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Multiscale Dynamics and Information in Data Collection and Assimilation **for Environmental Applications**

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14. ABSTRACT
Data assimilation or filtering involves blending information from observations of the actual system states with information from dynamical models to estimate the current system states or certain model parameters. The filtering problem relies on three fundamental ingredients, namely 1) sensor placement: where the sensors are placed in order to obtain the most useful information, 2) sensor fusion: how to combine the measurements from different sensors, and 3) estimation: how to use the measurements to obtain the best possible state estimates.
In this project, we considered the data assimilation problem for multi-timescale systems. An understanding of how scales interact with information led to the development of rigorous reduced-order data assimilation techniques for these high-dimensional problems.

This project developed new algorithms and tools for the collection, assimilation and harnessing of data by threading together ideas from random dynamical systems, information theory, and statistical learning. A new particle filtering algorithm based on the theoretical result that combines stochastic homogenization with filtering theory to construct a reduced-dimension nonlinear filter is presented. They are used for approximating the

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FINAL REPORT FOR AFOSR GRANT NO. FA9550-12-1-0390: "MULTISCALE DYNAMICS AND INFORMATION IN DATA COLLECTION AND ASSIMILATION FOR ENVIRONMENTAL APPLICATIONS"

PI: N. SRI NAMACHCHIVAYA

This is the final report for AFOSR Grant FA9550-12-1-0390, which was awarded in 07/01/2012. This research involves the work of the PI and graduate students at University of Illinois at Urbana-Champaign (UIUC) in collaboration with a graduate student of Humbolt University (HU), Berlin.

1. Summary of Project Activities and Findings

Data assimilation or filtering involves blending information from observations of the actual system states with information from dynamical models to estimate the current system states or certain model parameters. The filtering problem relies on three fundamental ingredients, namely 1) sensor placement: where the sensors are placed in order to obtain the most useful information, 2) sensor fusion: how to combine the measurements from different sensors, and 3) estimation: how to use the measurements to obtain the best possible state estimates. In this project, we considered the data assimilation problem for multi-timescale systems. An understanding of how scales interact with information led to the development of rigorous reduced-order data assimilation techniques for these high-dimensional problems.

The main results of the research project are:

(1) Rigorous mathematical development of a reduced-order particle filtering method for high-dimensional, multiscale random dynamical systems.
(2) Development of a particle filtering method adapted to high-dimensional, multiscale, chaotic systems

The research has led to three journal publications [1, 2, 3], four conference papers [4, 6, 7, 11], and two key note lectures one at an international research workshop [5] and the other at an international conference [10]. Two Ph.D students who were involved in the research are in the course of their graduate studies at UIUC, and one graduate student who collaborated in this project has completed his studies at HU, Berlin.

1.1. **Reduced-order filtering.**

Dynamical models for describing climate evolution consist of coupled ocean and atmospheric models, which are high-dimensional with multi-timescale dynamics, described by stochastic differential equations of the form:

$$
\begin{aligned}
dX_t^\varepsilon &= b(X_t^\varepsilon, Z_t^\varepsilon)dt + \sigma(X_t^\varepsilon, Z_t^\varepsilon)dW_t, \qquad X_0^\varepsilon = x \in \mathbb{R}^m,\\
dZ_t^\varepsilon &= \frac{1}{\varepsilon^2} f(X_t^\varepsilon, Z_t^\varepsilon)dt + \frac{1}{\varepsilon} g(X_t^\varepsilon, Z_t^\varepsilon)dV_t, \qquad Z_0^\varepsilon = z \in \mathbb{R}^m,
\end{aligned} \tag{1}
$$

where W and V are independent Wiener processes that model the uncertainties in the dynamical system and $\varepsilon << 1$ is a small parameter that characterizes the timescale separation between the fast and slow processes. In (1), the fast and slow processes are Z and X, respectively.

In the filtering framework, the objective is to obtain the best estimate of $(X^\varepsilon, Z^\varepsilon)$ from observations Y, which are described by

$$Y_t^\varepsilon = \int_0^T h(X_t^\varepsilon, Z_t^\varepsilon)ds + B_t, \qquad Y_0^\varepsilon = 0.$$

Formally, the objective is to find a normalized filter π_t^ε:

$$\pi_t^\varepsilon(\varphi) = \mathbb{E}\left[\varphi(X^\varepsilon, Z^\varepsilon | \mathcal{Y}_t^\varepsilon\right], \quad \mathcal{Y}_t^\varepsilon = \sigma\left(Y_s^\varepsilon : 0 \le s \le t\right) \vee \mathcal{N},$$

where $\varphi \in \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}$ is any bounded, measurable function and $\mathcal{Y}_t$ is the algebra generated by the observation Y up to time t.

