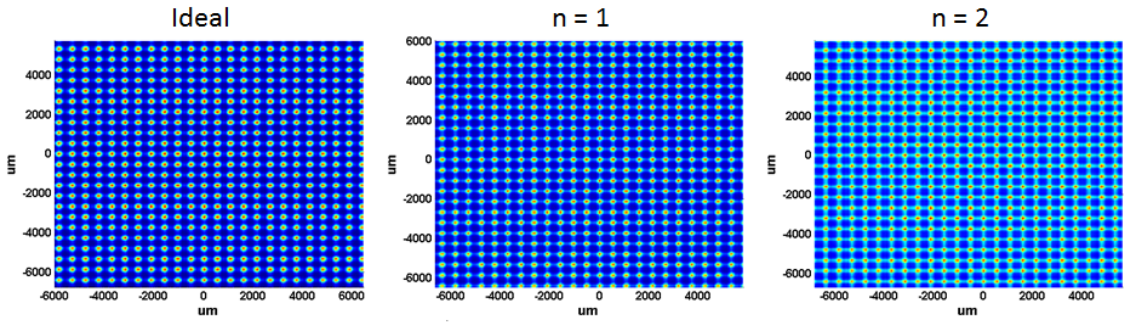

Figure 6: Rectangular arrays resulting from various orders of the transmission function given in equation 4.

Alternatively, the waveform modification in equation 3 can be achieved using a spatially varying polarization rotation. This enables 100% power transmission for the one-dimensional case while simultaneously providing an exact solution to equation 3, but requires a beam splitter to extract the two rectangular beam arrays. When extended to two dimensions, amplitude modulation is still required and the transmission efficiency of this technique is reduced to 50%.

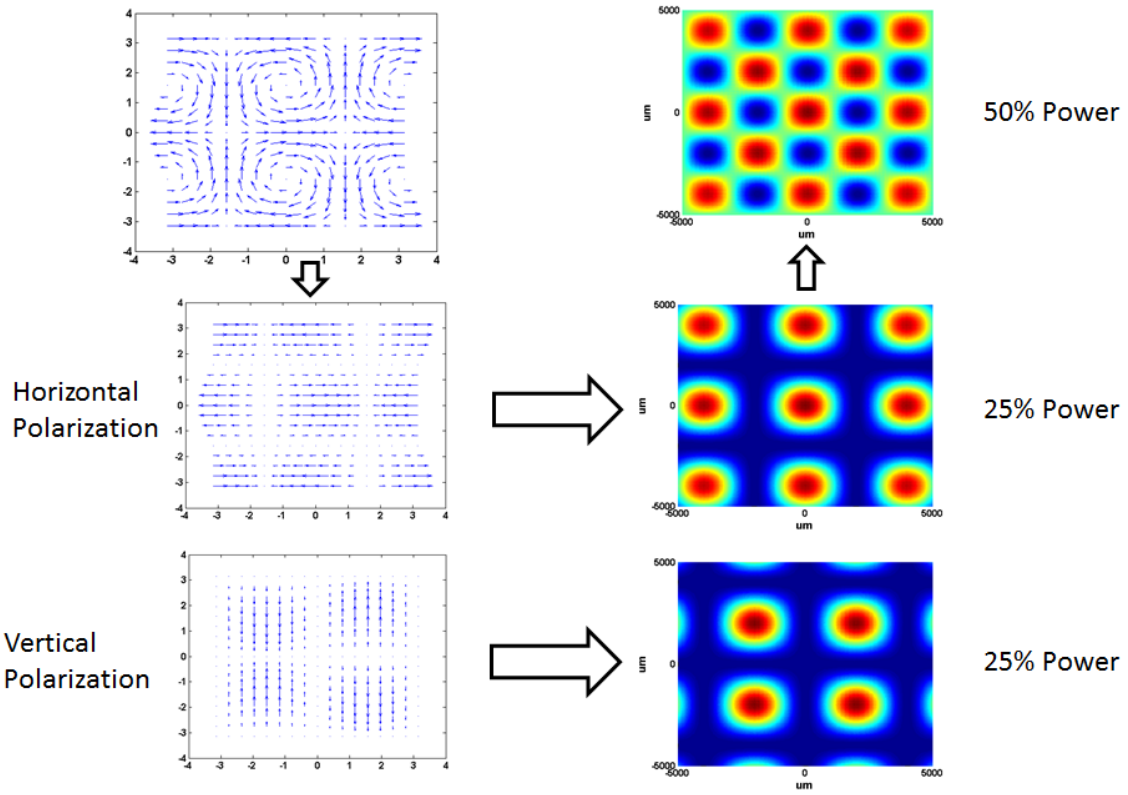

Figure 7: Spatially varying polarization rotation and simulations of the beam arrays resulting from such an arrangement.

Bibliography

[1] P. Polykin, M. Kolesik and J. Moloney, "Extended filamentation with temporally chirped femtosecond Bessel-Gauss beams in air," *Optics Express*, vol. 17, no. 2, pp. 575-584, 2009.