



**ENHANCED VEHICLE BEDDOWN  
APPROXIMATIONS FOR THE IMPROVED  
THEATER DISTRIBUTION MODEL**

THESIS

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## **Abstract**

Gathering insight into the theater distribution process can be a complex task, especially when estimating potential beddown solutions. Coming up with a low cost feasible mixture of cargo vehicles that will support distribution of military personnel and goods within theater is currently a high priority for force flow analysts at USTRANSCOM. In the past, analysts used a trial and error simulation process that was iterative and time consuming. Recent research has produced the Improved Theater Distribution Model (ITDM), which presents a less time consuming, more precise method to estimate beddown allocations.

Improving on this research, two linear programming methods are developed and added to the ITDM that reduce baseline beddown approximations. Because daily operational cost and initial beddown cost is included, this ultimately provides a realistically lower cost feasible solution when modeling theater distribution. The improved beddown solutions generated from post-processing results of the ITDM can be used as baselines for further distribution analysis. Within the construct of the model, precise set notation is carried over from the Improved Theater Distribution Model and slightly altered to reduce the generation of unnecessary variables and constraints with large-scale problems.

*To my brothers and sisters, who have been there since the beginning to push me and inspire me to be the best I can, and reach for great things.*

*To my parents, who taught me the value of hard work  
and to always finish what I start.*

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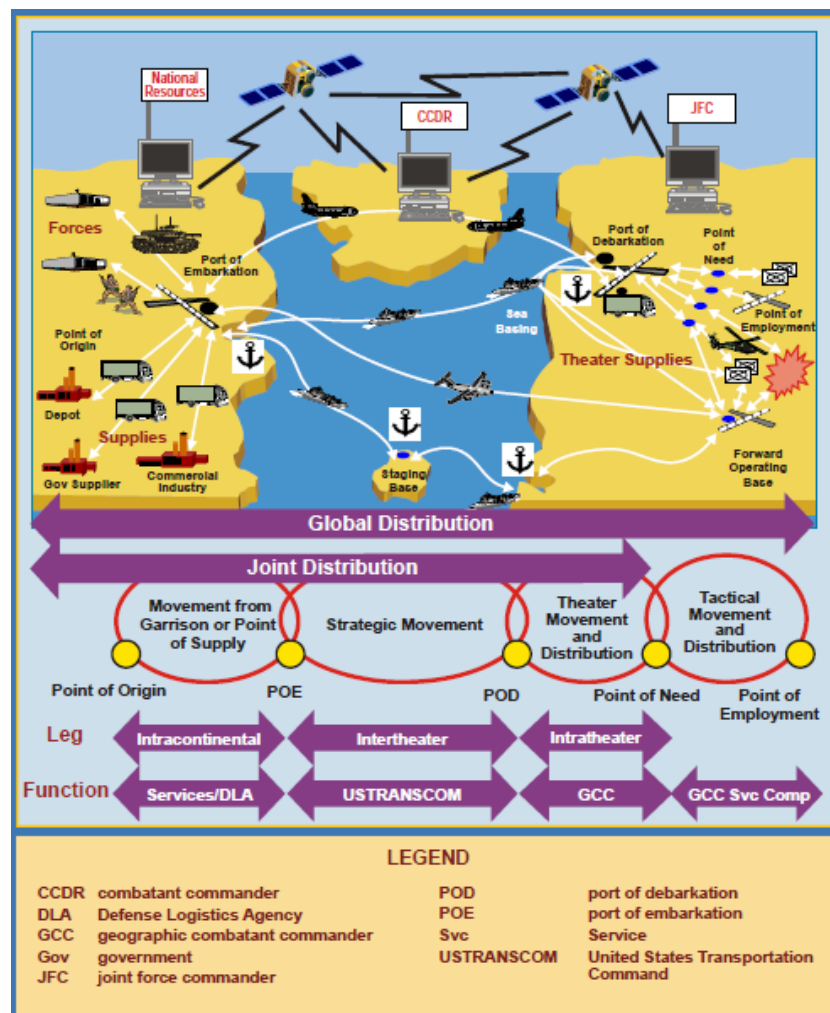
## **I. Introduction**

### **Background**

While many objectives exist within the US military to ensure the United States (US) remains the world's most prominent military force, one important and essential capability that is core to its success is the distribution of troops and needed supplies from the continental United States (CONUS) to overseas and deployed locations. Theater distribution is defined as the flow of personnel, equipment, and materiel within a given theater as necessitated by the geographic combatant commander to support theater missions (Joint Chiefs of Staff, 2010). United States Transportation Command (USTRANSCOM), which is responsible for movements of troops and supplies, ensures that these logistical needs are met. To meet these requirements, USTRANSCOM spends a considerable amount of time analyzing data, assessing simulations, and determining appropriate, feasible mixtures of vehicles to employ so that distribution of necessary supplies and troops is possible. The deployment process of goods and troops involves flow from a point of origin to a point of need. This point of need is typically the point at which a requirement exists.

USTRANSCOM breaks down this journey of supplies and personnel from the point of origin to the final point of need into three legs. The first leg involves movement from a point of origin to a Point of Embarkation (POE). This is usually from some starting CONUS base to a second en-route CONUS base. This leg is known as intercontinental movement. The second leg involves flow of goods from a POE to some Point of Debarkation (POD), also en-route. The POD can be thought of as the midway point, and this second leg typically encompasses movement from a CONUS location to a

distinct theater of operations. This leg is known as intertheater movement. The final leg, commonly referred to as intratheater movement, involves flow from a POD in theater to the final destination, or point of need, which constitutes the point at which the supplies are needed (Joint Chiefs of Staff, 2010). Figure 1 is included below to illustrate this process of intercontinental, intertheater, and intratheater distribution of personnel and supplies.



(Joint Chiefs of Staff, 2010, p. 12)

**Figure 1. Illustration of the Three Legs of Distribution Process**

Extensive research has been conducted on all three legs of the journey, but the leg that often poses the most challenge for USTRANSCOM force flow analysts is the

intratheater journey from POD to the final destination. Distribution within this area, called theater distribution, typically involves the movement of supplies from an aerial or sea port over a relatively short distance to a point in a combat zone or deployed location. Not only is it essential to get these supplies to their final destination, but they must also reach their destination in a timely manner.

Every grouping of supplies constitutes a requirement, and every requirement is accompanied by time windows within which it may be picked up and must be dropped off at its next destination. In addition, each requirement has differing due dates for each leg of its journey to final destination. For example, for a requirement to be dropped off at its POD there is a time window that has an Earliest Arrival Date (EAD) and a Latest Arrival Date (LAD). The EAD describes the earliest time that delivery of a requirement can occur at its POD and the LAD describes the latest point at which said requirement can be delivered to its POD. This creates a time window within which each requirement can be delivered on its first leg of the journey. There is also a Required Delivery Date (RDD) for the second leg that must be met for the requirement to be considered on time. The RDD is the latest date at which a requirement can reach its final destination or point of need. On top of this information, each requirement has an associated weight, measured in short tons.

Under the current system, USTRANCOM organizes all of this information in what is called a Time Phased Force Deployment Data (TPFDD) file. The TPFDD contains all necessary information to ensure force flow analysts at USTRANCOM can perform appropriate studies and determine a mix of vehicles that will ensure on time delivery of all requirements. One more measure that should be considered is the

Commander's Required Delivery Date (CRD). This date extends beyond the RDD and allows requirements to be delivered in a window between the RDD and CRD. It is the absolute delivery day, and is included so analysts may assess the impacts of a late delivery (Joint Chiefs of Staff, 2011a). These impacts will be discussed further in Chapters 3 and 4. Some sample data that is usually included in a TPFDD is shown in Table 1 below.

**Table1. Example of Data Included in a TPFDD**

| <b>Requirement</b> | <b>POE</b>     | <b>EAD</b> | <b>LAD</b> | <b>POD</b>     | <b>RDD</b> | <b>Destination</b> | <b>Total Short Tons</b> |
|--------------------|----------------|------------|------------|----------------|------------|--------------------|-------------------------|
| 1                  | KFFO (WPAFB)   | 5          | 8          | OAIX (baghran) | 10         | GHOS               | 200                     |
| 2                  | KDOV(DoverAFB) | 7          | 10         | OAKB (Kabul)   | 12         | BHEL               | 300                     |

As previously stated, USTRANSCOM takes particular interest in the final leg from POD to final destination. To meet war fighter needs, There is a progression of methods with which USTRANSOM handles the analysis of delivering troops and supplies over the last leg. Currently, analysts utilize various simulation software and tools in order to determine a feasible solution that meets both constraints on vehicles for the theater of interest, and constraints on vehicles selected for simulation. This method, however, only considers finding a feasible mix of vehicles to deliver requirements in a TPFDD, and does not prescribe an optimal scheduling based on certain military objectives. Realistically, military leadership will have several objectives they want to meet such as minimizing operational cost, minimizing the number of vehicles to meet requirements, and minimizing late deliveries. The method also involves an extremely time consuming, iterative process in which the operations plan (OPLAN) and TPFDD are continuously updated until a feasible vehicle schedule is developed. Longhorn and Kovich, while working for USTRANSCOM discovered that this process is inefficient and

may delay the formulation of an operations plan (OPLAN) and subsequently the delivery of essential supplies to the people who need them downrange. As these delays could negatively affect military operations and overall mission success, clearly, theater distribution acts as a crucial point in the delivery process.

In response, Longhorn and Kovich formulate a better system for determining an optimal mix of vehicles to meet TPFDD requirements. Their idea involves an integer programming optimization model that essentially provides a feasible schedule of vehicles for force flow analysts at USTRANSCOM to start with before performing simulation of distribution. Since analysts would start with this already feasible and optimal schedule of deliveries, the time consuming process of iteratively trying to determine a feasible schedule would theoretically be eliminated. This vehicle mixture would be used as input for simulation and should provide a feasible starting point for analysis. The model described in their detailed report is better known as the Theater Distribution Model (TDM) and will be referred to throughout this paper (Longhorn and Kovich). The issue with Longhorn and Kovich's model lies in the fact that it presents far too many decision variables and constraints to be computationally efficient. In other words, the way the TDM is formulated makes it more complicated and sparse than it really needs to be.

To reduce the size and complexity of the TDM, Micah Hafich, an Operations Research (OR) studying at the Air Force Institute of Technology (AFIT), proposed the Reduced Theater Distribution Model (RTDM). The RTDM, while still an integer programming mathematical model, reduces the size of the TDM by introducing sets that are not contained in the TDM while maintaining all of the original characteristics of the TDM. This improved formulation greatly reduces the computational needs to generate the

model, thus saving time and money for force flow analysts. The pure integer programming aspect of this model, however, limits its ability to allow more than one requirement to be scheduled to a single vehicle. For example, it would make operational sense to put two 20 ton requirements with the same final destination on one vehicle with a payload of 40 tons, thus reducing the necessity of using 2 or more vehicles to transport the requirements when they could have been moved with 1. To remedy this issue, Hafich further proposes the Improved Theater Distribution Model (ITDM). The ITDM, a Mixed Integer Programming (MIP) model, allows a single vehicle to be assigned to deliver more than 1 requirement if it is in fact feasible for both requirements to be placed on that vehicle. In addition, the ITDM addresses issues with modeling lateness present within the RTDM. For a model who's objective is finding a least cost, minimum lateness vehicle mixture solution, it is essential that lateness of requirements be modeled correctly to achieve a truly optimal or close to optimal solution. The ITDM takes care of both of these problems presented by the RTDM.

In addition to the formulation of both the RTDM and the ITDM, Hafich produces an excel based Decision Support System (DSS) which solves the MIP model of the ITDM and proposes a feasible and optimal vehicle mixture based on an inputted TPFDD file. The DSS is a macro embedded Microsoft Excel VBA program that uses the Lingo optimization software to determine the optimal mix; it then outputs that mix into an organized Excel file that force flow analysts at USTRANSCOM can easily decipher and use for further simulation. The RTDM, ITDM, and accompanying DSS tool fill the gap that existed after Longhorn and Kovich's formulation of the TDM, and gives analysts a

definitive means with which to more easily determine a feasible schedule of vehicles to meet all requirements specified in a given TPFDD.

One important take-away with the ITDM lies in the fact that a solution output by Hafich's DSS can be used to determine an appropriate vehicle beddown based on upcoming requirements within the theater of operations. A vehicle beddown involves the placement of various vehicles of varying mode and type at specific locations within theater. It can be reasonably assumed that the goal when determining an appropriate vehicle beddown at a POD would be to place the minimum number of vehicles necessary to meet delivery requirements outlined in a TPFDD, as there is typically a high cost associated with moving vehicles in theater and maintaining them while deployed there. Vehicle limitations exist as well. The outputs to Hafich's model can currently be used to estimate beddown needs at specific PODs within theater, therefore providing an efficient, effective tool for force flow analysis efforts at USTRANSCOM. The importance of the model and subsequent DSS tool revolves around its ability to produce a feasible vehicle mix quickly and consistently without the time consuming trial and error methods used by USTRANSCOM in years past. In addition to producing feasible vehicle mixtures when considering distinctive theater and vehicle constraints, the DSS tool gives an optimal solution with the main objective of producing a least cost on time solution. This research by Longhorn, Kovich and Hafich provides much needed support to USTRANSCOM force flow analysts, and will help supply war fighters with necessities when they need them.

## **Research Purpose and Objectives**

The purpose of this research is to serve as an improvement to the current cargo and personnel mobility planning practices at USTRANSCOM. The capabilities provided by Longhorn and Kovich's TDM model, and Hafich's RTDM and ITDM models, although great improvements upon the trial and error techniques used before, do not necessarily provide a realistically optimal mix of vehicles from a beddown standpoint. The ITDM provides a least cost on time solution with the objectives set forth in the model, but does not necessarily provide the real world least cost solution, as the task of moving large amounts of various types of vehicles into theater is usually accompanied with a high setup cost. In other words, the daily cost of operating and maintaining specific types of vehicles are considered in the objective, while cost associated with placing vehicles in theater is not.

Currently, the ITDM outputs a list of movements that are mostly on time. A shortfall within the model, however, is that it attempts to move large portions of certain requirements in one or two days instead of spreading delivery of these goods over several days, using the entire delivery time window. For example, consider a 100 ton requirement that must be transported from its POD to final destination with a 5 day time window to deliver these goods before its RDD. Consider further, that the route from POD to point of need only allows mode air for delivery, and C-130s with a payload of 10 short tons are prescribed by the ITDM to make the delivery. The solution to the ITDM will tend to move these supplies using a large number of C-130s in 1 Day, or over the course of 2 days. The 1 day movement would require 10 C-130s while a 2 day movement would require 5 C-130s. Although these allocations of vehicles minimize per vehicle cost to

transport the requirement on time, they do not account for the initial cost and logistical burden of the accompanying C-130 beddown. With this considered, transportation of the requirement using 2 C-130s over a 5 day period present a more desirable solution since it only requires a beddown of 2 aircraft in theater. The overall objective, then, will be to minimize the number of vehicles needed in a theater of operations to meet TPFDD requirements.

The first objective of this research is to test the ITDM and determine whether the results from this model will be adequate for approximating vehicle beddowns and locations within theater at a minimum setup cost. The current solutions do not tend to accomplish this.

The second objective of this research is to make any necessary additions and improvements to the ITDM formulation and DSS. In other words, the ITDM must be improved to take into account the cost of a feasible vehicle beddown for a particular POD. This will ensure that the objective of minimizing cost truly matches reality, taking into account both vehicle operation and beddown cost. It is noteworthy that cost may not necessarily be measured in currency. As a result, other costs such as a weighted penalty may need to be factored into the model.

The third objective of this research is to test the improvements made to the ITDM and determine if these improvements account for a beddown cost. This should minimize the number of vehicles necessary to meet all requirements outlined in a given TPFDD.

Fourthly, the research will attempt to give force flow analysts the ability to tailor the model based upon changing objectives such as minimizing late deliveries, changing the penalty within the model for each late delivery, minimizing beddown cost, and

minimizing vehicle operational costs. The improved system should model all of these objectives simultaneously, or allow the option to model just one or any combination of them, thus accommodating for real world objectives of decision makers at TRANSCOM.

Lastly, this research will strive to make improved beddown approximations at specific POD locations from the model results, with the purpose of lowering the logistical and monetary burden that the current solution to the ITDM prescribes.

The culmination of these research objectives will be in the greater scheme of things a better, lower cost method for USTRANSCOM to approximate beddown needs at locations within theater to support contingency operations. The research will improve the ITDM to model reality more closely and reduce the necessity of bedding down large numbers of vehicles at PODs. Force flow analysts should be able to more efficiently estimate a beddown, which should result in limited vehicle assets being available for use elsewhere. It will free up vehicles, and offer the opportunity for these vehicles to be used for other military objectives to support worldwide military operations. The model improvements should allow more flexibility to force flow analysts at USTRANSCOM, allowing them to change objectives based on preferences. Through this research, improved beddown approximations and model flexibility should improve efficiency when planning for operations in theater, and provide an improved least cost, on time model of reality to meet war fighter supply and resupply needs.

## **Organization**

The remainder of this thesis contains four additional chapters. Chapter II provides a literature review of airlift optimization modeling, the Pickup and Delivery Problem with Time Windows, and other relevant models focused on theater distribution and beddown

approximating. It also discusses Integer Programming and Mini/Max Programming. Additionally, the proposed ITDM is introduced and explained in detail. In Chapter III, the methodology utilized in this research is discussed. In particular, two models, the GIBR and the MPBR are introduced. Chapter IV shows the implementation of the methodology and demonstrates improvements over the ITDM. Chapter V offers concluding remarks and discusses how this work might be extended with further research.

## **II. Literature Review**

This chapter reviews pertinent and relevant literature, the purpose of which is to give a general understanding and background on the theater distribution problem and attempts at modeling beddown approximations. The information will provide the reader with a brief background on the issue, but is not an all-encompassing review of research in this area. The focus will be on four specific research interests in the air mobility and theater distribution field: a background on past theater distribution models, more specifically beddown planning efforts, the fixed charge assignment problem, goal programming, and an in depth investigation of the Improved Theater Distribution Model (ITDM) as formulated by Hafich. The ITDM represents the most recent effort to solve theater distribution and beddown planning issues, and is the primary motivation for this research. Thus, a detailed explanation of this model formulation is necessary.

### **Background**

Distribution planning is an important part of the overall joint operation planning process. It must include detailed analysis and evaluation of the distribution networks and functions supporting the end-to-end distribution process, as well as encompass the full range of activities necessary to plan for national mobilization, deployment, employment, sustainment, and redeployment requirements of forces and materiel (Joint Chiefs of Staff, 2010). This importance drives force flow analysis efforts at USTRANSCOM. The U.S military currently attempts to carry out mobility planning by using numerous simulation tools such as the Generalized Air Mobility Model (GAMM). Mckinzie and Barnes (2004) describe several of these tools and their importance in mobility scenario planning. Simulation, however, only tends to describe scenarios rather than prescribe a vehicle mix

to meet supply requirements. Although these tools help identify and describe limitations in the theater distribution, no currently used tool prescribes the number of mobility vehicles or mixture of vehicles to meet theater movement requirements (Longhorn and Kovich, 2012).

Force flow analysis revolves around planning for mobility requirements outlined in a TPFDD, and updating those plans as requirements change. Although optimization techniques in the past have been geared towards routing of vehicles, this is not a high priority for analysts because of the extremely variable conditions in a combat environment. For these reasons, the creation of individual vehicle routes and schedules is neither necessary nor desired for force flow analysis. Instead, analysts simply desire a baseline vehicle mixture that will successfully support distribution operations (Hafich, 2013). Longhorn and Kovich's RTDM and Hafich's subsequent ITDM represent two reasonably successful attempts at optimizing feasible baseline vehicle mixtures to successfully support distribution operations.

These efforts opened the door for follow-on research in several areas. Vehicle beddown planning for PODs in theater, the primary focus of this research, is one of those areas. Accurate beddown planning, as discussed in chapter I, is of utmost importance to the U.S. military and represents a key problem currently faced by USTRANSCOM. As a result, the drive for analysts is not only to find feasible mixtures of vehicles to meet requirements, but also to minimize the number of vehicles needed to do so, thus reducing beddown approximations at PODs of interest.

Although research in this specific area is minimal, Hafich along with the endeavors of Longhorn and Kovich, provide a basis. This chapter will provide an

overview of past optimization efforts and other approaches to theater distribution, several beddown approximation techniques, and will explain in detail the mathematical formulation of the ITDM and its beddown scheme. It will also outline the general form of a fixed charge assignment integer programming problem, and Mini/Max Programming techniques, the purpose of which is to provide an overview of approaches used in the methodology portion of this thesis to improve upon the beddown issue.

### **Theater Distribution Optimization**

Several attempts to optimize theater distribution were made in the 1990s. Rappoport, Levy, Toussaint, and Golden (1994) came up with an airlift planning tool for Military Airlift Command (MAC), predecessor to the US Air Force's Air Mobility Command (AMC), now headquartered at Scott AFB. The single transportation Mode (air) model assigned different airlift aircraft types and shipment days to specific requirements. Once these assignments were made, the results were preprocessed and then processed utilizing a heuristic routing and scheduling procedure the authors called the Airlift Planning Algorithm (APA). The linear programming model minimizes the costs of matching capacity to differing requirements. Although their model matched vehicle types to the shipments as a form of preprocessing, it does not prescribe a specific number of vehicles necessary to support distribution within the network.

Early optimization models also included THRUPUT II, developed at Naval Postgraduate School and discussed by Rosenthal et al. (1997). THRUPUT II, a linear programming model, described the entire distribution network in its formulation. The model takes given inputs of cargo and passengers to be moved, available airfields,

aircraft, and routes, and minimizes penalties for late deliveries subject to appropriate physical and policy constraints. Priorities are assigned to requirements when necessary.

Rink, Rodin, Sundarapandian, & Redfern (1999) applied a shortest path algorithm to the AMC aircraft routing problem. The model described utilizes a double-sweep algorithm to find the  $k$  – shortest paths between an onload Location and offload location provided in a TPFDD. Unlike THRUPUT II, however, this model does not consider lateness and its associated penalties. In addition, the shortest path may not be the best path, as unpredictable conditions such as weather, and enemy forces and other threats may hinder success of the delivery. Lastly, the shortest path does not account for outloading and unloading constraints within the network; there is no guarantee that enough resources will be available at certain airfields along the path.

In addition to the shortcomings described in these models, another important consideration is left out. Within a true theater distribution network, multiple modes of transportation such as air, road and rail are utilized to carry out delivery of goods and personnel. All airlift models discussed thus far only consider Mode air. Other Modes must be considered to have a realistic model of a transportation network. As such, beddown considerations should also include multiple modes of transportation as well.

### **Pickup and Delivery Problem with Time Windows**

Typically in theater distribution, the TPFDD outlines a delivery window within which a requirement can be picked up and must be delivered to its destination. The TPFDD gives a time window for both the pickup at a POD and the delivery at a final destination. As a result, The problem of theater distribution that the US military and USTRANSCOM faces can be related to a problem known as the Pickup and Delivery

Problem with Time Windows (PDPTW). Solutions to the PDPTW yield optimal routes for vehicles in which demand is met within the appropriate time windows while meeting capacity and precedence constraints (Dumas, Desrosiers, & Soumis, 1991).

Dumas et al. (1991) made an early attempt at formulating the PDPTW utilizing a column generation and shortest path sub problem. This model utilizes a homogeneous fleet of vehicles. Other endeavors into the PDPTW that utilize a homogeneous fleet include a Reactive Tabu Search method employed by Nanry & Barnes (2000) and a set partitioning formulation described by Baldacci, Bartolini, & Mingozzi (2011).

The homogeneous nature of these models makes them somewhat obsolete. Models that consider heterogeneous fleets of vehicles are far more realistic and useful, and have been researched. An exact algorithm for solving the PDPTW with multiple vehicle types was formulated by Lu & Dessouky (2004). The model, known as the Multiple Vehicle Pickup and Delivery Problem (MVPDP), does not necessarily include time windows. A more robust model, developed by Xu, Chen, Rajagopal, & Arunapuram (2003), considers, in addition to multiple vehicles, multiple time windows, compatibility constraints, and restrictions on travel time. This is known as the Practical Pickup and Delivery Problem (PPDP).

One important consideration to note is that the PDPTW usually assumes that vehicles are placed at centrally located bases from which vehicles begin their routes. A beddown, however, involves the placement of vehicles in a theater of operations to support transportation, and not exclusively the point at which vehicles begin their delivery routes, which could be anywhere, including CONUS locations. Specifically, this

idea of a beddown needs to be assessed for PODs in theater. This goal takes high priority for force flow analysts at USTRANSTOM.

### **Tabu Search Techniques to Solve Theater Distribution Problems**

Some of the most recent approaches to theater distribution problems involve the utilization of Tabu Search. Crino, Moore, Barnes, & Nanry (2004) approached the problem by employing Group Theoretic Tabu Search. This method outlines specific scheduling and routing of multiple modes of vehicles using Tabu Search. It takes into account delivery of goods within time windows. Similarly, Burks, Moore, Barnes, & Bell (2010) describe an implementation of an effective Adaptive Tabu Search (ATS) methodology for the Theater Distribution Problem (TDP). This methodology evaluates and provides a routing and scheduling of theater transportation assets at the individual asset level to ensure Time Definite Delivery (TDD) for all demands (Burks, Moore, Barnes, & Bell, 2010). It solves both the problem of depot location selection and specific vehicle routing to support delivery needs. Both of these models dictate vehicle routes as well as schedules at an individual vehicle level, while the optimization techniques discussed typically prescribed only one of these.

