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Coherent Risk-Adjusted Decisions Over Time: a Bilevel Programming Approach

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14. ABSTRACT

We developed a formal theory of time consistency of multistage systems of stochastic optimization models, analyzing and relating various relevant notions of time consistency. We proved that using multilevel optimization constraints to enforce time consistency results in NP-hard models, even in the simplest cases. However, we also found that a standard MIP solver could solve relatively small but realistic instances of such formulations in minutes. We developed and tested two techniques for approximating a time-inconsistent risk-averse objective function with a time-consistent one. We also investigated rolling-horizon applications of coherent risk measures and risk-averse control of Markov systems. We characterized the sets of optimal solutions to such risk-averse control problems, developing and testing multiple solution methods. We also examined risk-averse transient Markov models. Finally, we developed specialized risk measures for stochastic process control. By considering only random sequences that can actually occur in the controlled system, we were able to derive a much more refined structure than for general risk measures.

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Final Report:
Coherent Risk-Adjusted Decisions Over Time: a Bilevel Programming Approach

Principal Investigator:	Jonathan Eckstein
Co-Principal Investigator:	Andrzej Ruszczyński
Period of Performance:	July 15, 2011 – January 14, 2015
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Program Managers:	Donald Hearn, succeeded by Fariba Fahroo
Grant Number:	FA9550-11-1-0164

The central topic of this project was the risk-averse formulation of multistage optimization problems containing uncertainty. The key issue is developing time-consistent plans of action in situations when uncertainty is revealed at more than one point in time. Time consistency is essentially automatic for models that are completely risk-neutral or maximally risk averse, but becomes problematic in intermediate cases.

1. Computing global solutions of bilevel risk programs

We experimented extensively with solving relatively small but realistic multistage stochastic programming instances using time-inconsistent risk measures. Although we use objective functions that are not time consistent, we achieve model time consistency by using multilevel optimization constraints. In the first years of the project, we reformulated these multilevel models as MPEC's (Mathematical Programs with Equilibrium Constraints) and found local solutions using the LOQO Newton-barrier solver. In the last years, we further reformulated these MPEC models into forms recognizable by mixed-integer programming (MIP) solvers such as GuRoBi and CPLEX. To do this, we used two techniques, one with auxiliary binary variables and another using specially adapted special-ordered-set (SOS) constraints. We began our study with a class of simple supply-chain-based applications, using both mean-semideviation risk measures and a blended-CVaR risk measures (convex combinations of expected value and CVaR). We obtained the best run times using GuRoBi on SOS reformulations. In our experiments, GuRoBi able to find globally optimal solutions within minutes, except for one blended-CVaR instance. Although solution takes far longer than similar models with time-consistent objective functions, the run times are still acceptable, except for the one difficult instance. For higher levels of risk aversion, the identified solutions were also significantly different from those obtained using time-consistent objective functions. These results indicate that our multilevel optimization modeling approach is likely to have practical applications. In the last months of the project, we began working with a multistage portfolio application. We also obtained data for a hydropower system planning application.

We submitted a jounal paper containing most of these results, “Multilevel Optimization Modeling for Risk-Averse Stochastic Programming,” by Jonathan Eckstein, Deniz Eskandani, and Jingnan Fan. We have heard informally that this paper has been conditionally accepted to the *INFORMS Journal on Computing*.

2. Theoretical results on time consistency of models

We developed a formal abstract theory of time consistency of multistage systems of stochastic optimization models, where a system of models consists of an optimization model for each node of a scenario tree. We proved several theoretical results relating this form of consistency to existing notions of time consistency of risk measures, and showing that our proposed multilivel optimization modeling scheme produces time-consistent systems of models regardless of the objective functions used.

This work is included in the same paper described above, submitted to the *INFORMS Journal on Computing*.

3. Complexity Theory

We proved that the simplest subclasses of the kinds of models we propose are NP-hard. In particular, we proved that applying our techniques to three-stage problems with linear objective functions and constraints, and using the most common risk measures such as mean-semideviation or CVaR still produces an $\mathcal{NP}$-hard problem class. This work is included in the same paper described above, submitted to the *INFORMS Journal on Computing*, and also appears in several online working papers.

4. Time-consistent approximations of time-inconsistent models

With the goal of eventually creating specialized algorithms able to exploit the special structure of multistage stochastic programming problems with time-incosistent risk measures, we developed new techniques which iteratively construct time-consistent approximations to time-inconsistent formulations. Each approximation can be efficiently solved by a specialized decomposition method. The solution obtained is then used to refine the approximation. Repeating this process a number of times results in computable tight time-consistent upper bounds on the optimal value of the original problem.

We developed two versions of this approximate method: non-parametric and parametric. The non-parametric method finds the best bound among all time-consistent models, while the parametric method considers only models from a specific family of risk measures, allowing for easy representation of the approximations. We tested both methods extensively and established their efficiency.

To obtain universal bounds, that is, bounds that are valid for all possible decisions, not only the optimal one, we developed a specialized combinatorial method. It is capable of handling linear models with specific parametric families of risk measures. The method is computationally intensive. We compared it to our decomposition methods for bounding the optimal value. In summary, the bounds obtained by all methods are tight, and although the non-universal bounds are not guaranteed to hold away from the optimal point, they are of very good quality.

