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New Methods for Design and Computation of Free-form Optics

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14. ABSTRACT
Optical devices utilizing freeform optical surfaces must be used when the light sources and/or targets and input and/or output radiation patterns lack special symmetries. In these cases traditional designs constrained by use of a particular symmetry may lead to highly energy inefficient devcies. Freeform lenses and mirrors are surfaces designed without any a priori assumed symmetry. Under this effort a unified and rigorous approach to solving wide classes of optical design problems requiring freeform optical surfaces has been developed. Computational methods for such problems have also been introduced and tested.

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New Methods for Design and Computation of Freeform Optics

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Preface

The work presented in this report was performed at the Department of Mathematics and Computer Science, Emory University, Atlanta, Georgia 30322. This work was sponsored by the Air Force Office of Scientific Research during the period May 01, 2012 - April 30, 2015. The project Technical Monitor is Dr. Arje Nachman from the AFOSR.

During the reporting period in addition to the Principal Investigator Vladimir Oliker the following people were involved in the project at different stages:

Professors J. Rubinstein and G. Wolansky from Department of Mathematics of the Technion - Israel Institute of Technology, Professor L. Doskolovich from the Samara National Research University of Aeropace, Samara, Russia, - as research collaborators; Drs. Boris Cherkassky, Sergey Kochengin, and Laird Prussner provided programming support; Undergraduate students at Emory: Dallas Albritton and Mingjie Jiao helped with testing some of the developed codes.

Contents

1 Executive Summary

Most of the currently used conventional optical devices rely on lenses and mirrors which are rotationally or rectangularly symmetric or can be described by explicit formulas. By contrast, a freeform optical surface is a surface designed without any a priori assumptions of available symmetry or explicit formula. Consequently, freeform surfaces provide significantly more degrees of freedom to create optical systems for a much wider variety of applications. Many applications of optics - such as nanotechnology, photonics, optical photolithography, directed energy, laser beam shaping, antenna design, solar collection, optical engineering metrology, buildings and roads illumination - require such capabilities and will benefit from the use of freeform optics. In these applications, the optical surface (or a system of surfaces that may be lenses or mirrors) is required to redirect and reshape the radiation pattern of the source so that the output irradiance is concentrated onto a given target set and has a prescribed pattern. Such design problems are known as "prescribed irradiance problems"[18].

In problems of this kind the shape of a freeform optical surface has to be determined from the data describing the light beam incident on that surface and the prescribed intensity distribution to be produced by refracted (or reflected) light rays on a given target set. The required optical surface must transform the incident wavefront into a beam of light illuminating a given target set with pre-specified intensity. Thus, each such surface (if it exists) defines a transformation (called *ray mapping or refractor/reflector map*) between a cross section of the incoming bundle of rays and the target set. The Jacobian of this transformation is the ratio of input and output light intensities at corresponding points. Thus, to determine the required optical surface it is necessary to solve for the ray mapping the equation with the prescribed Jacobian. In general, this is a hard mathematical problem and no rigorous methods are available for its solution. In practice, such problems are approached via some kind of optimization of a heuristically constructed by the designer merit function. In most cases these functions are not convex and, at best, only a local optimum can be found with some iteration procedure without a guarantee of convergence or accuracy[9]. The published designs are usually difficult or impossible to reproduce.

A mathematically rigorous, physically meaningful and, as shown by our work, viable approach to such design problems is based on very different ideas. First of all, it has been shown[10, 12] that in many cases of interest to optical design the ray mapping has a scalar (quasi-)potential and the design problem can be formulated in terms of this potential as a partial differential equation(PDE) of second order with nonstandard boundary conditions. The solution to this PDE problem is a scalar function which determines the required optical surface. However, these PDE's are fully nonlinear, of Monge-Ampère type, and are notoriously hard to analyze theoretically and, even more so, to solve numerically. Except for very special cases, direct and reliable computational methods for their numerical solution are not known. In our approach we are not solving this equation directly. Instead, we construct a variational problem whose solution is also a weak solution of the required PDE. The construction of such variational problems is done by a procedure applicable in a unified way to any prescribed irradiance problem. The critical challenge of this approach is that from the point of view of numerics, to design the required freeform optical surface with precision sufficient for modern-day technologies one must determine reliable solutions to certain nonlinear systems with the number of variables of order $10^9 - 10^{12}$. Since the development of successful innovative approaches to solving such optical design problems will have a very high technological impact, overcoming that challenge is of great value.

Consequently, **the main efforts** of this project have been directed towards development of a revolutionary comprehensive approach to solving optical design problems in the general setting of freeform optics. Such approach, based on rigorous physical and geometric principles combined with recent developments in calculus of variations as well as on novel provably convergent computational methods that solve very large systems of nonlinear equations, has been developed and sucessfully validated with numerical experiments on concrete optical design problems. This work led to a deeper understanding of the mathematical and physical aspects of problems connected with optical designs utilizing freeform surfaces.

Our approach does not require any symmetry assumptions and produces a solution for a design problem that approximates the exact solution with any a priori given accuracy. By contrast with other approaches the solution obtained with our approach does not depend on ad hoc constructions or on optimization of ad hoc merit functions.

2 The approach and Main Results

The main ingredients of our approach are the Supporting Quadrics Method [10, 11, 12] and Monge-Kantorovich mass transport theory[13, 5, 14, 15, 16, 17].

