AFRL-OSR-VA-TR-2015-0126

# Wave-Based Algorithms and Bounds for Target Support Estimation

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05/21/2015
Final Report



Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ RTC
Arlington, Virginia 22203
Air Force Materiel Command

| REPORT DOCUMENTATION PAGE | *Form Approved*<br>*OMB No. 0704-0188* |
|---|---|



| **1. REPORT DATE** *(DD-MM-YYYY)* | **2. REPORT TYPE** | **3. DATES COVERED** *(From - To)* |
|---|---|---|
| 15-05-2015 | Final report | June 1 2012 - May 31 2015 |

**4. TITLE AND SUBTITLE**
Wave-Based Algorithms and Bounds for Target Support Estimation

**5a. CONTRACT NUMBER**

**5b. GRANT NUMBER**
FA9550-12-1-0285

**5c. PROGRAM ELEMENT NUMBER**

**6. AUTHOR(S)**
Dr. Edwin Marengo

**5d. PROJECT NUMBER**

**5e. TASK NUMBER**

**5f. WORK UNIT NUMBER**

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
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**8. PERFORMING ORGANIZATION REPORT NUMBER**

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
USAF, AFRL DUNS 143574726
AF OFFICE OF SCIENTIFIC RESEARCH
875 N. RANDOLPH ST. ROOM 3112
ARLINGTON VA 22203

**10. SPONSOR/MONITOR'S ACRONYM(S)**
AFOSR

**11. SPONSOR/MONITOR'S REPORT NUMBER(S)**

**12. DISTRIBUTION/AVAILABILITY STATEMENT**

**13. SUPPLEMENTARY NOTES**

**14. ABSTRACT**
In this research program we developed novel analytical and computational methods to estimate the support of radiating sources and scatterers from knowledge of the corresponding far field radiation pattern or scattering amplitude. The focus was the development of methods to estimate the so-called minimum source region or scattering support of a far field. The support of any source that generates the given far field must contain this minimum source region. The results were derived in the framework of the Helmholtz equation. The derived methodology consisted of two components, one for the estimation of convex support information, and a subsequent procedure to estimate the nonconvex support. This project also led to new theoretical and computational developments in estimation-theoretic assessment of radar target separation estimability from far field data as well as to a new change detection method called the optical theorem detector, which is relevant to detect unknown targets and changes in random and complex media.

**15. SUBJECT TERMS**

| **16. SECURITY CLASSIFICATION OF:** | | | **17. LIMITATION OF ABSTRACT** | **18. NUMBER OF PAGES** | **19a. NAME OF RESPONSIBLE PERSON** |
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# FINAL AFOSR GRANT REPORT

Wave-based Algorithms and Bounds for Target Support Estimation

Grant Number: FA9550-12-1-0285

Principal Investigator:
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## Abstract

In this research program we developed novel analytical and computational methods to estimate the support of radiating sources and scatterers from knowledge of the corresponding far field radiation pattern or scattering amplitude. The focus was the development of methods to estimate the so-called minimum source region or scattering support of a far field. The support of any source that generates the given far field must contain this minimum source region. The results were derived in the framework of the Helmholtz equation. The derived methodology consisted of two components, one for the estimation of convex support information, and a subsequent procedure to estimate the nonconvex support. The convex support information was obtained using an exterior inverse diffraction framework. This allows estimation of the minimum convex source region, which is the convex hull of the minimum source region. Nonconvex support information is extracted in subsequent steps via a complementary interior inverse diffraction approach. This gives nonconvex bounds for the minimum source region. In many practical problems, the latter coincides with the true source or scatterer support. The derived source and scatterer support estimation methods had a purely analytical form as well as a practical computational variant. These methods were validated with analytical and computational examples during the course of this research. Applications of the derived methods to imaging were also developed. In addition, this research also led to the investigation of the fun-

damental limits of radar target separation estimation, as measured via the Cramer-Rao bound, for multiple scattering systems in which multiple scattering is not negligible. In addition, this work led to a new change detection method to detect unknown targets or medium changes in arbitrary random and complex media. The method in question is based on the optical theorem and was termed the optical theorem detector. The method was found to be quite effective, performing better than alternative methods such as the energy detector.

