Wave and Circulation Prediction on Unstructured Grids

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LONG TERM GOALS

The long term goal of this research is to improve the accuracy of the computed multi-scale flow physics in geometrically and/or hydrodynamically complex oceanic and coastal ocean environments through refinements in the defined physics, domain definition and computational grid resolution. The focus is in particular on the improved coupling of wind generated short wave and circulation models within the framework of unstructured adaptive grid models for oceanic and coastal waters.

OBJECTIVES

The objective of this project is the dynamic coupling of the ADCIRC circulation model and the short wind wave model SWAN using unstructured computational meshes. In recent experience with coupling the continuous Galerkin based version of ADCIRC with short wind wave models, it has been found that wave transformation zones require high levels of resolution in order to correctly capture the wave radiation stress forcing to the circulation model. However the wave transformation zone tends to shift depending on the direction and period of the waves. Thus *h-p* adaptive discontinuous Galerkin (DG) based solutions are being developed to optimize the application of high resolution zones to correctly capture the wave coupling within these zones without over-resolving adjacent areas.

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APPROACH

We have focused on three areas to accomplish the defined objectives. The approach consists of the following:

- The first area is implementing and evaluating *h-p* adaptive schemes for Discontinuous Galerkin (DG) solutions to the shallow water equations (SWE) in anticipation of coupling with a mesh adaptive version of the SWAN wave model which is under development. This will improve the unstructured grid capabilities for the ADCIRC circulation model by implementing *hp*-adaptive methods to ensure that sufficient grid resolution is provided for all relevant flow scales. The present implementation of ADCIRC is based on both CG and DG finite element methods. Currently we are intensively developing the DG based algorithms. DG algorithms are particularly well suited for both propagation and advection dominated problems with or without sharp gradients in the forcing function, bathymetry, and/or flow. DG methods inherently preserve mass perfectly on an elemental level, which make them ideal for coupling flow and transport models. Example problems include flows with strong eddies such as those issuing from and through inlets; flows through rapidly varying bathymetry such as canyons or deep scour holes in inlets; and flows with sharp fronts such as tidal bores, ebb tide – waves interactions in inlets and wetting fronts. DG methods are conceptually similar to finite volume methods although DG methods are readily implemented with higher-order bases. DG methods are also ideal since non-conforming *h* and *p* refinement and adaptivity can be implemented.

- The second area is evaluating the influence of grid resolution in the SWE circulation model in relation to the wind wave model in the computed wave radiation stress set up. In addition, we are studying the sequencing of interpolation and the wave radiation stress gradient computation and the influence of both the wave and surge model grids in this context.

- The third area is to improve the wave – circulation model coupling which was bottle necked by our single processor interface for communication between the two models. We are designing all paradigms for very large scale parallelism on distributed memory machines, anticipating thousands of processors. We are migrating all interpolator functions to the individual processors and designing direct global to local and local to local processor communication between the wave model and circulation model processors. We are investigating the possibility of ADCIRC and SWAN running on identical grids on the same processors, entirely eliminating the inter-model processor to processor communication.

WORK COMPLETED

A DG model system has been developed, implemented, and tested for solving the two-dimensional shallow water equations, sediment transport, and passive scalar transport equations on unstructured triangular grids. The code features both standard *h*- (grid) and *p*- (polynomial order) refinement/adaptivity options (the current model is adaptive only in *p)*. The models have been rigorously verified on a series of idealized test cases using both the *hp*-refinement/adaptive strategies, and a recently parallelized version of the code is aiding in the validation process of the model on full-scale applications.

With regard to some specific algorithmic details of the model, through the use of an orthogonal, hierarchical basis and optimal triangular quadrature rules, an efficient, matrix-free DG method was developed and implemented as the unified solution technique for both the shallow water and transport models. The hierarchical nature of the basis that was employed gives the models a flexibility that allows for easy implementation of *p*-refinement and also *p*-adaptivity. Additionally, flexible programming of the models allows for the use of arbitrary stage and order strong-stability-preserving (SSP) Runge-Kutta time discretizations to be used in conjunction with the DG spatial discretization. This allowed for a detailed study of the computational efficiency of a number of different time stepping schemes.

In the critical area of inter-model communications we have also made substantial progress. Due to the bottlenecking that occurred in our preliminary implementation, with the circulation and wave models running on two sets of processors and inter-model communications and interpolation handled by one master processor, we have developed improved strategies. We have completed implementation of localized interpolation based on global input files together with on-the-fly globalization by the ADCIRC model. This allows for testing of difficult wind/wave coupling scenarios. However in anticipation of more complex scenarios we are developing processor to processor communications between the models. We are also investigating the possibility of the circulation and wave models running on identical grids on the same processors, entirely eliminating the inter-model processor to processor communication.

