



Applications of complex Huygens principles

**Gerald Kaiser
KAISER GERALD**

**06/22/2015
Final Report**

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ RTB
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to the Department of Defense, Executive Service Directorate (0704-0188). Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ORGANIZATION.

1. REPORT DATE (DD-MM-YYYY) 31/05/2015		2. REPORT TYPE Final Report		3. DATES COVERED (From - To) 31/03/2012-31/05/2015	
4. TITLE AND SUBTITLE Applications of the complex Huygens principle and Gaussian pulsed-beam wavelet representations in acoustics and electromagnetics				5a. CONTRACT NUMBER N/A	
				5b. GRANT NUMBER FA9550-12-1-0122	
				5c. PROGRAM ELEMENT NUMBER N/A	
6. AUTHOR(S) Dr. Gerald Kaiser				5d. PROJECT NUMBER N/A	
				5e. TASK NUMBER N/A	
				5f. WORK UNIT NUMBER N/A	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Center for Signals and Waves 3601 SW River Parkway #1108, Portland, OR 97239				8. PERFORMING ORGANIZATION REPORT NUMBER N/A	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Air Force Office of Scientific Research				10. SPONSOR/MONITOR'S ACRONYM(S) AFOSR	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S) N/A	
12. DISTRIBUTION/AVAILABILITY STATEMENT Distribution A - Approved for Public Release					
13. SUPPLEMENTARY NOTES None					
14. ABSTRACT The original goal was to apply the analytic Huygens representations of EM fields (developed jointly with Dr. Thorkild Hansen in 2009-2011) to scattering, communication and radar. However, in 2012 Dr. Richard Albanese urged me to clarify the concept of electromagnetic inertia which I had defined in 2011, and which posed some conceptual problems since it implied that electromagnetic energy in vacuum generally flows at speeds less than c. Thus I changed my goal to understanding EM inertia and relating it to the reactive field energy. But reactive EM energy is strictly linked to the complex Poynting theorem (CPT), which holds only for time-harmonic fields. Even there, no reactive energy density can be defined because this theorem cannot be interpreted as its conservation law. In 2014 I generalized the CPT to EM fields with arbitrary time dependence and showed that an analytic continuation of the generalized CPT does have an interpretation as a conservation law for reactive EM energy. I further defined a complex radiation impedance density analogous to the impedance $Z=R+iX$ of an RLC circuit. I hope to use this analogy to interpret the real and imaginary parts of the complex impedance density as the radiation resistance and radiation reactance densities of the EM field.					
15. SUBJECT TERMS Reactive energy, reactive power, complex Poynting theorem, energy conservation, reactive time					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT None	18. NUMBER OF PAGES 3	19a. NAME OF RESPONSIBLE PERSON Gerald Kaiser
a. REPORT Public	b. ABSTRACT Public	c. THIS PAGE Public			19b. TELEPHONE NUMBER (Include area code) (512) 300-8065

Reset

INSTRUCTIONS FOR COMPLETING SF 298

1. REPORT DATE. Full publication date, including day, month, if available. Must cite at least the year and be Year 2000 compliant, e.g. 30-06-1998; xx-06-1998; xx-xx-1998.

2. REPORT TYPE. State the type of report, such as final, technical, interim, memorandum, master's thesis, progress, quarterly, research, special, group study, etc.

3. DATES COVERED. Indicate the time during which the work was performed and the report was written, e.g., Jun 1997 - Jun 1998; 1-10 Jun 1996; May - Nov 1998; Nov 1998.

4. TITLE. Enter title and subtitle with volume number and part number, if applicable. On classified documents, enter the title classification in parentheses.

5a. CONTRACT NUMBER. Enter all contract numbers as they appear in the report, e.g. F33615-86-C-5169.

5b. GRANT NUMBER. Enter all grant numbers as they appear in the report, e.g. AFOSR-82-1234.

5c. PROGRAM ELEMENT NUMBER. Enter all program element numbers as they appear in the report, e.g. 61101A.

5d. PROJECT NUMBER. Enter all project numbers as they appear in the report, e.g. 1F665702D1257; ILIR.

5e. TASK NUMBER. Enter all task numbers as they appear in the report, e.g. 05; RF0330201; T4112.

5f. WORK UNIT NUMBER. Enter all work unit numbers as they appear in the report, e.g. 001; AFAPL30480105.

6. AUTHOR(S). Enter name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. The form of entry is the last name, first name, middle initial, and additional qualifiers separated by commas, e.g. Smith, Richard, J, Jr.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES). Self-explanatory.

