Dual, Hyperfast Spectral Decompositions for Sound Propagation on the Continental Shelf: Acoustic Lensing, Imaging, Communication and "Seeing" Inside Shadow Zones

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LONG-TERM GOALS

1. A ***nonlinear Fourier representation*** of sound propagation and internal soliton dynamics in the ocean is under development to analyze ***time series of internal wave data, sound speed profiles and to model sound waves*** propagating in the oceanic wave guide. The approach may be viewed as a generalization of linear Fourier analysis and is loosely referred to as Nonlinear Fourier Analysis or the ***Generalized Fourier Transform*** (GFT).

2. The present report focuses on new results for application of GFT methods to ***acoustic wave propagation***.

3. Application of the method as a tool for ***searching inside shadow zones*** and for ***acoustic communications*** is also underway.

Specifically I discuss two ***dual spectral decompositions*** for the sound speed field and for the acoustic field. Thus the same algorithm accomplishes ***simultaneous Fourier-type decompositions*** for both the sound speed field and for the acoustic field. This contrasts with standard eigenfunction analyses in which only the sound speed field has an eigenfunction decomposition. Additional work has been conducted on two new models for the propagation of sound waves in the oceanic environment: (1) the derivation and exploitation of ordinary differential equations for the evolution of sound waves and (2) the modeling of sound waves via the multi-dimensional Fourier approach. The methods used herein are based upon a generalization of ***inverse scattering theory*** that has been historically used for solving a wide variety of ***linear and nonlinear partial differential wave equations with particular boundary conditions***. Therefore in principle one also has all the advantages that we associate with this method, ***including (nonlinear) filtering, image processing of all types, power spectral analysis, coherence functions***, etc., but all generalized for the nonlinear nature of the scattering problem for acoustic waves.

OBJECTIVES

1. The long-term goal of the present research is the development of ***fast numerical techniques for the multidimensional Fourier method.*** This is a crucial step for the use of the method, especially for onboard computations (shipboard or unteathered vehicle imaging of underwater objects).

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2. Another major objective is to ***improve our understanding and predictive capability of sound propagation on the continental shelf in the presence of internal solitons***.

3. The primary approaches consist of (a) ***analysis of spatially distributed sound speed profile data*** and (b) ***hyperfast numerical modeling of sound speed propagation***.

APPROACH

The starting point of the present work is the ***parabolic wave equation (PE)*** [Hardin and Tappert, 1970], well known as one of the most important wave-theoretic, range-dependent propagation models [Jensen, Kuperman, Porter and Schmidt, 2000; Kuperman, 2001]. The PE is derived from the driven Helmholz equation in cylindrical coordinates (r,θ,z) without azimuthal dependence. The far field pressure field is given by $p(r,z)=\sqrt{2/\pi k_o r}\,\psi(r,z)e^{i(k_o r-\pi/4)}$. The paraxial (small-angle) approximation for the envelope of the pressure field, $\psi(r,z)$, gives the PE:

$$2ik_o\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial z^2}+k_o^2[n^2(r,z)-1]\psi=0 \tag{1}$$

In Eq. (1) $n(r,z)=c_o/c(r,z)$ is the index of refraction, $c(r,z)$ is the sound speed field, c_o is a reference sound speed. The fields $\psi(r,z)$, $n(r,z)$ and $p(r,z)$ are range (r) and depth (z) dependent. One views Eq. (1) as a Cauchy problem in the sense that given $\psi(0,z)$ (the source field, with say a gaussian depth profile and oscillation frequency, ω; $k_o=\omega/c_o$) one seeks the pressure envelope $\psi(r,z)$ over all range, r, and depth, z, given the environment (the sound speed field as a function of depth and range) defined by $n^2(r,z)$.

One of the most important methods for obtaining the numerical solutions of (1) is the split-step, range-marching algorithm developed by Tappert, where the FFT is applied to numerically integrate the sound field envelope, $\psi(r,z)$, given a set of measured or theoretical sound speed depth profiles specified over a given range distance. It is of course well-known that the physical applicability of the PE can be improved by modifying the equation for higher angle propagation and by modeling attenuation in the fluid and bottom, but these are not discussed in this brief report in terms of the input environment. The envelope, $\psi(r,z)$, and the sound speed profile field, $c(r,z)$, lie in the domains $0\le r\le R$ and $-z_{\max}\le z\le z_{\max}$ ($Z=2z_{\max}$). The usual domain of the model lies in $0\le z\le z_{\max}$. It is standard to reflect the sound speed field about the ocean surface, effectively doubling the domain size. One has $c(r,z)=c(r,-z)$, $U(r,z)=U(r,-z)$ so that the sound-speed profiles and the potential are ***symmetric about the sea surface***, and $\psi(r,z)=-\psi(r,-z)$ where the pressure envelope is ***antisymmetric about the sea surface***. I do not discuss the absorption of sound in the bottom layers, a standard topic in ocean acoustics, easily applied to the problem at hand, by suitable modification of the
.