RESULTS

The following results are noted regarding this work:

- The *h* and *p* convergence properties of the hp-DG circulation model were demonstrated for both linear and highly nonlinear problems that included the effects of source/sink terms, spatially varying bathymetry, and time varying boundary conditions. It was found that standard *h*-refinement for a fixed *p* leads to optimal convergence rates of *p*+1 in both surface elevation and velocity (see Figure 1), while exponential convergence rates are observed for *p*-refinement for a fixed *h* (see Figure 2).

- *h*-refinement versus *p*-refinement was investigated for a simple linear problem and a highly nonlinear problem with complex two-dimensional flow structure. In both cases, it was found that significant cost efficiency benefits could be obtained by using *p*-refinement instead of standard *h*-refinement. In the linear case, a quantitative analysis comparing model run times to error levels demonstrated convincingly the superiority of *p*-refinement for smooth problems. This fact is shown in Figure 3, which plots model run times versus error levels for a range of grids and polynomial orders, where it can clearly be seen that using *p*-refinement is more computationally efficient in this case. In the case of the nonlinear problem – the case of an idealized inlet (see Figure 4) – a qualitative comparison of the solutions for two grid refinements and a range of *p* approximations demonstrated the ability of *p*-refinement to resolve the formation and advection of strong eddies along the inlet channel even on very coarse grids. This is shown in Figure 5. Higher-order *p* approximations on the coarse grid gave very comparable results to the lower-order *p* approximations on a finer grid, the latter simulations taking two to four times longer than the former.

- The two-level *p*-adaptive strategy that was implemented for the shallow water model revealed the ease with which stable and accurate *p*-adaptive schemes can be implemented into the models, especially given the choice of basis functions that were implemented. Investigation of these schemes in the context of the idealized inlet problem also revealed the substantial efficiency advantages that are possible with these techniques, which ran in 36% to 40% of the CPU time as the same problem using the higher-order approximation for all the elements for all time, while obtaining qualitatively similar results (see Figure 6). Additionally, this paradigm stands to significantly simplify the design of large-domain unstructured grids that can adequately resolve the space and time varying flow features of a given problem. Starting with a computational grid that has been designed to adequately resolve the geometry of a given domain, adaptivity will then automatically adjust the elements locally, usually based on some local characteristic of the solution such as the gradient, in order to provide the necessary spatial resolution to accurately capture the flow field as it evolves in space and time. An example of a grid evolving is shown in Figure 7, which shows snapshots of the grid for the idealized inlet at different times during a simulation. It can be noted how the higher-order elements track the key features of the flow.

- In a one-dimensional setting, the efficiency of a number of SSP Runge-Kutta time discretizations was investigated. The class of SSP methods that was investigated is defined by the property that the number of stages s is greater than the order k of the method. From analysis, conditions on the time step (CFL conditions) for the linear stability of the methods defined using the $s > k$ SSP schemes were obtained that are less restrictive than those of the "standard" so-called RKDG methods that use $s = k$ SSP Runge-Kutta schemes. The improvement in the CFL conditions for linear stability of the methods more than offsets the additional work introduced by the increased number of stages. Thus, more efficient RKDG methods that possess the same favorable accuracy and stability properties of the "standard" RKDG methods were obtained. Numerical results verified the CFL conditions for stability obtained from analysis and demonstrated the efficiency advantages of these new RKDG methods. The theoretical efficiency advantages that are possible with these schemes over the standard schemes are summarized in Table 1. Numerical results showed excellent agreement with these theoretical results.

- The performance of the models was also demonstrated and evaluated for a range of different problems with hydraulic jumps and tidal bores (discontinuities or shocks) and advection dominance in both one- and two-dimensions. It was found that the models could accurately capture the steep fronts and discontinuities in surface elevation, and also concentration or bathymetry, without the introduction of spurious oscillations, as is common when using standard continuous Galerkin (CG) methods for these types of problems.

- The shallow water model and passive transport model were applied to a full-scale application at Shinnecock Inlet, New York. The models were shown to produce stable, non-oscillatory results, and, in particular, the circulation patterns observed in the vicinity of the inlet were shown to be consistent with observational data. The shallow water model accurately resolved the formation and advection of eddies, which were a key factor in determining the transport and fate of the contaminant using the passive transport model (see Figure 8). This problem is a good candidate for the *p*-adaptive strategy and will be tested in the near future.