Data assimilation for high-dimensional dynamical systems such as that described in (1) are faced with the computational complexities that arise from dimensionality issues. Part of the work performed under this research grant has been the development of a reduced-order particle filtering technique that enables data assimilation on a high-dimensional system by making use of an associated reduced-order, stochastically-averaged system.

In studying multiscale systems such as (1), we are usually interested in the statistics of the long-term ("slow") dynamics. Hence, we can take advantage of the timescale separation between the fast and slow processes and the appropriate stochastic properties that ensure the existence of a homogenized system associated with the original multiscale system (1). Under appropriate conditions on the coefficient functions of (1), it is known that, for generator $\mathcal{L}^\varepsilon$ associated with (1) and probability density p^ε,

$$\varepsilon \to 0 \implies (p^\varepsilon, \mathcal{L}^\varepsilon) \to (p^0, \bar{\mathcal{L}}),$$

where p^0 is a reduced dimension density function. We can then find a process associated with $\bar{\mathcal{L}}$ governed by SDE of the form:

$$dX_t^0 = \bar{b}(X_t^0)dt + \bar{\sigma}(X_t^0)dW_t, \qquad X_0^0 = x \in \mathbb{R}^m, \tag{2}$$

such that the process X^ε converges weakly to X^0 in the limit as $\varepsilon \to 0$. The fast process dynamics is averaged into the homogenized system (2), thus the dynamics of X^0 effectively captures the distribution of the coarse-grained dynamics of (1). The idea behind the development of a reduced-order filter for the multiscale systems is to make use of the existence of the process (2). Formally, the task is to show that π^ε is close to π^0 in L^p-sense as $\varepsilon \to 0$, where π^0 is the filter associated with the homogenized process (2). For purposes of filtering applications, this means that, for systems with large timescale separation, a reduced-order filter π^0 can be used in place of the original π^ε to estimate the coarse-grained dynamics. This dimensional reduction of the filtering problem translates to reduction in computational costs.

The important result of the analysis part of the project in the development of the reduced-order filtering technique is:

$$\boxed{\lim_{\varepsilon\to 0} \sup_{t\le T} \mathbb{E}\left[d\left(\pi_t^\varepsilon, \pi_t^0\right)\right] = 0, \quad \forall T > 0}. \tag{3}$$

The analysis and result for a one-dimensional system with timescale separation is shown in [12]. In the one-dimensional case, a stochastic partial differential equation (SPDE) driven by real observation was constructed for the unnormalized conditional density of the reduced-order filter. Convergence was shown by showing that the error between the solution of the Zakai equation, which governs the unnormalized conditional density of the multiscale system, and solution of the SPDE corresponding to the reduced-order filter vanishes as $\varepsilon \to 0$. The calculation required an application of Levy's Theorem for time-changed representation of Brownian motion, which is holds only in one dimension.

In the research under this grant, work was done on extending the result to the general multi-dimensional case. A SPDE driven by real observation was constructed for the unnormalized reduced-order filter. Solution to this SPDE is a measure-valued process, and the task was to show that the error relative to the solution of the measure-valued Zakai equation for the multiscale system is bounded and vanishes as $\varepsilon \to 0$. This was achieved by invoking dual representations of the unnormalized multiscale filter, v^ε, and the reduced-order filter driven by real observation, v^0. The dual processes satisfy backward stochastic partial differential equations (BSPDEs) that are related to the forward SPDEs of the corresponding filters. The outline of the convergence proof is as follows:

v^ε is asymptotically expanded into an order 1 term, a correction term of order ε, and a remainder order higher than ε. These expansion terms are governed by BSPDEs, and the BSPDE of the order 1 term corresponds to that of v^0. Hence, the correction and remainder terms correspond to the error between v^ε and v^0. Using BSPDE theory, probabilistic representations of the corrector and remainder are obtained in terms of solutions of backward "doubly" stochastic differential equations. From there, bounds of order $\sqrt{\varepsilon}$ were obtained for the corrector and remainder using existing probabilistic estimate results in literature. Hence, the error between v^ε and v^0 vanishes as $\varepsilon \to 0$. This convergence was translated back to convergence of the unnormalized filters by the dual relations, and boundedness of the measure change (in the Kallianpur-Striebel formula relating normalized and unnormalized filters) gives convergence of the normalized filters. This proof was completed at the end of the first year research. A preliminary result was presented in a conference and published in the proceedings [4], and the final results have been published in [1].