Vehicle routing and scheduling provide little practical insight for force flow analysis and beddown approximation, however, as the daily conditions in a wartime environment are so variable. This variability of conditions causes a day-to-day scheduling approximation to be much less useful and effective than a generalized approximation of vehicles to be placed in theater to support operations. Thus, Tabu Search does not provide a very useful solution for determining baseline multimodal vehicle beddown approximations.

## **Past Beddown Approximation Techniques**

Mobility aircraft, trucks and trains must often times compete with other types of vehicles for space at PODs in theater. As a result, many efforts have been made to solve the problem of approximating beddown needs for specific types of vehicles at strategic locations. These attempts involve both mathematically based and management based approaches. Hathaway et al. (1987) developed a method to make bed-down decisions utilizing an integer linear programming model to generate candidate basing (beddown) decisions for analysis and testing. Once determined, candidate beddown solutions are simulated in FLEETLIFT for further evaluation. This model captures the dynamic effects of the availability of material handling equipment (MHE), limited airfield ramp space, variable distance between network airfield locations, and variable combat attrition and planning factors such as limited aircrew and limited aircraft loading capacity (Hathaway, 1987) .

Zeisler et al. (2000) took a different approach by employing a greedy heuristic to solve AMC's intratheater airlift scenario as a multiple knapsack problem. Instead of prescribing specific vehicle mixtures to meet given TPFDD requirements, a generalized throughput assessment is given for a predetermined vehicle mixture and assignment scheme. This involves a trial and error process of testing different beddown mixtures to maximize throughput. This process of what-if analysis is time consuming and ineffective for force flow analysis. As a response to the need for a better system, Salmond et al. (2005) developed a decision analysis method for air mobility beddown planning scenarios. Instead of making beddown decisions through manual lookup, trial-and-error, and corporate knowledge, this research proposes a decision analysis tool that compares

hard requirements versus existing capabilities and through gap analysis, identifies infrastructure requirement shortfalls and associated costs to satisfy these shortfalls (Salmond, et al. 2005). This model, however, only outlines shortfalls for specific beddown decisions. Additionally, none of these methods prescribe a general fleet mix to support specific intratheater operational delivery needs and thus, are of little use to force flow analysts.

A few management based approaches have been utilized to aid in beddown scenario planning as well. Koewler et al. (2003) discusses improvements made to the Capabilities Based Logistics Planner (CBLP), a tool used by Air Force Studies and Analysis Agency to quickly estimate beddown plans. This is a homogeneous capabilities-based approach that assesses changes in airfield logistics capacity as more or less aircraft are beddown at specific airfields. Although a heuristic is developed to estimate the parking capability of airfields, this model is based upon very simple mathematics. Pennington et al. (2006) developed a Microsoft Access Based Cost Estimation Tool for Beddown Analysis (CETBA). The cost-based model is intended to provide the analyst with the maximum amount of both quantitative and qualitative input for any potential decision to quickly identify infrastructure requirement shortfalls and associated costs to satisfy those shortfalls (Koewler, 2003). Similar to Salmond's Decision Analysis approach, this tool involves the assessment of infrastructure shortfalls.

Although all of these models provide insight into the effectiveness, capabilities, and costs of specific beddown scenarios, they fail to provide information about a feasible mix of vehicles necessary to support theater distribution operations. USTRANSCOM requires beddown planning as a long term decision based on the specific requirements

given in a TPFDD. Therefore short term routing, scheduling, and beddown capabilities and cost analysis does not provide practical results to plan an in-theater beddown to support force flow.

### **Fixed-Charge Assignments Problem Overview**

Although fixed charge type approaches have rarely been applied specifically to the theater distribution problem, their applicability in this area is evident. Winston et al. (1991) describes a fixed-charge problem as an integer programming formulation where there is a cost associated with performing some activity at a non-zero level, independent of the level of the activity. These formulations are typically applied to production and location problems. In the production problem sense, if some product is produced, a one-time production setup cost is incurred no matter how many items of that product are manufactured. When applied to location problems, a decision is made on where to locate various facilities such as plants, warehouses, or business offices, and a fixed charge is associated with building or operating the facility.

For the purposes of this research, consider the classic Facility Location Problem. Given a set  $L$  of customer locations and a set  $F$  of candidate facility sites, you must decide which sites to build facilities on and assign coverage of customer demand to these sites so as to minimize cost. All customer demand  $d_i$  must be satisfied, and each facility has a demand capacity limit  $C$ . The total cost is the sum of the distances  $c_{ij}$  between facility  $j$  and its assigned customer  $i$ , plus a fixed charge  $f_j$  for building a facility at site  $j$ . This model can be formulated as the following integer linear program (SAS Institute Inc. 2010):

Decision Variables:

Let  $y_j = 1$  represent choosing site  $j$  to build a facility, and 0 otherwise.

Let  $x_{ij} = 1$  represent the assignment of customer  $i$  to facility  $j$ , and 0 otherwise.

$$\text{Minimize } \sum_{i \in L} \sum_{j \in F} c_{ij} x_{ij} + \sum_{j \in F} f_j y_j \quad (1)$$

Subject To

$$\sum_{j \in F} x_{ij} = 1 \quad \forall i \in L \quad (2)$$

$$\sum_{i \in L} d_i x_{ij} \leq y_j \quad \forall i \in L, \forall j \in F \quad (3)$$

$$\sum_{i \in L} d_i x_{ij} \leq C y_j \quad \forall j \in F \quad (4)$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in L, \forall j \in F \quad (5)$$

$$y_j \in \{0, 1\} \quad \forall j \in F \quad (6)$$

### Model 1. Generalized Fixed-Charge Location Problem (GFCLP)

The objective function seen in (1) minimizes cost. The constraint at (2) ensures that each customer is assigned to exactly one site. Constraint (3), known as the linking constraint, forces a facility to be built if any customer has been assigned to that facility. Lastly, constraint (4) enforces the capacity limit,  $C$  at each site. The beddown problem can be related to the GFCLP, only in terms of general integers rather than binary. Some of the basic model concepts will be utilized in Chapter III.

## Mini/max Goal Programming Overview

Mini/max goal programming is typically used to solve real world problems with multiple and often times competing objectives. This method attempts to meet some goal or set of goals rather than just minimize or maximize some objective, as traditional math programming models do. When integer variables are introduced into a goal programming model, it becomes an integer goal programming model; these models can contain zero-one integer decision variables, general integer variables, or a combination of both.

Several objectives can be utilized in goal programming formulations. Typically, goal programming seeks to minimize the sum of the deviations from all goals. Ragsdale et al.

formulates this objective as: Minimize  $\sum_i (d_i^- + d_i^+)$ , where  $d_i^-$  and  $d_i^+$  represent the

negative and positive deviations respectively from each goal  $i$ . One specific formulation deals with the Mini-Max objective, and is typically formulated as seen in model 2.

$$\text{Minimize: } Q \quad (7)$$

Subject to

$$d_1^- \leq Q \quad (8)$$

$$d_1^+ \leq Q \quad (9)$$

$$d_2^- \leq Q \quad (10)$$

Etc...

### Model 2. Mini-Max Goal Programming Formulation

Where  $Q$  represents the maximum deviation desired from each goal,  $i = 1, 2, \dots$  and  $d_i$  represents those deviations from each goal,  $i$ . Constraints (8), (9), and (10) ensure that no deviation, either positive or negative for each goal  $i$ , exceeds a set value  $Q$ . Note that both positive and negative deviations can be modeled, allowing more flexibility when setting goals. Mini/Max Goal programming's relevance to beddown planning within the ITDM will be discussed further in Chapter III, Methodology. Although the exact formulation is not used, some basic concepts are drawn from this model.

### **Improved Theater Distribution Model (ITDM)**

#### ***ITDM Overview.***

As previously discussed, Hafich (2013) improved upon the Longhorn and Kovich (2012) theater distribution model formulation, the TDM, by designing the RTDM to greatly reduce size and complexity, and subsequently the ITDM to model lateness of deliveries more realistically. Since the ITDM represents the most successful theater distribution modeling attempt to date, it is of particular interest for this research. The ITDM attempts to find an optimum allocation of requirements to vehicles in an on-time, least-cost manner, just as the RTDM does. The only difference being, that the ITDM is formulated as a mixed-integer linear programming model, while the RTDM is a pure integer programming model. This formulation is necessary as payloads for deliveries vary for each vehicle allocation. Thus, assigning the same penalty for two late deliveries with the same Type of cargo and differing payload sizes does not make practical sense. The late delivery containing more short tons of delivery should be assigned a higher penalty. For this reason, continuous decision variables that represent the number of short tons being delivered are introduced into the model. This difference in formulation ensures that

late deliveries of requirements given by a TPFDD are measured on a per short ton scale rather than per vehicle.

With the ITDM, users must select which modes of transportation and vehicle types they wish to enter into the model. The individual Modes  $m \in M$  will typically contain all or some elements of the set  $\{\text{Air, Road, Rail}\}$ . Vehicle types are selected by the user to form a set of vehicle Types  $K$ . Each vehicle Type  $k \in K$  is a specific vehicle (e.g. C-17) of a single Mode  $m$ , and has two input parameters associated with it. The first parameter is the daily cost of utilizing vehicle Type  $k$ ,  $b_k$ . This cost could be financial in nature, but it may also be utilized as an arbitrary cost in order to analyze the impact certain policy decisions have upon solutions. The second parameter is  $p_k$ , the average payload (measured in short tons) of a vehicle of Type  $k$  (Hafich, 2013). These parameters are essential to the model.

The ITDM draws in data from the TPFDD being used for force flow analysis. Each TPFDD will list a set of Requirements  $n \in N$  with  $N$  being the number of requirements listed in said TPFDD. Each Requirement  $n$  also has an associated POD  $i \in I$  and Destination  $j \in J$  where  $I$  and  $J$  represent the set of all PODs and destinations, respectively. Every delivery of Requirement  $n$  also has an associated delivery Day  $v$  on which it may be delivered to its final destination. The set  $V$  comprises the set of all possible delivery days on which Requirement  $n$  may be delivered to its specified Destination  $j$ . Each movement Requirement  $n$ , to be delivered from POD  $i$  to Destination  $j$ , has a requirement weight  $r_{nij}$  which is measured in short tons. Within the model, it is assumed that all requirements are standard cargo requirements. Passenger requirements and any potential restrictions on outsize or oversize cargo are ignored (Hafich, 2013).

Some vehicle Modes  $m$  may not have a direct path between POD  $i$  and Destination  $j$  supporting that vehicle mode. Thus,  $M_{ij}$ , the set of all Modes  $m$  with direct paths between POD  $i$  and Destination  $j$  is defined to account for those vehicle modes with no direct path between a certain POD  $i$  and Destination  $j$ . this set reduces the number of variables created by the model. Within the ITDM,  $K_m$  represents the set of all vehicle Types  $k$  which are also of Mode  $m$ . additionally, since the TPFDD outlines time windows within which Requirements  $n$  may be picked up at POD  $i$  and delivered to Destination  $j$ , not all days  $v$  within the set  $V$  are necessarily eligible delivery days for Requirement  $n$ . To reduce the number of variables further, the set  $N_{ijv}$  of Requirements  $n$  that are eligible to deliver from POD  $i$  to Destination  $j$  on Day  $v$  is defined. All of these sets described are decomposing sets within the model. These decomposing sets are easily determined with preprocessing and are of great value in reducing problem size by eliminating extraneous decision variable creation within constraints.

The delivery time windows and associated parameters for the EAD and RDD require further discussion. All of this information is also given by data in a TPFDD. The variable  $ad_n$  specifies the day in which Requirement  $n$  arrives at its given POD. It is assumed that Requirement  $n$  may not be picked up for delivery until the Day after  $ad_n$ . Thus, it is not possible for this requirement to be picked up until Day  $ad_n+1$ . Similarly, the variable  $rd_n$  specifies the Required Delivery Date (RDD), or the day in which Requirement  $n$  must be delivered to Destination  $j$ . The RDD however is not an absolute deadline for Requirement  $n$ . Thus, requirements may be delivered beyond their RDD. The parameter  $qd_n$  is defined as the maximum allowable extension days beyond the RDD in which Requirement  $n$  can be delivered to its given final Destination,

$j$ . Requirements delivered outside the time window created by parameters  $ad_n+1$  and  $rd_n$  incur a per short ton late penalty,  $g$ , which is user specified depending on preferences. Within the ITDM, the penalty variable  $g$  actually represents the late penalty per short ton per day delivered late.

The set  $V$  mentioned previously comprises all Days  $v$  within the time window described by the minimum of  $ad_n+1$  and the maximum of  $rd_n+qd_n$ . No deliveries within a given TPFDD may be made outside of this minimum-maximum window, and these extraneous decision variables should not be created by the model. This explains the reasoning behind defining the set of valid Days  $V$ . Additionally, the ITDM allows aggregation of requirements if they fall within the same delivery time window, doing away with the need for one vehicle to be assigned to each single requirement.

Most theater distribution models discussed thus far capture the limitations on daily outloading at PODs and unloading at destinations. The ITDM is no exception. In fact, variables are created to describe the maximum number of Mode  $m$  vehicles that can be outloaded at POD  $i$  on Day  $v$ , given by  $o_{imv}$ , and the maximum number of Mode  $m$  vehicles that can be unloaded at Destination  $j$  on Day  $v$ , given by  $u_{jmv}$ . This allows the user to define outload and unload restrictions at locations in theater based on real world scenarios and actualities, which can be provided by experts in the field. This allows flexibility as POD conditions certainly change over time. Since some PODs and destinations do not support certain Modes  $m$ ,  $o_{imv}$  and  $u_{jmv}$  will take on a value of zero in certain cases.

The decision variables in the ITDM are of two types. Variables  $x_{ijmkv}$  describe the number of vehicles of Mode  $m$ , Type  $k$  that are required on Day  $v$  to deliver any

requirements from POD  $i$  to Destination  $j$ . Notice that this general integer variable is not tied to any one Requirement  $n$ . Thus, the vehicle allocations dictated by decision variables  $x_{ijmkv}$  may embody the movement of one, or many different requirements (Hafich, 2013). To allow for aggregation of multiple requirements on a single vehicle, the decision variables  $y_{nijmkv}$  are introduced. They represent the number of short tons of requirement  $n$  delivered from POD  $i$  to Destination  $j$  on Mode  $m$ , Type  $k$  vehicle(s) on Day  $v$ . these variable are inherently related, because for every short ton of Requirement  $n$  delivered from POD  $i$  to Destination  $j$  on Mode  $m$ , Type  $k$  vehicle(s) on Day  $v$ , some vehicle must be assigned to make that delivery. The linking constraints described later ensure this requirement is met.

To fully understand the nature of the ITDM and its effectiveness at reducing the complexity of the Longhorn and Kovich TDM, its Function Derived Tuple Sets should be discussed. These are:  $VV$ ,  $VF$ ,  $LF$ ,  $VR$ ,  $VO$ , and  $VU$ . These tuple sets are derived from seven binary set defining functions, included in (11)-(17) below.

#### **ITDM Functions** (Hafich, 2013)

$$A(n, v) = \begin{cases} 1, & \text{if Movement } n \text{ delivered on Day } v \text{ would be on -- time} \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

$$B(n, v) = \begin{cases} 1, & \text{if Movement } n \text{ delivered on Day } v \text{ would be late} \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

$$C(m, k) = \begin{cases} 1, & \text{if vehicle of Type } k \text{ is a also a Mode } m \text{ vehicle} \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

$$D(n, i, j) = \begin{cases} 1, & \text{if Movement } n \text{ is to be delivered from POD } i \text{ to Destination } j \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

$$E(i, m, v) = \begin{cases} 1, & \text{if } \exists \text{ a requirement } n \text{ that may outload at POD } i \text{ onto a} \\ & \text{Mode } m \text{ vehicle on Day } v \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

$$F(j, m, v) = \begin{cases} 1, & \text{if } \exists \text{ a requirement } n \text{ that may unload at Destination } j \text{ onto a} \\ & \text{Mode } m \text{ vehicle on Day } v \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

$$G(i, j, v) = \begin{cases} 1, & \text{if } \exists \text{ a requirement } n, \text{ from POD } i \text{ to Destination } j \\ & \text{s. t. } adn + 1 \leq v \leq rdn + qdn \\ 0, & \text{otherwise} \end{cases} \quad (17)$$

The set of tuples in  $VV$ , where  $VV = \{(i, j, m, k, v) \mid G(i, j, v) \cdot C(m, k) = 1\}$  corresponds to valid vehicle variables that may take on a value. Thus, a vehicle variable is created only when the 5-tuple  $(i, j, m, k, v)$  corresponds to a theoretically possible vehicle assignment (Hafich, 2012).

The continuous decision variables associated with flows of goods and personnel,  $y_{nijmkv}$ , motivate the necessity for two tuple sets in order to reduce the number of variables. The first, Valid Flows, defined as  $VF = \{(n, i, j, m, k, v) \mid A(n, v) \cdot C(m, k) \cdot D(n, i, j) + B(n, v) \cdot C(m, k) \cdot D(n, i, j)\}$  corresponds to decision variables that are defined only if they are valid on-time or late flows. The second, Late Flows correspond to valid flow decision variables that are associated with late shipments in theater to Destination  $j$ .

The three remaining tuple sets are Valid Routes (VR), Valid Outload (VO), and Valid Unload (VU). Since only one valid route exists for each Requirement  $n$  moving from POD  $i$  to Destination  $j$ , only a single 3-tuple exists for said requirement in the set  $VR = \{(n, i, j) \mid D(n, i, j) = 1\}$ .  $VO = \{(i, m, v) \mid E(i, m, v) = 1\}$  describes the set of 3-tuples that are defined only if Requirement  $n$  may outload at POD  $i$  onto vehicle Mode  $m$  on Day  $v$ . finally, the set VU, defined mathematically by  $VU = \{(j, m, v) \mid F(j, m, v) = 1\}$  is very similar to the Function Derived Tuple Set VO, with the difference being unloading at a Destination  $j$ . now that all parameters, sets, and decision variables have been described, the model formulation follows. Table 1 - Table 4 below summarize the sets, parameters, and variables utilized in the ITDM's pure integer programming formulation.

**Table 2. ITDM Basic Sets** (Hafich, 2013)

| <b>Set</b> | <b>Description</b>                                                                              |
|------------|-------------------------------------------------------------------------------------------------|
| $N$        | Set of all Movement Requirements $n$                                                            |
| $I$        | Set of all PODs $i$                                                                             |
| $J$        | Set of all Destinations $j$                                                                     |
| $M$        | Set of all vehicle Modes $m$                                                                    |
| $K$        | Set of all vehicle Types $k$                                                                    |
| $V$        | Set of all possible delivery Days $v$                                                           |
| $M_{ij}$   | Set of all Modes $m$ with direct paths between POD $i$ and Destination $j$                      |
| $K_m$      | Set of all vehicle Types $k$ which are of Mode $m$                                              |
| $N_{ijv}$  | Set of Requirements $n$ that are eligible to deliver from POD $i$ to Destination $j$ on Day $v$ |

**Table 3. ITDM Function Derived Tuple Sets (Hafich, 2013)**

| Set | Description      | Mathematical Notation                                                                                             |
|-----|------------------|-------------------------------------------------------------------------------------------------------------------|
| VV  | Valid Vehicle    | $\{(i, j, m, k, v) \mid G(i, j, v) \cdot C(m, k) = 1\}$                                                           |
| VF  | Valid Flows      | $\{(n, i, j, m, k, v) \mid A(n, v) \cdot C(m, k) \cdot D(n, i, j) + B(n, v) \cdot C(m, k) \cdot D(n, i, j) = 1\}$ |
| LF  | Late Flows       | $\{(n, i, j, m, k, v) \mid B(n, v) \cdot C(m, k) \cdot D(n, i, j) = 1\}$                                          |
| VR  | Valid Routes     | $\{(n, i, j) \mid D(n, i, j) = 1\}$                                                                               |
| VO  | Valid Outloading | $\{(i, m, v) \mid E(i, m, v) = 1\}$                                                                               |
| VU  | Valid Unloading  | $\{(j, m, v) \mid F(j, m, v) = 1\}$                                                                               |

**Table 4. ITDM Parameters (Hafich, 2013)**

| Parameter  | Description                                                                                                               |
|------------|---------------------------------------------------------------------------------------------------------------------------|
| $b_k$      | Daily operating cost for a Type $k$ vehicle                                                                               |
| $p_k$      | Average payload of Type $k$ vehicle                                                                                       |
| $r_{nij}$  | Total weight (in short tons) of Requirement $n$ to be delivered from POD $i$ to Destination $j$                           |
| $ad_n$     | Day in which Requirement $n$ arrives at its given POD                                                                     |
| $rd_n$     | Required Delivery Date (RDD) at the given Destination for Requirement $n$                                                 |
| $qd_n$     | Maximum allowable extension days beyond RDD in which Requirement $n$ can be delivered to given Destination (with penalty) |
| $g$        | Late penalty per Short Ton late per day                                                                                   |
| $o_{imv}$  | Maximum number of Mode $m$ vehicles that can be outloaded at POD $i$ on Day $v$                                           |
| $u_{jmv}$  | Maximum number of Mode $m$ vehicles that can be unloaded at Destination $j$ on Day $v$                                    |
| $w_{ijmk}$ | Daily cycles for a Mode $m$ , Type $k$ vehicle delivering from POD $i$ to Destination $j$                                 |

**Table 5. ITDM Decision Variables (Hafich, 2013)**

| Variables    | Description                                                                                                                      |
|--------------|----------------------------------------------------------------------------------------------------------------------------------|
| $x_{ijmk}$   | Number of vehicles of Mode $m$ , Type $k$ that are required on Day $v$ to deliver requirement(s) from POD $i$ to Destination $j$ |
| $y_{nijmkv}$ | Short tons of Requirement $n$ delivered from POD $i$ to Destination $j$ on Mode $m$ , Type $k$ vehicle(s) on Day $v$             |

The mixed integer programming formulation of the Improved Theater Distribution Model (ITDM) follows below in Model 3.

$$\text{Minimize } \sum_{(i,j,m,k,v) \in VV} b_k x_{ijmkv} + \sum_{(n,i,j,m,k,v) \in LF} g(v - rd_n) y_{nijmkv} \quad (18)$$

Subject to

$$\sum_{M_{ij}} \sum_{K_m} \sum_{v=ad_n+1}^{rd_n+q_n} y_{nijmkv} = r_{nij} \quad \forall (n,i,j) \in VR \quad (19)$$

$$\sum_J \sum_{K_m} w_{ijmk} x_{ijmkv} \leq o_{imv} \quad \forall (i,m,v) \in VO \quad (20)$$

$$\sum_I \sum_{K_m} w_{ijmk} x_{ijmkv} \leq u_{jmv} \quad \forall (j,m,v) \in VU \quad (21)$$

$$\sum_{N_{jv}} y_{nijmkv} \leq x_{ijmkv} w_{ijmk} p_k \quad \forall (i,j,m,k,v) \in VV \quad (22)$$

$$y_{nijmkv} \geq 0 \quad \forall (n,i,j,m,k,v) \in VF \quad (23)$$

$$x_{ijmkv} \in \{0\} \cup \mathbb{Z}^+ \quad \forall (i,j,m,k,v) \in VV \quad (24)$$

### Model 3. Improved Theater Distribution Model (ITDM) (Hafich, 2013)

The ITDM represents a significant improvement upon previous theater distribution modeling, specifically when it comes to integer programming formulations. The objective function at (18) utilizes the integer variables  $x_{ijmkv}$ , which describe the number of vehicles

prescribed by the model, and the continuous flow variables  $y_{nijmkv}$ , which describe the number of short tons of a requirement delivered on a specific day. This objective utilizes these variables to minimize not only the cost of operating the vehicles prescribed, but also the total penalty costs accrued from late deliveries, measured in per short ton late as opposed to per vehicle. This per short ton measurement introduced by the continuous variables certainly allows for more realistic modeling of theater distribution because it does not tie vehicles to specific requirements. In other words, a single vehicle can carry portions of several requirements on one trip instead of being tied to just one requirement.

Constraints at (19) ensure that the total sum of the weight (short ton flow variables) associated with a requirement equals the actual weight of that requirement. Constraints at (20) and (21) ensure that the user specified outloading and unloading restrictions at certain locations are not exceeded.

Constraints at (22) link the continuous flow variables with the integer vehicle variables. This constraint ensure that for all decisions corresponding to matching  $(i,j,m,k,v)$  values, enough vehicles are allocated to provide transportation capacity for appropriate requirements included as part of those flows (Hafich, 2013). As mentioned before, this formulation allows vehicles to hold cargo from several requirements if necessary. In addition, it allows late cargo from requirements to be delivered with on-time cargo from requirements. Finally, constraints at (23) ensure that flow variables take on a nonnegative value, while constraints at (24) ensure vehicle variables take on a nonnegative, integer value.

### ***ITDM Beddown Approximation***

As previously discussed, from the outputs of the ITDM, beddown approximations

can be made for PODs in theater. This is accomplished by post-processing the solutions. Mathematically, the beddown of vehicles of Mode  $m$ , Type  $k$ , needed at POD  $i$  can be approximated by: (Hafich, 2013)

$$Beddown_{imk} = \max_{v \in V} \left( \sum_j x_{ijmkv} \right) \quad (25)$$

This measure finds the maximum value of each vehicle Mode  $m$  and Type  $k$  allocation over all days on the outputted delivery schedule, which represents the number of this type of vehicles needed at POD  $i$  to make all deliveries. This measure assumes within the model that for vehicles utilized on Day  $v$ , these same vehicles will be available for use on all days following Day  $v$ . That is, they complete a full cycle encompassing the trip from POD  $i$  to Destination  $j$  and back to POD  $i$  again. Vehicles will thus be ready for use on the day following a delivery. This approach will be utilized for beddown analysis in the methodology and results sections of this research.