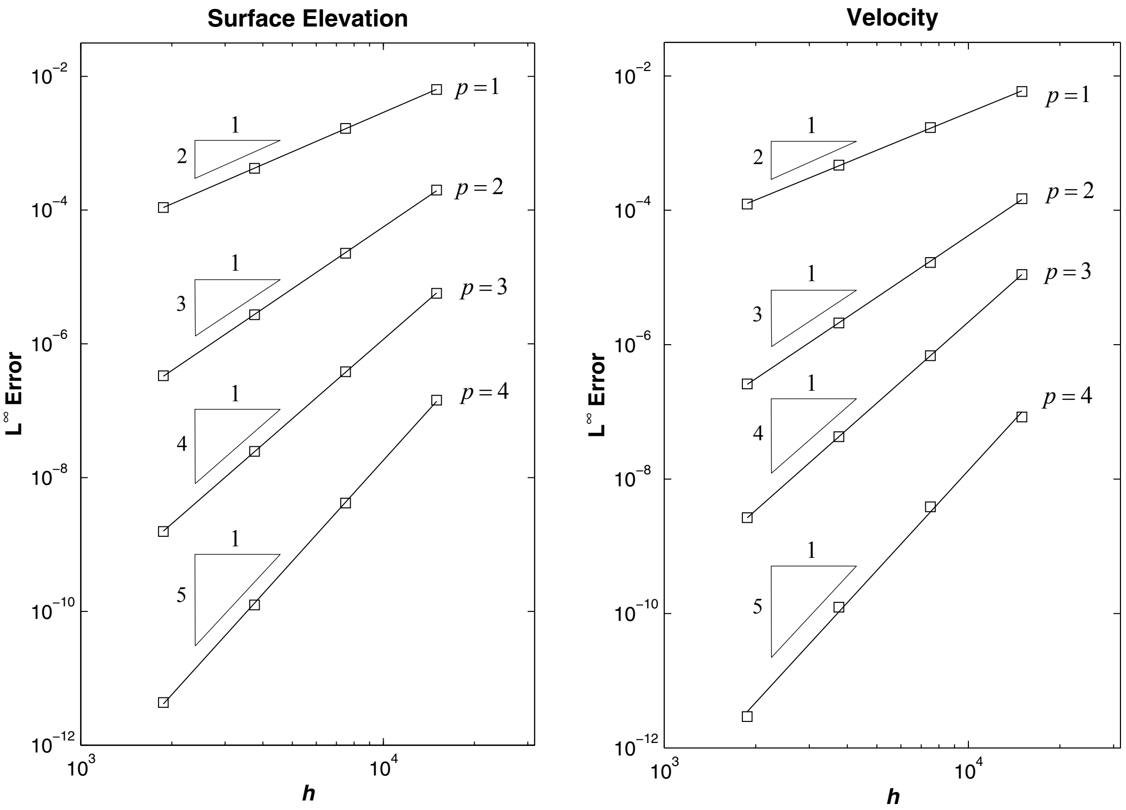


Figure 1: h convergence of the DG shallow water model in surface elevation and velocity for an idealized harbor problem.