# 1 Introduction

The problem of estimating the support or shape of a scattering target from knowledge of its far zone radiation pattern (or far field scattering amplitude) is relevant to many radar problems including target location, detection, classification, and discrimination. Target support information is also useful as a prior for the estimation of target material properties, which is relevant for radar inverse scattering and imaging applications. In this project we developed and validated novel computational methods based on multipole expansions which allow one to estimate the support of an unknown scattering target from knowledge of the far field radiation pattern. In this approach, only one probing field is required, and the computed support information includes both the conventional convex support as well as the possibly nonconvex support of the target. Methods based on field inversion (inverse diffraction) as well as on source inversion (inverse source problem) were developed and validated. Application of the developed target support estimation approach to inverse source and scattering problems was also demonstrated. The corresponding results of this main focus area were published in two journal papers [1],[2] and were further disseminated in several conference papers and invited talks in academia and a national laboratory.

In addition, during the course of this project we developed fundamental Cramer-Rao bound characterizations providing insight about the estimability of target separation information. This problem is pertinent to the estimation of the separation of two targets or the size of an extended target from far field radar data. The results on this companion problem were reported in a journal paper [3]. These findings provide fundamental performance bounds on target support estimability and complement prior research on the related problem of target position estimation developed by the PI's group under another project [4].

In addition, we also developed a novel wave physics-based approach for the detection of unknown targets or medium changes from active sensor wavefield data. This detection methodology is relevant to target or change detection in rather arbitrary unknown backgrounds including random and complex media. The physical foundation for this novel approach is a new theory of the optical theorem in arbitrary media, which was developed during this project within the full vector electromagnetic formalism in [5]. This theory leads to three main variants of the optical theorem detector, in particular, three alternative indicators or statistics for change detection: one based on real average power, one based on reactive power associated to energy storage, and another based on the total or apparent power. The particular form of this new approach for the context of one-dimensional wave propagation systems, which is relevant to transmission lines and wave systems that can be approximately modeled in a single dimension, such as, e.g., canonical radar, sonar, and lidar models, was presented in [6]. This development provides proof that for a single-frequency and single receiver system the apparent power form of the optical theorem detector is equivalent to the conventional energy detector. On the other hand, the two approaches differ for multi-dimensional data such as in multi-frequency and multi-sensor systems. The application to change detection is formulated and validated in [7] with particular emphasis on the detection of a target embedded in enclosed environments and in random media. Further results pertinent to the detection of targets made of active media (which is essential for active cloaking-covered targets) were presented in a recent paper [8]. Another paper is under preparation which further expands the applicability for transient pulse change detection of arbitrary nonlinear-media and time-varying targets [9].

This report provides a summary of the main results obtained in this research project, along with references indicating where these results can be found in further detail in the published literature. We provide a summary of the publication record associated to this grant. We discuss the main findings resulting from this project. We also provide suggestions for future research.

# 2 Publications

The research resulting from this grant led to 7 journal papers [1]-[3],[5]-[8] and 6 conference papers [10]-[15], including a paper in review at the time of submission of this report [8]. In addition, a 14th paper resulting from this grant is in preparation [9]. The results of this research effort were further disseminated through invited talks, including one at Lawrence Livermore National Laboratory [16], two talks at AFOSR meetings including one in 2012 [17], and a number of talks at universities [18]-[21].

# 3 Main Findings

The focus of this project was the development of algorithms for the estimation of the shape of scatterers from active wave sensing far field data. The goal was to demonstrate successful estimation of both convex and nonconvex minimum source regions which are contained inside the scatterer and which in most inverse problems coincide with the true support of the scatterer. This main goal was the focus of our research associated to this grant. It was accomplished successfully, and the main results were published in a number of journal and conference papers, being of particular relevance the two journal papers [1],[2]. In those papers we demonstrated by means of analytical multipole theory methods and computational electromagnetic methods a new methodology to estimate the minimum convex source region and the (possibly nonconvex) support of a scattering target from knowledge of its far scattering amplitude. The results include many numerical examples validating the proposed new approach to estimate the localization and shape of targets from the far field data, which is important for radar imaging. Applications to imaging were demonstrated in the papers. The problem under consideration and the main components of the formulation and associated computational developments are summarized next. In the following, we borrow mostly from the key paper [2].