### ***ITDM Conclusion.***

The ITDM was formulated as a response to the TDMs shortfalls and overly complex formulation. Its purpose was to greatly reduce the size of the model by eliminating unnecessary variables and constraints, and to model reality more efficiently by measuring flow of goods in terms of short tons delivered. Both of these objectives were met successfully. However, the ITDM is lacking in beddown approximation efficiency. As stated, USTRANSCOM desires a tool to effectively estimate long term beddowns of airlift aircraft and ground vehicles at strategic theater locations. As it stands, test runs for the ITDM usually prescribe a large number of vehicles to accomplish delivery of all requirements in a given TPFDD. This translates to undesirably large beddown estimations,

as costs associated with vehicle beddown can be significant. While the ITDM is certainly an improvement on the TDM and other beddown approximation models discussed, there is room for further research in this area.

## **Conclusion**

Many of the approaches to solving the theater distribution problem mentioned such as prior airlift optimization techniques, the PDPTW, and Tabu search, although they have expanded research and provided insight, are simply not what force flow analysts at USTRANSCOM currently desire. To provide a useful tool for approximating beddown, a model should output vehicle types and the specific numbers of these vehicles necessary to support operations. Most modeling attempts prior to the ITDM focused mainly on the specific routing and scheduling of vehicles. As previously mentioned, changing battlefield conditions hinder this approach from being of use for analysis efforts, especially for beddown analysis. Thus, vehicles and transportation capability as output rather than input is much more useful for estimating beddown needs. This is precisely the difference between the ITDM and past modeling techniques, and the reason why it is a much more effective tool.

Most of these methods were formulated with the purpose of solving several aspects of the theater distribution problem simultaneously. Several attempts at providing models which solve the beddown issue specifically have been made. Many of these models, however, are focused on the effectiveness, capabilities, and costs of specific beddown scenarios already outlined rather than a mix of vehicles to support theater distribution. In addition, the management based beddown approaches discussed focus mainly on capabilities and infrastructure shortfalls given an inputted fleet of vehicles at specific

bases. Although in some cases, this approach could be helpful, analysts at force flow conferences are not particularly concerned with the costs of infrastructure shortfalls.

Although the ITDM provides exactly what USTRANSCOM needs in terms of output, further research into the beddown issue is necessary. As it stands, the number of vehicles needed to support theater distribution operations must be reduced in order to achieve a minimal beddown approximation. The methodologies outlined in this thesis aim to improve the mixed integer programming formulation of the ITDM by providing a reduced feasible vehicle output that will support TPFDD requirements. Details on the methodology of the improved model are included in Chapter III.

### **III. Methodology**

#### **Introduction**

This research is organized into three sections presenting different ways to modify the ITDM through additions to the objective function and the set of constraints in order to improve beddown approximations. Firstly, work is done to formulate the General Integer Beddown Reduction (GIBR) addition to the ITDM. This problem is centered on the creation of general integer variables within the objective function, and necessary constraints are outlined. Additionally, solutions are further improved by separately applying the Mini/Max Programming Beddown Reduction (MPBR). This approach tends to smooth vehicle allocation solutions when the ITDM is applied to a given TPFDD, providing a feasible, reduced across-the-board beddown approximation. The MPBR formulation is tested individually, and results are compared with those of the original ITDM for verification purposes. Lastly, MPBR is reformulated with all original mathematical notations to provide a multi-objective concept for force flow analysts. Finally, analysis is conducted on these proposed additions to the ITDM.

#### **Assumptions**

A large number of the model assumptions are outlined by Longhorn and Kovich (2012) and Hafich (2013). Interested readers are referred to their research for these assumptions. Several, however, are worth mentioning here. First, it is assumed that once a vehicle is moved into theater at a certain POD, it is available for use on all subsequent days. This applies to all vehicles at all PODs within the theater of interest. It is also assumed that vehicles utilized for delivery of requirements return only to their original

beddown location. That is, vehicles may not be placed at multiple PODs, nor will they deliver to multiple destinations within a single trip. In reality, vehicles may be able to deliver to multiple destinations before returning to a POD, but this will not be considered for the purposes of this research. Additionally, beddowns at destinations are not considered with this research. It is assumed that the cost of placing a vehicle in theater is much greater than this vehicle's daily operating cost, which generally holds true. Almost trivially, it is assumed that when a vehicle is prescribed to perform delivery of a Requirement  $n$ , this vehicle must already be located at the originating POD. Thus, the number of vehicles of a certain Mode  $m$ , Type  $k$  must be sufficient to make all deliveries outlined in the ITDM solution. Simply put, distribution of requirements is not possible unless the necessary vehicles are beddown at each POD considered in the TPFDD.

### **General Integer Beddown Reduction (GIBR)**

#### ***GIBR Motivation.***

Recall that the ITDM seeks to provide an optimal cost of delivering all Requirements  $n$  in a TPFDD by minimizing the combination of vehicle operating costs and late delivery costs. The ITDM does not, however, attempt to reduce the addition of excess vehicles on days subsequent to Day  $v$ . Current solutions tend to prescribe delivery of requirements utilizing a varied number of vehicles, which often times increase as the time window progresses. Realistically, once vehicles are beddown at a particular POD, they should be utilized as much as possible as long as requirements are available. Instead of adding more vehicles of the same or even a different Type  $k$ , these vehicles should be utilized over and over. From a beddown standpoint, efficiency means utilizing vehicles repetitively. The ITDM simply does not recognize the profitability to be gained by

utilizing the same vehicles repeatedly and thus reducing the number of vehicles and maintenance personnel necessary to meet delivery needs.

To illustrate this concept, consider the objective function of the original ITDM from Chapter II outlined in (18).

$$\text{Minimize } \sum_{(i,j,m,k,v) \in VV} b_k x_{ijmkv} + \sum_{(n,i,j,m,k,v) \in LF} g(v - rd_n) y_{nijmkv} \quad (18)$$

Recall that the first portion of the objective function (18),

$$\sum_{(i,j,m,k,v) \in VV} b_k x_{ijmkv}$$

attempts to minimize the cumulative operating cost of all vehicles,  $x_{ijmkv}$ , by utilizing the daily vehicle operating cost,  $b_k$ , for a vehicle Type  $k$ . Daily operation costs for a vehicle of Type  $k$  are constant regardless of day. Thus,  $b_k$  is the same on Day  $v$  as it is on Day  $v+1$ ,  $v+2$ , and all subsequent days. To reduce beddown increases and to ensure that vehicles are utilized efficiently once in theater, some penalty should be incurred for increasing vehicles of Mode  $m$ , Type  $k$  after Day  $v$  within a TPFDD time window. For example, suppose  $x_{i1-j1-AIR-C130-1}$  is prescribed by the ITDM as 10. That is, 10 Mode *AIR*, type *C-130* are required to deliver any available requirements from POD *i1* to Destination *j1* on Day 1. Now, since these 10 C-130s are flown from POD *i1* on day 1, they should be utilized at POD *i1* as much as possible on day 2, day 3, and so forth as long as Requirements  $n$  exist to be delivered. If for instance  $x_{i1-j1-AIR-C130-1}$ , which is a Day 2 allocation, is prescribed as 12, the penalty is incurred. If this penalty is large enough, the result should be reduction or elimination of vehicle beddown additions. This reutilization

of vehicles should result in more efficiency on Day  $v+1$  and subsequent days, and ultimately require a smaller beddown.

Recall also that the second part of the objective function at (18),

$$\sum_{(n,i,j,m,k,v) \in LF} g(v - rd_n) y_{nijmkv}$$

seeks to minimize the costs incurred by delivering portions of requirements late. If anything, this drives the model to deliver requirements quickly to avoid incurring any late penalties,  $g$ , especially if  $g$  is large. The issue is that the ITDM *might* avoid increasing beddown of vehicle Mode  $m$ , Type  $k$  at POD  $i$ , but nothing in the objective function pushes it to do so. Since neither portion of the objective does this, solutions often show increases in vehicles on days subsequent to Day  $v$ . The ITDM lacks a key portion in the objective function that will help reduce the number of beddown increases once a beddown is set. The GIBR addition to Hafichs ITDM addresses inefficient use of extra vehicles of Mode  $m$ , Type  $k$  once a beddown is estimated on Day  $v$ . It also ensures that quicker beddown approximation can be taken directly from vehicles prescribed by the ITDM on Day  $v$ , the first day within a given time window ( $ad_n+1$  to  $rd_n + qd_n$ ), instead of calculating them. The next subsection will explain concepts developed for the GIBR before addition to the ITDM model formulation is given.

### ***GIBR Overview.***

To accomplish efficient utilization of vehicles it must be ensured that once a beddown of vehicles is estimated for a certain Day  $v$ , that for every subsequent Day ( $v+1$ ,  $v+2 \dots$ ), these same vehicles are utilized to deliver more Requirements  $n$  or portions of requirements. This ensures reuse of vehicles that are already in theater, thus reducing the

necessity of adding more to a given PODs beddown. In general, a beddown should be a long term solution that supplies enough vehicles to meet all delivery needs outlined in a given TPFDD.

The first improvement approach outlined in this work is an addition to the ITDM called the General Integer Beddown Reduction (GIBR). The GIBR involves the introduction of decision variables,  $z_{imkv}$ , which represents the number of vehicles of Mode  $m$ , Type  $k$  that are required to be beddown at POD  $i$  on Day  $v$  in order to move any eligible requirements. These variables stem directly from the ITDM integer variables,  $x_{ijmkv}$ , and define precisely what this research seeks to improve. As mentioned, one way to help ensure that in-theater vehicles are utilized on subsequent days is to define a penalty, or cost for situations when the number of beddown vehicles  $z_{imkv}$  is larger than the number of vehicles  $z_{imk(v-1)}$  on a previous day. This can be represented mathematically by positive values of the expression  $(z_{imkv} - z_{imk(v-1)})$ . The GIBR has a penalty for this situation defined by  $c_{mk}$ , which is the cost of bedding down an extra vehicle of mode  $m$ , type  $k$  at any POD on any day. It is important to note here that  $c_{mk}$  is constant for all PODs  $i$ . When added to the objective of the ITDM, the penalty for increasing a beddown can be shown as  $c_{mk}(z_{imkv} - z_{imk(v-1)})$ . Since the objective seeks to minimize values, when  $c_{mk}$  is large enough, the ITDM tends toward minimizing the value of  $(z_{imkv} - z_{imk(v-1)})$ . Thus, with this extra portion to the objective applied to the ITDM, increases in the beddown of vehicles of Mode  $m$ , Type  $k$  on days subsequent to Day  $v$  are reduced and possibly eliminated.

Because of its straightforward and elegant nature, the entire initial formulation of the ITDM is preserved and utilized in formulation of the MPBR. Readers are encouraged

to review Chapter II of this research for a thorough explanation of the ITDM and its mathematical components. It is important to realize that the GIBR is not a new formulation of the mixed integer programming model. It is primarily an addition to the objective function and inclusion of three new constraints involving relationships between the ITDM defined decision variables  $x_{ijmkv}$  and the GIBR defined decision variables  $z_{imkv}$ . All original aspects of the ITDM including parameters, decision variables, set defining binary functions, basic sets, and function derived tuple sets remain the same. In fact, several of these aspects are utilized when defining the formulation of the GIBR addition. The three sets utilized within the GIBR are the function derived tuple sets of Valid Vehicles,  $VV$ , and Valid Outloading,  $VO$ , and the basic set  $K_m$ . Recall that the in order to determine  $VV$ , the ITDM describes the set defining binary functions in (13) and (17). These functions are included below.

$$C(m, k) = \begin{cases} 1, & \text{if vehicle of Type } k \text{ is a also a Mode } m \text{ vehicle} \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

$$G(i, j, v) = \begin{cases} 1, & \text{if } \exists \text{ a requirement } n, \text{ from POD } i \text{ to Destination } j \\ & \text{s. t. } ad_n + 1 \leq v \leq rd_n + qd_n \\ 0, & \text{otherwise} \end{cases} \quad (17)$$

These binary functions are crucial in the creation of vehicle decision variables within the ITDM, which populate the set  $VV$ . The set of tuples in  $VV$ , where  $VV = \{(i, j, m, k, v) \mid G(i, j, v) \cdot C(m, k) = 1\}$ , includes those tuples which correspond to valid vehicle variables that may take on value within the mixed integer program. The

set of vehicle decision variables  $\forall(i, j, m, k, v) \in VV$  is used to describe summations in the new objective function as well as which constraints the updated model includes as valid. The set  $VO = \{(i, m, v) \mid E(i, m, v) = 1\}$  describes the set of 3-tuples that are defined only if requirement  $n$  may outload at POD  $i$  onto vehicle Mode  $m$  on any day. Detail on how this set is derived is given in (15) below.

$$E(i, m, v) = \begin{cases} 1, & \text{if } \exists \text{ a requirement } n \text{ that may outload at POD } i \text{ onto a} \\ & \text{Mode } m \text{ vehicle on Day } v \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

In addition, the GIBR employs one of the ITDM basic sets,  $K_m$ , the set of all vehicles that are of Mode  $m$ , Type  $k$ . The set  $K_m$  is included for describing summations in the new constraints and the objective function. It is important to note the beddown decision variables  $z_{imkv}$  need not be defined by specific POD  $i$ . They can also be described across the entire theater of operations including all PODs within a TPFDD utilized for analysis. In other words, if analysts are interested in estimating beddowns by Location  $i$ , the variables  $z_{imkv}$  would be included in the model. Further, if they are interested in the estimation of a theater wide beddown, the variables  $z_{mkv}$  are included. This describes the number of vehicles of Mode  $m$ , Type  $k$  that are required to be beddown in a specific theater (all PODs in TPFDD) on Day  $v$  to deliver any eligible requirements.

Several different variations to the formulation exist such as including variables  $z_{imv}$ , which represents the number of vehicles of Mode  $m$  required to be beddown at POD  $i$  on Day  $v$  to deliver eligible requirements. There is also  $z_{im}$ , which represents the number of vehicles of Mode  $m$  required to be beddown at POD  $i$  on any day to deliver any

eligible requirements. Redefining these decision variables depends on analysts desires.

Note that as the decision variables are indexed differently, the penalty parameter must be indexed differently as well. For the purpose of this research, only the formulation including the decision variables  $z_{imkv}$  and cost parameter  $c_{mk}$  is included and described in this chapter. The ITDM remains a mixed integer programming model, and any additions made to it are outlined and described in Models 3 and 4 below.

### **General Integer Beddown Reduction (GIBR) Addition.**

#### ***Initial ITDM Formulation.***

The ITDM, which is the initial model, is shown below in Model 3.

$$\text{Minimize } \sum_{(i,j,m,k,v) \in VV} b_k x_{ijmkv} + \sum_{(n,i,j,m,k,v) \in LF} g(v - rd_n) y_{nijmkv} \quad (18)$$

Subject to

$$\sum_{M_{ij}} \sum_{K_m} \sum_{v=ad_n+1}^{rd_n+q_n} y_{nijmkv} = r_{nij} \quad \forall (n,i,j) \in VR \quad (19)$$

$$\sum_J \sum_{K_m} w_{ijmk} x_{ijmkv} \leq o_{imv} \quad \forall (i,m,v) \in VO \quad (20)$$

$$\sum_I \sum_{K_m} w_{ijmk} x_{ijmkv} \leq u_{jmv} \quad \forall (j,m,v) \in VU \quad (21)$$

$$\sum_{N_{ijv}} y_{nijmkv} \leq x_{ijmkv} w_{ijmk} p_k \quad \forall (i,j,m,k,v) \in VV \quad (22)$$

$$y_{nijmkv} \geq 0 \quad \forall (n,i,j,m,k,v) \in VF \quad (23)$$

$$x_{ijmkv} \in \{0\} \cup \mathbb{Z}^+ \quad \forall (i,j,m,k,v) \in VV \quad (24)$$

### **Model 3. Improved Theater Distribution Model (ITDM) (Hafich, 2013)**

***ITDM With GIBR Addition (By Location  $i$ , Mode  $m$ , Type  $k$ , Day  $v$ ).***

The General Integer Beddown Reduction variation of the ITDM, which describes beddown decision variables by Location  $i$  is formulated below in Model 4. Tables 6 and 7 outline new parameters and decision variables. Any additions (26)-(29) to the ITDM are followed by an asterisk.

**Table 6. GIBR Parameters**

| <b>Parameter</b> | <b>Description</b>                                                                  |
|------------------|-------------------------------------------------------------------------------------|
| $c_{mk}$         | Penalty for bedding down extra vehicle of Mode $m$ , Type $k$ at any POD on any day |

**Table 7. GIBR Decision Variables**

| <b>Variable</b> | <b>Description</b>                                                                                                                |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------|
| $z_{imkv}$      | Number of vehicles of Mode $m$ , Type $k$ that are required to be beddown at POD $i$ on Day $v$ to move any eligible requirements |

***GIBR Formulation.***

The GIBR, which is the initial ITDM model with additions, is shown below in Model 4.

$$\begin{aligned} \text{Minimize } & \sum_{(i,j,m,k,v) \in VV} b_k x_{ijmkv} + \sum_{(n,i,j,m,k,v) \in LF} g(v - rd_n) y_{nijmkv} + \\ & \sum_{K_m} \sum_{(i,m,v) \in VO} c_{mk} (z_{imkv} - z_{imk(v-1)}) \quad * \end{aligned} \quad (26)$$

Subject to

$$\sum_{M_{ij}} \sum_{K_m} \sum_{v=ad_n+1}^{rd_n+q_n} y_{nijmkv} = r_{nij} \quad \forall (n,i,j) \in VR \quad (19)$$

$$\sum_J \sum_{K_m} w_{ijmk} x_{ijmkv} \leq o_{imv} \quad \forall (i,m,v) \in VO \quad (20)$$

$$\sum_I \sum_{K_m} w_{ijmk} x_{ijmkv} \leq u_{jmv} \quad \forall (j,m,v) \in VU \quad (21)$$

$$\sum_{N_{ijv}} y_{nijmkv} \leq x_{ijmkv} w_{ijmk} p_k \quad \forall (i,j,m,k,v) \in VV \quad (22)$$

$$\sum_{j:(i,j,m,k,v) \in VV} x_{ijmkv} \leq z_{imkv} \quad \forall (i,m,v) \in VO, (\forall k \in K_m)^* \quad (27)$$

$$z_{imkv} \geq z_{imk(v-1)} \quad \forall (i,m,v) \in VO, (\forall k \in K_m)^* \quad (28)$$

$$y_{nijmkv} \geq 0 \quad \forall (n,i,j,m,k,v) \in VF \quad (23)$$

$$z_{imkv} \in \{0\} \cup \mathbb{Z}^+ \quad \forall (i,m,v) \in VO, (\forall k \in K_m)^* \quad (29)$$

$$x_{ijmkv} \in \{0\} \cup \mathbb{Z}^+ \quad \forall (i,j,m,k,v) \in VV \quad (24)$$

**Model 4. ITDM With GIBR Addition (By Location  $i$ , Mode  $m$ , Type  $k$ , Day  $v$ ).**

The GIBR presents a significant improvement over ITDM beddown approximations. First, addition of the cost parameter  $c_{mk}$  ensures that some penalty is assessed in the objective function (26) when a Day  $v$  beddown,  $z_{imkv}$ , is increased on any day subsequent to  $v$ . If  $c_{mk}$  is set large enough, this penalty ensures that a beddown of vehicles of Mode  $m$ , Type  $k$  will not increase throughout a time window. Because a beddown will not increase, the model will utilize the vehicles already in theater more often, providing improved vehicle utilization and a more consistent vehicle beddown. It also provides simpler beddown estimation, as the maximum of vehicles would be the number of each type of vehicle prescribed on the first day in the time window.

The objective function is now threefold. It attempts to simultaneously minimize vehicle usage costs, late deliveries penalties, and beddown increase penalties. Later this research shows how manipulating the costs  $b_k$  and penalties  $g$  and  $c_{mk}$  can cause the model to achieve certain desired solutions. Again, this formulation of the GIBR is indexed by POD  $i$ , rather than across the whole theater of operations. Thus, it seeks to improve beddowns by specific location rather than by theater.

Three additional constraints are added to the model as well. Constraints at (27) ensure that whatever the number of vehicles prescribed by the model, the beddown is sufficient to meet these needs. Thus, beddown of vehicles at every POD  $i$  must be sufficient to meet delivery needs.

Constraints at (28) ensure that on Day  $v+1$  that there are at least  $z_{imkv}$  vehicles available. Any increase in  $z_{imkv}$  on days after  $v+1$  will be penalized by the objective function. That is, beddown of vehicle Mode  $m$  Type  $k$  will be consistent at a Location  $i$  throughout a given time window. Constraints at (29) ensure that beddown variables are

nonnegative and integer. This is necessary because a partial vehicle does not make sense. Thus, a beddown must be a positive integer.

### ***GIBR Addition Conclusion.***

The GIBR is the first of two main contributions to this thesis. The formulation, with the use of new parameters and decision variables, forces the ITDM to more efficiently utilize vehicles over a time window. The model now attempts to minimize spikes in the number of a certain type of vehicles later in a TPFDD time window, and to utilize vehicles already in theater to move requirements in that TPFDD. It also tells analysts when the best time to add vehicles to a beddown if in fact it becomes necessary to add vehicles. Thus, the GIBR provides beddown reductions for force flow analysis and can help save valuable vehicle resources which may be utilized elsewhere.

### **Mini/Max Programming Beddown Reduction (MPBR) Addition**

#### ***MPBR Motivation.***

The Mini/Max formulation is mathematically equivalent to the GIBR. Further, given the same penalty, it will have the same effect on the ITDM objective when solutions are found. The exception to this is when there are multiple optimal solutions. In this case, the exact same solution may not be output. The difference between the two models, however, is the number of variables needed to formulate the linear program since the MPBR problem is not indexed by  $v$ .

For this reason, some addition that minimizes the maximum of the number of vehicles with fewer variables is worth investigating. Traditionally, Mini/Max Programming is applied to problems such as this. Thus, an improvement known as the MPBR is developed to reduce beddown approximations and vehicle allocations. This addition is similar to the

GIBR in that it involves a single addition to the ITDM objective function as well as a few constraints.

Recall that the ITDM seeks to minimize the cost of delivery of all Requirements  $n \in N$  in a TPFDD by minimizing the addition of vehicle operating costs and late delivery costs. The ITDM does not, however, seek to reduce the size of the fleet necessary to accomplish this distribution. Current solutions tend to prescribe delivery of requirements quickly using a large fleet, instead of utilizing the full allowable time window and fulfilling requirements with a smaller fleet. The ITDM simply does not recognize the profitability to be gained by reducing the overall number of vehicles necessary to meet delivery needs. The result is solutions and subsequent beddown approximations that are often much larger than they need to be.

To illustrate this concept, consider again the objective function of the ITDM from Chapter II outlined in (18).

$$\text{Minimize } \sum_{(i,j,m,k,v) \in VV} b_k x_{ijmkv} + \sum_{(n,i,j,m,k,v) \in LF} g(v - rd_n) y_{nijmkv} \quad (18)$$

As previously discussed, the first portion of the objective function (18),

$$\sum_{(i,j,m,k,v) \in VV} b_k x_{ijmkv}$$

seeks to minimize the overall cost of utilizing all vehicles  $x_{ijmkv}$  by utilizing the daily vehicle operating cost  $b_k$  for a vehicle  $k$ . Daily operation costs for a vehicle Type  $k$  are constant regardless of day. Thus,  $b_k$  is the same on Day  $v$  as it is on Day  $v+1$  and all subsequent days.

Consider a delivery schedule that requires a Mode  $m$ , Type  $k$  vehicle 10 times with an allowable time window ( $ad_n+1$  to  $rd_n+qd_n$ ) of 5 days for delivery. Now, since  $b_k$  is constant across all Days  $v$ , the timeline of this delivery is irrelevant as long as all short tons of Requirements  $n \in N$  are delivered within the allowable time window. Assuming no late deliveries, there is no difference in operating cost for these requirements when delivering over 2 days using 5 vehicles, as opposed to delivering it over 5 days using 2 vehicles. As a result, the first part of the objective does not seek to minimize vehicles used overall, but seeks to minimize total cost of trips made by vehicles. The true difference is that realistically, a 5 vehicle beddown costs more than a 2 vehicle beddown of the same mode and type.

Recall again that the second part of the objective function (18),

$$\sum_{(n,i,j,m,k,v) \in LF} g(v - rd_n) y_{nijkv}$$

seeks to minimize the costs incurred by delivering portions of requirements late. If anything, this drives the model to deliver requirements as quickly as possible to avoid incurring any late penalties  $g$ , especially if  $g$  is large. The issue is that the ITDM *might* spread delivery over a given time window, but still, nothing in the objective function pushes it to do so. The MPBR addition to Hafich's ITDM addresses high cost beddown solutions with large numbers of vehicles. Keep in mind the formulation of the MPBR produces the same beddown solution as the GIBR, but eliminating subscript  $v$  in decision variables of the MPBR decreases the number of variables produced when building the model. In other words, these models are simply two formulations that accomplish the same solution. The next subsection explains concepts developed for the MPBR before addition

to the ITDM model formulation is given and described.

### ***MPBR Overview.***

The MPBR introduces a new cost parameter and a new set of decision variables in order to smooth vehicle solutions to the ITDM. Instead of assessing vehicle costs based solely on daily operation, a beddown cost,  $d_m$ , indexed by vehicle Mode  $m$  is utilized. As described previously, these extra costs provide a more realistic assessment of overall costs. In addition, decision variables  $Q_{im}$ , which represent the upper bound on the number of Mode  $m$  vehicles to be beddown at POD  $i$  on any day within the allotted time window are introduced. As with the GIBR, the MPBR is not a new formulation of the ITDM. It simply involves the addition of a portion to the objective function along with a few necessary constraints.