1.2. **Reduced-order particle filter adapted to chaotic systems.** Based on the analytical results previously described, a reduced-order filtering algorithm, the Homogenized Hybrid Particle Filter (HHPF) was developed, described in detail in [13] for the continuous time case.

In the second year under the research grant, work was done to apply the HHPF algorithm to high-dimensional chaotic systems. Due to the sensitivity to initial conditions nature of chaotic systems, the regular HHPF was found to be inadequate for estimating the system states accurately and efficiently. The discrete-time filtering problem was considered in the attempt to adapt the reduced-order filtering algorithm to chaotic systems. A discrete-time version of the HHPF was developed, which incorporates the technique of Sequential Importance Sampling (SIS) for particle filtering. The idea behind importance sampling is to represent a target probability density that is difficult to sample from using an alternate proposal density. The discrepancy from not using the true density is accounted for by appropriately weighting the proposal density.

In the filtering problem, the objective of importance sampling is to represent the posterior density of the state given observations using a proposal density. Using SIS, a proposal density can be generated using appropriately weighted particles, where particle weights are updated sequentially. For the discrete-time case, the optimal proposal density (that minimizes the variance of particle weights) at a discrete timestep k, $q^{\text{opt}}(x_k|x_{k-1}, y_k)$, is a known result that is the normalized product of the prior density based on signal dynamics and the likelihood of observation given the state, with corresponding sequential particle weight update expression. Consider the discrete-time signal dynamics and observation

$$\begin{aligned} X_k &= B(X_{k-1}) + \sigma_x \Delta W_k, \\ Y_k &= H(X_k) + \sigma_y \Delta V_k. \end{aligned} \tag{4}$$

For the case of linear observation function H,

$$\begin{aligned} q^{\text{opt}}(x_k|x_{k-1}, y_k) &= \mathcal{N}(B(x_{k-1}) + \alpha(x_{k-1}, y_k), \hat{Q}), \\ \hat{Q} &= \left(Q^{-1} + H^T R^{-1} H\right)^{-1}, \\ \alpha(x_{k-1}, y_k) &= \hat{Q} H^T R^{-1}(y_k - HF(x_{k-1})), \end{aligned} \tag{5}$$

where $Q \stackrel{\text{def}}{=} \sigma_x \sigma_x^T$, $R \stackrel{\text{def}}{=} \sigma_y \sigma_y^T$. The proposal density (5) can be generated by propagating particles using an augmented signal dynamics. For particle i, the original dynamics (4) is modified into

$$X_k = B(X_{k-1}) + \alpha(X_{k-1}, Y_k) + \hat{\sigma}_x W_k \tag{6}$$

where $\hat{\sigma}_x$ is such that $\hat{\sigma}_x \hat{\sigma}_x^T = \hat{Q}$. Then X_k^i behaves like a particle sampled from $q^{\text{opt}}(\cdot|x_{k-1}^i, y_k)$.

A similar augmented signal dynamics for particle propagation result was also obtained by considering the discrete-time problem using a stochastic optimal control approach. The problem setup was to consider particle dynamics governed by the stochastic difference equation of the form (6), where $\alpha(X_{k-1}^i, Y_k)$ is now an unknown "control" that is to be applied to steer particles towards the true signal based on observations. This is a one-step-ahead control problem. The control term was obtained by minimizing a quadratic cost function that penalizes the amount of control applied over a timestep and the distance of a particle from the location prescribed by the observations. This augmentation of signal dynamics for particles propagation can be related to existing techniques widely used in geophysical data assimilation such as the 4D-VAR technique.