## 3.1 Problem Statement and Relevance

In this project we considered the problem of estimating the *minimum source region* of a given far field. This problem was formulated in the framework of the scalar Helmholtz equation which captures the essence of most electromagnetic, acoustic, and optical wave phenomena. The minimum source region has the following meaning: The support of any source that produces the far field in question must contain this region. This region is also sufficiently large so that there is a source supported in it that generates the given far field. Thus this region is optimal: it is the smallest support of any source that can produce the far field in question.

Previous works addressing this problem considered the computation of the convex hull of the minimum source region, also called the *convex scattering support* [22] or the *minimum convex source region* [23]. Our research in this grant led to new theory and algorithms to compute the actual minimum source region including nonconvex minimum source regions that cannot be estimated with the previous methods. We also derived new formulas for the minimum source region. They are based on the multipole expansion [24]. They complement the related past work on plane-wave-theory-based approach to estimate the minimum convex source region [23]. The derived formulas allow computation of the minimum source region for far fields that are known in closed form. This is of interest in antenna synthesis. It is also relevant for solving certain inverse scattering problems analytically, which is a topic of modern interest in the inverse problems community.

The derived theory and computational methods for support estimation apply to three-dimensional (3D) and two-dimensional (2D) wave propagation systems. The latter are relevant to targets that vary essentially only in two transverse coordinates and whose length in the other dimension is much bigger than the wavelength. In the first paper [1] we considered in detail both 2D and 3D problems, while in the second paper [2] we emphasized mostly the 2D context. In both papers we provided 2D examples. In the following, the key aspects of the theory are summarized only for the 2D context. The 3D theory is given in the above papers.

The principles to determine the minimum source region of a far field can be traced back to the work of Müller ([26], ths. 26, 27, 29) who used the spherical wave or multipole expansion to show that, for a fixed origin $O$, there is a minimum spherical source region of radius $R_{min}$, $B_{min}(O) = \{\mathbf{r} \in \mathbb{R}^3 : r \equiv |\mathbf{r}| \leq R_{min}(O)\}$, such that in order for a source to produce the given far field, it must lie in the interior of a spherical volume centered about the origin and having a radius that is at least as large as $R_{min}$. Furthermore, the radiated field outside $B_{min}$ is an analytic function that can be continued up to the boundary of a minimum source region $V_{min}$ located inside $B_{min}$ [27], p.143. Thus the far field can be inverse-diffracted up to the boundary of $V_{min} \subseteq B_{min}$. If one considers another origin $O'$ one can find another minimum spherical volume $B_{min}(O')$, and so on. Then if one considers a set of, say, $n$ origins and $n$ associated minimum spherical volumes, the intersection of all such regions, say $\mathcal{B}_{min}$, establishes a sharper bound on the minimum source region $V_{min} \subseteq \mathcal{B}_{min}$ which lies inside such intersection. By means of this procedure, it is possible to uniquely define and compute the convex scattering support or minimum convex source region, say $B_{min,conv} \supseteq V_{min}$, which is a subset of the convex hull of any source radiating the given far field. This minimum convex source region is the smallest convex support of any source producing that field, and represents important source localization information that can be inverted uniquely from the far field data despite the nonuniqueness of the full inverse source problem of reconstructing the actual source that generated the given far field [29, 30].

Previous work on the scattering support is given in [22, 28, 31, 25, 32]. Of much relevance to this project was the past work in [28] which demonstrated that the convex scattering support can be defined via a Picard test. In addition, Sylvester [25] demonstrated the existence of a unique union of well-supported convex sets (UWSC sets) for the given far field. A source of that support exists that generates the given far field. Also, this unique UWSC sets is a subset of the UWSC-support of any source that produces the same far field.

The computation of the minimum convex source region was also addressed in previous work ([23]; see also [27], p. 143-146) by means of a simpler methodology based on the plane wave expansion, rendering an alternative approach to estimate the minimum convex source region of a given far field. Unlike the multipole expansion or circular Paley-Wiener theorem approaches, or other range or Picard tests, the plane wave expansion method does not depend on the coordinate system. On the other hand, this method is useful only if the far field is known in closed form. For real data and computations, one needs to adopt alternative approaches such as those in [22, 28] and other related studies [31, 32]. On the other hand, there is a lot of interest in the analytical approach since it allows handling of problems where the far field is known in closed form, which cannot be studied in rigor with computational approaches. The methods developed in this research program included both purely analytical approaches and practical computational methods. The analytical part is the multipole theory counterpart of the plane wave expansion method in [23]. This included the methods to estimate the convex support as well as the nonconvex support. The analytical component includes the multipole theory counterpart of the key formula (eq. 6) in [23], for estimation of the convex support as well as the respective formula for the nonconvex support. The practical computational implementation for both convex and nonconvex support estimation was also developed and illustrated with examples.