All original aspects of the ITDM including parameters, decision variables, set defining binary functions, basic sets, and function derived tuple sets remain the same. In fact, three of these aspects are utilized when defining the formulation of the MPBR addition. The tuple sets utilized by the MPBR are the set of Valid Outloading,  $VO$ , and the set of Valid Vehicles,  $VV$ . Detail on how these sets are derived is given in (15) previously. The basic set  $K_m$  is used in describing constraints.

In addition, a new tuple set Valid Beddown,  $VB$ , is defined in order to describe the added summation in the objective function. The set  $VB = \{(i, m) \mid H(i, m) = 1\}$  describes the set of 2-tuples that are defined only if Requirement  $n$  may outload at POD  $i$  onto vehicle Mode  $m$  on any day. Detail on how this set is derived is given in (30) below.

$$H(i, m) = \begin{cases} 1, & \text{if } \exists \text{ a requirement } n \text{ that may outload at POD } i \text{ onto a} \\ & \text{Mode } m \text{ vehicle on any day} \\ 0, & \text{otherwise} \end{cases} \quad (30)$$

The addition of the MPBR bears similarity to the GIBR addition. Both attempt to accomplish the same effects on the ITDM, and both involve a single addition to the objective function and a few extra constraints. Again, definition of the beddown Mini/Max decision variables  $Q_{im}$  need not be defined by specific POD  $i$ . They can also be described across the entire theater of operations including all PODs in a TPFDD utilized for analysis. In other words, if analysts are interested in minimizing the maximum on beddowns of vehicle Mode  $m$  by Location  $i$ , the variables  $Q_{im}$  would be included in the model. Further, if they are interested in doing this on a theater wide beddown of vehicle Mode  $m$ , the variables  $Q_m$  are included. This describes the upper bound of vehicles of Mode  $m$  to be beddown in a specific theater (all PODs in TPFDD) on any day. Several more variations of the Mini/Max Programming decision variables exist, such as  $Q_{imk}$  which minimizes the upper bound on the number of vehicles of Mode  $m$ , Type  $k$  at POD  $i$  on any day, and  $Q_{mk}$ , which minimizes the upper bound on the number of vehicles of Mode  $m$ , Type  $k$  at any POD on any day. Note that as the decision variables are indexed differently, the cost parameter  $d$  must be indexed differently as well. For the purposes of this research only the formulations including variables  $Q_{im}$  and  $Q_m$  and cost parameter  $d_m$  will be described. The  $Q_{im}$  defined model is given first. Any additions made to the ITDM is outlined and described in Model 5 below. See Model 3 for information on and formulation of the ITDM.

### Mini/Max Programming Beddown Reduction (MPBR)

The Mini/Max Programming Beddown Reduction variation of the ITDM which describes goal variables by Location  $i$  and Mode  $m$  is formulated below in Model 5.

Tables 8 and 9 outline new parameters and decision variables. Additions to the ITDM (31)-(33) are followed by an asterisk.

#### *ITDM With MPBR Addition (by Location $i$ , Mode $m$ ).*

**Table 8. MPBR Parameters**

| Parameter | Description                                           |
|-----------|-------------------------------------------------------|
| $d_m$     | Cost of bedding down a vehicle of Mode $m$ at any POD |

**Table 9. MPBR Decision Variables**

| Variable | Description                                                                    |
|----------|--------------------------------------------------------------------------------|
| $Q_{im}$ | Upper bound on number of Mode $m$ vehicles to be beddown at POD $i$ on any day |

#### ***MPBR Formulation.***

The MPBR, which is the ITDM model with Min/Max additions, is shown below in Model 5.

$$\begin{aligned} \text{Minimize} \quad & \sum_{(i,j,m,k,v) \in VV} b_k x_{ijmkv} + \sum_{(n,i,j,m,k,v) \in LF} g(v - rd_n) y_{nijmkv} \\ & \sum_{(i,m) \in VB} d_m Q_{im} \quad * \end{aligned} \quad (31)$$

Subject to

$$\sum_{M_{ij}} \sum_{K_m} \sum_{v=ad_n+1}^{rd_n+q_n} y_{nijmkv} = r_{nij} \quad \forall (n,i,j) \in VR \quad (19)$$

$$\sum_J \sum_{K_m} w_{ijmk} x_{ijmkv} \leq o_{imv} \quad \forall (i,m,v) \in VO \quad (20)$$

$$\sum_I \sum_{K_m} w_{ijmk} x_{ijmkv} \leq u_{jmv} \quad \forall (j,m,v) \in VU \quad (21)$$

$$\sum_{N_{ijv}} y_{nijmkv} \leq x_{ijmkv} w_{ijmk} p_k \quad \forall (i,j,m,k,v) \in VV \quad (22)$$

$$\sum_{j:(i,j,m,k,v) \in VV} x_{ijmkv} \leq Q_{im} \quad \forall (i,m,v) \in VO, (\forall k \in K_m) \quad * \quad (32)$$

$$y_{nijmkv} \geq 0 \quad \forall (n,i,j,m,k,v) \in VF \quad (23)$$

$$Q_{im} \in \{0\} \cup Z^+ \quad \forall (i,m) \in VB \quad * \quad (33)$$

$$x_{ijmkv} \in \{0\} \cup Z^+ \quad \forall (i,j,m,k,v) \in VV \quad (24)$$

### Model 5. ITDM With MPBR Addition (By Location $i$ , Mode $m$ )

Addition of the MPBR presents another significant improvement on the beddown approximations of the ITDM, which stem directly from its vehicle allocation solutions. First, an addition to the objective function (31) ensures that a cost is assessed whenever a vehicle of Mode  $m$  is beddown at a POD. When this cost is appropriately set, this portion of the objective function ensures a smooth mix of vehicle allocations across a given time window. The objective function is now threefold. It attempts to simultaneously minimize vehicle usage costs, late delivery penalties, and maximum beddown costs of vehicle Mode  $m$ . We will see later on how manipulating the costs  $b_k$  and  $d_m$ , and penalty  $g$  can cause the model to achieve certain desired solutions.

Two sets of constraints are added to the ITDM. Constraints at (32) ensure that the number of vehicles required to make all deliveries from a POD  $i$  is less than the maximum of vehicles at a specific location. When these constraints are combined with (30) in the objective function, the model attempts to minimize the maximum number of vehicles of Mode  $m$  at POD  $i$  prescribed in a solution, resulting in a reduced beddown approximation. Constraints at (33) ensure that maximum goal variables are non-negative and integer, and are required.

### ***MPBR Conclusion.***

The MPBR is the second contribution of this thesis. When combined with the initial formulation of the ITDM, the MPBR provides better beddown solutions for force flow analysts to work with. Instead of solutions accounting only for daily vehicle costs, results are now based on the simultaneous minimization of daily costs and beddown costs. The model now attempts to give a minimal vehicle solution based on two cost objectives as well as the reduction of late deliveries. Thus, a more realistic theater distribution

modeling affect is achieved. While the MPBR and GIBR have different formulations, both achieve the same effect. For this reason, only MPBR test results are included in Chapter 4 of this research.

### **ITDM With MPBR By Location and Theater (A Multi-Objective Approach)**

#### ***MPBR Multi-Objective Approach Overview***

As discussed previously, the MPBR decision variables  $Q$  may be indexed by specific Location,  $Q_{im}$ , or by the entire theater of interest,  $Q_m$ . It turns out that both of these minimizations can be achieved simultaneously. This allows force flow analysts to set limits on the number vehicles of a specific Mode  $m$  at each POD  $i$ , as well as in theater,  $(\forall i \in I)$ .

As with previous models, all original aspects of the ITDM including parameters, decision variables, set defining binary functions, basic sets, and function derived tuple sets remain the same. In fact, several of these aspects are utilized when defining this formulation of the MPBR addition. One function derived tuple set utilized in the MPBR is the set of Valid Beddowns,  $VB$ , which appears in the objective function. Since the variables  $Q_m$  are defined across all PODs  $i$ , A new function derived tuple set is created for constraint formulation. The set  $VM = \{(m,v) \mid L(m,v) = 1\}$  describes the set of 2-tuples that are defined only if Requirement  $n$  may outload at any POD in theater onto vehicle Mode  $m$  on day  $v$ . Detail on how this set is derived is given in (34) below.

$$L(m, v) = \begin{cases} 1, & \text{if } \exists \text{ a requirement } n \text{ that may outload at any POD in theater onto a} \\ & \text{Mode } m \text{ vehicle on Day } v \\ 0, & \text{otherwise} \end{cases} \quad (34)$$

In addition, the formulation employs two of the ITDM basic sets. The basic sets

utilized in the MPBR addition are the set of all vehicles of Mode  $m$  that are also Type  $k$ ,  $K_m$ , and the set of all vehicles of Mode  $m$ ,  $M$ . The sets  $M$  and  $VB$  are employed in describing summations in the objective function while the sets  $K_m$  and  $VV$  help formulate necessary constraints. Two costs are now defined,  $d_{im}$  and  $d_m$ , which are indexed by location *and* theater, respectively. This Multi-Objective approach provides an example of how versatile the MPBR can be in terms of modeling policy driven preferences. Model formulation including both the MPBR objective by Location  $i$  and whole theater is shown below in Model 6. Tables 8 and 10 outline the parameters and decision variables.

**Table 8. MPBR Parameters**

| Parameter | Description                                                    |
|-----------|----------------------------------------------------------------|
| $d_m$     | Cost of bedding down a vehicle of Mode $m$ Type $k$ at any POD |
| $d_{im}$  | Cost of bedding down a vehicle of Mode $m$ Type $k$ at POD $i$ |

**Table 10. MPBR Decision Variables (Multi-Objective)**

| Variable | Description                                                                               |
|----------|-------------------------------------------------------------------------------------------|
| $Q_{im}$ | Upper bound on number of Mode $m$ vehicles to be beddown at POD $i$ on any day            |
| $Q_m$    | Upper bound on number of Mode $m$ vehicles to be beddown in theater (all PODs) on any day |

***MPBR Multi-Objective Formulation.***

The MPBR, the ITDM model with Min/Max additions by location *and* theater is shown below in Model 6.

$$\begin{aligned} \text{Minimize } & \sum_{(i,j,m,k,v) \in VV} b_k x_{ijmkv} + \sum_{(n,i,j,m,k,v) \in LF} g(v - rd_n) y_{nijmkv} \\ & \sum_{(i,m) \in VB} d_{im} Q_{im} + \sum_M d_m Q_m \quad * \end{aligned} \quad (35)$$

Subject to

$$\sum_{M_{ij}} \sum_{K_m} \sum_{v=ad_n+1}^{rd_n+q_n} y_{nijmkv} = r_{nij} \quad \forall (n,i,j) \in VR \quad (19)$$

$$\sum_J \sum_{K_m} w_{ijmk} x_{ijmkv} \leq o_{imv} \quad \forall (i,m,v) \in VO \quad (20)$$

$$\sum_I \sum_{K_m} w_{ijmk} x_{ijmkv} \leq u_{jmv} \quad \forall (j,m,v) \in VU \quad (21)$$

$$\sum_{N_{ijv}} y_{nijmkv} \leq x_{ijmk} w_{ijmk} p_k \quad \forall (i,j,m,k,v) \in VV \quad (22)$$

$$\sum_{j:(i,j,m,k,v) \in VV} x_{ijmkv} \leq Q_{im} \quad \forall (i,m,v) \in VO, (\forall k \in K_m)^* \quad (32)$$

$$\sum_{i,j:(i,j,m,k,v) \in VV} x_{ijmkv} \leq Q_m \quad \forall (m,v) \in VM, (\forall k \in K_m)^* \quad (36)$$

$$y_{nijmkv} \geq 0 \quad \forall (n,i,j,m,k,v) \in VF \quad (23)$$

$$Q_{im} \in \{0\} \cup \mathbb{Z}^+ \quad \forall (i,m) \in VB \quad * \quad (33)$$

$$Q_m \in \{0\} \cup \mathbb{Z}^+ \quad \forall m \in M \quad * \quad (37)$$

$$x_{ijmkv} \in \{0\} \cup \mathbb{Z}^+ \quad \forall (i,j,m,k,v) \in VV \quad (24)$$

### Model 6. ITDM With MPBR Addition By Location and Theater

The MPBR formulation with multi-objective addition to the ITDM presents a technique that models two important aspects of the research described to this point. With

the addition of both objectives simultaneously seen in (35), and the new constraints at (32)-(33), (36)-(37), the ITDM provides a detailed mixed integer programming model for force flow analysts at USTRANSCOM to utilize. Objective additions at (35) ensure vehicle solutions are minimized by mode at each POD and by mode for the whole theater. The parameter  $d_{im}$  is indexed by Location  $i$  and allows the model to define differing beddown costs based on POD. This helps when a user desires to use fewer vehicles of a certain Mode  $m$  at a specific POD  $i$ .

Two sets of constraints are added to the ITDM. Constraints at (32) ensure that the number of vehicles required to make deliveries in a TPFDD by location is less than the maximum number of vehicles at that location. Constraints at (36) ensure that the number of vehicles required to make all deliveries in a TPFDD is less than the maximum on vehicles for the theater. These constraints help minimize location and theater wide beddown approximations, and allow analysts to set limits on measures. Constraints at (33), (37) ensure that decision variables are non-negative and integer.

As with other models presented in this research, this model can be manipulated to provide certain deliberate solutions based on current policy and changing objectives. Manipulation can be achieved by adjusting the values of  $b_k$ ,  $g$ , and  $d_m$ . The ITDM now attempts to reduce beddown by location, and seeks to minimize the maximum allocation of vehicles in theater as described in the MPBR addition. The formulation and basic mathematics of this cumulative model remain unchanged from the original MPBR model. It simply expands upon it. The goal here is to provide a tool that optimizes several realistic aspects of theater distribution simultaneously. Different variations of Min/Max decision variables could be easily added to the model, depending on vehicle needs and constraints.

## **Conclusion**

This chapter has extensively detailed the model additions developed in this research, namely the GIBR, the MPBR, and the Min/Max Multi-Objective approach. The improvements that each addition makes on the ITDM were also discussed. Approximating measures for beddowns are carried over from the original ITDM and remain unchanged. These measures will be utilized in depth in Chapter 4 when comparing original solutions to the ITDM with improved solutions. The next chapter of this thesis will entail implementation of the MPBR model addition on several different test cases.

## **IV. Implementation and Results**

### **Implementation.**

The ITDM developed by Hafich was implemented using both Microsoft Excel 2007 and the optimization software LINGO 13 (Lindo Systems Inc, 2012). This was done with a decision Support System (DSS) built in the excel domain. The DSS was organized such that a user uploads a TPFDD and enters all input parameters necessary to define the model. Once a TPFDD is selected and all parameters entered, the DSS uses Visual Basic for Applications (VBA) code to process data and write the mixed integer programming model in the LINGO 13 environment. The model is then solved by LINGO 13 and solution data is passed back to the Excel environment in a readable format. All original elements of this DSS were developed by Hafich with the assistance of Dr. Jeffery Weir of the Air Force Institute of Technology. This version of the DSS is relatively unchanged when solutions to the ITDM are referenced for comparison testing. Readers are encouraged to see Appendix I for ITDM VBA code updates utilized in this research.

To test whether the newly developed models in this research produce better solutions, VBA code additions were made to the DSS and implemented. These updates align directly with the MPBR mathematical changes made to the ITDM. Thus, the process of obtaining solutions to the math programming model via Excel VBA 2007 and LINGO 13 remained the same. The differences were the formulation of the model in the LINGO interface, and the resulting solutions from this formulation. All testing was conducted on a Lenovo Think Center M58 computer running Windows Vista (Service Pack 2) with two Intel Celeron 2.6GHz processors and 4 GB of RAM.

Because the MPBR increases model size and complexity, it often increases the time to find an optimal solution. This is because the Mini/Max Programming model tends to bounce back and forth between solutions within the branch and bound process. A relative optimality tolerance was set to encourage faster solutions, and the solver was set to search for solutions within 5% of the true optimal for one minute. If an optimal solution was not found within one minute, a feasible solution within 5% of the Linear Program Relaxation lower bound was reported as globally optimal. For consistency, the same relative optimality tolerance was used when testing both the ITDM and MPBR. Other settings imposed on LINGO 13 for this chapter are available for review in Appendix A.

### **Model Testing.**

For this analysis, ITDM beddown solutions were tested and compared with MPBR solutions for 6 test cases. All test cases were notional. The first four test cases involved varying vehicle mode and type constraints within the DSS. The fifth test case was carried out to analyze the effect of widening time windows in a notional TPFDD, while the last case looked at equal operating costs per short ton and possible policy driven solutions. Most solutions of the ITDM were found quickly, while the majority of MPBR solutions took the entire minute to solve.

For each test case, a smaller notional TPFDD was used and solutions compared. This was the exact TPFDD and data used as an example in the internal research paper by Longhorn & Kovich (2012). For Test Case 6 a similar but larger notional TPFDD was also implemented and results compared. For each case, information regarding beddown solutions was collected and reported. This beddown information was taken directly from

the nonzero vehicle allocation decision variables  $x_{ijmkv}$ . The smaller TPFDD used is shown in Table 11 below.

**Table 11. Smaller Notional TPFDD**

| Requirement | POD | Destination | Short Tons | EAD | RDD |
|-------------|-----|-------------|------------|-----|-----|
| 1           | i1  | j1          | 500        | 2   | 4   |
| 2           | i1  | j1          | 250        | 3   | 5   |
| 3           | i1  | j1          | 750        | 4   | 6   |
| 4           | i1  | j1          | 200        | 5   | 7   |
| 5           | i1  | j1          | 100        | 6   | 8   |
| 6           | i1  | j2          | 600        | 2   | 5   |
| 7           | i1  | j2          | 400        | 3   | 6   |
| 8           | i1  | j2          | 200        | 4   | 7   |
| 9           | i1  | j2          | 300        | 5   | 8   |
| 10          | i1  | j2          | 500        | 6   | 9   |
| 11          | i2  | j1          | 500        | 4   | 5   |
| 12          | i2  | j1          | 400        | 5   | 6   |
| 13          | i2  | j1          | 300        | 6   | 7   |
| 14          | i2  | j2          | 1000       | 3   | 5   |
| 15          | i2  | j2          | 200        | 5   | 7   |
| 16          | i2  | j2          | 500        | 7   | 9   |

The TPFDD lists 16 movement Requirements, two PODs, and two Destinations. Note that the Short Tons column gives the  $r_{nij}$  values, the EAD column gives the  $ad_n$  values, and the RDD column gives the  $rd_n$  values. Note also that the possible delivery days, including extension days, (i.e. the set  $V$ ) ranges between Day 3 and Day 10 (Hafich). The larger TPFDD, although too large to include here, is very similar to the smaller one but contains 64 Requirements, two PODs, and two Destinations.

Some constant assumptions were made throughout for consistency in model testing. First, it was assumed that  $w_{ijmk} = 1$  in all cases. That is, cycle values were always

set to one, meaning each vehicle could make a single pickup and delivery per day. For example, if  $w_{i1_j1\_AIR\_C-5} = 1$ , then it is possible for a C-5 to make a single delivery from  $i1$  to  $j1$ , return to  $i1$  the same day, and be available for use the following day. In addition, for most testing, an arbitrarily large bound (1000) was set on outloading and unloading parameters while bounds of 30 was set for others. Setting these bounds directly implies the beddown upper bound limits at each Location  $i$ . All of this information was clearly defined. For the purpose of this research, problem size (the number of variables and constraints) and objective function values were considered irrelevant and were not recorded. Additionally, all solutions produced were 100% on time as a result of a large late penalty value.

In all tests, it was assumed that requirements arrived at the POD on the EAD stated in the TPFDD and require one day of processing. Thus, for each requirement,  $ad_n$  is set to the requirement's EAD + 1 day. Additionally, every requirement was given a single extension day within all test cases. That is,  $qd_n = 1$  for all requirements (Hafich).

### **Deriving Beddown Solutions.**

Potential beddown solutions, as discussed in Chapter II, were derived from vehicle allocation variables using the equation

$$Beddown_{imk} = \max_{v \in V} \left( \sum_j x_{ijmkv} \right) \quad (25)$$

All results reported in this chapter were obtained by post-processing solutions from the DSS by way of equation (25).

**Test Case 1: Comparison Utilizing All Modes, All Types.**

As with all test cases, the first test case utilized the same TPFDD from the Longhorn & Kovich paper shown in Table 11. Here, all Modes (AIR, RAIL, ROAD), and all Types (C-130, C-17, C-5, HEMTT, M1083, M35, DODX, ITTX, and FTTX) of vehicles are used to investigate how the models react. The penalty per day per late short ton was set to  $g = 10,000$ . Daily cost and payload data were given by USTRANSCOM directly and are notional. The payload, cost, and unloading parameters used are shown in Appendix B. After the ITDM and ITDM with MPBR addition were tested on this case, model outputs were post-processed for beddown approximations and these solutions included in Tables 12 and 13 below.

**Table 12. Beddowns by Vehicle Type, POD For Test Case 1(Outload/Unload1000)**

| Model |           | C-130 | C-17 | C-5 | HEMT | M1083 | M35 | DODX | ITTX | FTTX | TOTAL      |
|-------|-----------|-------|------|-----|------|-------|-----|------|------|------|------------|
| ITDM  |           |       |      |     |      |       |     |      |      |      |            |
|       | <i>i1</i> | 0     | 0    | 0   | 0    | 0     | 131 | 0    | 0    | 0    | <b>131</b> |
|       | <i>i2</i> | 0     | 0    | 0   | 0    | 0     | 188 | 0    | 0    | 0    | <b>188</b> |
| MPBR  |           |       |      |     |      |       |     |      |      |      |            |
|       | <i>i1</i> | 1     | 1    | 1   | 1    | 1     | 1   | 0    | 1    | 4    | <b>11</b>  |
|       | <i>i2</i> | 1     | 2    | 5   | 1    | 1     | 0   | 0    | 1    | 4    | <b>15</b>  |

**Table 13. Beddowns by Vehicle Type, POD For Test Case 1 (Outload/Unload 30)**

| Model |           | C-130 | C-17 | C-5 | HEMT | M1083 | M35 | DODX | ITTX | FTTX | TOTAL     |
|-------|-----------|-------|------|-----|------|-------|-----|------|------|------|-----------|
| ITDM  |           |       |      |     |      |       |     |      |      |      |           |
|       | <i>i1</i> | 30    | 8    | 0   | 0    | 0     | 30  | 0    | 0    | 0    | <b>68</b> |
|       | <i>i2</i> | 30    | 12   | 0   | 0    | 0     | 30  | 0    | 0    | 0    | <b>72</b> |
| MPBR  |           |       |      |     |      |       |     |      |      |      |           |
|       | <i>i1</i> | 1     | 1    | 1   | 1    | 1     | 1   | 0    | 1    | 4    | <b>11</b> |
|       | <i>i2</i> | 1     | 2    | 5   | 1    | 1     | 0   | 0    | 1    | 4    | <b>15</b> |

Note that for both unload/outload bound settings (1000 and 30), solutions to the ITDM and MPBR are drastically different in terms of total vehicle beddowns. In the first case, the ITDM produced an all M35 solution with 131 M35s at POD  $i1$  and 188 M35s at POD  $i2$ . Meanwhile, the MPBR outputs a smooth distribution of vehicles with a total beddown of 11 vehicles at POD  $i1$  and 15 at POD  $i2$ , presenting a significant improvement over the ITDM. Delivery of requirements in the notional TPFDD is spread more efficiently over the allowable time window. For this test case, when outload/unload parameters are constrained at 30 as shown in Table 13, the MPBR still produced significantly reduced beddown results. Notice that the ITDM reached its upper bound on both C-130s and M35s. These unnecessarily large vehicle beddowns for the ITDM result from respective vehicle operating costs  $b_k$ . This is because the ITDM selects the cheapest vehicles in terms of  $b_k$  and attempts to move as many requirements as possible using these types of vehicles. Model output solutions from this case are included for reference in Figures 2 and 3 below.

|     |        |         |     |    |     |             |    |    |     |   |        |         |            |             |    |
|-----|--------|---------|-----|----|-----|-------------|----|----|-----|---|--------|---------|------------|-------------|----|
| 63  | M35(s) | leaving | POD | I1 | for | destination | J1 | on | day | 4 | (ROAD) | 500.00  | Short Tons | of Movement | 1  |
|     |        |         |     |    |     |             |    |    |     |   |        | 4.00    | Short Tons | of Movement | 2  |
| 50  | M35(s) | leaving | POD | I1 | for | destination | J2 | on | day | 4 | (ROAD) | 400.00  | Short Tons | of Movement | 7  |
| 31  | M35(s) | leaving | POD | I1 | for | destination | J1 | on | day | 5 | (ROAD) | 246.00  | Short Tons | of Movement | 2  |
|     |        |         |     |    |     |             |    |    |     |   |        | 2.00    | Short Tons | of Movement | 3  |
| 100 | M35(s) | leaving | POD | I1 | for | destination | J2 | on | day | 5 | (ROAD) | 600.00  | Short Tons | of Movement | 6  |
|     |        |         |     |    |     |             |    |    |     |   |        | 200.00  | Short Tons | of Movement | 8  |
| 94  | M35(s) | leaving | POD | I1 | for | destination | J1 | on | day | 6 | (ROAD) | 748.00  | Short Tons | of Movement | 3  |
|     |        |         |     |    |     |             |    |    |     |   |        | 4.00    | Short Tons | of Movement | 4  |
| 37  | M35(s) | leaving | POD | I1 | for | destination | J2 | on | day | 6 | (ROAD) | 296.00  | Short Tons | of Movement | 9  |
| 37  | M35(s) | leaving | POD | I1 | for | destination | J1 | on | day | 7 | (ROAD) | 196.00  | Short Tons | of Movement | 4  |
|     |        |         |     |    |     |             |    |    |     |   |        | 100.00  | Short Tons | of Movement | 5  |
| 63  | M35(s) | leaving | POD | I1 | for | destination | J2 | on | day | 7 | (ROAD) | 4.00    | Short Tons | of Movement | 9  |
|     |        |         |     |    |     |             |    |    |     |   |        | 500.00  | Short Tons | of Movement | 10 |
| 63  | M35(s) | leaving | POD | I2 | for | destination | J1 | on | day | 5 | (ROAD) | 500.00  | Short Tons | of Movement | 11 |
| 125 | M35(s) | leaving | POD | I2 | for | destination | J2 | on | day | 5 | (ROAD) | 1000.00 | Short Tons | of Movement | 14 |
| 50  | M35(s) | leaving | POD | I2 | for | destination | J1 | on | day | 6 | (ROAD) | 400.00  | Short Tons | of Movement | 12 |
| 25  | M35(s) | leaving | POD | I2 | for | destination | J2 | on | day | 6 | (ROAD) | 200.00  | Short Tons | of Movement | 15 |
| 38  | M35(s) | leaving | POD | I2 | for | destination | J1 | on | day | 7 | (ROAD) | 300.00  | Short Tons | of Movement | 13 |
| 63  | M35(s) | leaving | POD | I2 | for | destination | J2 | on | day | 9 | (ROAD) | 500.00  | Short Tons | of Movement | 16 |