The supporting quadric method (SQM) is a general strategy for constructing weak solutions to nonlinear partial differential equations arising in design problems involving freeform optical surfaces[10], including design of reflector antennas[10] and lenses[11]. It can also be used for investigating properties of eikonal functions associated with optical systems, including applications to diffractive optical elements[2] as well as determination of dependence of a solution on the data which is important for tolerancing considerations. Its earlier versions gained wide popularity among optical engineers; see, for example, [3, 8, 7, 4, 1], and other references there.

It is important to note that SQM is a general framework for formulating optical design problems, rather than a recipe for solving a specific problem. Because of such generality it can be applied to almost any problem of optical design but additional efforts are required to adapt SQM to a particular problem. For some problems such application of SQM is a research project in itself (cf.[11]).

In principle, the SQM is constructive but its numerical implementaion is not simple because of its geometric nature. It is well known that geometric methods usually require highly sophisticated algorithms stable with respect to perturbation of data and capable of handling very large amount of data. In our approach, we use SQM to formulate and mathematically rigorously analyse weak solutions of the corresponding Monge-Ampère equations. Our computational strategy, on the other hand, begins with converting the Monge-Ampère PDE into a variational form which turned out to be much more convenient for numerics. The combination of these two strategies allows us to avoid many difficulties connected with numerical implementation of SQM and develop computational methods capable of solving efficiently and accurately large problems of optical design.

Let us describe this process in more detail. First of all, for a given design problem the underlying physics in combination with the SQM allows us to replace the fully nonlinear PDE of Monge-Ampère type associated with the design problem by an equation in measures. Using the SQM framework we can prove existence of weak solutions to such an equation and investigate uniqueness of weak solutions. The next step, directed towards a computational solution, consists in constructing a variational problem associated with the optical problem. In optical design problems the functionals of the corresponding variational problems are defined on sets of surfaces which are envelopes of families of special optical surfaces each of which has a unique focal point. Such optical surfaces are typically quadrics but, in general, can be any optical surface with known focal properties. The SQM is used here as an effective tool for determination of the type and description of these quadrics.

Our procedure for deriving a suitable variational problem is very general. In fact, we were able to show that a variational problem can be associated with any optical design problem of prescribed irradiance type. In [13] this was shown for several classes of problems of optical design (primarily, for problems dealing with the far-field case and for beam shaping problems arising in applications of lasers). Currently, these results have been extended to the near-field case. Moreover, this procedure can be carried out in a unified way applicable to wide classes of optical problems. The corresponding functional is a generalization of the classical Fermat principle, and therefore physically motivated. On the other hand, it is also the cost of "transporting" (=transforming) intensity from the input beam into the required distribution on the prescribed target. This property, allows us also to show that the solution obtained with SQM is a solution of the generalized Euler-Lagrange equation for the corresponding variational problem. This part of our work is also of independent interest as it shows deep connections between problems of optical design and contemporary Monge-Kantorovich theory of optimal mass transport[17].

The functional in the variational problem requires special treatment because it contains nonstandard nonlinearities and methods for dealing with such nonlinearities had to be developed. Nevertheless, using the variational form we were able to avoid most of the geometric difficulties and develop algorithms for efficient solution of systems of very large size mentioned earlier.

It is worthwhile noting that the quadrics mentioned above (or other special optical surfaces as required in concrete designs problems) can be viewed as generalizations of fundamental solutions in PDE theory. However, because the underlying equations are highly nonlinear PDE's there is no complete analogy with the classical fundamental solutions.

3 Conclusions

Overall, the developed approach for treating prescribed irradiance problems is a unified mathematical and algorithmic framework for understanding and solving numerous optical design problems concerned with control of optical radiation. This framework, providing rigorous mathematical formulations of optical design problems and provably convergent computational methods for their solution, has evolved into an interdisciplinary research program on *development of mathematical and physical theories and computational methods for design of freeform optical surfaces*. Several critical parts of this program have been completed during the reporting period by the present author and his collaborators[5, 14, 12, 15, 16, 11] working in related areas of fully nonlinear partial differential equations, optics, and computational methods for optimization under extremely large number of nonlinear constraints.

Principally, using the SQM and variational methods, we were able to develop a unified theory for formulating and solving wide classes of optical design problems. Within our approach we derived effective descriptions of classes of admissible solutions and established conditions for existence of solutions to corresponding optical problems. Fast computational methods based on convex optimization and linear and nonlinear programming were developed for solving these problems numerically. In contrast with other methods, optical designs within our framework have the following characteristics:

- No a priori symmetry assumptions are imposed,
- No initial solution is required,
- Designed freeform lenses and mirrors are convex or concave surfaces,
- Designed optics accepts the entire input beam and redistributes light with ≈100% efficiency (excluding losses unavoidable for physical reasons),
- Superresolution of the radiation intensity on the target,
- Arbitrary target sets in 3D space may be used as image sets; for example, images on curved surfaces can be produced with high accuracy,
- Design of miniaturized optics is possible,
- Rigorous theory of design with explicit conditions on the data for design feasibility,
- Provably convergent computational methods,
- Design methods usable in visible, UV and IR parts of the spectrum.

The work completed so far shows that our rigorous approach to design of freeform optics consisting of a combination of the SQM and Monge-Kantorovich mass transport thery is feasible and reliable and the unified framework for its application is versatile and robust.

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