It is important to comment on the fundamental and practical relevance of minimum source regions. There are many useful methods for imaging and shape reconstruction of scatterers from scattering data. The research developed thanks to this grant on "minimum source regions" is an important addition to the *state of the art* in this important area. First of all, the definition of the minimum source region is rigorous. It renders key insight on the information about the support of a source or scatterer that is contained in far fields of sources and scatterers. In addition, despite the nonuniqueness of the inverse problem, the minimum source region is a piece of information about the source or scatterer that can be deduced uniquely from the field data. Moreover, this approach has the particular appeal that it applies in theory to the extraction of information about the support of a scatterer using a single probing field. Most of the imaging methods used in qualitative inverse scattering apply to multiple excitation fields. In addition, this method performs robustly, even though it defines only a bound for the true scatterer support. In particular, usually the true support is similar to the estimated minimum source region. Another aspect that makes the developed target support estimation theory and algorithms highly relevant is the fact that it is essentially based on a Picard test so that it can be extended readily to other inverse problems.

## 3.2 Support Estimation Theory and Algorithms

We consider electromagnetic or acoustic radiation and scattering that can be described in the framework of the scalar Helmholtz equation. To simplify the presentation we consider next the basic 2D context, however the general results in [1],[2] hold for both 2D and 3D problems. We consider next the 2D context considered in [2] which is relevant to transverse electromagnetic fields. In this 2D context

$$(\nabla^2 + k^2)E_z(\mathbf{r}) = -i\omega\mu I_z(\mathbf{r}), \tag{1}$$

where $E_z$ represents the field, $I_z$ is source distribution which in the scattering context is the source induced in the scatterer upon the given probing field, $\omega$ is the angular oscillation frequency, $\mu$ is the free space permeability, and $k$ is the wavenumber, defined by

$$k = \omega\sqrt{\mu\epsilon}, \tag{2}$$

where $\epsilon$ is the free space permittivity. In Cartesian coordinates $\mathbf{r} = (x, y)$ while in cylindrical coordinates $\mathbf{r} = (\rho, \phi)$.

The radiated field is given by ([33], p.598)

$$E_z(\mathbf{r}) \quad = \quad -\frac{\omega\mu}{4}\int d\mathbf{r}' H_0^{(1)}(k|\mathbf{r} - \mathbf{r}'|)I_z(\mathbf{r}') \tag{3}$$

where $H_0^{(1)}$ is the Hankel function of the first kind and order 0. From the addition theorem

$$H_0^{(1)}(k|\mathbf{r}-\mathbf{r}'|) \;=\; \sum_{m=-\infty}^{\infty} J_m(k\rho_<)H_m^{(1)}(k\rho_>)e^{-im(\phi-\phi')} \tag{4}$$

where $\rho_< = \min(\rho,\rho')$ and $\rho_> = \max(\rho,\rho')$ where $\rho$ is the distance from $\mathbf{r}$ to the origin of coordinates and $\rho'$ is the distance from $\mathbf{r}'$ to the origin.

For a source whose support $V$ is contained in a ball of radius $R$ centered about the origin $O$ we have in view of (3,4)

$$E_z(\mathbf{r}=(\rho,\phi)) = -\frac{\omega\mu}{4}\sum_{m=-\infty}^{\infty} a_m H_m^{(1)}(k\rho)e^{-im\phi} \quad \rho > R \tag{5}$$

where the multipole moments

$$a_m = \int_0^R d\rho'\rho' \int_0^{2\pi} d\phi' J_m(k\rho')e^{im\phi'}I_z(\rho',\phi'). \tag{6}$$