**Figure 2. ITDM Case 1 Solution Test Case 1**

|           |             |    |     |             |    |    |     |          |                   |             |    |
|-----------|-------------|----|-----|-------------|----|----|-----|----------|-------------------|-------------|----|
| 1 C130(s) | leaving POD | I1 | for | destination | J2 | on | day | 3 (AIR)  |                   |             |    |
|           |             |    |     |             |    |    |     |          | 12.00 Short Tons  | of Movement | 6  |
| 1 C5(s)   | leaving POD | I1 | for | destination | J1 | on | day | 4 (AIR)  | 50.00 Short Tons  | of Movement | 1  |
|           |             |    |     |             |    |    |     |          | 10.00 Short Tons  | of Movement | 2  |
| 1 C5(s)   | leaving POD | I1 | for | destination | J2 | on | day | 5 (AIR)  | 59.00 Short Tons  | of Movement | 6  |
|           |             |    |     |             |    |    |     |          | 1.00 Short Tons   | of Movement | 8  |
| 1 C5(s)   | leaving POD | I1 | for | destination | J1 | on | day | 6 (AIR)  | 60.00 Short Tons  | of Movement | 3  |
| 1 C17(s)  | leaving POD | I1 | for | destination | J2 | on | day | 7 (AIR)  | 4.00 Short Tons   | of Movement | 9  |
|           |             |    |     |             |    |    |     |          | 31.00 Short Tons  | of Movement | 10 |
| 1 C130(s) | leaving POD | I1 | for | destination | J2 | on | day | 8 (AIR)  | 12.00 Short Tons  | of Movement | 10 |
| 2 C5(s)   | leaving POD | I2 | for | destination | J2 | on | day | 4 (AIR)  | 120.00 Short Tons | of Movement | 14 |
| 2 C17(s)  | leaving POD | I2 | for | destination | J1 | on | day | 5 (AIR)  | 50.00 Short Tons  | of Movement | 11 |
| 1 C17(s)  | leaving POD | I2 | for | destination | J2 | on | day | 5 (AIR)  | 35.00 Short Tons  | of Movement | 14 |
| 1 C5(s)   | leaving POD | I2 | for | destination | J2 | on | day | 5 (AIR)  | 60.00 Short Tons  | of Movement | 14 |
| 1 C130(s) | leaving POD | I2 | for | destination | J1 | on | day | 6 (AIR)  | 12.00 Short Tons  | of Movement | 12 |
| 1 C17(s)  | leaving POD | I2 | for | destination | J1 | on | day | 6 (AIR)  | 35.00 Short Tons  | of Movement | 12 |
| 1 C5(s)   | leaving POD | I2 | for | destination | J1 | on | day | 6 (AIR)  | 60.00 Short Tons  | of Movement | 12 |
| 1 C5(s)   | leaving POD | I2 | for | destination | J2 | on | day | 7 (AIR)  | 60.00 Short Tons  | of Movement | 15 |
| 5 C5(s)   | leaving POD | I2 | for | destination | J2 | on | day | 8 (AIR)  | 300.00 Short Tons | of Movement | 16 |
| 1 C17(s)  | leaving POD | I2 | for | destination | J2 | on | day | 9 (AIR)  | 35.00 Short Tons  | of Movement | 16 |
| 3 FTTX(s) | leaving POD | I1 | for | destination | J2 | on | day | 3 (RAIL) | 450.00 Short Tons | of Movement | 6  |
| 3 FTTX(s) | leaving POD | I1 | for | destination | J1 | on | day | 4 (RAIL) | 450.00 Short Tons | of Movement | 1  |
| 1 FTTX(s) | leaving POD | I1 | for | destination | J2 | on | day | 4 (RAIL) | 150.00 Short Tons | of Movement | 7  |
| 1 FTTX(s) | leaving POD | I1 | for | destination | J1 | on | day | 5 (RAIL) | 60.00 Short Tons  | of Movement | 2  |
|           |             |    |     |             |    |    |     |          | 90.00 Short Tons  | of Movement | 3  |
| 1 ITTX(s) | leaving POD | I1 | for | destination | J1 | on | day | 5 (RAIL) | 180.00 Short Tons | of Movement | 2  |
| 2 FTTX(s) | leaving POD | I1 | for | destination | J2 | on | day | 5 (RAIL) | 63.00 Short Tons  | of Movement | 6  |
|           |             |    |     |             |    |    |     |          | 237.00 Short Tons | of Movement | 7  |
| 4 FTTX(s) | leaving POD | I1 | for | destination | J1 | on | day | 6 (RAIL) | 600.00 Short Tons | of Movement | 3  |
| 2 FTTX(s) | leaving POD | I1 | for | destination | J1 | on | day | 7 (RAIL) | 200.00 Short Tons | of Movement | 4  |
|           |             |    |     |             |    |    |     |          | 100.00 Short Tons | of Movement | 5  |

**Figure 2. ITDM Case 1 Solution Test Case 1 (cont.)**

|            |             |    |                 |    |                 |                   |             |    |
|------------|-------------|----|-----------------|----|-----------------|-------------------|-------------|----|
| 2 FTTX(s)  | leaving POD | I1 | for destination | J2 | on day 7 (RAIL) | 199.00 Short Tons | of Movement | 8  |
|            |             |    |                 |    |                 | 101.00 Short Tons | of Movement | 9  |
| 1 ITTX(s)  | leaving POD | I1 | for destination | J2 | on day 8 (RAIL) | 180.00 Short Tons | of Movement | 9  |
| 3 FTTX(s)  | leaving POD | I1 | for destination | J2 | on day 9 (RAIL) | 450.00 Short Tons | of Movement | 10 |
| 4 FTTX(s)  | leaving POD | I2 | for destination | J2 | on day 4 (RAIL) | 600.00 Short Tons | of Movement | 14 |
| 1 ITTX(s)  | leaving POD | I2 | for destination | J2 | on day 4 (RAIL) | 180.00 Short Tons | of Movement | 14 |
| 3 FTTX(s)  | leaving POD | I2 | for destination | J1 | on day 5 (RAIL) | 450.00 Short Tons | of Movement | 11 |
| 2 FTTX(s)  | leaving POD | I2 | for destination | J1 | on day 6 (RAIL) | 293.00 Short Tons | of Movement | 12 |
| 2 FTTX(s)  | leaving POD | I2 | for destination | J1 | on day 7 (RAIL) | 300.00 Short Tons | of Movement | 13 |
| 1 FTTX(s)  | leaving POD | I2 | for destination | J2 | on day 7 (RAIL) | 140.00 Short Tons | of Movement | 15 |
| 1 FTTX(s)  | leaving POD | I2 | for destination | J2 | on day 8 (RAIL) | 150.00 Short Tons | of Movement | 16 |
| 1 M35(s)   | leaving POD | I1 | for destination | J2 | on day 3 (ROAD) | 8.00 Short Tons   | of Movement | 6  |
| 1 M1083(s) | leaving POD | I1 | for destination | J2 | on day 4 (ROAD) | 5.00 Short Tons   | of Movement | 7  |
| 1 M35(s)   | leaving POD | I1 | for destination | J2 | on day 5 (ROAD) | 8.00 Short Tons   | of Movement | 6  |
| 1 M35(s)   | leaving POD | I1 | for destination | J2 | on day 6 (ROAD) | 8.00 Short Tons   | of Movement | 7  |
| 1 M35(s)   | leaving POD | I1 | for destination | J2 | on day 7 (ROAD) | 8.00 Short Tons   | of Movement | 9  |
| 1 HEMTT(s) | leaving POD | I1 | for destination | J2 | on day 8 (ROAD) | 7.00 Short Tons   | of Movement | 9  |
| 1 HEMTT(s) | leaving POD | I1 | for destination | J2 | on day 9 (ROAD) | 7.00 Short Tons   | of Movement | 10 |
| 1 M1083(s) | leaving POD | I2 | for destination | J2 | on day 5 (ROAD) | 5.00 Short Tons   | of Movement | 14 |
| 2 M1083(s) | leaving POD | I2 | for destination | J2 | on day 8 (ROAD) | 10.00 Short Tons  | of Movement | 16 |
| 1 HEMTT(s) | leaving POD | I2 | for destination | J2 | on day 9 (ROAD) | 5.00 Short Tons   | of Movement | 16 |

**Figure 3. ITDM with MPBR Solution Test Case 1**

It is clear from the MPBR model output that requirement distribution has been smoothly spread over the time window for each requirement, ultimately resulting in a

reduced vehicle solution. Similar smoothing of delivery will be seen throughout this chapter when the MPBR addition is implemented in the remaining test cases.

***Test Case 2: Comparison Using All Modes, Single Type.***

The second test case involved constraining both models to include all Modes (AIR, RAIL, ROAD) but only a single Type (C-130, HEMTT, and DODX) within each Mode. The penalty per day per late short ton was set to  $g = 10,000$ . The same daily cost and payload data used for Test Case 1 is used here and will remain constant until Test Case 6 is presented. The payload, cost, and unloading parameters used are attached in Appendix C. After the ITDM and ITDM with MPBR addition were tested on this case, model outputs were post-processed for beddown approximations and these solutions included in Table 14 below.

**Table 14. Beddowns by All Mode, Type, POD Test Case 2 (Outload/Unload 1000)**

| Model |           | C-130 | HEMT | DODX | TOTAL      |
|-------|-----------|-------|------|------|------------|
| ITDM  |           |       |      |      |            |
|       | <i>i1</i> | 0     | 156  | 0    | <b>156</b> |
|       | <i>i2</i> | 0     | 214  | 0    | <b>214</b> |
| MPBR  |           |       |      |      |            |
|       | <i>i1</i> | 0     | 1    | 4    | <b>5</b>   |
|       | <i>i2</i> | 1     | 20   | 3    | <b>24</b>  |

When the ITDM and MPBR are tested with all modes by a single vehicle type with arbitrarily high outloading and unloading values (1000), beddown approximations are once again improved greatly. The ITDM proposes an all Road (HEMTT) solution as a result of this vehicle being the least expensive option in terms of operating costs  $b_k$ . POD *i1* requires 156 total vehicles to deliver requirements while POD *i2* requires a total of 214

vehicles. When compared to the MPBR beddown approximation of 5 total vehicles at POD *i1* and 24 vehicles at POD *i2*, it is clear that this beddown solution for all modes and a single type is largely reduced from that of the ITDM. Further, delivery of requirements in the notional TPFDD is spread more efficiently over the allowable time window. Model output solutions are included for reference in Appendix H.

***Test Case 3: Comparison Using a Single Mode.***

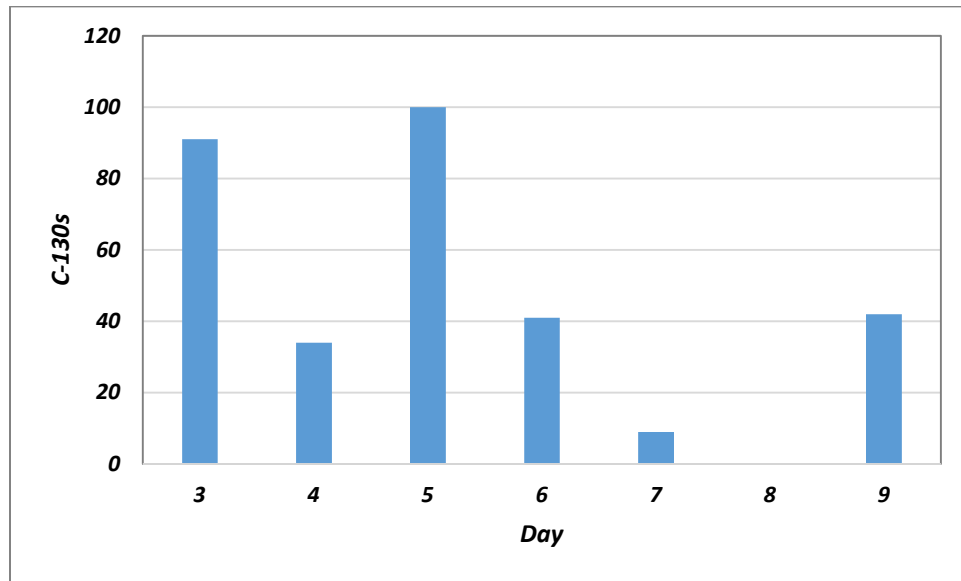
The third test case involved constraining both models to include a single Mode (AIR) and all Types (C-130, C-17, and C-5) within Mode AIR. The penalty per day per late short ton was still set to  $g = 10,000$ . Payload, cost, and unloading parameters used are shown in Appendix D. After the ITDM and ITDM with MPBR addition were tested on this case, model outputs were post-processed for beddown approximations and these solutions included in Table 15 below.

**Table 15. Beddowns by Single Mode, POD For Test Case 3(Outload/Unload 1000)**

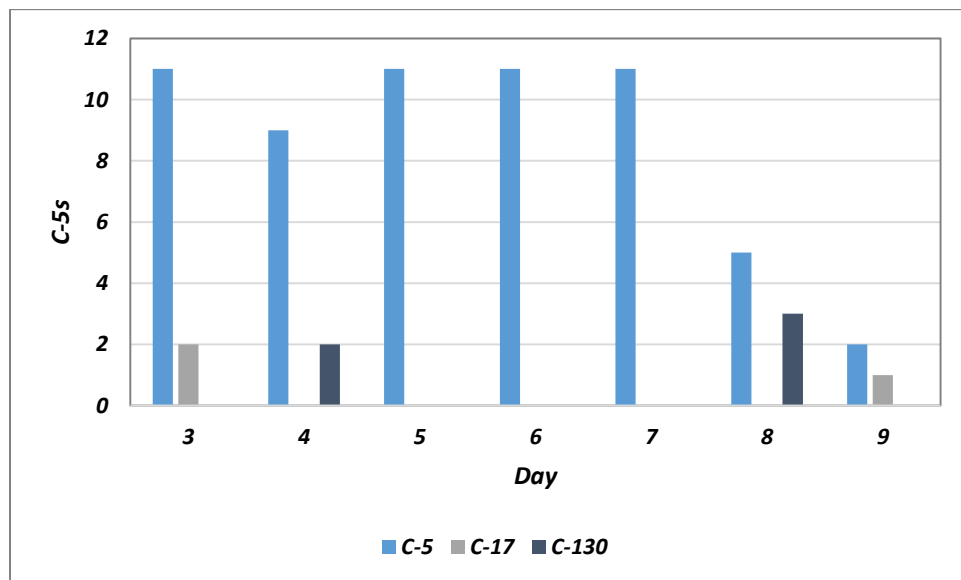
| Model |           | C-130 | C-17 | C-5 | TOTAL      |
|-------|-----------|-------|------|-----|------------|
| ITDM  |           |       |      |     |            |
|       | <i>i1</i> | 100   | 0    | 0   | <b>100</b> |
|       | <i>i2</i> | 126   | 0    | 0   | <b>126</b> |
| MPBR  |           |       |      |     |            |
|       | <i>i1</i> | 3     | 3    | 11  | <b>17</b>  |
|       | <i>i2</i> | 5     | 8    | 11  | <b>24</b>  |

The ITDM proposes an all C-130 solution as a result of this being the least expensive option in terms of operating costs  $b_k$ . POD *i1* requires 100 total AIR vehicles to deliver requirements while POD *i2* requires a total of 126 AIR vehicles. When compared to the MPBR beddown approximation of 17 AIR vehicles at POD *i1* and 24 AIR vehicles at POD *i2*, it can be inferred that this Mode AIR beddown solution is more

efficient and incurs a lower overall cost than that of the ITDM. To illustrate this fact, Figures 4-5 and Table 16 are included below. Table 16 shows total vehicle trips (sum of bars in Figures 4 and 5), which differs from the beddown approximations.



**Figure 4. ITDM C-130 Allocations By Day at POD *i1***



**Figure 5. MPBR Mode AIR Allocations By Day at POD *i1***

**Table 16. Vehicle Allocations and Cost Information  
at POD *i1* For Test Case 3**

| <b>Model</b> | <b>Type<br/><math>k</math></b> | <b>Daily Cost<br/><math>b_k</math></b> | <b>Total Vehicle<br/>Trips <math>\forall v</math><br/><math>\sum x_{ijmkv}</math></b> | <b>Total Operation<br/>Cost<br/><math>\sum b_k x_{ijmkv}</math></b> |
|--------------|--------------------------------|----------------------------------------|---------------------------------------------------------------------------------------|---------------------------------------------------------------------|
| ITDM         |                                |                                        |                                                                                       |                                                                     |
|              | C-130                          | 3                                      | 317                                                                                   | <b>951</b>                                                          |
|              | C-17                           | 9                                      | 0                                                                                     |                                                                     |
|              | C-5                            | 16                                     | 0                                                                                     |                                                                     |
| MPBR         |                                |                                        |                                                                                       |                                                                     |
|              | C-130                          | 3                                      | 5                                                                                     | <b>1,011</b>                                                        |
|              | C-17                           | 9                                      | 4                                                                                     |                                                                     |
|              | C-5                            | 16                                     | 60                                                                                    |                                                                     |

It remains to determine whether total cost in terms of vehicle trips and beddowns is reduced by the MPBR. Comparison of Figures 4 and 5 reveals that spikes in vehicle allocations is greatly reduced. There is a trade off, however, in terms of operating costs evidenced by Table 28. Although total operating costs increased slightly with the MPBR, its solution is more desirable as the theater wide beddown of aircraft was cut from 226 C-130s with ITDM to 41 various aircraft as seen in Table 15. Note, however, that the cost to beddown a C-130 is almost certainly cheaper than this cost for a C-5 or C-17. Thus, an all C-130 beddown solution should come with a lower cost than a mixed AIR beddown approximation equivalent in value. As with Test Cases 1 and 2, delivery of requirements in the notional TPFDD is spread more effectively over the allowable time window. Similar results can be expected for testing with other Modes (ROAD, RAIL). Model outputs are included for reference in Appendix H.

***Test Case 4: Comparison Using a Single Mode, Single Type.***

The fourth test case involved constraining both models to include a single Mode (AIR) and a single Type (C-5) within Mode AIR. The penalty per day per late short ton was again set to  $g = 10,000$ . The payload, cost, and unloading parameters used are shown in Appendix E. After the ITDM and ITDM with MPBR addition were tested on this case, model outputs were post-processed for beddown approximations and these solutions are included in Table 17 below.

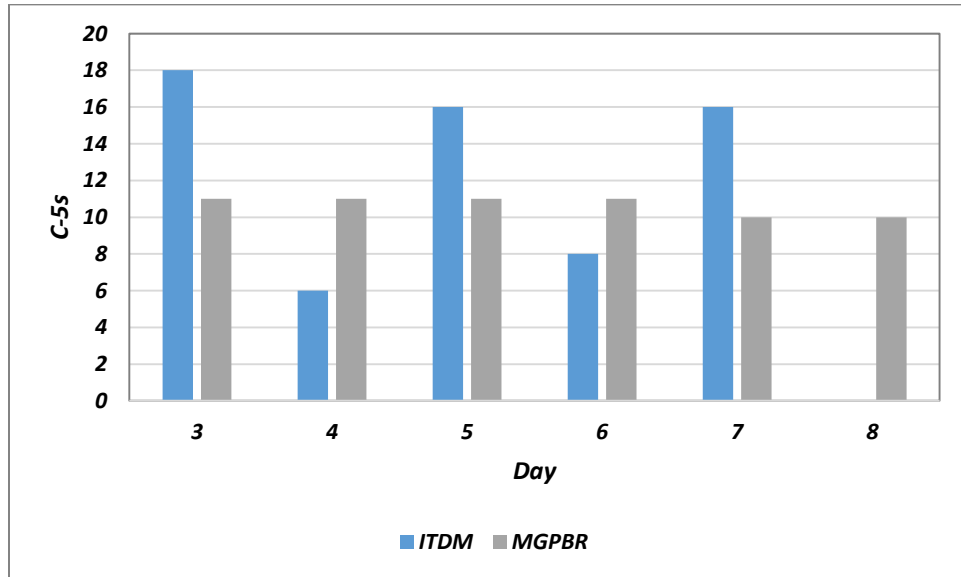
**Table 17. Beddowns by Single Type, POD For Test Case 4 (Outload/Unload 1000)**

| Model |           | C-5 | TOTAL     | THEATER   |
|-------|-----------|-----|-----------|-----------|
| ITDM  |           |     |           |           |
|       | <i>i1</i> | 18  | <b>18</b> | <b>44</b> |
|       | <i>i2</i> | 26  | <b>26</b> |           |
| MPBR  |           |     |           |           |
|       | <i>i1</i> | 11  | <b>11</b> | <b>24</b> |
|       | <i>i2</i> | 13  | <b>13</b> |           |

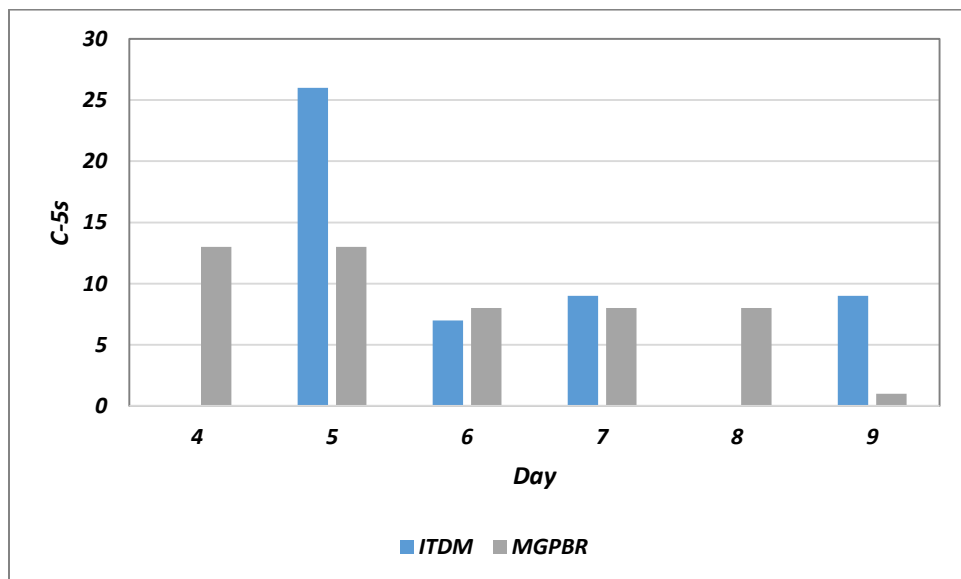
These results present perhaps the most undeniable validation that the MPBR reduces overall theater distribution cost. Solutions of previous test cases showed a mixture of vehicles with differing daily operating costs, while the solution to Test Case 4 is limited to Type C-5 aircraft having constant operating costs. The MPBR produces a 7 vehicle reduction at POD *i1* while it shows a 13 vehicle reduction at POD *i2*, making the theater wide beddown improvement 20 C-5s for this small test case. Even larger reductions can be expected for larger TPFDDs.

To illustrate that the MPBR allocates vehicles in a cheaper and smoother manner,

figures 6 and 7 are included below. The “peanut butter” spread effect of Mini/Max Programming is clearly evident with investigation of these figures.



**Figure 6. C-5 Allocations By Day at POD i1**



**Figure 7. C-5 Allocations By Day at POD i2**

It is seen from the figures that at both POD *i1* and *i2*, the MPBR allocates vehicles more smoothly over the time window than the ITDM in the single vehicle type case. In addition, total C-5 trips for both models at POD *i1* is equal with exactly 64, while total C-5 trips at POD *i2* matches for the two models as well with 51. This indicates that total operating costs to deliver requirements are equivalent at Location *i1* as well as *i2*. However, cost savings come from the significant theater wide beddown reduction of 20 C-5s seen in Table 17. Thus, results here can be translated directly to a truly lower cost solution in terms of both vehicle operation and beddowns. These results indicate that the MPBR optimizes cumulative costs when a single type is utilized within the math programming models. Similar results can be expected when a single type of a different mode (ROAD, RAIL) is tested. Model output solutions are included for reference in Appendix H.

***Test case 5: Analysis of TPFDD with Wider Time Windows.***

It still remains to validate that when the MPBR is given the opportunity, it will spread delivery of requirements even more efficiently over a wider time window. That is, the wider requirement time windows are within a TPFDD, the fewer the number of vehicles required to deliver its requirements will be. This is a feat that the ITDM is not able to accomplish. Up to this point, testing has been done on a TPFDD with relatively narrow time windows averaging 3 days. Test Case 5 involves comparing solutions of the MPBR using the original notional TPFDD to solutions using the same TPFDD with wider requirement delivery time windows. The only changes made to the updated TPFDD are increases of 7 in  $(ad_n + 1 \text{ to } rd_n + qd_n)$  for each respective Time Window.