By implementing SIS with the optimal proposal density in the HHPF, we have a discrete-time version of the HHPF that is adapted to systems with high sensitivity to initial conditions. For numerical experiments, the optimized HHPF was applied to a 396 dimensions (36 slow, 360 fast) Lorenz '96 model that mimics midlatitude dynamics of an atmospheric variable. The Lorenz '96 model is a popular testbed for numerical statistical analysis. The purpose of the numerical experiments is to study the performance of the optimized HHPF in estimating the slow state variables based on complete and incomplete (not every dimension of the true signal is observed) noisy observations of the true signal. The optimized HHPF is compared with the regular HHPF (uses prior as proposal), an existing weights variance-reduction particle filtering technique from geophysical data assimilation literature, and the Ensemble Kalman Filter (EnKF). The optimized HHPF is found to outperform the regular HHPF using the same particles sample size, since the best possible proposal density is used in SIS. For the same particles sample sizes, the optimized HHPF does not perform as well as the weights variance-reduction particle filter and the EnKF in terms of estimation accuracy. By increasing the the particles sample size, the optimized HHPF can achieve the same level of accuracy as the other two filters. The advantage of the optimized HHPF is in terms of computation time, even with larger sample size than the weights variance-reduction particle filter and EnKF, due to the incorporation of a homogenization scheme that overcomes the need to fully simulate the fast dynamics. A typical experiment of the optimized HHPF using 100 particles, the weights variance-reduction particle filter and the EnKF, both using 20 particles, all filters displaying the same level of accuracy, has runtimes of 134, 1757, and 540 seconds for each respective filter. Some comparison of the filter performances are shown in the proceeding figures.

Results of the numerical experiments has been published [2].

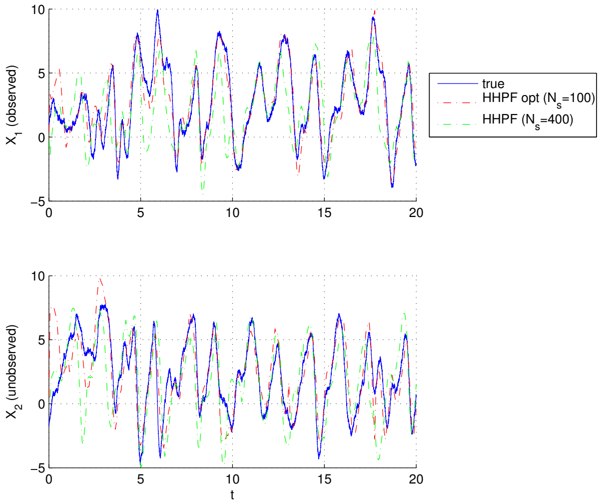

FIGURE 1. Sparse observations, optimized ($N_s = 100$) and direct ($N_s = 400$) HHPFs. The upper plot shows the estimates of an observed state, the lower plot shows that of an unobserved state. The unobserved state was estimated well by the optimized HHPF with $N_s = 100$. Even with $N_s = 400$, The direct HHPF captures the fluctuations in the truth but did not follow the trajectory well.

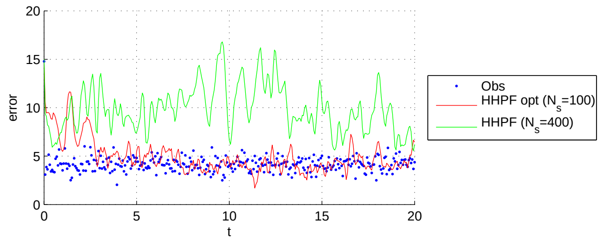

FIGURE 2. Sparse observations, estimation error of the observed states for the optimized ($N_s = 100$) and direct ($N_s = 400$) HHPFs compared with observation error. The optimized HHPF estimates were as good as observations (but the optimized HHPF also provided estimates of the unobserved states so more information is gained by using the filter instead of just observation).

The next step was to consider the setting where observations are sparse in time, i.e. observations are collected at regular intervals that are large compared to the numerical integration timestep. In this setting, for chaotic systems, error in between observation times can grow significantly if the observation interval is large compared to the system's error doubling time. Using the optimal control approach of the previous part, the problem can be treated as a continuous-time optimal control problem with time horizon equal to the length of the observation interval. The cost function consists of a running cost that is a quadratic function of the control energy, and a terminal cost that is a quadratic function of particle distance from the location prescribed by the observation at the end of the observation interval. The minimum cost satisfies the Hamilton-Jacobi-Bellman (HJB) equation. The optimal control as a function of the derivative of the minimum cost, hence computation of the optimal control requires solving the HJB equation. Using a log transformation, the HJB equation was converted into a linear partial differential equation, which solution can be obtained using the Feynman-Kac formula. The derivative of the Feynman-Kac representation involves the derivative of a stochastic path, which is obtained using Malliavin calculus.