Using the large argument approximation for the Hankel function

$$H_m^{(1)}(k\rho) \sim \sqrt{\frac{2}{\pi k\rho}}e^{i[k\rho - m(\pi/2)-\pi/4]} \tag{7}$$

we get from (14) the far zone relation

$$E_z(\rho,\phi) \sim -\frac{\omega\mu}{4}\sqrt{\frac{2}{\pi k\rho}}e^{i(k\rho-\pi/4)}f(\phi) \tag{8}$$

where the far field radiation pattern

$$f(\phi) = \sum_{m=-\infty}^{\infty} i^{-m}a_m e^{-im\phi} \tag{9}$$

so that in view of the orthogonality of the complex exponentials

$$a_m = \frac{i^m}{2\pi}\int_0^{2\pi} d\phi e^{im\phi}f(\phi). \tag{10}$$

In addition, for nonzero far fields, there exists a source inside the minimum source region $V_{min} \subseteq V$ radiating the given field, corresponding to the moments $a_m$. By applying the above results

$$E_z(\mathbf{r} = (\rho, \phi)) = -\frac{\omega\mu}{4} \sum_{m=-\infty}^{\infty} a_m H_m^{(1)}(k\rho) e^{-im\phi} \quad \rho > R_{min} \tag{11}$$

where $R_{min}$ is the radius of the smallest ball that is centered about the same origin and contains the minimum source region $V_{min} \subseteq V$ of the given far field.

We showed in the key papers [1],[2] that $R_{min}$ is defined by

$$R_{min} = \sup\{b \in \mathbb{R}^+ : \lim_{m\to\infty} \Gamma(m) \left(\frac{2}{kb}\right)^m |a_m| \neq 0\}. \tag{12}$$

where $\Gamma$ is the gamma function and sup means supremum. This result is the mathematical basis of the method developed in those papers to estimate the convex support of the source or scatterer. We also developed and demonstrated the associated computational implementation with discrete data. The derived methodology to estimate the convex target support can be summarized as follows.

The result (eq.(12)) holds for any origin $O$. The intersection of the minimum spherical source regions $B_{min} = \{\mathbf{r} \in \mathbb{R}^2 : \rho \leq R_{min}\}$ for different origins defines a convex region that contains the minimum source region. Using sufficiently many such origins one can define the minimum convex source region $B_{min,conv}$ of the given far field. The method consists of the following steps: 1) Consider a number $n$ of origins $O_\alpha, \alpha = 1, 2, \cdots, n$. 2) For each origin, compute via the test (12) the associated minimum radius $R_{min}(O_\alpha)$ and associated minimum circular region $B_{min}(O_\alpha) = \{\mathbf{r} \in \mathbb{R}^2 : |\mathbf{r} - O_\alpha| \leq R_{min}(O_\alpha)\}$. 3) The intersection of the regions $B_{min}(O_\alpha), \alpha = 1, 2, \cdots, n$ defines a convex region $\mathcal{B}_{min}$ bounding the minimum convex source region $B_{min,conv}$, i.e., $B_{min,conv} \subseteq \mathcal{B}_{min}$. The estimate $\mathcal{B}_{min}$ becomes closer to $B_{min,conv}$ as one uses more sample origins and their minimum circular regions.

Since (11) can be used for any of the test origins $O_\alpha$, it follows that we can uniquely associate, to the given far field, a valid radiated field that is well defined everywhere outside the respective convex region $\mathcal{B}_{min}$. This is exploited in the estimation of the possibly nonconvex support of the target as we outline in the following.

If the origin $O$ is such that $V$ is contained outside a circle of radius $R^{int}$ centered at the origin, then according to (3,4) the field in the interior of this circle of radius $R^{int}$ is given by

$$E_z(\mathbf{r}=(\rho,\phi)) = -\frac{\omega\mu}{4}\sum_{m=-\infty}^{\infty} g_m J_m(k\rho)e^{-im\phi} \quad \rho < R^{int} \tag{13}$$

where

$$g_m = \int_{R^{int}}^{\infty} d\rho'\rho' \int_0^{2\pi} d\phi' H_m^{(1)}(k\rho')e^{im\phi'} I_z(\rho',\phi'). \tag{14}$$

Moreover, there is a source inside $V_{min} \subseteq V$ radiating the same field outside $V$, i.e.,

$$E_z(\mathbf{r}=(\rho,\phi)) = -\frac{\omega\mu}{4}\sum_{m=-\infty}^{\infty} g_m J_m(k\rho)e^{-im\phi} \quad \rho < R_{min}^{int} \tag{15}$$

where $R_{min}^{int}$ is the radius of the largest ball centered about the same origin that is disjoint to the minimum source region $V_{min}$ and is tangential to it in a way that defines a concave boundary of $V_{min}$.