We validated our procedures by testing them on a real-world portfolio optimization problem.

These results are presented in the paper “Time-Consistent Approximations of Risk-Averse Multistage Stochastic Optimization Problems,” by Tsvetan Asamov and Andrzej Ruszczyński, which has been published online in *Mathematical Programming* (with print version in process).

5. Rolling-horizon applications

We experimented with application of dynamic measures of risk to portfolio management on a rolling-horizon basis. We compared first- and higher-order semideviation risk measures, and confirmed that the use of higher-order dynamic risk measures gave superior portfolio performance.

We presented these results in the paper “Two-Stage Portfolio Optimization with Higher-Order Conditional Measures of Risk” by Sitki Gulten and Andrzej Ruszczyński, which has been published online in *Annals of Operations Research* (with print version in process).

6. Risk-averse control of Markov systems

Markov control models are more appropriate for decision problems with a long or perhaps infinite time horizon. We refined our earlier theory of Markov risk measures to accommodate undiscounted problems, such as risk-averse stochastic shortest paths. The resulting theory, in the form of risk-averse dynamic programming equations, fully characterizes the set of optimal solutions and provides insight into its properties. Moreover, we have proposed and investigate several new numerical methods for solving risk-averse dynamic programming models: value iteration, policy iteration, and both Newton and convex programming approaches to policy evaluation. We found that policy iteration with convex programming for policy evaluation is the most efficient method, with Newton policy evaluation nearly as good.

This work is included in paper by Ozlem Çavuş and Andrzej Ruszczyński, “Computational Methods for Risk-Averse Undiscounted Transient Markov Models,” *Operations Research* 62(2) (2014) 401–417.

7. Development of the theory of risk-averse dynamic optimization problems for transient models.

We formulated the concept of a risk-transient model and developed corresponding dynamic programming equations for risk-averse optimal control. In this setting, we demonstrated that randomized control may better optimize risk than deterministic control. As special cases, we solved several examples of risk-averse optimal stopping and stochastic shortest path models.

This work is included in the paper by Ozlem Çavuş and Andrzej Ruszczyński, “Risk-Averse Control of Undiscounted Transient Markov Models,” *SIAM Journal on Control and Optimization* 52(6) (2014) 3935–3966, and the aforementioned paper by Ozlem Çavuş and Andrzej Ruszczyński in *Operations Research*.

8. Process-based risk measures

We created a new, refined theory of process-based risk measures, suitable for use with Markov and non-Markov control models. We also developed a corresponding theory of time consistency. Our main results in this area are derived needing to measure risk only for random sequences that can actually occur in the controlled system. As a result, one can deduce a much more refined structure for the risk measures than in more general settings.

The results are included in a manuscript “Process-Based Risk Measures for Observable and Partially Observable Discrete-Time Controlled Systems,” by Jingnan Fan and Andrzej Ruszczyński, which has been submitted for publication.

We acknowledged AFOSR support in all submitted manuscripts.

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Program Manager

The AFOSR Program Manager currently assigned to the award

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Abstract

This project addressed the risk-averse formulation of optimization problems containing uncertainty. The key issue is developing time-consistent plans of action in situations when uncertainty is revealed at more than one point in time. Time consistency is essentially automatic for models that are completely risk-neutral or maximally risk averse, but becomes problematic in intermediate cases.

We developed a formal theory of time consistency of multistage systems of stochastic optimization models with an optimization model for each node of a scenario tree, analyzing and relating various relevant notions of time consistency. We proved that using multilevel optimization constraints to enforce time consistency results in NP-hard models, even in the simplest cases. We experimented with solving relatively small but realistic instances of such formulations by applying standard MIP software, finding that all but one instance we formulated could be solved to global optimality in minutes on a standard workstation. To aid in the development of specialized approximation and implicit enumeration algorithms, we developed and tested two techniques, one parametric and

one non-parametric, for approximating a time-inconsistent risk-averse objective function with a time-consistent one.

We also investigated rolling-horizon applications of coherent risk measures and risk-averse control of Markov systems. We characterized the sets of optimal solutions to risk-averse Markov control problems, developing and testing multiple solution methods, finding optimization-based policy evaluation methods to be the most efficient. We also examined risk-averse transient Markov models. Finally, we developed specialized risk measures for stochastic process control. By considering only random sequences that can actually occur in the controlled system, we were able to derive a much more refined structure than for general risk measures.

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Ozlem Çavuş and Andrzej Ruszczyński, “Risk-Averse Control of Undiscounted Transient Markov Models,” SIAM Journal on Control and Optimization 52(6) (2014) 3935–3966

Sitki Gulten and Andrzej Ruszczyński, “Two-Stage Portfolio Optimization with Higher-Order Conditional Measures of Risk” Annals of Operations Research (online first).

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Our original program manager was Donald Hearn. He was replaced by Fariba Fahroo.

Extensions granted or milestones slipped, if any:

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Reporting Period

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Program Officer

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