One of the unique features of the method presented herein (the GFT) is that I have derived ***unique spectral decompositions*** not only for the complex sound speed profile, $\psi(r,z)$, (as is standard in eigenfunction analysis) but also simultaneously for the ***potential function of the PE*** (not normally done in eigenfunction analysis). This is a (pleasantly) surprising result and is instrumental for all applications discussed herein. For the acoustic propagation problem as just proposed the ***potential function*** of the PE (1) (the coefficient in front of the last term of (1)) the ***nonlinear Fourier decomposition*** is:

$$V(r,z) = K_o^2[n^2(r,z)-1] = 2\frac{\partial^2}{\partial x^2}\ln\theta(r/2K_o,z\,|\tilde{\mathbf{B}}) \tag{2}$$

where the ***generalized Fourier series***, $\theta(x,t\,|\tilde{\mathbf{B}})$, is given by the expression

$$\theta(r/2K_o,z\,|\,\mathbf{B}) = \sum_{m_1=-\infty}^{\infty}\sum_{m_2=-\infty}^{\infty}\cdots\sum_{m_N=-\infty}^{\infty} e^{i\sum_{n=1}^{N} m_n\left(\frac{k_n r}{2K_o}+l_n z+\phi_n\right)-\frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N} m_m m_n B_{mn}} \tag{3}$$

In the algebraic geometry literature the function $\theta(x,t\,|\tilde{\mathbf{B}})$ is referred to as a ***Riemann theta function***. For our purposes it can be viewed as a ***multidimensional Fourier series*** in which $\mathbf{B}$ is the Riemann matrix, k_n are the spectral wave numbers in the range coordinate, l_n are the spectral wave numbers in the depth coordinate, ω_n are spectral frequencies, ϕ_n are phases. Each diagonal element of the Riemann matrix describes one of the ***nonlinear modes of the sound speed field***. Eq. (3) is a generalized Fourier, nonlinear superposition law. The number of modes is given by *N*, which determines the number of nested sums in (3). The envelope of the pressure field as a solution of the PE (1) is also computable by similar methods, but I omit the formulas here. A deeper discussion of the boundary conditions awaits a later report.

The analytic solution of the complex envelope solution of the PE (2) can be written in terms of the multi-dimensional Fourier series:

$$\psi(r,z) = \left[\frac{\theta\left(\int_{P_\infty}^{P}\boldsymbol{\omega} + \mathbf{K}_r r + \mathbf{K}_z z + \mathbf{D}\middle|\mathbf{B}\right)\theta(\mathbf{D}|\mathbf{B})}{\theta\left(\int_{P_\infty}^{P}\boldsymbol{\omega} + \mathbf{D}\middle|\mathbf{B}\right)\theta(\mathbf{K}_r r + \mathbf{K}_z z + \mathbf{D}|\mathbf{B})}\right]e^{i\Omega_1 r + i\Omega_2 z} \tag{4}$$

Thus we have the construction of the envelope of the pressure field in terms of Riemann theta functions. The $\boldsymbol{\omega}$ are Abelian differentials, the K_r, K_z are wave numbers, the $\mathbf{D}$ are phases, the $\Omega_i(P)$ are Abelian integrals of the second kind. Details of the determination of the parameters will be given elsewhere.

The goal and scope of the present research is to develop a basis for using (2)-(4) to better understand acoustic wave propagation as described by the PE (1) and its generalizations.

It is important to note that ***the work described here is not the usual modal analysis*** typically applied to ocean acoustic problems, but is a ***simultaneous nonlinear Fourier representation*** (given the appropriate Riemann matrix) for the potential $V(r,z) = K_o^2[n^2(r,z)-1]$ (and therefore also for the sound speed profiles, $n(r,z)=\sqrt{1+V(r,z)/K_o^2}$) and for the envelope of the sound pressure field solution of the PE, $\psi(r,z)$.