We demonstrated in [1],[2] that $R_{min}^{int}$ is defined by

$$R_{min}^{int} = \inf\{a \in \mathbb{R}^+ : \lim_{m\to\infty} \left(\frac{ka}{2}\right)^m |g_m|/\Gamma(m+1) \neq 0\}. \tag{16}$$

The procedure to estimate the minimum source region associated to the far field of the radiating source or scatterer under consideration consists of two parts: First an estimate of the convex support is obtained. This is followed by refinement of the target support estimate by means of the nonconvex support method based on (16) or its computational counterpart in the case of real and discrete data. The steps of the support reconstruction process can be summarized as follows:

1) Apply exterior inverse diffraction and compute the convex region $\mathcal{B}_{min}$ where $V_{min} \subseteq B_{min,conv} \subseteq \mathcal{B}_{min}$. For sufficiently many computational origins it is possible that $\mathcal{B}_{min} = B_{min,conv}$.

2) Consider next reference origins outside $\mathcal{B}_{min}$, say $O_\alpha, \alpha = 1, 2, \cdots, n$, and compute the field (via exterior inverse diffraction) over circles centered about those origins and having radius as large as possible but not intersecting the convex region $\mathcal{B}_{min}$. Let us assume such computational circles correspond to radii $b_\alpha$, with centers $O_\alpha$.

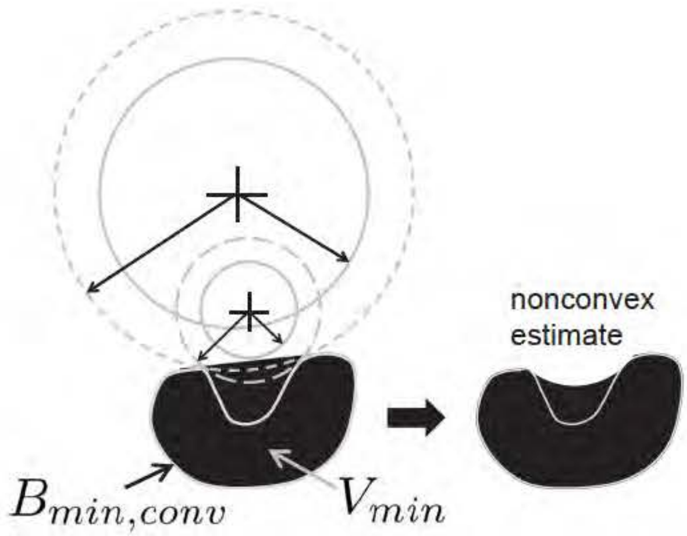


Figure 1: Successive estimation of the critical ball of radius $R_{min}^{int}$, based on field information available from previous iterations. This method allows the estimation of the minimum source region $V_{min}$ associated to the far field of the given radiating source or scatterer. (Source: [2].)

3) Using the value of the field on such circles compute the respective $R_{min}^{int}(\alpha)$.
4) Compute the union $U^{int}$ of all the regions $B_{min}^{int}(\alpha) = \{\mathbf{r} \in R^2 : |\mathbf{r} - O_\alpha| \leq R_{min}^{int}(\alpha)$. Compute the complement $\bar{U}^{int} = \{\mathbf{r} \in \mathbb{R}^2 : \mathbf{r} \notin U^{int}\}$.
5) Compute the intersection of $\bar{U}^{int}$ and $\mathcal{B}_{min}$, which defines the revised estimate $V_{min,est} \supseteq V_{min}$ for the minimum source region:

$$V_{min,est} = \mathcal{B}_{min} \cap \bar{U}^{int} \tag{17}$$

6) The information employed in the preceding steps is the field outside $\mathcal{B}_{min}$. As the estimate $V_{min,est}$ becomes closer to $V_{min}$ than $\mathcal{B}_{min}$ we repeat steps 2-4 for computational circles located outside the available estimate $V_{min,est}$. Computation of the field outside the available estimate relies on interior inverse diffraction. The counterpart of step 5 associated to the second and following iterations is $V_{min,est}(\text{current}) = V_{min,est}(\text{previous}) \cap \bar{U}^{int}$. By repeating this process (see figure 1) it is possible to obtain $V_{min,est} = V_{min}$.