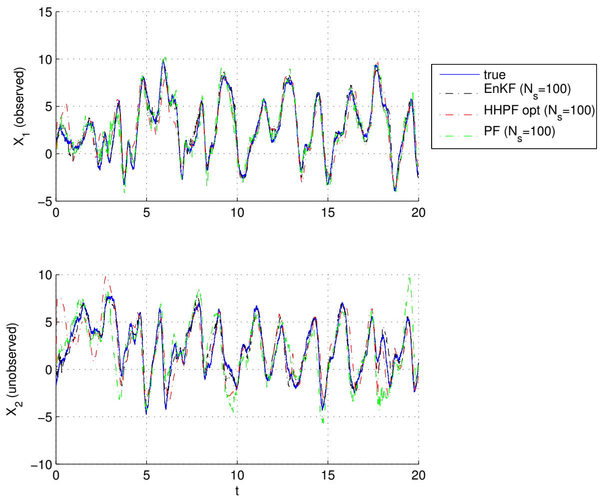

FIGURE 3. Sparse observations, EnKF, weights variance-reduction PF and optimized HHPF comparison at fixed $N_s = 100$. The optimized HHPF performed as well as the PF, but in shorter time than both the EnKF and PF.

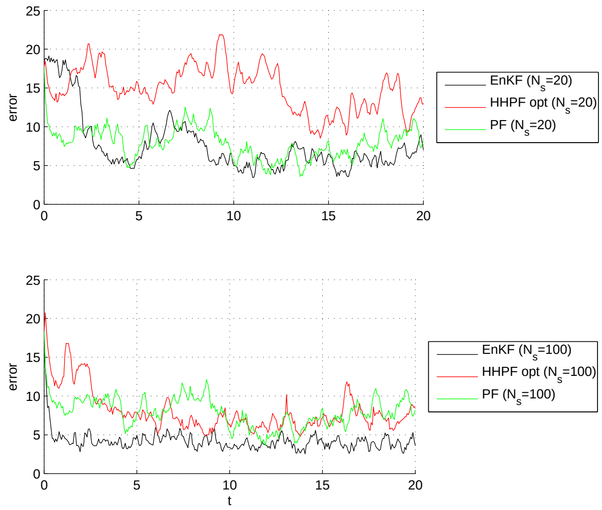

FIGURE 4. Sparse observations, comparison of estimation errors of the EnKF, weights variance-reduction PF and optimized HHPF. Estimation error of the optimized HHPF and the PF are of the same magnitude at $N_s = 100$. The optimized HHPF still required less computation time than the EnKF and PF even with the EnKF and PF being implemented using minimum N_s possible.

Augmenting the control modifies the signal dynamics for the particles. By the principle of SIS, this has to be accounted for by weighting the particles appropriately relative to the original signal dynamics. The corresponding weights are determined by an application of Girsanov's Theorem to relate the probability measures of the original and controlled signals. The relation corresponds to shifting the mean of the Brownian motion in the original signal by the same amount as the control. This weighting procedure is appended to the particle weights based on observation in the particle filtering algorithm.

In practice, evaluation of the optimal control based on the Feynman-Kac representation and Malliavin derivative can become computationally overwhelming for nonlinear signals. However, the optimal control solution can be obtained explicitly for linear systems. We implemented the linear control strategy as a suboptimal control solution on particle filtering for the 3-dimensional Lorenz '63 system. The filtering results are shown in Figures 5 and 6. Implementation of the suboptimal control was sufficient to ensure consistent tracking of the signal, even when the intervals when no observation is available is large.

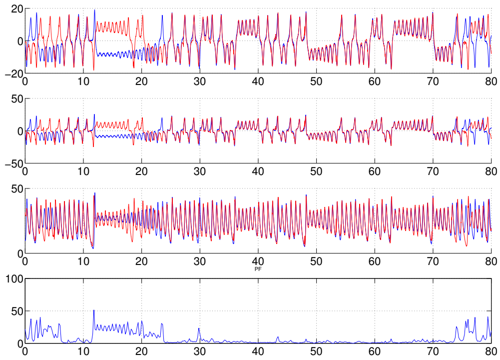

FIGURE 5. Regular particle filter; sparse observations collected at intervals equal to error doubling time