8. PERFORMING ORGANIZATION REPORT NUMBER. Enter all unique alphanumeric report numbers assigned by the performing organization, e.g. BRL-1234; AFWL-TR-85-4017-Vol-21-PT-2.

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES). Enter the name and address of the organization(s) financially responsible for and monitoring the work.

10. SPONSOR/MONITOR'S ACRONYM(S). Enter, if available, e.g. BRL, ARDEC, NADC.

11. SPONSOR/MONITOR'S REPORT NUMBER(S). Enter report number as assigned by the sponsoring/monitoring agency, if available, e.g. BRL-TR-829; -215.

12. DISTRIBUTION/AVAILABILITY STATEMENT. Use agency-mandated availability statements to indicate the public availability or distribution limitations of the report. If additional limitations/ restrictions or special markings are indicated, follow agency authorization procedures, e.g. RD/FRD, PROPIN, ITAR, etc. Include copyright information.

13. SUPPLEMENTARY NOTES. Enter information not included elsewhere such as: prepared in cooperation with; translation of; report supersedes; old edition number, etc.

14. ABSTRACT. A brief (approximately 200 words) factual summary of the most significant information.

15. SUBJECT TERMS. Key words or phrases identifying major concepts in the report.

16. SECURITY CLASSIFICATION. Enter security classification in accordance with security classification regulations, e.g. U, C, S, etc. If this form contains classified information, stamp classification level on the top and bottom of this page.

17. LIMITATION OF ABSTRACT. This block must be completed to assign a distribution limitation to the abstract. Enter UU (Unclassified Unlimited) or SAR (Same as Report). An entry in this block is necessary if the abstract is to be limited.

Final Report
AFOSR Grant # FA9550-12-1-0122
31 March 2012–31 May 2015

Gerald Kaiser
Center for Signals and Waves
Portland, OR
kaiser@wavelets.com

May 29, 2015

While preparing my grant proposal in 2011, the main objective was to further develop and apply the newly discovered *analytic Huygens wavelet representations* [HK9, HK11, K12a] of electromagnetic fields. One of their main attractions is the potential for *compressing* data, based on the high directivity of these wavelets which makes it possible to ignore components outside any directional cone of interest with minimal loss of accuracy. Dr. Hansen and I believe that such representations may offer a computationally attractive alternative to the Fast Method of Moments. This has, in fact, been the direction of Dr. Hansen's recent work.

My focus changed after discovering [K11a, K11b, K12b] that a generic electromagnetic field (including any field with spatially bounded sources) has a positive *inertia density* throughout its entire space-time region of support. This is a classical version of the well-known fact in QED that virtual photons (i.e., photons which are emitted and/or absorbed by charged matter) have a non-vanishing mass. Only *idealized* photons and EM fields (e.g., traveling plane waves in vacuum) have vanishing inertia, yet this is the basis for the common statement that photons and EM fields are *massless*. Since positive mass is associated with propagation at speeds less than c , the conclusion that EM fields have positive inertia seems to contradict the fact that they propagate in vacuum at speed c . This led to intense and challenging discussions with Dr. Richard Albanese throughout 2011. To clarify the situation, I applied my theory to several concrete examples: standing plane waves and electric or magnetic dipoles with general time dependence. In each case I showed that the local EM energy flows at $v < c$, although the *fields* themselves propagate at c as expected. Such discrepancies are due to *interference* between local wave components propagating (at c) in different directions. The clearest example is a standing plane

wave, where the energy executes elastic, singular oscillations between adjacent nodal planes [K11b, Section 4]. The relation between the EM fields and their energy-momentum densities is closely analogous to that between quantum wave functions and their observables, with $\mathbf{F} = \mathbf{E} + i\mathbf{B}$ playing the role of the complex wave function. Observables are *sesquilinear* in the fields (wave functions) and independent of their overall phase, and consequently they obey different equations (conservation laws) than the wave functions (Maxwell's equations).

Aside from the trivial example of traveling plane waves, only very special EM fields have a vanishing inertia density. If a charge-current density can be found whose field has vanishing inertia density, it would be an *ideal radiator*, propagating all of its EM energy at speed c without a near-field and hence without any self-interference. In [K11a] I constructed a class of such solutions which I called *coherent electromagnetic wavelets*. However, the sources of these wavelets consist of a relativistically spinning disk and its entire spin axis, hence they are unbounded and therefore unrealizable. I also wrote a short paper [K12c] proposing *regularized* coherent wavelets as models for *quasars*, where ‘thickened’ versions of the singular source disk and its spin axis represent the accretion disk and the jets of the quasar, respectively. The accretion disk is a thin oblate spheroid, and the jets are thin orthogonal hyperboloids. Although the jets are unbounded, they are a reasonable model for quasar jets since these can extend for billions of light years before being absorbed.