The numerical implementation of this method employs discrete data. For a reference origin, for example, we compute the far field radiation pattern and the corresponding multipole moments. The number of significant multipole moments corresponds approximately to the familiar truncation formula $a_m \simeq 0, m > kR$ where $R$ is the radius of the smallest circle that circumcribes the original source support and is centered at the origin. The reconstruction algorithm is implemented computationally as a Picard test. In particular, we require that the functional energy of the reconstructed field exhibit a reasonable finite value. A methodology resembling the familiar L curve approach is developed and illustrated in [2] which performs this task effectively.

Figure 2, from the key paper [2], illustrates the iterative process to construct the estimate of the convex support of a source consisting of a point source plus a perfect electric conducting cylinder, as shown in the figure. Figure 3, from [2], illustrates the nonconvex support iterations leading to a better estimate of the total source support. Figure 4, also from [2], shows a source support estimate obtained with 4 convex bounding regions and 4 nonconvex bounding regions. It was found that the performance of the developed methodology is affected by the regularization procedure, which establishes a tradeoff between accuracy and stability. It represents the practical way to handle numerical noise issues while maintaining sufficient target information. After exploring different regularization alternatives we found that the method in [2] was quite effective and rendered estimates consistent with conventional imaging considerations discussed in [1],[2] and in conference papers associated to this grant.

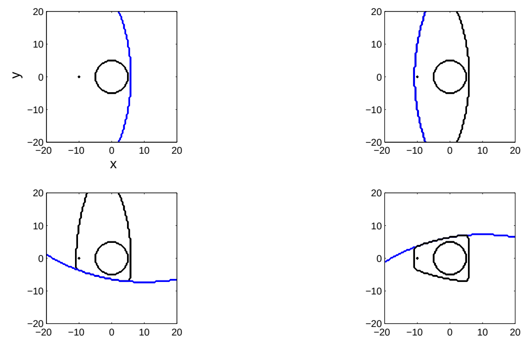


Figure 2: Minimum circles and cummulative estimated convex boundary of the minimum source region for 4 origins: $(-50, 0)$ (top left), $(50, 0)$ (top right), $(10, 50)$ (bottom left), and $(10, -50)$ (bottom right). Also shown is the correct support of the total source, consisting of a point source and a perfect electric conducting cylinder. (Source: [2].)

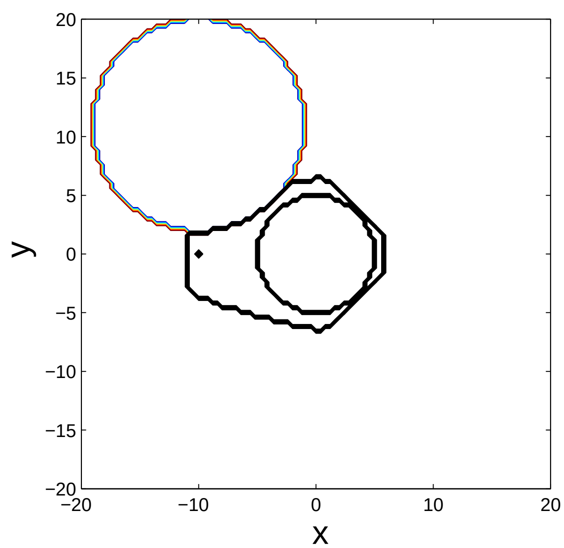


Figure 3: Cummulative estimated nonconvex support, based on the estimated circular boundary corresponding to a given test origin. (Source: [2].)