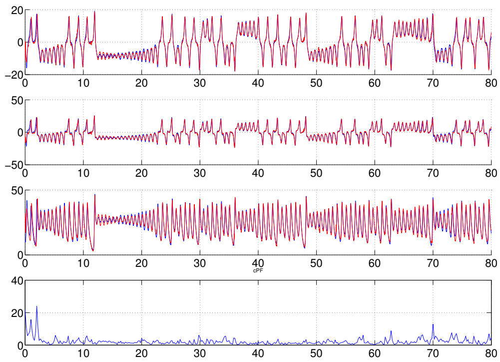

FIGURE 6. Particle filter with suboptimal particle control; sparse observations collected at intervals equal to error doubling time

2. Education and Academic Outreach

Results from the development of the reduced-order particle filtering technique for high-dimensional, multi-timescale systems were presented for the first time at the IUTAM Symposium on 50 Years of Chaos: Applied and Theoretical at Kyoto University, Kyoto, Japan. The results from an improved version of the algorithm that was capable of assimilating data on high-dimensional chaotic systems were presented at the IUTAM Symposium on Multiscale Problems in Stochastic Mechanics at Karlsruhe Institute of Technology, Karlsruhe, Germany. Two Ph.D students, Nishanth Lingala and Hoong Chieh Yeong in the Department of Aerospace Engineering at UIUC were supported by this grant. Nicolas Perkowski in the Department of Mathematics at HU, Berlin, has been working on this research along with his graduate studies and was supported by the grant during his visits to UIUC. Two other MS students, Yingtian Jiang and Mahmoud Mamlouk, who have also been involved part time, will graduate from the Department of Aerospace Engineering at UIUC in Fall 2013.

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Publications:

1. Peter Imkeller, N. Sri Namachchivaya, Nicolas Perkowski, and Hoong C. Yeong, "A Homogenization Approach to Multi-scale Filtering," in 50 Years of Chaos: Applied and Theoretical, Procedia IUTAM Volume 5, 2012, pp. 34-45.
2. Leo Dostal, Edwin Kreuzer and N. Sri Namachchivaya, "Almost Sure Stability Analysis of Parametric Roll in Random Seas Based on Top Lyapunov Exponent," *Proceedings in Applied Mathematics and Mechanics* (PAMM), Vol. 12, No. 1, 2012, pp. 607 - 609.
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4. Leo Dostal, Edwin Kreuzer and N. Sri Namachchivaya, "Non-standard Stochastic Averaging of Large Amplitude Ship Rolling in Random Seas," *Proceedings of the Royal Society, A*, Vol. 468, No. 2148, 2012, pp. 4146-4173.
5. Peter Imkeller, N. Sri Namachchivaya, Nicolas Perkowski and Hoong Chieh Yeong, "Dimensional Reduction in Nonlinear Filtering: A Homogenization Approach," *Annals of Applied Probability*, Volume 23, Number 6, December 2013, pp. 2290-2326.
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7. Dostal L., Kreuzer, E., Namachchivaya N. Sri, Stochastic Averaging of Roll-Pitch and Roll-Heave Motion in Random Seas. *Proceedings, IUTAM Symposium on Multiscale Problems in Stochastic Mechanics*, Volume 6, 132–140, 2013. 8. Kristjan Onu, Nishanth Lingala and N. Sri Namachchivaya "Random Vibration of a Nonlinear Auto-parametric System," in *Theory and Applications in Nonlinear Dynamics* (eds. Visarath In, Patrick Longhini and Antonio Palacios), Springer Verlag, Series 5394, pp. 11-24, 2014.

9. Nishanth Lingala, Nicolas Perkowski, Hoong C Yeong, N. Sri Namachchivaya, and Zoi Rapti, "Optimal Nudging in Particle Filters," *Probabilistic Engineering Mechanics*, Volume 37, Number 6, July 2014, pp. 160-169.
10. Prince Singh, Hoong C Yeong, Huiqing Zhang, Zoi Rapti, and N. Sri Namachchivaya, "Stochastic Dynamics of a Two-dimensional Structurally Nonlinear Airfoil in Turbulent Flow," (submitted).

AFOSR Deliverables Submission Survey

Response ID:4829 Data

1.

1. Report Type

Final Report

Primary Contact E-mail
Contact email if there is a problem with the report.

navam@illinois.edu

Primary Contact Phone Number
Contact phone number if there is a problem with the report

2172440683

Organization / Institution name

University of Illinois at Urbana - Champaign

Grant/Contract Title
The full title of the funded effort.