I began to suspect in 2011 that the inertia density of an EM field is related to its *reactive energy* [K11b, K12b]. In the literature, the concept of reactive EM energy is based on the complex Poynting theorem (CPT), which applies only to time-harmonic fields. The *real* and *imaginary* parts of the CPT describe the flows in space and time of *active* and *reactive* energy, respectively.¹ The real part of the CPT can be interpreted as a *conservation law* for the period-averaged active energy, and this gives an expression for the period-averaged active local energy *density* of the field.² However, the imaginary part *cannot* be interpreted as a conservation law, hence no expression can be defined for the local *reactive* energy density. In 2014, I generalized the CPT to EM fields with arbitrary time dependence and showed that an analytic continuation of this generalized CPT does have an interpretation as a conservation law for reactive EM energy.³ Furthermore, I defined a *complex radiation impedance density* $\mathcal{Z} = \mathcal{R} + i\mathcal{X}$ for a general EM field, analogous to the complex impedance $Z = R + iX$ of an RLC circuit. \mathcal{Z} is related to the inertia density \mathcal{I} of the field by $\mathcal{I} = |\mathcal{Z}|$. I hope to use this analogy to interpret \mathcal{R} and \mathcal{X} as the distributed radiation resistance density and radiation reactance density of the EM field, respectively. However, these interpretations are rather speculative at this point and I'm not sure how

¹Active energy can perform work, and reactive energy cannot.

²The ordinary (time-domain) Poynting theorem gives the *instantaneous* active energy density of general fields, while the real part of the CPT gives the *period-averaged* active energy density of time-harmonic fields.

³The reactive energy is conserved in *imaginary time*, which I interpreted as the ‘reactive time’ tracking the lags and leads associated with capacitive and inductive reactive energy.

to substantiate them. For this reason, I have decided not to apply for another grant at this time. I am discussing these ideas with Professor David Griffiths at nearby Read College, and will present my latest results on these topics at the IEEE/URSI Joint Meeting this July 2015 [K15].

Acknowledgements

I am grateful to Dr. Arje Nachman for supporting my research through AFOSR grants for the past 20 years and to Dr. Richard Albanese for his encouragement, stimulating discussions, and moral support during the same period.

References

- [HK9] T Hansen and G Kaiser, [Generalized Huygens principle with pulsed-beam wavelets](#). *J. Phys. A: Math. Theor.* 42 (2009) 475403 (33 pages)
- [HK11] T Hansen and G Kaiser, Huygens' Principle for Complex Spheres. *IEEE Transactions on Antennas and Propagation* Vol. 59 # 10, October 2011
- [K11a] G Kaiser, [Coherent electromagnetic wavelets and their twisting null congruences](#). Preprint, 2011
- [K11b] G Kaiser, [Electromagnetic inertia, reactive energy, and energy flow velocity](#). *J. Phys. A: Math. Theor.* 44 (2011) 345206
- [K12a] G Kaiser, [Huygens' principle in classical electrodynamics: a distributional approach](#). *Advances in Applied Clifford Algebras* Vol. 22, # 3 (2012), Pages 703–720
- [K12b] G Kaiser, [The Reactive Energy of Transient EM Fields](#). IEEE International Symposium on Antennas and Propagation and USNC-URSI National Radio Science Meeting, Session 563, July 13, 2012
- [K12c] G Kaiser, [Electromagnetic helicity wavelets: a model for quasar engines?](#) Preprint, 2012
- [K14] G Kaiser, [Completing the complex Poynting theorem: Conservation of reactive energy in reactive time](#). Preprint, 2014
- [K15] G Kaiser, [Conservation of reactive EM energy in reactive time](#). Invited paper, Special Session on Fundamental Considerations of Electromagnetic Energy and Interactions: Theory and Applications, IEEE AP-S Symposium on Antennas and Propagation and URSI CNC/USNC Joint Meeting, Vancouver, BC, July 2015

1.

1. Report Type

Final Report

Primary Contact E-mail

Contact email if there is a problem with the report.

kaiser@wavelets.com

Primary Contact Phone Number

Contact phone number if there is a problem with the report

512-300-8065

Organization / Institution name

Center for Signals and Waves

Grant/Contract Title

The full title of the funded effort.

Applications of the complex Huygens principle and Gaussian pulsed- beam wavelet representations in acoustics and electromagnetics

Grant/Contract Number

AFOSR assigned control number. It must begin with "FA9550" or "F49620" or "FA2386".