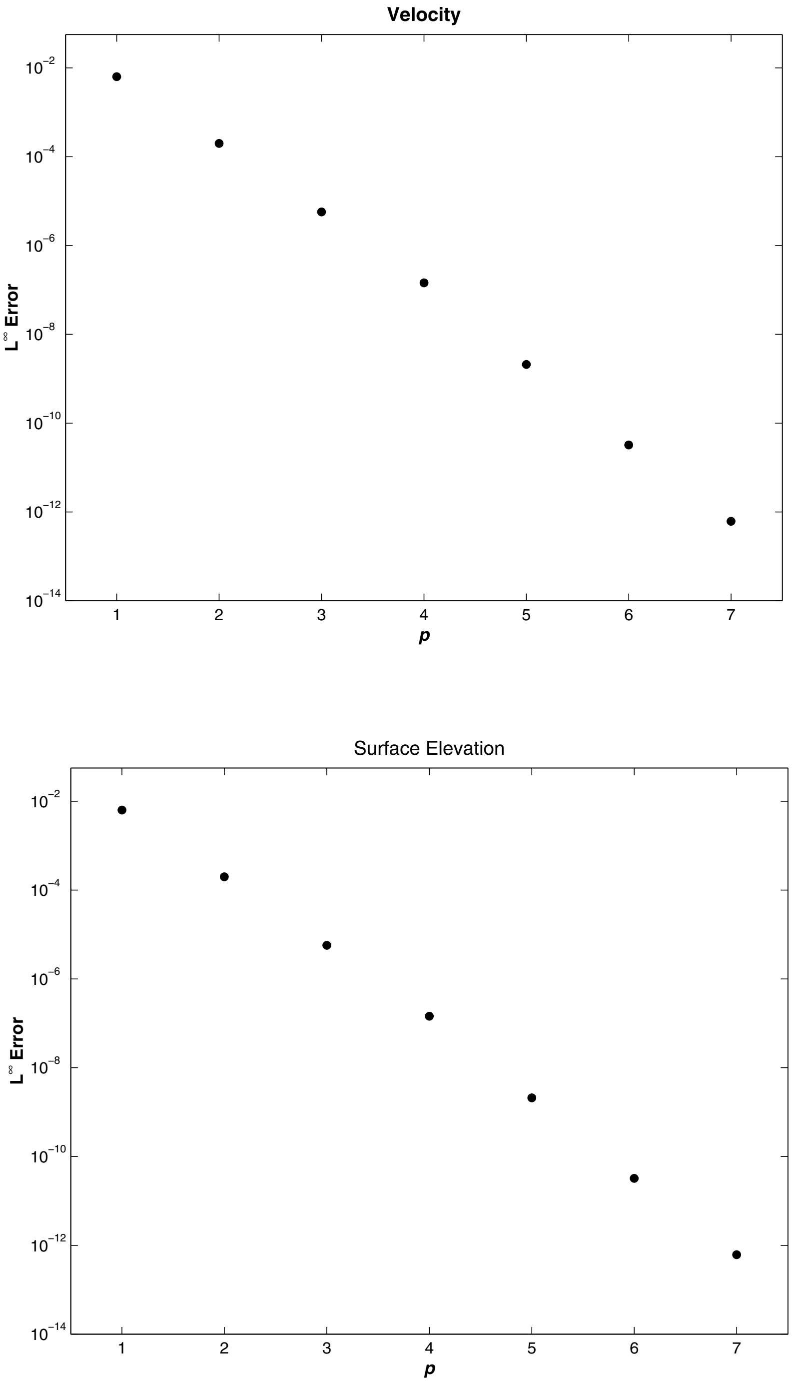


Figure 2: p convergence of the DG shallow water model in surface elevation and velocity for an idealized harbor problem.

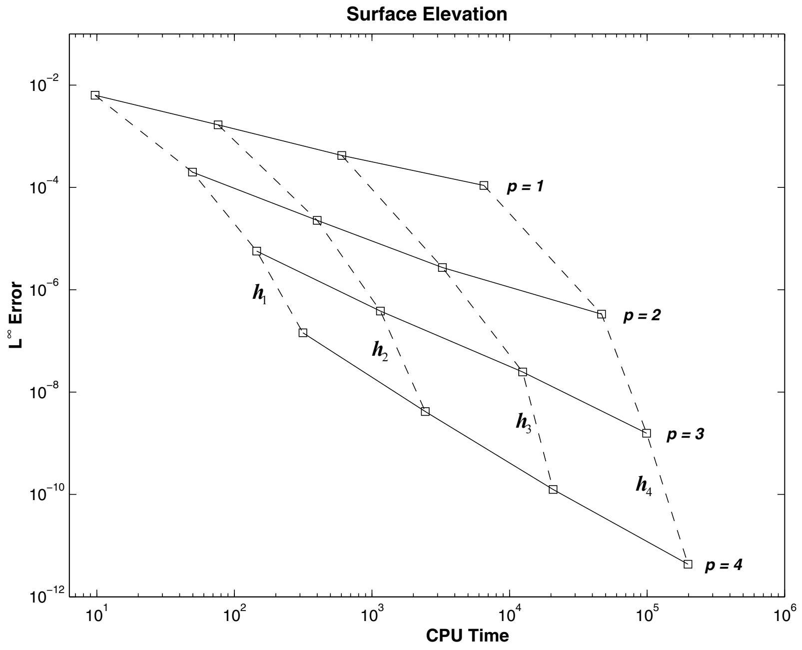


Figure 3: Error in surface elevation versus CPU times for a range of grid and polynomial orders