This is done by increasing the value of the Required Delivery Date  $rd_n$  by 7 throughout the TPFDD. This TPFDD is included in Table 18 below.

**Table 18. Smaller Notional TPFDD  
With Large Time Windows**

| Requirement | POD | Destination | Short Tons | EAD | RDD |
|-------------|-----|-------------|------------|-----|-----|
| 1           | i1  | j1          | 500        | 2   | 11  |
| 2           | i1  | j1          | 250        | 3   | 12  |
| 3           | i1  | j1          | 750        | 4   | 13  |
| 4           | i1  | j1          | 200        | 5   | 14  |
| 5           | i1  | j1          | 100        | 6   | 15  |
| 6           | i1  | j2          | 600        | 2   | 12  |
| 7           | i1  | j2          | 400        | 3   | 13  |
| 8           | i1  | j2          | 200        | 4   | 14  |
| 9           | i1  | j2          | 300        | 5   | 15  |
| 10          | i1  | j2          | 500        | 6   | 16  |
| 11          | i2  | j1          | 500        | 4   | 12  |
| 12          | i2  | j1          | 400        | 5   | 13  |
| 13          | i2  | j1          | 300        | 6   | 14  |
| 14          | i2  | j2          | 1000       | 3   | 12  |
| 15          | i2  | j2          | 200        | 5   | 14  |
| 16          | i2  | j2          | 500        | 7   | 16  |

As with all test cases, the fifth test case utilized the same TPFDD from the Longhorn & Kovich paper shown in Table 11. It also implements the modified version of the notional TPFDD at Table 15. Here, all Modes (AIR, RAIL, ROAD), and all Types (C-130, C-17, C-5, HEMTT, M1083, M35, DODX, ITTX, and FTTX) of vehicles are used to investigate how the MPBR model reacts. The penalty per day per late short ton was set to  $g = 10,000$  once again. Payload, cost, and unloading parameters used are

shown in Appendix F. After the ITDM and ITDM with MPBR addition were tested on this case, model outputs were post-processed for beddown approximations and these solutions included in Table 19 below.

**Table 19. Beddowns by Vehicle Type, POD-Test Case 5 (Outload/Unload 1000)**

| <b>Model</b>  |                  | <b>C-130</b> | <b>C-17</b> | <b>C-5</b> | <b>HEMT</b> | <b>M1083</b> | <b>M35</b> | <b>DODX</b> | <b>ITTX</b> | <b>FTTX</b> | <b>TOTAL</b> |
|---------------|------------------|--------------|-------------|------------|-------------|--------------|------------|-------------|-------------|-------------|--------------|
| MPBR          |                  |              |             |            |             |              |            |             |             |             |              |
| Narrow Window | <i><b>i1</b></i> | 1            | 1           | 1          | 1           | 1            | 1          | 0           | 1           | 4           | <b>11</b>    |
|               | <i><b>i2</b></i> | 1            | 2           | 5          | 1           | 1            | 0          | 0           | 1           | 4           | <b>15</b>    |
| MPBR          |                  |              |             |            |             |              |            |             |             |             |              |
| Wide Window   | <i><b>i1</b></i> | 0            | 0           | 1          | 0           | 0            | 0          | 0           | 0           | 2           | <b>3</b>     |
|               | <i><b>i2</b></i> | 0            | 0           | 0          | 0           | 0            | 1          | 0           | 0           | 2           | <b>3</b>     |

Note that the Narrow Window Solution matches the MPBR solution to Test Case 1, as nothing changed for the input parameters and within the TPFDD used. However, the Wide Window Solution requires significantly fewer vehicles. All requirements within the TPFDD can be moved with 6 vehicles in theater, while the narrow window solution requires 26. This follows intuitively, as fewer vehicles should be required when more time is allotted to distribute requirements. Thus, the MPBR model formulation acts as desired, and tends to spread delivery as much as possible over a time window. This presents a highly desirable result for force flow analysis at USTRANSCOM because in most realistic, operational TPFDDs, time windows are much wider than those contained in the notional TPFDD. Model output solutions are included for reference in Appendix H.

### ***Test Case 6: Investigating Equal Operating Costs Per Short Ton***

Test Cases 1-5 involved setting daily operating costs  $b_k$  to USTRANSCOM provided preferences. In every case, when the ITDM was tested, there was a clear lowest cost vehicle for the ITDM mixed integer program to select. It is interesting to point out that the ITDM selects vehicles based on lowest operating cost per short ton, not just lowest operating cost. That is, the ratio of operating cost to average pay load,  $b_k/p_k$ , for a Type  $k$  vehicle actually determines whether a vehicle is the cheapest within the model. Since all ITDM test cases prescribed large numbers of vehicles with characteristic lowest cost per short ton, it remains to investigate the effect of setting  $b_k/p_k$  constant for all vehicles  $k$ .

The final test case looked at the effects of standardizing operating costs per short ton of payload. The results of the ITDM were compiled and compared with MPBR findings. First, all Modes (AIR, RAIL, ROAD), and all Types (C-130, C-17, C-5, HEMTT, M1083, M35, DODX, ITTX, and FTTX) of vehicles were used to investigate how both models react with the larger notional TPFDD. Both models are then constrained to include a single Mode (AIR) and all Types (C-130, C-17, and C-5) within Mode AIR, and tested on the smaller notional TPFDD. The penalty per day per late short ton was again set to  $g = 10,000$ . The payload and cost parameters are included in Table 20 below. Unloading and outloading parameters are shown in Appendix F for reference. After the ITDM and ITDM with MPBR addition were tested on both the smaller and larger TPFDD, model outputs were post-processed for beddown approximations and these solutions included in Tables 21 and 22 below.

**Table 20. Vehicle Parameters for Test Case 6**

| Type  | Average Payload<br>$p_k$ | Daily Cost<br>$b_k$ | Cost Per Ton<br>$b_k/p_k$ | Beddown Cost<br>$d_m$ |
|-------|--------------------------|---------------------|---------------------------|-----------------------|
| C-130 | 12                       | 8.4                 | 0.7                       | 5                     |
| C-17  | 35                       | 24.5                | 0.7                       | 5                     |
| C-5   | 35 60                    | 42                  | 0.7                       | 5                     |
| HEMTT | 35 7                     | 4.9                 | 0.7                       | 5                     |
| M1083 | 635 5                    | 3.5                 | 0.7                       | 5                     |
| M35   | 60 8                     | 5.6                 | 0.7                       | 5                     |
| DODX  | 200                      | 140                 | 0.7                       | 5                     |
| FTTX  | 20 200                   | 105                 | 0.7                       | 5                     |
| ITTX  | 5180                     | 126                 | 0.7                       | 5                     |

**Table 21. Beddowns by Vehicle Type, POD For Test Case 1 (Larger TPFDD)**

| Model |           | C-130 | C-17 | C-5 | HEMT | M1083 | M35 | DODX | ITTX | FTTX | TOTAL |
|-------|-----------|-------|------|-----|------|-------|-----|------|------|------|-------|
| ITDM  |           |       |      |     |      |       |     |      |      |      |       |
|       | <i>i1</i> | 0     | 0    | 1   | 1    | 1     | 13  | 10   | 3    |      | 29    |
|       | <i>i2</i> | 0     | 2    | 1   | 1    | 0     | 15  | 3    | 8    |      | 32    |
| MPBR  |           |       |      |     |      |       |     |      |      |      |       |
|       | <i>i1</i> | 0     | 0    | 0   | 0    | 0     | 12  | 5    | 4    |      | 21    |
|       | <i>i2</i> | 1     | 1    | 0   | 0    | 0     | 15  | 6    | 4    |      | 31    |

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**Table 22. Beddowns by Vehicle Type, POD For Test Case 1 (Smaller TPFDD)**

| Model |           | C-130 | C-17 | C-5 | TOTAL |
|-------|-----------|-------|------|-----|-------|
| ITDM  |           |       |      |     |       |
|       | <i>i1</i> | 25    | 15   | 17  | 57    |
|       | <i>i2</i> | 25    | 20   | 5   | 50    |
| MPBR  |           |       |      |     |       |
|       | <i>i1</i> | 3     | 3    | 11  | 17    |
|       | <i>i2</i> | 5     | 8    | 11  | 24    |

Table 21 shows results from testing a larger TPFDD containing 64 requirements. Setting the ratio of operating cost per short ton to a constant value across all vehicle Types  $k$  clearly improves ITDM beddown approximations in terms of total vehicles. Note, however, that this solution prescribes primarily RAIL vehicles, which realistically can carry a high initial beddown cost. In fact, the MPBR prescribes a largely RAIL solution as well. It seems here that when  $b_k/p_k$  is constant for all Types  $k$  both the ITDM and MPBR tend to select vehicles capable of delivering higher payloads. The MPBR still requires fewer vehicles to deliver requirements with a total in theater beddown of 52 vehicles. This is a 9 vehicle improvement over the ITDM solution of 61 vehicles. Thus, even with constant costs per short ton the Mini/Max approximation provides a reduced beddown approximation.

More significant improvements are seen in the single mode (AIR) test. Testing was performed on the smaller notional TPFDD to show that the MPBR present a reduced mix of AIR vehicles across the board. Although the ITDM does select a mixture of the vehicles available (50 C-130s, 35 C-17s, 22 C-5s) instead of strictly C-130s, the numbers are unnecessarily high. This fact is validated by the MPBR solution of AIR vehicles (8 C-130s, 11 C-17s, 22 C-5s). Since model input remained unchanged, this solution is exactly the same as the MPBR results from Test Case 3. Thus, although setting operating costs per short ton equal does improve upon single mode beddown approximations given by the ITDM, it does not improve single mode solutions from the MPBR. Additionally, because of vehicle beddown reductions, the MPBR provides a better solution than the ITDM here.

## **Verification and Validation**

Any time one builds a new model, it must be verified and validated before analysis can be considered complete. Verification ensures that the model is being built the right way. Validation ensures that the right model is being built. These techniques are discussed for the two models presented and tested in Chapter 4 of this research.

### ***Verification.***

The ITDM is easily verified by the results from Test Case 1. First, Since the ITDM formulation and accompanying DSS were taken directly from research presented by Hafich (2013), its solutions within this research are exactly the same as solutions given by Hafich, assuming identical input parameters. Furthermore, before testing was done, one of the ITDM solutions given by Hafich was identically reproduced with the model used for this research. Thus, the ITDM is verified. The MPBR is verified by the fact that in Test Case 4, the exact same number of total vehicle allocations was output. It is also verified in seeing that vehicle allocations are similar to the ITDM in Test Case 6, although significantly reduced due to the Min/Max formulation. Additionally, the ITDM maintains its mixed integer programming formulation with inclusion of the MPBR. It also produced feasible solutions to this model in LINGO 13 for all input settings investigated in Test Cases 1-6.

### ***Validation.***

Test Case 1 offers one reason why the ITDM cannot be the best model for approximating beddowns in theater. An unnecessarily large number of vehicles is prescribed to deliver the 16 requirements. With the MPBR addition, the model avoids allocation of unnecessary vehicles, resulting in a greatly reduced beddown solution. Test

Case 1 validates MPBR effectiveness when all modes and types are utilized, while Test Cases 2-4 validate other mode/type combinations. The ITDM fails to provide a “peanut butter spread” of vehicle allocations throughout a given time window. The MPBR corrects this issue. The most notable validation of this fact is seen in Test Case 5, where fewer vehicles were allocated to deliver requirements with wider time windows.

The fact that ITDM solutions are affected so much by variations in operating costs demonstrates that it is not entirely useful as formulated. As operating costs are greater for larger vehicles, and vary with USTRANSCOM policy, an effective model is one that is not overly sensitive to changes in these costs. The MPBR addresses this, as its solutions react to beddown costs as well as operating costs. This fact is validated by results from Test case 6.

## **V. Conclusions and Future Research**

### **Conclusions**

The three model variations presented in this research involved additions to the objective function and constraints of the ITDM. First, the GIBR added a penalty for increasing beddown size throughout a time window. It also constrained the number of vehicles used on a certain day to being no more than the number of vehicles used on the previous day. Next, the MPBR applied a penalty to the beddown of each mode of vehicles. It also constrained the number of vehicle allocations to being less than a certain number, which was minimized in the objective function. In terms of beddown approximating, the GIBR and MPBR produce the same results. However, it is unclear whether they will produce the exact same vehicle allocation solutions. Finally, the Multi-Objective Approach provided is just one of many possible existing variations to the MPBR. It focuses on minimizing beddowns at each POD as well as the total in theater beddown.

The GIBR and the MPBR improve upon the ITDM making it a much more useful tool for theater distribution analysis. As the ITDM, the baseline model, was tested and analyzed, it became clear that beddown approximations would be much too large to be realistically feasible. To address this, the GIBR was developed, which although never tested, provided an improved formulation of the ITDM. This formulation was expected to spread requirement distribution more efficiently over a given time window. However, the GIBR created excess decision variables and constraints, as its accompanying decision variables were indexed by day. Thus, the MPBR was developed, which provided the same modeling effects of the GIBR, but decision variables that were not defined by day. This

reduces the problem size and increases solution speed over the GIBR.

The MPBR gives force flow analysts a much better approximation of vehicles needed for theater distribution. In terms of problem size, the MPBR increases the size of the ITDM, but not significantly. Solutions take longer to produce, but a relative optimality tolerance can easily be set to encourage faster results. Additionally, the integrality property of decision variables can be relaxed to speed up the model. The ITDM forces unpredictable spikes in vehicle allocations, even when equivalent operating costs per short ton are used as input. When applied to the ITDM, the MPBR uses an iterative process to take these vehicle allocation spikes on certain days and transfer deliveries to other days where fewer allocations were made. This is because a onetime penalty is applied to the maximum number of vehicles of each mode prescribed to distribute requirements. Thus, the model attempts to minimize high vehicle allocations. The result is a much smoother and more desirable delivery schedule, as beddown approximations are greatly reduced.

The ITDM with MPBR is able to find feasible vehicle mixtures that minimize operational cost, late deliveries, and beddown size simultaneously. Since costs are user defined, solutions can be vectored toward a vehicle mixture that aligns with current policy. Furthermore, because various cost settings drive different vehicle mixtures, results may be sensitive to alternate optimal solutions. Thus, one beddown solution could be desired over another even though both are reported as optimal cost wise.

The MPBR has the potential to provide reduced vehicle mixtures when post-processing results of the ITDM and analyzing possible beddowns, which can result in lower cost starting points in terms of vehicles to support theater distribution. Through this addition to the ITDM and associated Decision Support System, force flow analysts at

USTRANSCOM possess a highly flexible tool to assist in theater distribution analysis. The MPBR portion of the formulation can be easily altered based on preferences and existing policy. This use of the updated ITDM to model theater distribution and estimate beddowns has the potential to save increasingly valuable vehicle resources and DOD funds.

### **Future Research**

There are many areas in which this research can be furthered to improve upon results, organize testing procedures, and model operational realities. The greatest potential for future research is to investigate how variations in the weighting of costs within the objective function affect solutions. Through the research process, it became clear that model output was highly sensitive to user defined costs including daily operating cost, the late delivery penalty, and beddown cost. Although solutions were somewhat sensitive to the operational costs and the late penalty, they were most affected by variations in the beddown cost. This was evident when multiple mode and type mixtures were used as input. For example, in running two tests with the exact same model inputs, even a small change in the beddown penalty could produce significantly different vehicle allocations. Research into the effects of varying the different model costs, and organizing these effects in some standardized format should prove fruitful. It would also allow force flow analysts to predict model output based on model input, which could save significant time when a specific vehicle solution is needed.

An investigation of how vehicle batching affects solutions of the MPBR formulation should prove to be practical research. Operational realities and current policy often requires vehicles to be deployed into theater in batches. This is because vehicles typically relocate or deploy as a unit containing a set number of this type of vehicle. For

example, trucks usually move into theater with the United States Army (USA) as a Battalion or Company, while aircraft typically move with the United States Air Force (USAF) as a wing or squadron. This would constrain the beddown of vehicles of a certain type to be a multiple of the number of vehicles contained in that unit type's typical batch size. Investigating this issue by coordinating with USTRANSCOM to determine batch size should add to the realistic effects of the research.

In addition, variations in the formulation of the MPBR such as the Multi-Objective Approach should be tested. It should be determined if several Mini/Max beddown objectives can be achieved at the same time, as higher leadership may have multiple preferences for vehicle beddown numbers.

An exploration of how the General Integer Beddown Reduction formulation presented in this thesis can affect solutions would provide insight into whether or not this beddown improvement technique is better than the MPBR. Although the same modeling effect is expected in terms of beddown approximations, the GIBR may provide vehicle allocation solutions throughout a given time window that are better in terms of operational costs. This would involve updating the VBA code within the ITDM DSS to match the modeling changes of the GIBR.

Lastly, as mentioned in the research by Hafich, further research into defining cycle values should be conducted. Instead of relying on input from a user to determine feasible cycles, a tool could be developed that accounts for operational capabilities such as vehicle speeds and outload/unload times, and returns a specific cycle value. This pre-processing result could then be used as input before implementing the DSS.

## **Appendix A. LINGO 13 Settings File Contents**

The LINGO.CNF file contains settings which have been changed from their default values within LINGO 13. The contents of the LINGO.CNF file as utilized in this thesis appear below (Hafich).

```
Lingo CNF info:  
! LINGO Custom Configuration Data:  
MXMEMB= 25000  
ABSINT= 0.10000000E-11  
IPTOLR= 0.50000000E-01  
TIM2RL= 120  
LINLEN= 150  
DUALCO= 0  
PRECIS= 12
```

## Appendix B. Additional Model Inputs for Test Case 1

**Table 23. Vehicle Parameters for Test Case 1**

| Type  | Average Payload<br>$p_k$ | Daily Cost<br>$b_k$ | Beddown Cost<br>$d_m$ |
|-------|--------------------------|---------------------|-----------------------|
| C-130 | 6                        | 3                   | 10                    |
| C-17  | 12                       | 9                   | 10                    |
| C-5   | 35                       | 16                  | 10                    |
| HEMTT | 35                       | 1                   | 10                    |
| M1083 | 60                       | 1                   | 10                    |
| M35   | 60                       | 1                   | 10                    |
| DODX  | 7200                     | 60                  | 10                    |
| FTTX  | 20350                    | 42                  | 10                    |
| ITTX  | 5180                     | 52                  | 10                    |

**Table 24. Outloading Parameters for Test Case 1**

| POD | Mode | Outload Capacity<br>$o_{inv}$ |
|-----|------|-------------------------------|
| 180 | Air  | 1000                          |
| 180 | Road | 1000                          |
| 180 | Rail | 1000                          |
| 180 | Air  | 1000                          |
| 180 | Road | 1000                          |
| 180 | Rail | 1000                          |

\*Note, for each POD/Mode pair, the outload capacity is assumed constant for all  $v$ .

**Table 25. Unloading Parameters for Test Case 1**

| Destination | Mode | Unload Capacity<br>$u_{jmv}$ |
|-------------|------|------------------------------|
| j1          | Air  | 1000                         |
|             | Road | 1000                         |
|             | Rail | 1000                         |
| j2          | Air  | 1000                         |
|             | Road | 1000                         |
|             | Rail | 1000                         |

\*Note, for each Destination/Mode pair, the unload capacity is constant for all  $v$ .

## Appendix C. Additional Model Inputs for Test Case 2

**Table 26. Vehicle Parameters for Test Case 2**

| Type  | Average Payload<br>$p_k$ | Daily Cost<br>$b_k$ | Beddown Cost<br>$d_m$ |
|-------|--------------------------|---------------------|-----------------------|
| C-130 | 12                       | 3                   | 10                    |
| HEMTT | 7                        | 1                   | 10                    |
| DODX  | 35 <sub>200</sub>        | 60                  | 10                    |

99

**Table 27. Outloading Parameters for Test Cases 2**

| POD | Mode | Outload Capacity<br>$o_{inv}$ |
|-----|------|-------------------------------|
| i1  | Air  | 1000                          |
| 200 | Road | 1000                          |
| 200 | Rail | 1000                          |
| 150 | Air  | 1000                          |
| 200 | Road | 1000                          |
| 150 | Rail | 1000                          |

\*Note, for each POD/Mode pair, the outload capacity is assumed constant for all Days  $v$ .

**Table 28. Unloading Parameters for Test Case 2**

| Destination | Mode | Unload Capacity<br>$u_{jmv}$ |
|-------------|------|------------------------------|
| j1          | Air  | 1000                         |
|             | Road | 1000                         |
|             | Rail | 1000                         |
| j2          | Air  | 1000                         |
|             | Road | 1000                         |
|             | Rail | 1000                         |

\*Note, for each Destination/Mode pair, the unload capacity is assumed constant for all Days  $v$ .

## Appendix D. Additional Model Inputs for Test Case 3

**Table 29. Vehicle Parameters for Test Case 3**

| Type | Average Payload<br>$p_k$ | Daily Cost<br>$b_k$ | Beddown Cost<br>$d_m$ |
|------|--------------------------|---------------------|-----------------------|
| C130 | 12                       | 3                   | 5                     |
| C-17 | 35                       | 9                   | 5                     |
| C-5  | 35<br>60                 | 16                  | 5                     |

**Table 30. Outloading Parameters for Test Cases 3**

| POD | Mode | Outload Capacity<br>$o_{imv}$ |
|-----|------|-------------------------------|
| 8   |      |                               |
| 8   | Air  | 1000                          |
| 8   | Air  | 1000                          |

\*Note, for each POD/Mode pair, the outload capacity is assumed constant for all days  $v$

**Table 31. Unloading Parameters for Test Case 3**

| Destination | Mode | Unload Capacity<br>$u_{jmv}$ |
|-------------|------|------------------------------|
| j1          | Air  | 1000                         |
| j2          | Air  | 1000                         |

\*Note, for each Destination/Mode pair, the unload capacity is assumed constant for all days  $v$ .

## Appendix E. Additional Model Inputs for Test Case 4

**Table 32. Vehicle Parameters for Test Case 4**

| Type | Average Payload<br>$p_k$ | Daily Cost<br>$b_k$ | Beddown Cost<br>$d_m$ |
|------|--------------------------|---------------------|-----------------------|
| C-5  | 60                       | 16                  | 5                     |

635

60

**Table 33. Outloading Parameters for Test Cases 4**

| POD | Mode | Outload Capacity<br>$o_{imv}$ |
|-----|------|-------------------------------|
| 5   |      |                               |
| i1  | Air  | 1000                          |
| i2  | Air  | 1000                          |

\*Note, for each POD/Mode pair, the outload capacity is assumed constant for all days  $v$ .

150

180

**Table 34. Unloading Parameters for Test Case 4**

| Destination | Mode | Unload Capacity<br>$u_{jmv}$ |
|-------------|------|------------------------------|
| j1          | Air  | 1000                         |
| j2          | Air  | 1000                         |

\*Note, for each Destination/Mode pair, the unload capacity is assumed constant for all days  $v$ .

## Appendix F. Additional Model Inputs for Test Case 5

**Table 35. Vehicle Parameters for Test Case 5  
TPFDD With Wide Time Windows**

| Type  | Average Payload<br>$p_k$ | Daily Cost<br>$b_k$ | Beddown Cost<br>$d_m$ |
|-------|--------------------------|---------------------|-----------------------|
| C130  | 12                       | 3                   | 25                    |
| C-17  | 35                       | 9                   | 25                    |
| C-5   | 35 60                    | 16                  | 25                    |
| HEMTT | 35 7                     | 1                   | 25                    |
| M1083 | 65 5                     | 1                   | 25                    |
| M35   | 60 8                     | 1                   | 25                    |
| DODX  | 35 200                   | 60                  | 25                    |
| FTTX  | 20 25 100                | 42                  | 25                    |
| ITTX  | 60 5 180                 | 52                  | 25                    |

**Table 36. Vehicle Parameters for Test Case 5  
TPFDD With Original Time Windows**

| Type  | Average Payload<br>$p_k$ | Daily Cost<br>$b_k$ | Beddown Cost<br>$d_m$ |
|-------|--------------------------|---------------------|-----------------------|
| C130  | 12                       | 3                   | 10                    |
| C-17  | 35                       | 9                   | 10                    |
| C-5   | 35 60                    | 16                  | 10                    |
| HEMTT | 35 7                     | 1                   | 10                    |
| M1083 | 65 5                     | 1                   | 10                    |
| M35   | 60 8                     | 1                   | 10                    |
| DODX  | 35 200                   | 60                  | 10                    |
| FTTX  | 20 25 100                | 42                  | 10                    |
| ITTX  | 60 5 180                 | 52                  | 10                    |

**Table 37. Outloading Parameters for Test Case 5**

| POD | Mode | Outload Capacity<br>$O_{imv}$ |
|-----|------|-------------------------------|
| i1  | Air  | 1000                          |
|     | Road | 1000                          |
|     | Rail | 1000                          |
| i2  | Air  | 1000                          |
|     | Road | 1000                          |
|     | Rail | 1000                          |

\*Note, for each POD/Mode pair, the outload capacity is assumed constant for all days  $v$ .

**Table 38. Unloading Parameters for Test Case 5**

| Destination | Mode | Unload Capacity<br>$U_{jmv}$ |
|-------------|------|------------------------------|
| j1          | Air  | 1000                         |
|             | Road | 1000                         |
|             | Rail | 1000                         |
| j2          | Air  | 1000                         |
|             | Road | 1000                         |
|             | Rail | 1000                         |

\*Note, for each Destination/Mode pair, the unload capacity is constant for all  $v$ .