FA9550-12-1-0122

Principal Investigator Name

The full name of the principal investigator on the grant or contract.

Gerald Kaiser

Program Manager

The AFOSR Program Manager currently assigned to the award

Dr. Arje Nachman

Reporting Period Start Date

04/01/2012

Reporting Period End Date

05/31/2015

Abstract

The original goal was to apply the analytic Huygens representations of EM fields (developed jointly with Dr. Thorkild Hansen in 2009-2011) to scattering, communication and radar. However, in 2012 Dr. Richard Albanese urged me to clarify the concept of electromagnetic inertia which I had defined in 2011, and which posed some conceptual problems since it implied that electromagnetic energy in vacuum generally flows at speeds less than c . Thus I changed my goal to understanding EM inertia and relating it to the reactive field energy. But reactive EM energy is strictly linked to the complex Poynting theorem (CPT), which holds only for time-harmonic fields. Even there, no reactive energy density can be defined because this theorem cannot be interpreted as its conservation law. In 2014 I generalized the CPT to EM fields with arbitrary time dependence and showed that an analytic continuation of the generalized CPT does have an interpretation as a conservation law for reactive EM energy. I further defined a complex radiation impedance density analogous to the impedance $Z=R+iX$ of an RLC circuit. I hope to use this analogy to interpret the real and imaginary parts of the complex impedance density as the radiation resistance and radiation reactance densities of a general EM field.

Distribution Statement

This is block 12 on the SF298 form.

Distribution A - Approved for Public Release

Explanation for Distribution Statement

If this is not approved for public release, please provide a short explanation. E.g., contains proprietary information.

SF298 Form

Please attach your [SF298](#) form. A blank SF298 can be found [here](#). Please do not password protect or secure the PDF. The maximum file size for an SF298 is 50MB.

[SF298_Compl.pdf](#)

Upload the Report Document. File must be a PDF. Please do not password protect or secure the PDF. The maximum file size for the Report Document is 50MB.

[Final_Report.pdf](#)

Upload a Report Document, if any. The maximum file size for the Report Document is 50MB.

Archival Publications (published) during reporting period:

2012: G. Kaiser, "The Reactive Energy of Transient EM Fields."

IEEE International Symposium on Antennas and Propagation and USNC/URSI National Radio Science Meeting, Session 563, July 13, 2012. <http://arxiv.org/abs/1201.6575>

2012: G. Kaiser, "Huygens' principle in classical electrodynamics: a distributional approach." Advances in Applied Clifford Algebras, Vol. 22, # 3

(2012), Pages 703-720. <http://arxiv.org/abs/0906.4167>

2012: G. Kaiser, "Electromagnetic helicity wavelets: a model for quasar engines?" Preprint, 2012.

<http://arxiv.org/abs/1209.2913>

2014: G. Kaiser, "Completing the complex Poynting theorem: Conservation of reactive energy in reactive time." Preprint, 2014. <http://arxiv.org/abs/1412.3850>

2015: G. Kaiser, Conservation of reactive EM energy in reactive time. Invited paper, Special Session on Fundamental Considerations of Electromagnetic Energy and Interactions: Theory and Applications, IEEE AP-S Symposium on Antennas and Propagation and URSI CNC/USNC Joint Meeting, Vancouver, BC, July 2015

<http://arxiv.org/abs/1501.01005>

Changes in research objectives (if any):

The original goal was to apply the analytic Huygens wavelet representations of acoustic and electromagnetic fields developed in 2009-2011 with Dr. Thorkild Hansen. Then Dr. Richard Albanese suggested in 2012 that I change the goal to clarifying the properties of the EM inertia density I had discovered in 2011. This led in 2014 to a generalization of the complex Poynting theorem from time-harmonic to arbitrary EM fields and its extension to complex time. That in turn led to a definition of the complex radiation impedance density of a general EM field, whose real and imaginary parts I believe to be related to the radiation resistance and radiation reactance densities of the field.

Change in AFOSR Program Manager, if any:

None

Extensions granted or milestones slipped, if any:

No-Cost Extension granted from 31 March 2015 to 31 May 2015.

AFOSR LRIR Number

LRIR Title

Reporting Period

Laboratory Task Manager

Program Officer

Research Objectives

Technical Summary

Funding Summary by Cost Category (by FY, \$K)

	Starting FY	FY+1	FY+2
Salary			
Equipment/Facilities			
Supplies			
Total			

Report Document

Report Document - Text Analysis

Report Document - Text Analysis

Appendix Documents

2. Thank You

E-mail user

May 29, 2015 14:44:45 Success: Email Sent to: kaiser@wavelets.com