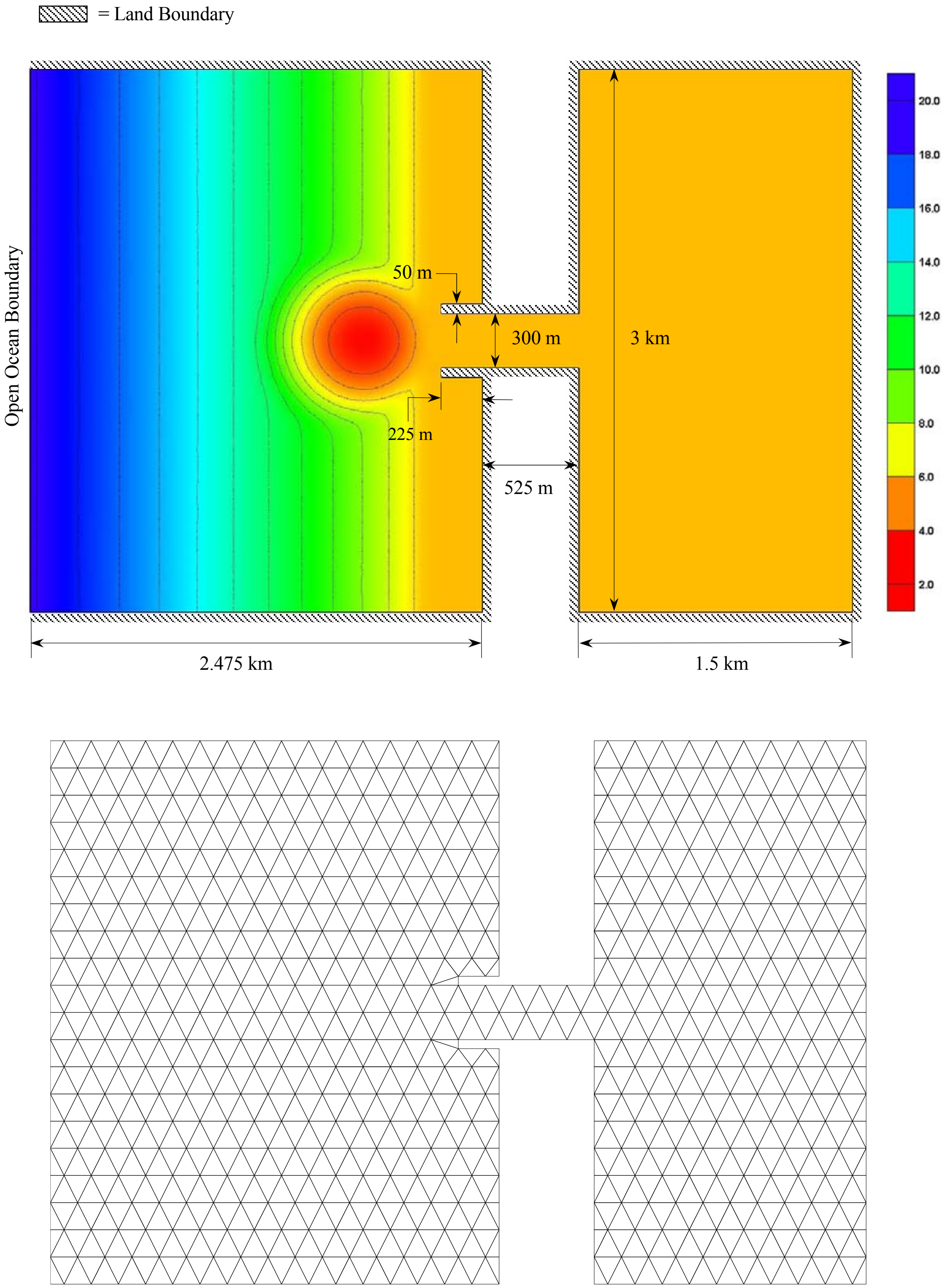
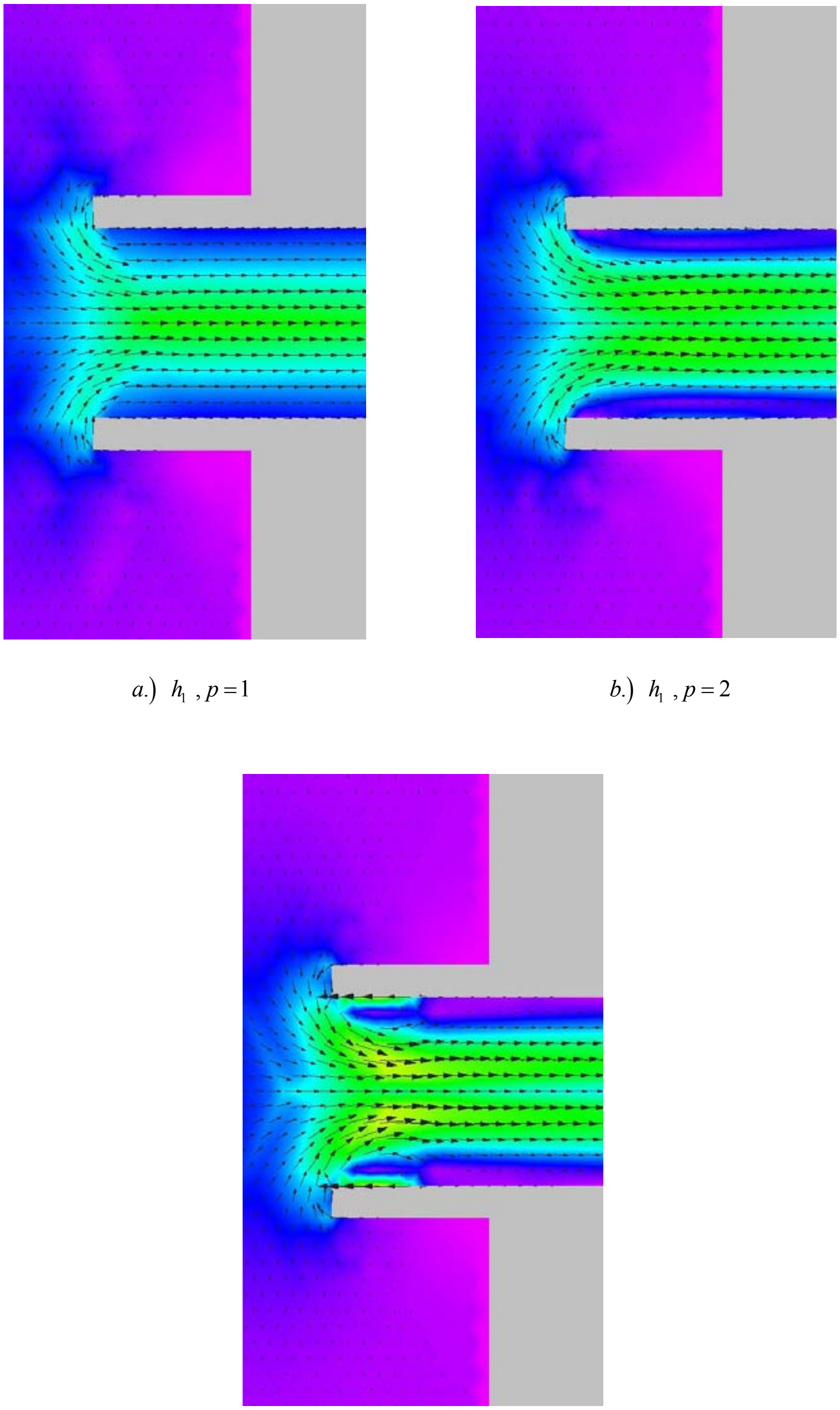