## Appendix G. Additional Model Inputs For Test Case 6

**Table 39. Outloading Parameters for Test Case 6**

| POD | Mode | Outload Capacity<br>$O_{imv}$ |
|-----|------|-------------------------------|
| i1  | Air  | 1000                          |
|     | Road | 1000                          |
|     | Rail | 1000                          |
| i2  | Air  | 1000                          |
|     | Road | 1000                          |
|     | Rail | 1000                          |

\*Note, for each POD/Mode pair, the outload capacity is assumed constant for all days  $v$

**Table 40. Unloading Parameters for Test Case 6**

| Destination | Mode | Unload Capacity<br>$U_{jmv}$ |
|-------------|------|------------------------------|
| j1          | Air  | 1000                         |
|             | Road | 1000                         |
|             | Rail | 1000                         |
| j2          | Air  | 1000                         |
|             | Road | 1000                         |
|             | Rail | 1000                         |

\*Note, for each Destination/Mode pair, the unload capacity is constant for all  $v$  .

## Appendix H. Model Output Solutions Test Cases 2-5

|              |             |    |                 |    |                 |                   |             |    |
|--------------|-------------|----|-----------------|----|-----------------|-------------------|-------------|----|
| 71 HEMTT(s)  | leaving POD | I1 | for destination | J1 | on day 3 (ROAD) | 497.00 Short Tons | of Movement | 1  |
| 85 HEMTT(s)  | leaving POD | I1 | for destination | J2 | on day 3 (ROAD) | 595.00 Short Tons | of Movement | 6  |
| 36 HEMTT(s)  | leaving POD | I1 | for destination | J1 | on day 4 (ROAD) | 3.00 Short Tons   | of Movement | 1  |
|              |             |    |                 |    |                 | 249.00 Short Tons | of Movement | 2  |
| 57 HEMTT(s)  | leaving POD | I1 | for destination | J2 | on day 4 (ROAD) | 399.00 Short Tons | of Movement | 7  |
| 108 HEMTT(s) | leaving POD | I1 | for destination | J1 | on day 5 (ROAD) | 1.00 Short Tons   | of Movement | 2  |
|              |             |    |                 |    |                 | 749.17 Short Tons | of Movement | 3  |
| 29 HEMTT(s)  | leaving POD | I1 | for destination | J2 | on day 5 (ROAD) | 5.00 Short Tons   | of Movement | 6  |
|              |             |    |                 |    |                 | 196.00 Short Tons | of Movement | 8  |
| 28 HEMTT(s)  | leaving POD | I1 | for destination | J1 | on day 6 (ROAD) | 0.83 Short Tons   | of Movement | 3  |
|              |             |    |                 |    |                 | 195.00 Short Tons | of Movement | 4  |
| 43 HEMTT(s)  | leaving POD | I1 | for destination | J2 | on day 6 (ROAD) | 1.00 Short Tons   | of Movement | 7  |
|              |             |    |                 |    |                 | 300.00 Short Tons | of Movement | 9  |
| 15 HEMTT(s)  | leaving POD | I1 | for destination | J1 | on day 7 (ROAD) | 5.00 Short Tons   | of Movement | 4  |
|              |             |    |                 |    |                 | 100.00 Short Tons | of Movement | 5  |
| 1 HEMTT(s)   | leaving POD | I1 | for destination | J2 | on day 7 (ROAD) | 4.00 Short Tons   | of Movement | 8  |
|              |             |    |                 |    |                 | 3.00 Short Tons   | of Movement | 10 |
| 71 HEMTT(s)  | leaving POD | I1 | for destination | J2 | on day 8 (ROAD) | 497.00 Short Tons | of Movement | 10 |
| 1 HEMTT(s)   | leaving POD | I2 | for destination | J2 | on day 4 (ROAD) | 6.00 Short Tons   | of Movement | 14 |
| 72 HEMTT(s)  | leaving POD | I2 | for destination | J1 | on day 5 (ROAD) | 500.00 Short Tons | of Movement | 11 |
| 142 HEMTT(s) | leaving POD | I2 | for destination | J2 | on day 5 (ROAD) | 994.00 Short Tons | of Movement | 14 |
| 58 HEMTT(s)  | leaving POD | I2 | for destination | J1 | on day 6 (ROAD) | 400.00 Short Tons | of Movement | 12 |
| 1 HEMTT(s)   | leaving POD | I2 | for destination | J2 | on day 6 (ROAD) | 4.00 Short Tons   | of Movement | 15 |
| 43 HEMTT(s)  | leaving POD | I2 | for destination | J1 | on day 7 (ROAD) | 300.00 Short Tons | of Movement | 13 |
| 28 HEMTT(s)  | leaving POD | I2 | for destination | J2 | on day 7 (ROAD) | 196.00 Short Tons | of Movement | 15 |
| 72 HEMTT(s)  | leaving POD | I2 | for destination | J2 | on day 8 (ROAD) | 500.00 Short Tons | of Movement | 16 |

**Figure 10. ITDM Solution Test Case 2**

|           |             |    |                 |    |                 |                   |             |    |
|-----------|-------------|----|-----------------|----|-----------------|-------------------|-------------|----|
| 1 C130(s) | leaving POD | I2 | for destination | J2 | on day 4 (AIR)  | 11.00 Short Tons  | of Movement | 14 |
| 1 C130(s) | leaving POD | I2 | for destination | J1 | on day 5 (AIR)  | 9.00 Short Tons   | of Movement | 11 |
| 1 C130(s) | leaving POD | I2 | for destination | J2 | on day 6 (AIR)  | 11.00 Short Tons  | of Movement | 15 |
| 1 C130(s) | leaving POD | I2 | for destination | J1 | on day 7 (AIR)  | 9.00 Short Tons   | of Movement | 13 |
| 1 C130(s) | leaving POD | I2 | for destination | J2 | on day 8 (AIR)  | 8.00 Short Tons   | of Movement | 16 |
| 1 C130(s) | leaving POD | I2 | for destination | J2 | on day 9 (AIR)  | 12.00 Short Tons  | of Movement | 16 |
| 2 DODX(s) | leaving POD | I1 | for destination | J1 | on day 3 (RAIL) | 400.00 Short Tons | of Movement | 1  |
| 2 DODX(s) | leaving POD | I1 | for destination | J2 | on day 3 (RAIL) | 400.00 Short Tons | of Movement | 6  |
| 1 DODX(s) | leaving POD | I1 | for destination | J1 | on day 4 (RAIL) | 100.00 Short Tons | of Movement | 1  |
|           |             |    |                 |    |                 | 100.00 Short Tons | of Movement | 2  |
| 3 DODX(s) | leaving POD | I1 | for destination | J1 | on day 5 (RAIL) | 150.00 Short Tons | of Movement | 2  |
|           |             |    |                 |    |                 | 450.00 Short Tons | of Movement | 3  |
| 1 DODX(s) | leaving POD | I1 | for destination | J2 | on day 5 (RAIL) | 200.00 Short Tons | of Movement | 6  |
| 2 DODX(s) | leaving POD | I1 | for destination | J1 | on day 6 (RAIL) | 300.00 Short Tons | of Movement | 3  |
|           |             |    |                 |    |                 | 100.00 Short Tons | of Movement | 4  |
| 2 DODX(s) | leaving POD | I1 | for destination | J2 | on day 6 (RAIL) | 400.00 Short Tons | of Movement | 7  |
| 1 DODX(s) | leaving POD | I1 | for destination | J1 | on day 7 (RAIL) | 100.00 Short Tons | of Movement | 4  |
|           |             |    |                 |    |                 | 100.00 Short Tons | of Movement | 5  |
| 1 DODX(s) | leaving POD | I1 | for destination | J2 | on day 7 (RAIL) | 200.00 Short Tons | of Movement | 8  |
| 2 DODX(s) | leaving POD | I1 | for destination | J2 | on day 8 (RAIL) | 300.00 Short Tons | of Movement | 9  |
|           |             |    |                 |    |                 | 100.00 Short Tons | of Movement | 10 |
| 2 DODX(s) | leaving POD | I1 | for destination | J2 | on day 9 (RAIL) | 400.00 Short Tons | of Movement | 10 |
| 3 DODX(s) | leaving POD | I2 | for destination | J2 | on day 4 (RAIL) | 600.00 Short Tons | of Movement | 14 |
| 2 DODX(s) | leaving POD | I2 | for destination | J1 | on day 5 (RAIL) | 400.00 Short Tons | of Movement | 11 |
| 1 DODX(s) | leaving POD | I2 | for destination | J2 | on day 5 (RAIL) | 200.00 Short Tons | of Movement | 14 |

**Figure 10. ITDM Solution Test Case 2 (cont.)**

|             |             |    |                 |    |                 |                   |             |    |
|-------------|-------------|----|-----------------|----|-----------------|-------------------|-------------|----|
| 2 DODX(s)   | leaving POD | I2 | for destination | J1 | on day 6 (RAIL) | 400.00 Short Tons | of Movement | 12 |
| 1 DODX(s)   | leaving POD | I2 | for destination | J1 | on day 7 (RAIL) | 200.00 Short Tons | of Movement | 13 |
| 1 DODX(s)   | leaving POD | I2 | for destination | J2 | on day 9 (RAIL) | 200.00 Short Tons | of Movement | 16 |
| 20 HEMTT(s) | leaving POD | I2 | for destination | J2 | on day 4 (ROAD) | 140.00 Short Tons | of Movement | 14 |
| 13 HEMTT(s) | leaving POD | I2 | for destination | J1 | on day 5 (ROAD) | 91.00 Short Tons  | of Movement | 11 |
| 7 HEMTT(s)  | leaving POD | I2 | for destination | J2 | on day 5 (ROAD) | 49.00 Short Tons  | of Movement | 14 |
| 20 HEMTT(s) | leaving POD | I2 | for destination | J2 | on day 6 (ROAD) | 140.00 Short Tons | of Movement | 15 |
| 13 HEMTT(s) | leaving POD | I2 | for destination | J1 | on day 7 (ROAD) | 91.00 Short Tons  | of Movement | 13 |
| 7 HEMTT(s)  | leaving POD | I2 | for destination | J2 | on day 7 (ROAD) | 49.00 Short Tons  | of Movement | 15 |
| 20 HEMTT(s) | leaving POD | I2 | for destination | J2 | on day 8 (ROAD) | 140.00 Short Tons | of Movement | 16 |
| 20 HEMTT(s) | leaving POD | I2 | for destination | J2 | on day 9 (ROAD) | 140.00 Short Tons | of Movement | 16 |

**Figure 11. ITDM with MPBR Solution Test Case 2**

|    |         |             |    |                 |    |                   |                    |             |    |
|----|---------|-------------|----|-----------------|----|-------------------|--------------------|-------------|----|
| 41 | C130(s) | leaving POD | I1 | for destination | J1 | on day 3 (AIR)    | 492.00 Short Tons  | of Movement | 1  |
| 50 | C130(s) | leaving POD | I1 | for destination | J2 | on day 3 (AIR)    | 600.00 Short Tons  | of Movement | 6  |
| 1  | C130(s) | leaving POD | I1 | for destination | J1 | on day 4 (AIR)    | 8.00 Short Tons    | of Movement | 1  |
|    |         |             |    |                 |    | 4.00 Short Tons   | of Movement        | 2           |    |
| 33 | C130(s) | leaving POD | I1 | for destination | J2 | on day 4 (AIR)    | 396.00 Short Tons  | of Movement | 7  |
| 83 | C130(s) | leaving POD | I1 | for destination | J1 | on day 5 (AIR)    | 246.00 Short Tons  | of Movement | 2  |
|    |         |             |    |                 |    | 750.00 Short Tons | of Movement        | 3           |    |
| 17 | C130(s) | leaving POD | I1 | for destination | J2 | on day 5 (AIR)    | 4.00 Short Tons    | of Movement | 7  |
|    |         |             |    |                 |    | 200.00 Short Tons | of Movement        | 8           |    |
| 16 | C130(s) | leaving POD | I1 | for destination | J1 | on day 6 (AIR)    | 192.00 Short Tons  | of Movement | 4  |
| 25 | C130(s) | leaving POD | I1 | for destination | J2 | on day 6 (AIR)    | 300.00 Short Tons  | of Movement | 9  |
| 9  | C130(s) | leaving POD | I1 | for destination | J1 | on day 7 (AIR)    | 8.00 Short Tons    | of Movement | 4  |
|    |         |             |    |                 |    | 100.00 Short Tons | of Movement        | 5           |    |
| 42 | C130(s) | leaving POD | I1 | for destination | J2 | on day 9 (AIR)    | 500.00 Short Tons  | of Movement | 10 |
| 42 | C130(s) | leaving POD | I2 | for destination | J1 | on day 5 (AIR)    | 500.00 Short Tons  | of Movement | 11 |
| 84 | C130(s) | leaving POD | I2 | for destination | J2 | on day 5 (AIR)    | 1000.00 Short Tons | of Movement | 14 |
| 34 | C130(s) | leaving POD | I2 | for destination | J1 | on day 6 (AIR)    | 400.00 Short Tons  | of Movement | 12 |
| 25 | C130(s) | leaving POD | I2 | for destination | J1 | on day 7 (AIR)    | 300.00 Short Tons  | of Movement | 13 |
| 17 | C130(s) | leaving POD | I2 | for destination | J2 | on day 7 (AIR)    | 200.00 Short Tons  | of Movement | 15 |
| 42 | C130(s) | leaving POD | I2 | for destination | J2 | on day 9 (AIR)    | 500.00 Short Tons  | of Movement | 16 |

**Figure 12. ITDM Solution Test Case 3**

|    |         |             |    |                 |    |                |                   |             |    |
|----|---------|-------------|----|-----------------|----|----------------|-------------------|-------------|----|
| 1  | C5(s)   | leaving POD | I1 | for destination | J1 | on day 3 (AIR) | 60.00 Short Tons  | of Movement | 1  |
| 10 | C5(s)   | leaving POD | I1 | for destination | J2 | on day 3 (AIR) | 600.00 Short Tons | of Movement | 6  |
| 2  | C130(s) | leaving POD | I1 | for destination | J1 | on day 4 (AIR) | 20.00 Short Tons  | of Movement | 1  |
|    |         |             |    |                 |    |                | 4.00 Short Tons   | of Movement | 2  |
| 7  | C5(s)   | leaving POD | I1 | for destination | J1 | on day 4 (AIR) | 420.00 Short Tons | of Movement | 1  |
| 2  | C5(s)   | leaving POD | I1 | for destination | J2 | on day 4 (AIR) | 120.00 Short Tons | of Movement | 7  |
| 9  | C5(s)   | leaving POD | I1 | for destination | J1 | on day 5 (AIR) | 246.00 Short Tons | of Movement | 2  |
|    |         |             |    |                 |    |                | 294.00 Short Tons | of Movement | 3  |
| 2  | C5(s)   | leaving POD | I1 | for destination | J2 | on day 5 (AIR) | 100.00 Short Tons | of Movement | 7  |
|    |         |             |    |                 |    |                | 20.00 Short Tons  | of Movement | 8  |
| 8  | C5(s)   | leaving POD | I1 | for destination | J1 | on day 6 (AIR) | 456.00 Short Tons | of Movement | 3  |
|    |         |             |    |                 |    |                | 24.00 Short Tons  | of Movement | 4  |
| 3  | C5(s)   | leaving POD | I1 | for destination | J2 | on day 6 (AIR) | 180.00 Short Tons | of Movement | 7  |
| 4  | C5(s)   | leaving POD | I1 | for destination | J1 | on day 7 (AIR) | 176.00 Short Tons | of Movement | 4  |
|    |         |             |    |                 |    |                | 64.00 Short Tons  | of Movement | 5  |
| 7  | C5(s)   | leaving POD | I1 | for destination | J2 | on day 7 (AIR) | 180.00 Short Tons | of Movement | 8  |
|    |         |             |    |                 |    |                | 240.00 Short Tons | of Movement | 10 |
| 3  | C130(s) | leaving POD | I1 | for destination | J1 | on day 8 (AIR) | 36.00 Short Tons  | of Movement | 5  |
| 3  | C17(s)  | leaving POD | I1 | for destination | J2 | on day 8 (AIR) | 105.00 Short Tons | of Movement | 10 |
| 5  | C5(s)   | leaving POD | I1 | for destination | J2 | on day 8 (AIR) | 300.00 Short Tons | of Movement | 9  |
| 1  | C17(s)  | leaving POD | I1 | for destination | J2 | on day 9 (AIR) | 35.00 Short Tons  | of Movement | 10 |
| 2  | C5(s)   | leaving POD | I1 | for destination | J2 | on day 9 (AIR) | 120.00 Short Tons | of Movement | 10 |
| 3  | C17(s)  | leaving POD | I2 | for destination | J2 | on day 4 (AIR) | 105.00 Short Tons | of Movement | 14 |
| 11 | C5(s)   | leaving POD | I2 | for destination | J2 | on day 4 (AIR) | 655.00 Short Tons | of Movement | 14 |
| 4  | C17(s)  | leaving POD | I2 | for destination | J1 | on day 5 (AIR) | 140.00 Short Tons | of Movement | 11 |
| 6  | C5(s)   | leaving POD | I2 | for destination | J1 | on day 5 (AIR) | 360.00 Short Tons | of Movement | 11 |
| 4  | C5(s)   | leaving POD | I2 | for destination | J2 | on day 5 (AIR) | 240.00 Short Tons | of Movement | 14 |

**Figure 12. ITDM Solution Test Case 3 (cont.)**

|   |         |             |    |                 |    |                |                   |             |    |
|---|---------|-------------|----|-----------------|----|----------------|-------------------|-------------|----|
| 5 | C130(s) | leaving POD | I2 | for destination | J1 | on day 6 (AIR) | 60.00 Short Tons  | of Movement | 12 |
| 8 | C17(s)  | leaving POD | I2 | for destination | J1 | on day 6 (AIR) | 280.00 Short Tons | of Movement | 12 |
| 1 | C5(s)   | leaving POD | I2 | for destination | J1 | on day 6 (AIR) | 60.00 Short Tons  | of Movement | 12 |
| 5 | C5(s)   | leaving POD | I2 | for destination | J1 | on day 7 (AIR) | 300.00 Short Tons | of Movement | 13 |
| 4 | C17(s)  | leaving POD | I2 | for destination | J2 | on day 7 (AIR) | 140.00 Short Tons | of Movement | 15 |
| 1 | C5(s)   | leaving POD | I2 | for destination | J2 | on day 7 (AIR) | 60.00 Short Tons  | of Movement | 15 |
| 5 | C130(s) | leaving POD | I2 | for destination | J2 | on day 9 (AIR) | 60.00 Short Tons  | of Movement | 16 |
| 4 | C17(s)  | leaving POD | I2 | for destination | J2 | on day 9 (AIR) | 140.00 Short Tons | of Movement | 16 |
| 5 | C5(s)   | leaving POD | I2 | for destination | J2 | on day 9 (AIR) | 300.00 Short Tons | of Movement | 16 |

**Figure 13. ITDM with MPBR Solution Test Case 3**

|    |       |             |    |                 |    |                |                    |             |    |
|----|-------|-------------|----|-----------------|----|----------------|--------------------|-------------|----|
| 8  | C5(s) | leaving POD | I1 | for destination | J1 | on day 3 (AIR) | 480.00 Short Tons  | of Movement | 1  |
| 10 | C5(s) | leaving POD | I1 | for destination | J2 | on day 3 (AIR) | 590.00 Short Tons  | of Movement | 6  |
| 4  | C5(s) | leaving POD | I1 | for destination | J1 | on day 4 (AIR) | 20.00 Short Tons   | of Movement | 1  |
|    |       |             |    |                 |    |                | 220.00 Short Tons  | of Movement | 2  |
| 2  | C5(s) | leaving POD | I1 | for destination | J2 | on day 4 (AIR) | 120.00 Short Tons  | of Movement | 7  |
| 13 | C5(s) | leaving POD | I1 | for destination | J1 | on day 5 (AIR) | 30.00 Short Tons   | of Movement | 2  |
|    |       |             |    |                 |    |                | 750.00 Short Tons  | of Movement | 3  |
| 3  | C5(s) | leaving POD | I1 | for destination | J2 | on day 5 (AIR) | 10.00 Short Tons   | of Movement | 6  |
|    |       |             |    |                 |    |                | 170.00 Short Tons  | of Movement | 8  |
| 3  | C5(s) | leaving POD | I1 | for destination | J1 | on day 6 (AIR) | 180.00 Short Tons  | of Movement | 4  |
| 5  | C5(s) | leaving POD | I1 | for destination | J2 | on day 6 (AIR) | 280.00 Short Tons  | of Movement | 7  |
|    |       |             |    |                 |    |                | 10.00 Short Tons   | of Movement | 8  |
|    |       |             |    |                 |    |                | 10.00 Short Tons   | of Movement | 9  |
| 2  | C5(s) | leaving POD | I1 | for destination | J1 | on day 7 (AIR) | 20.00 Short Tons   | of Movement | 4  |
|    |       |             |    |                 |    |                | 100.00 Short Tons  | of Movement | 5  |
| 14 | C5(s) | leaving POD | I1 | for destination | J2 | on day 7 (AIR) | 20.00 Short Tons   | of Movement | 8  |
|    |       |             |    |                 |    |                | 290.00 Short Tons  | of Movement | 9  |
|    |       |             |    |                 |    |                | 500.00 Short Tons  | of Movement | 10 |
| 9  | C5(s) | leaving POD | I2 | for destination | J1 | on day 5 (AIR) | 500.00 Short Tons  | of Movement | 11 |
| 17 | C5(s) | leaving POD | I2 | for destination | J2 | on day 5 (AIR) | 1000.00 Short Tons | of Movement | 14 |
| 7  | C5(s) | leaving POD | I2 | for destination | J1 | on day 6 (AIR) | 400.00 Short Tons  | of Movement | 12 |
| 5  | C5(s) | leaving POD | I2 | for destination | J1 | on day 7 (AIR) | 300.00 Short Tons  | of Movement | 13 |
| 4  | C5(s) | leaving POD | I2 | for destination | J2 | on day 7 (AIR) | 200.00 Short Tons  | of Movement | 15 |
| 9  | C5(s) | leaving POD | I2 | for destination | J2 | on day 9 (AIR) | 500.00 Short Tons  | of Movement | 16 |

**Figure 14. ITDM Solution Test Case 4**

|    |       |         |     |    |     |             |    |    |     |         |                   |             |    |
|----|-------|---------|-----|----|-----|-------------|----|----|-----|---------|-------------------|-------------|----|
| 1  | C5(s) | leaving | POD | I1 | for | destination | J1 | on | day | 3 (AIR) | 60.00 Short Tons  | of Movement | 1  |
| 10 | C5(s) | leaving | POD | I1 | for | destination | J2 | on | day | 3 (AIR) | 600.00 Short Tons | of Movement | 6  |
| 10 | C5(s) | leaving | POD | I1 | for | destination | J1 | on | day | 4 (AIR) | 440.00 Short Tons | of Movement | 1  |
|    |       |         |     |    |     |             |    |    |     |         | 160.00 Short Tons | of Movement | 2  |
| 1  | C5(s) | leaving | POD | I1 | for | destination | J2 | on | day | 4 (AIR) | 60.00 Short Tons  | of Movement | 7  |
| 8  | C5(s) | leaving | POD | I1 | for | destination | J1 | on | day | 5 (AIR) | 90.00 Short Tons  | of Movement | 2  |
|    |       |         |     |    |     |             |    |    |     |         | 390.00 Short Tons | of Movement | 3  |
| 3  | C5(s) | leaving | POD | I1 | for | destination | J2 | on | day | 5 (AIR) | 180.00 Short Tons | of Movement | 7  |
| 7  | C5(s) | leaving | POD | I1 | for | destination | J1 | on | day | 6 (AIR) | 360.00 Short Tons | of Movement | 3  |
|    |       |         |     |    |     |             |    |    |     |         | 60.00 Short Tons  | of Movement | 4  |
| 4  | C5(s) | leaving | POD | I1 | for | destination | J2 | on | day | 6 (AIR) | 160.00 Short Tons | of Movement | 7  |
|    |       |         |     |    |     |             |    |    |     |         | 80.00 Short Tons  | of Movement | 9  |
| 3  | C5(s) | leaving | POD | I1 | for | destination | J1 | on | day | 7 (AIR) | 140.00 Short Tons | of Movement | 4  |
|    |       |         |     |    |     |             |    |    |     |         | 40.00 Short Tons  | of Movement | 5  |
| 7  | C5(s) | leaving | POD | I1 | for | destination | J2 | on | day | 7 (AIR) | 200.00 Short Tons | of Movement | 8  |
|    |       |         |     |    |     |             |    |    |     |         | 220.00 Short Tons | of Movement | 9  |
| 1  | C5(s) | leaving | POD | I1 | for | destination | J1 | on | day | 8 (AIR) | 60.00 Short Tons  | of Movement | 5  |
| 9  | C5(s) | leaving | POD | I1 | for | destination | J2 | on | day | 8 (AIR) | 500.00 Short Tons | of Movement | 10 |
| 13 | C5(s) | leaving | POD | I2 | for | destination | J2 | on | day | 4 (AIR) | 780.00 Short Tons | of Movement | 14 |
| 9  | C5(s) | leaving | POD | I2 | for | destination | J1 | on | day | 5 (AIR) | 500.00 Short Tons | of Movement | 11 |
| 4  | C5(s) | leaving | POD | I2 | for | destination | J2 | on | day | 5 (AIR) | 220.00 Short Tons | of Movement | 14 |
| 7  | C5(s) | leaving | POD | I2 | for | destination | J1 | on | day | 6 (AIR) | 400.00 Short Tons | of Movement | 12 |
| 1  | C5(s) | leaving | POD | I2 | for | destination | J2 | on | day | 6 (AIR) | 20.00 Short Tons  | of Movement | 15 |
| 5  | C5(s) | leaving | POD | I2 | for | destination | J1 | on | day | 7 (AIR) | 300.00 Short Tons | of Movement | 13 |
| 3  | C5(s) | leaving | POD | I2 | for | destination | J2 | on | day | 7 (AIR) | 180.00 Short Tons | of Movement | 15 |
| 8  | C5(s) | leaving | POD | I2 | for | destination | J2 | on | day | 8 (AIR) | 480.00 Short Tons | of Movement | 16 |
| 1  | C5(s) | leaving | POD | I2 | for | destination | J2 | on | day | 9 (AIR) | 20.00 Short Tons  | of Movement | 16 |