WORK COMPLETED

Let us now discuss some aspects of the numerical analysis of the problem. This means that the above formulation, the dual theta function decomposition, needs to be modified to include the boundary conditions. First, in order for the potential to be symmetric about the sea surface we require

$$U(r,z)=2\partial_{zz}\ln\theta_{sym}(r,z\,|\,\mathbf{B},\phi)$$

where

$$\theta_{sym}(r,z\,|\,\mathbf{B},\phi)=\frac{1}{2}\left[\theta(r,z\,|\,\mathbf{B},\phi)+\theta(r,-z\,|\,\mathbf{B},\phi)\right]$$

The complex pressure field must be asymmetric about the sea surface, therefore:

$$\psi(r,z)=a\left[\frac{\theta_{asym}\left(r,z\middle|\mathbf{B},\delta\right)}{\theta_{sym}\left(r,z\middle|\mathbf{B},\phi\right)}\right]e^{i\Omega_1 r+i\Omega_2 z}$$

where

$$\theta_{asym}(r,z\,|\,\mathbf{B},\phi)=\frac{1}{2}\left[\theta(r,z\,|\,\mathbf{B},\phi)-\theta(r,-z\,|\,\mathbf{B},\phi)\right]$$

One of the most important aspects of this approach is that the above formulation has the same Riemann matrix for the spectral decompositions of both the sound speed field and the complex acoustic field. Numerically this means that one first obtains the Riemann matrix purely from the sound speed field. It is then used in the spectral decomposition of the sound speed field. Note that the phases are different for the two spectral decompositions! This allows a whole range of novel ways to analyze the acoustic situation in the ocean from aspects of imaging, lensing and observing inside shadow zones.

There are several reasons way the approach is so novel:

(1) The dual nature of the spectral composition in terms of the sound speed field and in terms of the complex acoustic field,

(2) the phases are unique for these two fields and (3) the nonlinear nature of the spectral decomposition allows for unique lensing via nonlinear filtering techniques. The combination of these aspects of the method provide ways to address ocean acoustics in ways not addressable by any other approach.

RESULTS

A major focus of this year's work has been the continued development and understanding of the above theoretical formulation and the coding of over 25,000 lines of FORTRAN programs to implement the method. Here is a list of capability completed in these codes:

(1) The PE (1) has been reprogramed in terms of its ***exact analytical solution using generalized Fourier analysis***. The solution assumes that the sound field is simultaneously a function of range, r, and depth, z. ***Thus we have developed a range dependent generalized Fourier analysis for all nonsingular solutions of the PE arising from a particular sound field,*** $c(r,z)$**.**

(2) ***Two new numerical methods*** have been programed for ***integrating solutions of the PE for any input sound speed field,*** $c(r,z)$**.** The first method is an order of magnitude faster than conventional methods, the second, based upon initial estimates, will be about 1000 times faster.

(3) ***The theta function approach has been extended to fully three-dimensional propagation***, which includes the cross range.

(4) ***The full nonlinear Fourier analysis of the PE and its nonlinear generalizations can be exploited.*** Indeed the important applications of the approach (is ***signal processing and nonlinear filtering of acoustic signals in the ocean environment in real time***. We have begun to address using the method for ***seeing into shadow zones in the acoustic field and for encrypted communications using sound waves.***

(5) The numerical methods seem to be sufficiently fast that ***algorithms for use in real time could be placed on autonomous, unmanned vehicles or on board ships or submarines***.

IMPACT/APPLICATION

The impact of this research will occur in general for the nonlinear Fourier analysis of acoustic wave and internal wave dynamics.

TRANSITIONS

Because of the potential speed-up in the computation of solutions to the nonlinear Fourier methods discussed herein one can conjecture that future applications could center around use on board vessels, including UUVs, ships and submarines. Maneuvers in shadow zones, acoustic communications and acoustic encryption are also other areas where transitions can happen.

RELATED PROJECTS

An intimate relationship between our results and other projects exists because of the relationship to internal wave propagation, since the acoustic sound channel depends crucially on the configuration of internal solitons and other internal wave dynamics, particularly on the continental shelf.

REFERENCES

F. B. Jensen, W. A. Kuperman, M. B. Porter and H. Schmidt, *Computation Ocean Acoustics*, Springer, 2000.

PUBLICATIONS

A. R. Osborne, Nonlinear Ocean Waves and the Inverse Scattering Transform, in *Scattering*, edited by R. Pike and P. Sabatier, Academic Press, 2002.

A. R. Osborne, M. Onorato and M. Serio, Searching for shadow zones and communicating and imaging in the oceanic acoustic wave guide, proceedings of the conference 14 International Symposium on Unmanned Untethered Submersible Technology, August 21-24, 2005, University of New Hampshire, Durham, New Hampshire.

A. R. Osborne, Dual Spectral Decompositions for Acoustic Wave Propagation in Media with Spatially Variable Index of Refraction I: Ordinary Linear Fourier Analysis, submitted, 2006.

A. R. Osborne, Dual Spectral Decompositions for Acoustic Wave Propagation in Media with Spatially Variable Index of Refraction II: Spectral Decomposition in Terms of Riemann Theta Functions, submitted, 2006.

A. R. Osborne, M. Onorato and M. Serio, New Approaches for the Study of Imaging, Ocean Acoustics, in preparation, 2006.