Figure 4: The computational domain and grid of the idealized inlet test case.



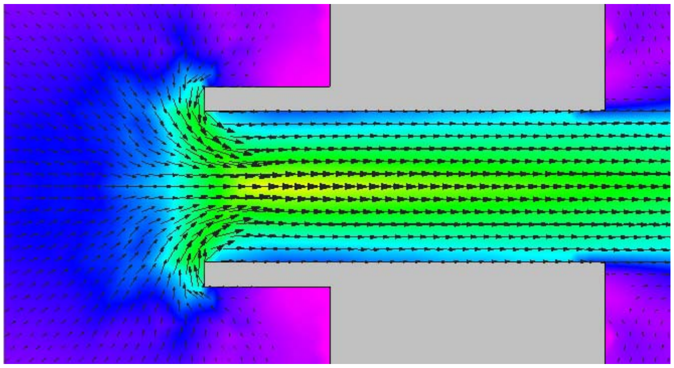
a.) h_1 , $p = 1$

b.) h_1 , $p = 2$

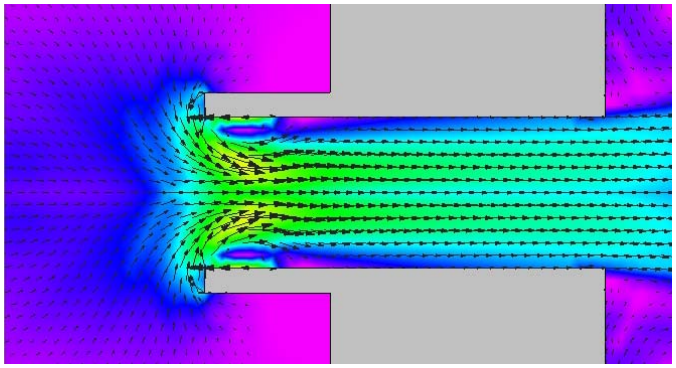
c.) h_1 , $p = 3$

Figure 5: Computed velocity contours and vectors for the idealized inlet test case using a coarse grid, h1 with polynomial approximations of $p = 1,\ 2,$ and 3

$p = 1$:



$p = 3$:



Adaptive Method, $p = 1 / p = 3$:

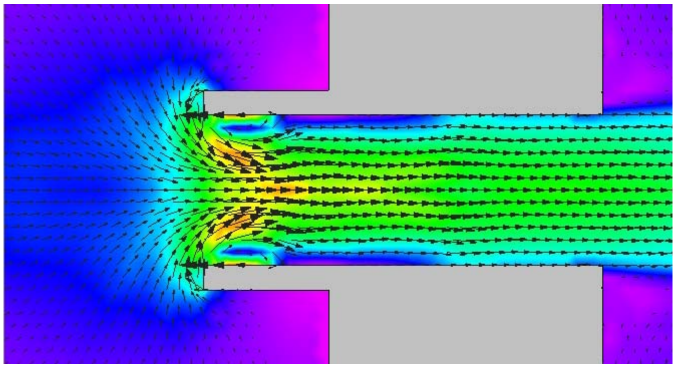


Figure 6: Comparison of the $p = 1$***,*** $p = 3$ ***and adaptive*** $p = 1 / p = 3$ ***solutions for the idealized inlet.***

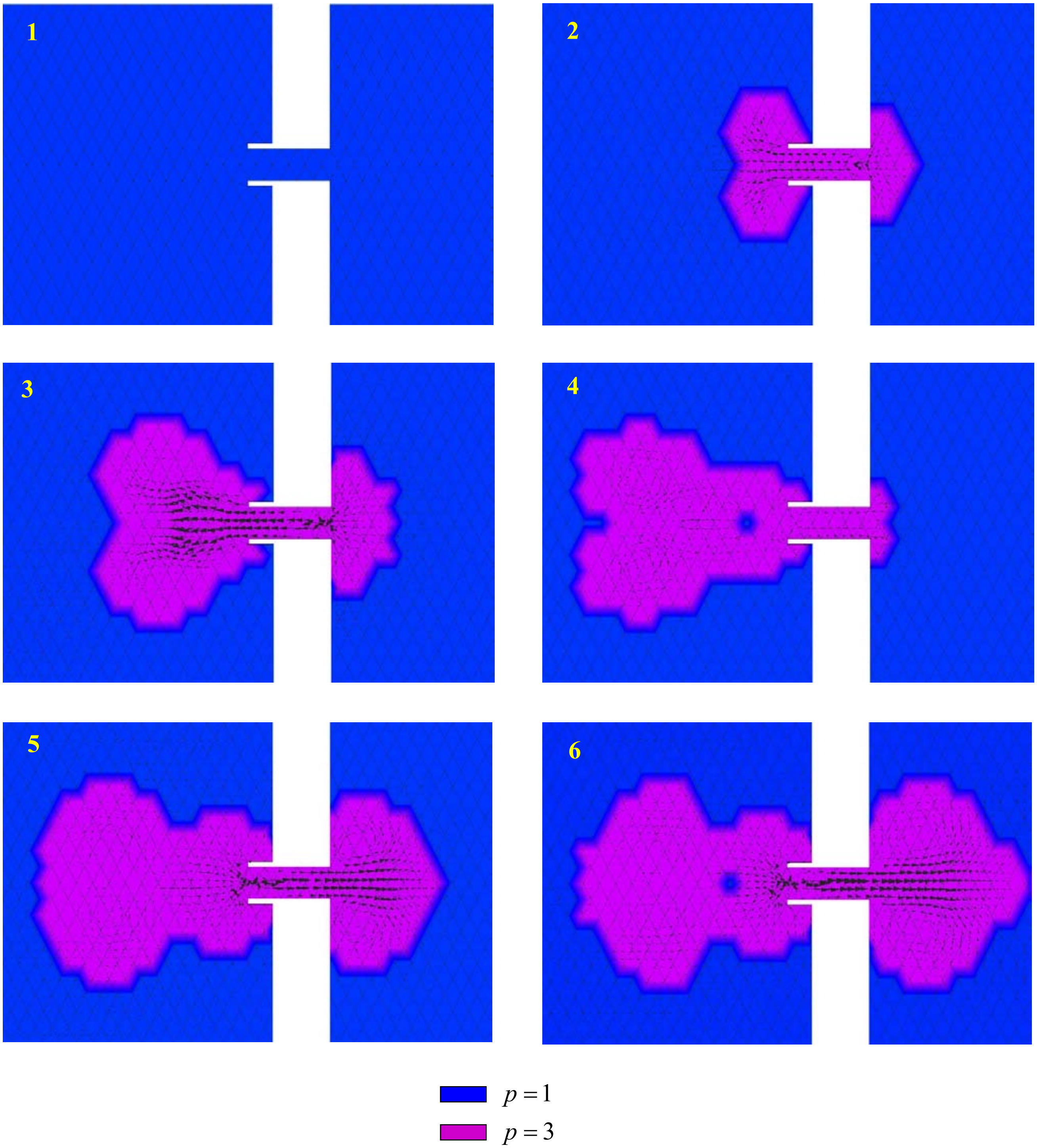


Figure 7: Dynamic evolution of the elements using the p-adaptive algorithm.