**Figure 15. ITDM with MPBR Solution Test Case 4**

|           |             |    |                 |    |                 |                   |             |    |
|-----------|-------------|----|-----------------|----|-----------------|-------------------|-------------|----|
| 1 C130(s) | leaving POD | I1 | for destination | J2 | on day 3 (AIR)  |                   |             |    |
|           |             |    |                 |    |                 | 12.00 Short Tons  | of Movement | 6  |
| 1 C5(s)   | leaving POD | I1 | for destination | J1 | on day 4 (AIR)  | 50.00 Short Tons  | of Movement | 1  |
|           |             |    |                 |    |                 | 10.00 Short Tons  | of Movement | 2  |
| 1 C5(s)   | leaving POD | I1 | for destination | J2 | on day 5 (AIR)  | 59.00 Short Tons  | of Movement | 6  |
|           |             |    |                 |    |                 | 1.00 Short Tons   | of Movement | 8  |
| 1 C5(s)   | leaving POD | I1 | for destination | J1 | on day 6 (AIR)  | 60.00 Short Tons  | of Movement | 3  |
| 1 C17(s)  | leaving POD | I1 | for destination | J2 | on day 7 (AIR)  | 4.00 Short Tons   | of Movement | 9  |
|           |             |    |                 |    |                 | 31.00 Short Tons  | of Movement | 10 |
| 1 C130(s) | leaving POD | I1 | for destination | J2 | on day 8 (AIR)  | 12.00 Short Tons  | of Movement | 10 |
| 2 C5(s)   | leaving POD | I2 | for destination | J2 | on day 4 (AIR)  | 120.00 Short Tons | of Movement | 14 |
| 2 C17(s)  | leaving POD | I2 | for destination | J1 | on day 5 (AIR)  | 50.00 Short Tons  | of Movement | 11 |
| 1 C17(s)  | leaving POD | I2 | for destination | J2 | on day 5 (AIR)  | 35.00 Short Tons  | of Movement | 14 |
| 1 C5(s)   | leaving POD | I2 | for destination | J2 | on day 5 (AIR)  | 60.00 Short Tons  | of Movement | 14 |
| 1 C130(s) | leaving POD | I2 | for destination | J1 | on day 6 (AIR)  | 12.00 Short Tons  | of Movement | 12 |
| 1 C17(s)  | leaving POD | I2 | for destination | J1 | on day 6 (AIR)  | 35.00 Short Tons  | of Movement | 12 |
| 1 C5(s)   | leaving POD | I2 | for destination | J1 | on day 6 (AIR)  | 60.00 Short Tons  | of Movement | 12 |
| 1 C5(s)   | leaving POD | I2 | for destination | J2 | on day 7 (AIR)  | 60.00 Short Tons  | of Movement | 15 |
| 5 C5(s)   | leaving POD | I2 | for destination | J2 | on day 8 (AIR)  | 300.00 Short Tons | of Movement | 16 |
| 1 C17(s)  | leaving POD | I2 | for destination | J2 | on day 9 (AIR)  | 35.00 Short Tons  | of Movement | 16 |
| 3 FTTX(s) | leaving POD | I1 | for destination | J2 | on day 3 (RAIL) | 450.00 Short Tons | of Movement | 6  |
| 3 FTTX(s) | leaving POD | I1 | for destination | J1 | on day 4 (RAIL) | 450.00 Short Tons | of Movement | 1  |
| 1 FTTX(s) | leaving POD | I1 | for destination | J2 | on day 4 (RAIL) | 150.00 Short Tons | of Movement | 7  |
| 1 FTTX(s) | leaving POD | I1 | for destination | J1 | on day 5 (RAIL) | 60.00 Short Tons  | of Movement | 2  |
|           |             |    |                 |    |                 | 90.00 Short Tons  | of Movement | 3  |
| 1 ITTX(s) | leaving POD | I1 | for destination | J1 | on day 5 (RAIL) | 180.00 Short Tons | of Movement | 2  |
| 2 FTTX(s) | leaving POD | I1 | for destination | J2 | on day 5 (RAIL) | 63.00 Short Tons  | of Movement | 6  |
|           |             |    |                 |    |                 | 237.00 Short Tons | of Movement | 7  |
| 4 FTTX(s) | leaving POD | I1 | for destination | J1 | on day 6 (RAIL) | 600.00 Short Tons | of Movement | 3  |
| 2 FTTX(s) | leaving POD | I1 | for destination | J1 | on day 7 (RAIL) | 200.00 Short Tons | of Movement | 4  |
|           |             |    |                 |    |                 | 100.00 Short Tons | of Movement | 5  |

**Figure 16. ITDM with MPBR Solution Narrow Time Window Test Case 5**

|            |             |    |                 |    |                 |                   |             |    |
|------------|-------------|----|-----------------|----|-----------------|-------------------|-------------|----|
| 2 FTTX(s)  | leaving POD | I1 | for destination | J2 | on day 7 (RAIL) | 199.00 Short Tons | of Movement | 8  |
|            |             |    |                 |    |                 | 101.00 Short Tons | of Movement | 9  |
| 1 ITTX(s)  | leaving POD | I1 | for destination | J2 | on day 8 (RAIL) | 180.00 Short Tons | of Movement | 9  |
| 3 FTTX(s)  | leaving POD | I1 | for destination | J2 | on day 9 (RAIL) | 450.00 Short Tons | of Movement | 10 |
| 4 FTTX(s)  | leaving POD | I2 | for destination | J2 | on day 4 (RAIL) | 600.00 Short Tons | of Movement | 14 |
| 1 ITTX(s)  | leaving POD | I2 | for destination | J2 | on day 4 (RAIL) | 180.00 Short Tons | of Movement | 14 |
| 3 FTTX(s)  | leaving POD | I2 | for destination | J1 | on day 5 (RAIL) | 450.00 Short Tons | of Movement | 11 |
| 2 FTTX(s)  | leaving POD | I2 | for destination | J1 | on day 6 (RAIL) | 293.00 Short Tons | of Movement | 12 |
| 2 FTTX(s)  | leaving POD | I2 | for destination | J1 | on day 7 (RAIL) | 300.00 Short Tons | of Movement | 13 |
| 1 FTTX(s)  | leaving POD | I2 | for destination | J2 | on day 7 (RAIL) | 140.00 Short Tons | of Movement | 15 |
| 1 FTTX(s)  | leaving POD | I2 | for destination | J2 | on day 8 (RAIL) | 150.00 Short Tons | of Movement | 16 |
| 1 M35(s)   | leaving POD | I1 | for destination | J2 | on day 3 (ROAD) | 8.00 Short Tons   | of Movement | 6  |
| 1 M1083(s) | leaving POD | I1 | for destination | J2 | on day 4 (ROAD) | 5.00 Short Tons   | of Movement | 7  |
| 1 M35(s)   | leaving POD | I1 | for destination | J2 | on day 5 (ROAD) | 8.00 Short Tons   | of Movement | 6  |
| 1 M35(s)   | leaving POD | I1 | for destination | J2 | on day 6 (ROAD) | 8.00 Short Tons   | of Movement | 7  |
| 1 M35(s)   | leaving POD | I1 | for destination | J2 | on day 7 (ROAD) | 8.00 Short Tons   | of Movement | 9  |
| 1 HEMTT(s) | leaving POD | I1 | for destination | J2 | on day 8 (ROAD) | 7.00 Short Tons   | of Movement | 9  |
| 1 HEMTT(s) | leaving POD | I1 | for destination | J2 | on day 9 (ROAD) | 7.00 Short Tons   | of Movement | 10 |
| 1 M1083(s) | leaving POD | I2 | for destination | J2 | on day 5 (ROAD) | 5.00 Short Tons   | of Movement | 14 |
| 2 M1083(s) | leaving POD | I2 | for destination | J2 | on day 8 (ROAD) | 10.00 Short Tons  | of Movement | 16 |
| 1 HEMTT(s) | leaving POD | I2 | for destination | J2 | on day 9 (ROAD) | 5.00 Short Tons   | of Movement | 16 |

**Figure 16. ITDM with MPBR Solution Narrow Time Window Test Case 5 (cont.)**

|           |             |    |                 |    |        |           |                   |             |    |
|-----------|-------------|----|-----------------|----|--------|-----------|-------------------|-------------|----|
| 1 C5(s)   | leaving POD | I1 | for destination | J2 | on day | 3 (AIR)   |                   |             |    |
|           |             |    |                 |    |        |           | 60.00 Short Tons  | of Movement | 6  |
| 1 C5(s)   | leaving POD | I1 | for destination | J2 | on day | 4 (AIR)   |                   |             |    |
|           |             |    |                 |    |        |           | 60.00 Short Tons  | of Movement | 6  |
| 1 C5(s)   | leaving POD | I1 | for destination | J2 | on day | 5 (AIR)   |                   |             |    |
|           |             |    |                 |    |        |           | 30.00 Short Tons  | of Movement | 6  |
|           |             |    |                 |    |        |           | 30.00 Short Tons  | of Movement | 7  |
| 1 C5(s)   | leaving POD | I1 | for destination | J2 | on day | 6 (AIR)   |                   |             |    |
|           |             |    |                 |    |        |           | 60.00 Short Tons  | of Movement | 6  |
| 1 C5(s)   | leaving POD | I1 | for destination | J2 | on day | 7 (AIR)   |                   |             |    |
|           |             |    |                 |    |        |           | 60.00 Short Tons  | of Movement | 10 |
| 1 C5(s)   | leaving POD | I1 | for destination | J1 | on day | 8 (AIR)   |                   |             |    |
|           |             |    |                 |    |        |           | 60.00 Short Tons  | of Movement | 3  |
| 1 C5(s)   | leaving POD | I1 | for destination | J1 | on day | 9 (AIR)   |                   |             |    |
|           |             |    |                 |    |        |           | 60.00 Short Tons  | of Movement | 3  |
| 1 C5(s)   | leaving POD | I1 | for destination | J1 | on day | 10 (AIR)  |                   |             |    |
|           |             |    |                 |    |        |           | 60.00 Short Tons  | of Movement | 2  |
| 1 C5(s)   | leaving POD | I1 | for destination | J1 | on day | 11 (AIR)  |                   |             |    |
|           |             |    |                 |    |        |           | 60.00 Short Tons  | of Movement | 3  |
| 1 C5(s)   | leaving POD | I1 | for destination | J1 | on day | 12 (AIR)  |                   |             |    |
|           |             |    |                 |    |        |           | 60.00 Short Tons  | of Movement | 4  |
| 1 C5(s)   | leaving POD | I1 | for destination | J2 | on day | 13 (AIR)  |                   |             |    |
|           |             |    |                 |    |        |           | 60.00 Short Tons  | of Movement | 10 |
| 1 FTTX(s) | leaving POD | I1 | for destination | J2 | on day | 3 (RAIL)  |                   |             |    |
|           |             |    |                 |    |        |           | 140.00 Short Tons | of Movement | 6  |
| 2 FTTX(s) | leaving POD | I1 | for destination | J2 | on day | 4 (RAIL)  |                   |             |    |
|           |             |    |                 |    |        |           | 300.00 Short Tons | of Movement | 7  |
| 2 FTTX(s) | leaving POD | I1 | for destination | J1 | on day | 5 (RAIL)  |                   |             |    |
|           |             |    |                 |    |        |           | 300.00 Short Tons | of Movement | 1  |
| 2 FTTX(s) | leaving POD | I1 | for destination | J1 | on day | 6 (RAIL)  |                   |             |    |
|           |             |    |                 |    |        |           | 30.00 Short Tons  | of Movement | 1  |
|           |             |    |                 |    |        |           | 270.00 Short Tons | of Movement | 3  |
| 2 FTTX(s) | leaving POD | I1 | for destination | J2 | on day | 7 (RAIL)  |                   |             |    |
|           |             |    |                 |    |        |           | 70.00 Short Tons  | of Movement | 7  |
|           |             |    |                 |    |        |           | 230.00 Short Tons | of Movement | 9  |
| 2 FTTX(s) | leaving POD | I1 | for destination | J1 | on day | 8 (RAIL)  |                   |             |    |
|           |             |    |                 |    |        |           | 170.00 Short Tons | of Movement | 1  |
|           |             |    |                 |    |        |           | 130.00 Short Tons | of Movement | 2  |
| 2 FTTX(s) | leaving POD | I1 | for destination | J2 | on day | 9 (RAIL)  |                   |             |    |
|           |             |    |                 |    |        |           | 250.00 Short Tons | of Movement | 6  |
|           |             |    |                 |    |        |           | 50.00 Short Tons  | of Movement | 8  |
| 1 FTTX(s) | leaving POD | I1 | for destination | J1 | on day | 10 (RAIL) |                   |             |    |
|           |             |    |                 |    |        |           | 60.00 Short Tons  | of Movement | 2  |
|           |             |    |                 |    |        |           | 90.00 Short Tons  | of Movement | 4  |
| 1 FTTX(s) | leaving POD | I1 | for destination | J2 | on day | 10 (RAIL) |                   |             |    |
|           |             |    |                 |    |        |           | 150.00 Short Tons | of Movement | 8  |
| 2 FTTX(s) | leaving POD | I1 | for destination | J1 | on day | 11 (RAIL) |                   |             |    |
|           |             |    |                 |    |        |           | 300.00 Short Tons | of Movement | 3  |
| 1 FTTX(s) | leaving POD | I1 | for destination | J1 | on day | 12 (RAIL) |                   |             |    |
|           |             |    |                 |    |        |           | 50.00 Short Tons  | of Movement | 4  |
|           |             |    |                 |    |        |           | 100.00 Short Tons | of Movement | 5  |

**Figure 17. ITDM with MPBR Solution Wider Time Window Test Case 5**

|           |             |    |                 |    |                  |                   |             |    |
|-----------|-------------|----|-----------------|----|------------------|-------------------|-------------|----|
| 1 FTTX(s) | leaving POD | I1 | for destination | J2 | on day 12 (RAIL) | 70.00 Short Tons  | of Movement | 9  |
|           |             |    |                 |    |                  | 80.00 Short Tons  | of Movement | 10 |
| 2 FTTX(s) | leaving POD | I1 | for destination | J2 | on day 13 (RAIL) | 300.00 Short Tons | of Movement | 10 |
| 1 FTTX(s) | leaving POD | I2 | for destination | J2 | on day 4 (RAIL)  | 144.00 Short Tons | of Movement | 14 |
| 2 FTTX(s) | leaving POD | I2 | for destination | J2 | on day 5 (RAIL)  | 300.00 Short Tons | of Movement | 14 |
| 2 FTTX(s) | leaving POD | I2 | for destination | J2 | on day 6 (RAIL)  | 300.00 Short Tons | of Movement | 14 |
| 2 FTTX(s) | leaving POD | I2 | for destination | J1 | on day 7 (RAIL)  | 300.00 Short Tons | of Movement | 11 |
| 2 FTTX(s) | leaving POD | I2 | for destination | J1 | on day 8 (RAIL)  | 200.00 Short Tons | of Movement | 11 |
|           |             |    |                 |    |                  | 100.00 Short Tons | of Movement | 12 |
| 2 FTTX(s) | leaving POD | I2 | for destination | J2 | on day 9 (RAIL)  | 232.00 Short Tons | of Movement | 14 |
|           |             |    |                 |    |                  | 68.00 Short Tons  | of Movement | 16 |
| 2 FTTX(s) | leaving POD | I2 | for destination | J2 | on day 11 (RAIL) | 184.00 Short Tons | of Movement | 15 |
|           |             |    |                 |    |                  | 116.00 Short Tons | of Movement | 16 |
| 2 FTTX(s) | leaving POD | I2 | for destination | J1 | on day 12 (RAIL) | 300.00 Short Tons | of Movement | 12 |
| 2 FTTX(s) | leaving POD | I2 | for destination | J1 | on day 13 (RAIL) | 300.00 Short Tons | of Movement | 13 |
| 2 FTTX(s) | leaving POD | I2 | for destination | J2 | on day 14 (RAIL) | 300.00 Short Tons | of Movement | 16 |
| 1 M35(s)  | leaving POD | I2 | for destination | J2 | on day 7 (ROAD)  | 8.00 Short Tons   | of Movement | 14 |
| 1 M35(s)  | leaving POD | I2 | for destination | J2 | on day 8 (ROAD)  | 8.00 Short Tons   | of Movement | 14 |
| 1 M35(s)  | leaving POD | I2 | for destination | J2 | on day 9 (ROAD)  | 8.00 Short Tons   | of Movement | 14 |
| 1 M35(s)  | leaving POD | I2 | for destination | J2 | on day 10 (ROAD) | 8.00 Short Tons   | of Movement | 15 |
| 1 M35(s)  | leaving POD | I2 | for destination | J2 | on day 11 (ROAD) | 8.00 Short Tons   | of Movement | 15 |
| 1 M35(s)  | leaving POD | I2 | for destination | J2 | on day 12 (ROAD) | 8.00 Short Tons   | of Movement | 16 |
| 1 M35(s)  | leaving POD | I2 | for destination | J2 | on day 13 (ROAD) | 8.00 Short Tons   | of Movement | 16 |

**Figure 17. ITDM with MPBR Solution Wider Time Window Test Case 5 (cont.)**

## Appendix I. MPBR VBA Code Updates

### Objective Function.

'Build Q variables (beddown upper bound)

```
For i = 1 To UBound(POD)
    For m = 1 To UBound(mode)
        coeff = 10 'g * 3
        Print #1, " + " & coeff & " * " & "Q_" & _
            POD(i) & "_" & _
            mode(m);
    Next m
Next i
```

Print #1, ",";

'Update Status Form

OBJFUNC.LabelProgress.Width = 0

OBJFUNC.Label1.Caption = "Generating Requirement Constraints..."

## Constraints.

\*\*\*\*\*Build New Q/X Constraint \*\*\*\*\*

RowCount = 1

'initialize current w value based off of RowCount=1

curr\_i = w(RowCount, 1)

curr\_j = w(RowCount, 2)

curr\_m = w(RowCount, 3)

curr\_k = w(RowCount, 4)

daynum = 1

xRowCount = 1

LHScout = 0

Print #1, "[MinMax\_" & constraintnum & "]" ,

While RowCount <= UBound(w)

  If daynum >= w(RowCount, 6) And daynum <= w(RowCount, 7) Then

    While xRowCount <= UBound(w)

      If w(xRowCount, 1) = curr\_i And \_

      w(xRowCount, 3) = curr\_m And \_

      daynum >= w(xRowCount, 6) And daynum <= w(xRowCount, 7) Then

        Print #1, " + X\_" & \_

          w(xRowCount, 1) & " " & \_

          w(xRowCount, 2) & " " & \_

          w(xRowCount, 3) & " " & \_

          w(xRowCount, 4) & " " & \_

          daynum;

        LHScout = LHScout + 1

      End If

      xRowCount = xRowCount + 1

    Wend

  'do RHS

  If LHScout > 0 Then

    Print #1, " <= " & " Q\_" & \_

      w(RowCount, 1) & " " & \_

      w(RowCount, 3);

    Print #1, ",";

    constraintnum = constraintnum + 1

    If RowCount = UBound(w) And daynum = absmaxday Then

      'nada

    Else

```
Print #1, "[MinMax_" & constraintnum & "]", 'print for next constraint if  
not on last vehicle constraint
```

```
End If  
End If
```

```
xRowCount = 1  
daynum = daynum + 1  
LHScount = 0
```

```
ElseIf daynum < w(RowCount, 6) Then  
daynum = daynum + 1
```

```
Else 'Implies daynum > w(RowCount, 7) Then  
daynum = 1  
RowCount = RowCount + 1  
If RowCount <= UBound(w) Then  
curr_i = w(RowCount, 1)  
curr_j = w(RowCount, 2)  
curr_m = w(RowCount, 3)  
curr_k = w(RowCount, 4)  
End If  
End If
```

```
If RowCount Mod 10 = 0 Then
```

```
PctDone = RowCount / UBound(w)  
With OBJFUNC  
.FrameProgress.Caption = Format(PctDone, "0%")  
.LabelProgress.Width = PctDone * (.FrameProgress.Width - 10)  
End With
```

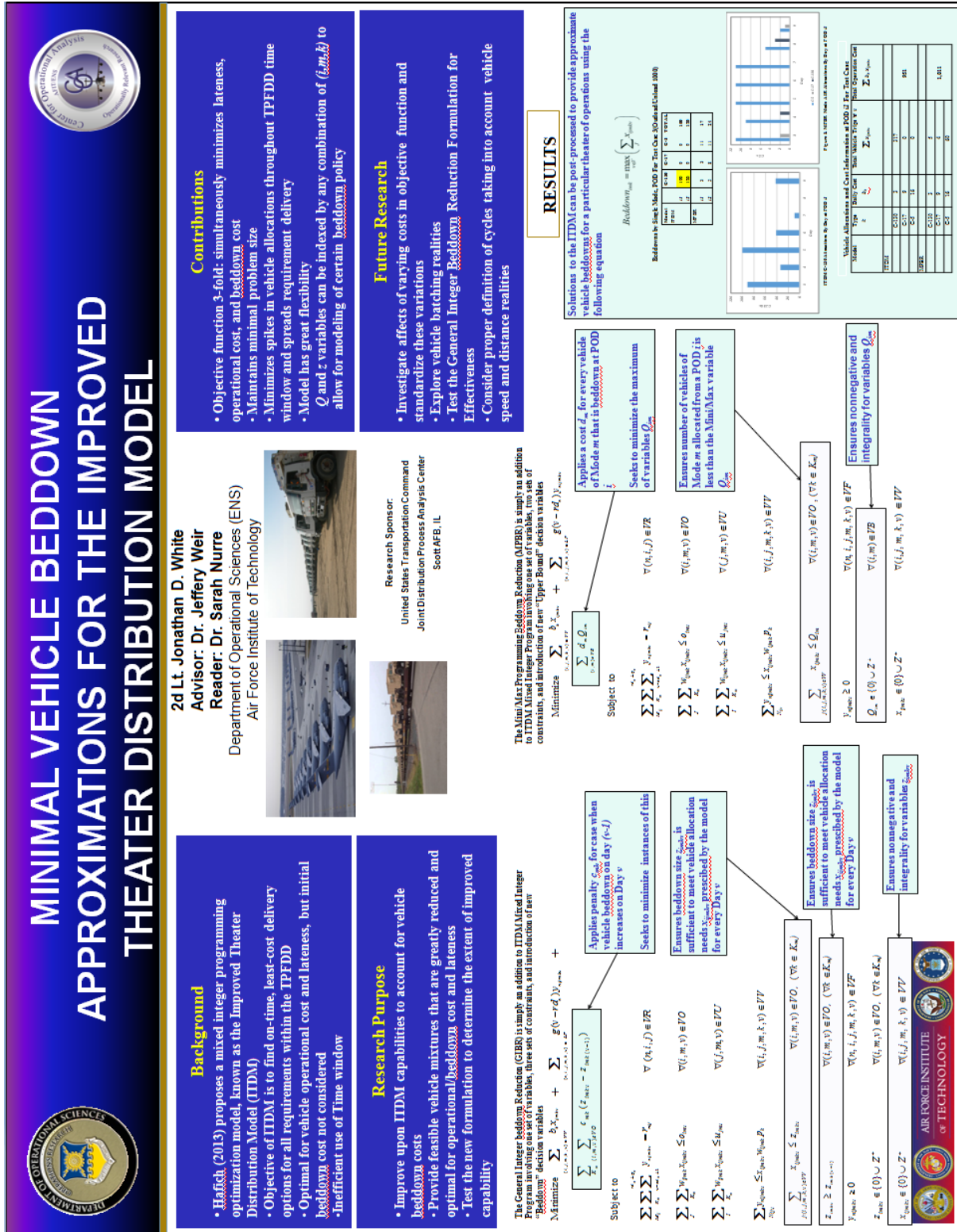
```
'The DoEvents statement is responsible for the form updating
```

```
DoEvents  
End If
```

```
Wend
```

```
'update status form  
OBJFUNC.LabelProgress.Width = 0  
OBJFUNC.Label1.Caption = "Generating Outloading Constraints..."
```

# Appendix J. Research Summary Chart



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| 14. ABSTRACT<br>Gathering insight into the theater distribution process can be a complex task, especially when estimating potential beddown solutions. Coming up with a low cost feasible mixture of cargo vehicles that will support distribution of military personnel and goods within theater is currently a high priority for force flow analysts at USTRANSCOM. In the past, analysts used a trial and error simulation process that was iterative and time consuming. Recent research has produced the Improved Theater Distribution Model (ITDM), which presents a less time consuming, more precise method to estimate beddown allocations. Improving on this research, two linear programming methods were developed and added to the ITDM that reduce baseline beddown approximations. Because daily usage cost and initial beddown cost was included, this ultimately presented a lower cost feasible solution when modeling theater distribution. The improved beddown solutions generated from post-processing results of the ITDM can be used as baselines for further distribution analysis. Within the construct of the model, precise set notation is carried over from the Improved Theater Distribution Model and slightly altered to reduce the generation of unnecessary variables and constraints with large-scale problems. |                  |                                   |                                      |                                                              |                                                                                          |
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