Multiscale Dynamics and Information in Data Collection and Assimilation for Environmental Applications

Grant/Contract Number
AFOSR assigned control number. It must begin with "FA9550" or "F49620" or "FA2386".

FA9550-12-1-0390

Principal Investigator Name
The full name of the principal investigator on the grant or contract.

Navaratnam Sri Namachchivaya

Program Manager
The AFOSR Program Manager currently assigned to the award

Fariba Fahroo

Reporting Period Start Date

08/01/2012

Reporting Period End Date

07/31/2015

Abstract

Data assimilation or filtering involves blending information from observations of the actual system states with information from dynamical models to estimate the current system states or certain model parameters. The filtering problem relies on three fundamental ingredients, namely 1) sensor placement: where the sensors are placed in order to obtain the most useful information, 2) sensor fusion: how to combine the measurements from different sensors, and 3) estimation: how to use the measurements to obtain the best possible state estimates. In this project, we considered the data assimilation problem for multi-timescale systems. An understanding of how scales interact with information led to the development of rigorous reduced-order data assimilation techniques for these high-dimensional problems.

This project developed new algorithms and tools for the collection, assimilation and harnessing of data by threading together

ideas from random dynamical systems, information theory, and statistical learning. A new particle filtering algorithm based on the theoretical result that combines stochastic homogenization with filtering theory to construct a reduced-dimension nonlinear filter is presented. They are used for approximating the real time filtering of chaotic signals.

The main results of the research project are: Rigorous mathematical development of a reduced-order particle filtering method for high-dimensional, multiscale random dynamical systems; Development of a particle filtering method adapted to high-dimensional, multiscale, chaotic systems

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Archival Publications (published) during reporting period:

1. Peter Imkeller, N. Sri Namachchivaya, Nicolas Perkowski, and Hoong C. Yeong, ``A Homogenization Approach to Multi-scale Filtering," in
50 Years of Chaos: Applied and Theoretical, Procedia IUTAM Volume 5, 2012, pp. 34-45.
2. Leo Dostal, Edwin Kreuzer and N. Sri Namachchivaya, ``Almost Sure Stability Analysis of Parametric Roll in Random Seas Based on Top Lyapunov Exponent," Proceedings in Applied Mathematics and Mechanics, Vol. 12, No. 1, 2012, pp. 607 - 609.
3. Nishanth Lingala, N. Sri Namachchivaya, Nicolas Perkowski and Hoong C Yeong ``Particle Filtering in High-Dimensional Chaotic Systems,"
Chaos: An Interdisciplinary Journal of Nonlinear Science}, Vol. 22(4), 2012, http://dx.doi.org/10.1063/1.4766595 (18 pages).
4. Leo Dostal, Edwin Kreuzer and N. Sri Namachchivaya, ``Non-standard Stochastic Averaging of Large Amplitude Ship Rolling in Random Seas,"
Proceedings of the Royal Society, A}, Vol. 468, No. 2148, 2012, pp. 4146-4173.
5. Peter Imkeller, N. Sri Namachchivaya, Nicolas Perkowski and Hoong Chieh Yeong, ``Dimensional Reduction in Nonlinear Filtering: A Homogenization Approach," Annals of Applied Probability, Volume 23, Number 6, December 2013, pp. 2290-2326.
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7. Dostal L., Kreuzer, E., Namachchivaya N. Sri, Stochastic Averaging of Roll-Pitch and Roll-Heave Motion in Random Seas. Proceedings, IUTAM Symposium on Multiscale Problems in Stochastic Mechanics, Volume 6, 132--140, 2013.
8. Kristjan Onu, Nishanth Lingala and N. Sri Namachchivaya ``Random Vibration of a Nonlinear Auto-parametric System," in Theory and Applications in Nonlinear Dynamics, (eds. Visarath In, Patrick Longhini and Antonio Palacios), Springer Verlag, Series 5394, pp. 11-24, 2014.
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Changes in research objectives (if any):

Change in AFOSR Program Manager, if any:

Extensions granted or milestones slipped, if any:

AFOSR LRIR Number

LRIR Title

Reporting Period

Laboratory Task Manager

Program Officer

Research Objectives

Technical Summary

Funding Summary by Cost Category (by FY, $K)

	Starting FY	FY+1	FY+2
Salary			
Equipment/Facilities			
Supplies			
Total			

Report Document

Report Document - Text Analysis

Report Document - Text Analysis

Appendix Documents

2. Thank You

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