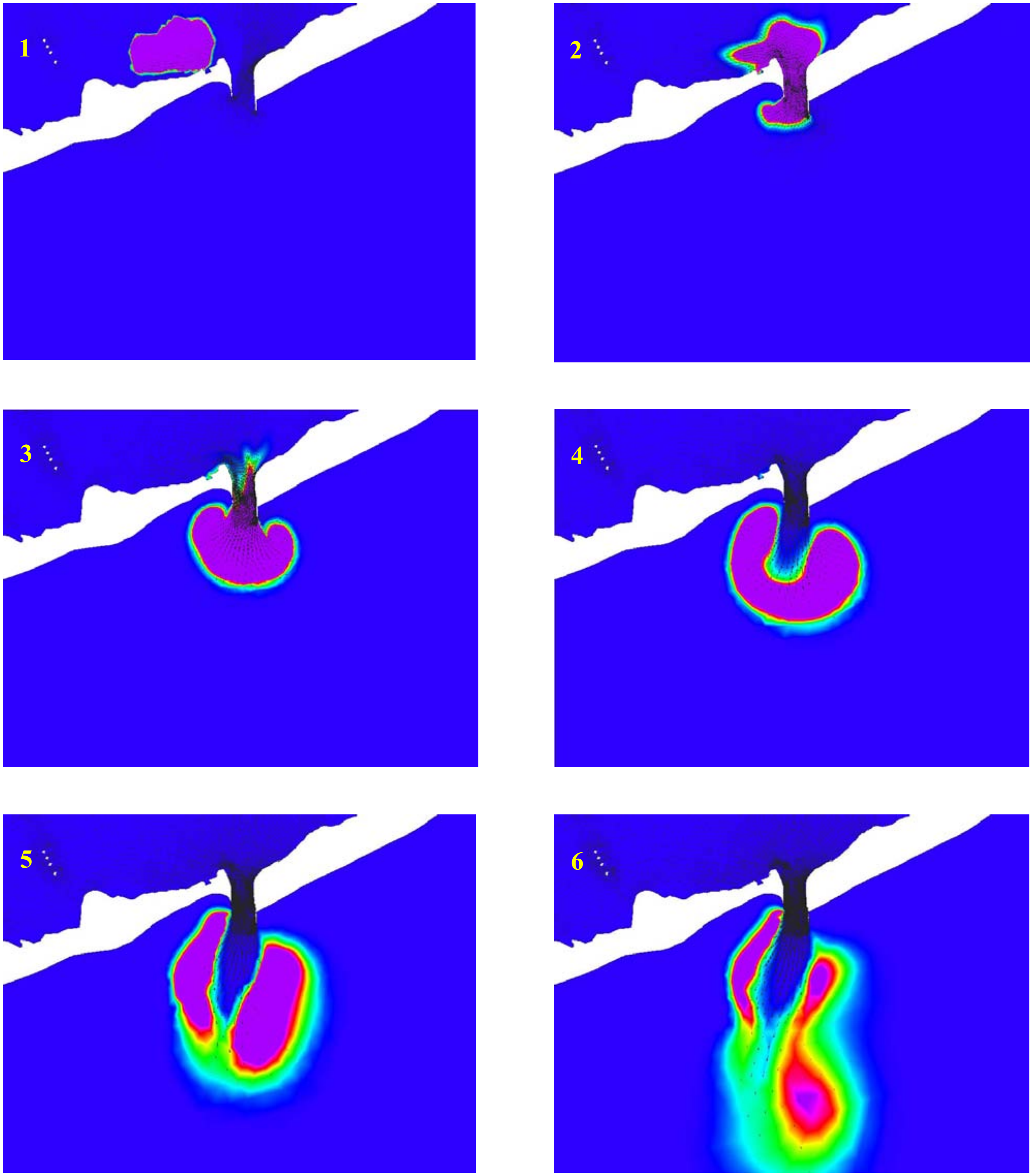


Figure 8: Transport of a passive quantity at Shinnecock Inlet at various times.

Table 1: CFL conditions and relative efficiencies of the SSP(s, k) RKDG methods (optimal schemes are boxed)

Stages, *s*	SSP(*s*,2)		SSP(*s*,3)		SSP(*s*,4)	
	CFL	RE (%)	CFL	RE (%)	CFL	RE (%)
2	0.3333	---	---	---	---	---
3	0.5882	$\boxed{15.00}$	0.2000	---	---	---
4	0.7611	12.42	0.3061	12.88	0.1696	---
5	0.8966	7.06	0.4060	$\boxed{17.90}$	0.2152	21.19
6	1.0089	0.89	0.4842	17.39	0.2747	25.19
7	1.1052	-5.09	0.5667	17.65	0.3214	26.12
8	1.1895	-12.08	0.6444	17.24	0.3707	$\boxed{26.80}$