

# Predictability in Unstable, Continuous Systems

Roger M. Samelson  
College of Oceanic and Atmospheric Sciences  
Oregon State University  
104 Ocean Admin Bldg  
Corvallis, OR 97331-5503  
phone: (541) 737-4752 fax: (541) 737-2064 email: [rsamelson@oce.orst.edu](mailto:rsamelson@oce.orst.edu)  
Award Number N00014-98-1-0813  
<http://www.oce.orst.edu/faculty/samelson.html>

## LONG-TERM GOALS

My long-term goal in this project is to improve our ability to predict environmental conditions using dynamical models.

## OBJECTIVES

The central objective of my research in this project is to understand the mathematical and physical connections between the bred-growing-mode technique recently developed for numerical weather prediction, the Lyapunov vectors and exponents of dynamical systems theory, and the local instability theories of geophysical fluid dynamics. My intent is to gain insight into fundamental mathematical and physical aspects of predictability in unstable (irregular, chaotic) continuous systems.

## APPROACH

I am using a combination of analytical and numerical methods to study a variety of simplified mathematical and physical models.

## WORK COMPLETED

I have completed an initial study of Lyapunov and singular vectors in the framework of the periodic orbit analysis of a coupled system of ordinary and partial differential equations describing wave-mean interaction in a weakly nonlinear, baroclinically unstable, quasi-geostrophic flow. The leading unstable periodic orbits of the weakly nonlinear baroclinic system have been computed numerically in asymptotically periodic and chaotic regimes. In the case of periodic orbits, the Lyapunov vectors are computed using Floquet theory. Floquet and singular vectors have been computed and compared for the leading periodic orbit in asymptotically periodic and chaotic regimes. In addition, I have carried out some closely related computations as part of an incipient collaboration with Eli Tziperman (Weizmann Institute, Israel) on periodic orbits and predictability in the Cane-Zebiak coupled tropical ocean-atmosphere model.

This project has also provided partial support for several other efforts, including a set of lectures given at the 1999 CalTech Summer School on Lagrangian Transport, Stirring, and Mixing in Geophysical Flows, a collaboration with Roger Temam and Shouhong Wang (Indiana University) on the

# Report Documentation Page

Form Approved  
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE <b>30 SEP 1999</b>		2. REPORT TYPE		3. DATES COVERED <b>00-00-1999 to 00-00-1999</b>	
4. TITLE AND SUBTITLE <b>Predictability in Unstable, Continuous Systems</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Oregon State University, College of Oceanic and Atmospheric Sciences, 104 Oceanography Admin Bldg, Corvallis, OR, 97331</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

mathematical properties of the planetary geostrophic equations (Samelson et al. 1998, 1999, submitted; Samelson, 1999a), and observations and modeling of the lower atmosphere along the Oregon coast during summer 1999 that involved collaboration with and support of the remote sensing, aircraft, and modeling activities of Melanie Wetzel (DRI, Reno), Gabor Vali (U. Wyoming), and William Thompson (NRL Monterey) associated with the COSAT experiment.

## **RESULTS**

The Floquet vectors of the leading periodic orbits tend to fall into two classes, the first dominated by baroclinic wave dynamics and the second associated with the decay of higher meridional modes of the zonal mean flow. The leading singular vector tends to contain contributions primarily from the first class of Floquet vectors, indicating that the presence of decaying zonal modes does not generally lead to artificial singular vectors associated with rapidly decaying modes that are unlikely to have substantial initial excitations. The analysis of leading periodic orbits and their Floquet eigenvectors gives a useful and efficient description of the dynamical evolution when the attractor geometry is sufficiently simple. The analysis also shows that in general it is necessary to know the amplitude of the disturbance to (or error in) the initial state, and the geometry of the attractor, to conclude whether it is appropriate to compute singular vectors in the subspace of the unstable Lyapunov vectors, an approach that has been advocated by other investigators.

## **IMPACT/APPLICATIONS**

Results from this theoretical and numerical modeling effort will contribute to the overall goal of improving atmospheric and oceanic prediction models through ensemble prediction techniques.

## **TRANSITIONS**

## **RELATED PROJECTS**

This work is part of the ONR Predictability DRI. The coastal meteorological research (Samelson, 1999b; Burk et al., 1999) has been supported primarily by the NOPP project "Prediction of Wind-Driven Circulation" (PI's J. S. Allen and J. Barth) and by the ONR project "Dynamics of Forced Coastal-Trapped Disturbances" (PI A. Rogerson).

## **REFERENCES**

Burk, S., T. Haack, and R. Samelson, 1999. Mesoscale simulation of supercritical, subcritical, and transcritical flows along coastal topography. *J. Atmos. Sci.*, 56, 2780-2795.

Samelson, R., 1999a. Internal boundary layer scaling in "two-layer" solutions of the thermocline equations. *J. Phys. Oceanogr.*, 29, 2099-2102.

Samelson, R., 1999b. The vertical structure of linear coastal-trapped disturbances. *Mon. Wea. Rev.*, 127, 201-213.

Samelson, R., R. Temam, and S. Wang, 1998. Some mathematical properties of the planetary geostrophic equations for large-scale ocean circulation. *Appl. Anal.*, 70, 147-173.

Samelson, R., R. Temam, and S. Wang, 1999. Remarks on the planetary geostrophic model of gyre-scale ocean circulation. *Diff. Int. Eq.*, in press.

## **PUBLICATIONS**

Samelson, R., R. Temam, and S. Wang, Smooth solutions and attractor dimension bounds for planetary geostrophic ocean models. *Q. J. Roy. Met. Soc.*, submitted.