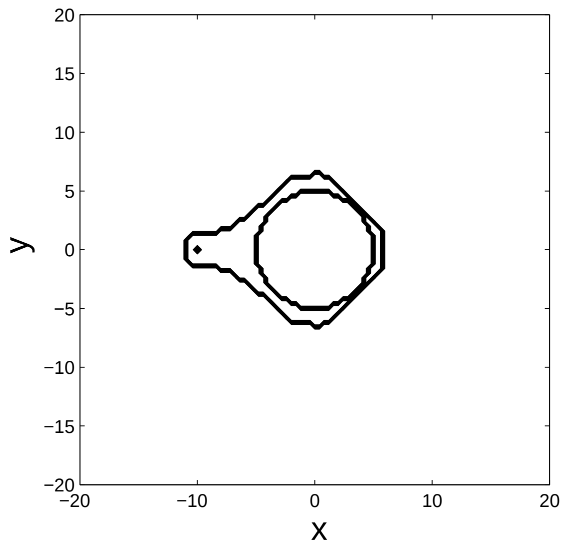


Figure 4: Cummulative estimated nonconvex support based on 8 test origins for the convex part and 4 origins for the nonconvex part. (Source: [2].)

## 3.3 Summary and Future Directions

In summary, in this main focus area of this project, we developed the required wave theory and computational methodology to exactly and approximately estimate the minimum source region of a given far field. This is relevant to the imaging and localization of sources and scatterers. In the scatterers' case, the probing involves a single incident field. Our developments provide the multipole theory counterpart of the plane wave expansion approach considered before [23]. The proposed methodology complements the previous efforts in this area which had emphasized only the minimum nonconvex source region. The method derived in this project allows the formulation of problems in which the far field is expressed as a multipole series expansion, which cannot be handled via the prior method in [23]. This holds for free space but can be further generalized to nonhomogeneous media via the plane wave and multipole expansion generalizations presented in [34].

We demonstrated the proposed target support methods with both analytical and computational examples. The computational implementation involves regularization to make the data essentially finite-dimensional. This regularization makes the support estimates more conservative and more stable than the analytical, infinite-dimensional counterpart. We found that for certain scatterers and under homogeneous plane wave excitation the analytical results essentially define only the center of the scatterer. On the other hand, under near field excitation, the boundary of the minimum source region associated to the induced source in the same scatterer tends to be better defined by means of this methodology. This appears to be due to the greater information content about the target that is available in the far zone when the target is probed with near fields. Due to reciprocity the same results are obtained if one reverses the role of the transmitter and the receiver, i.e., the probing fields may be due to far zone sources while the scattering fields are sensed in the near zone, with identical results. In addition, in this research we also demonstrated the use of the support estimation step as a prior for inverse source problems, with encouraging results (a detailed example is given in [2]).

Possible future directions include a purely computational variant of the methods derived in this project, which employs manifolds of realizable fields, corresponding to bounded energy sources. Another interesting area is extending the results of this grant to partially correlated fields which are relevant in optics. Another important extension of the results obtained in this grant is the generalization to nonhomogeneous media which can be accomplished in principle via the generalized expansion methods proposed in [34].

## 3.4 Other Related Research

This research program also led to the characterization, via the Cramer-Rao bound, of the estimability of target separation information. The focus was multiple scattering systems in which multiple scattering is not negligible so that linearizing approximations, in particular, the Born approximation leading to the $\lambda/2$ rule of thumb for the imaging resolution limit, are not applicable. The estimation theoretic results and associated computer illustrations were published in [3].

In addition, this grant led to a new change detection radar method based on statistics associated to physical electromagnetic energy, as governed by the optical theorem. This can be considered an entirely new field, with ramifications to all forms of active wave sensing, as well as rather arbitrary media and targets. The detailed results were presented in [5],[7],[6], in upcoming papers [8],[9], and in many conference papers and invited presentations.

In particular, we developed the fundamental physical principles [5],[6],[9] applied detection theoretic formulation [7], computer simulation tools including multiple scattering [7],[8], and performance receiver operator characteristic (ROC) curves [7],[8] of a new physics-based coherent change detection approach that is based on a statistic that has the meaning of physical power. We termed this new approach the optical theorem detector. It is based on a fundamental wave energy conservation result called the optical theorem and reciprocity relations of wave phenomena. This radar concept is relevant to detect unknown targets or changes in an unknown background medium and is thus broadly applicable to many practical military and national security situations. The concept in question is particularly useful for highly reverberating environments such as enclosed environments (caves, tunnels, indoor facilities, urban canyon, ocean waveguide, and so on). It was validated with numerical simulations corresponding to canonical waveguide-like environments as well as for random multiple scattering media. The computer simulation results suggest that this method performs similar to the matched filter even though it does not require prior knowledge of the sought target. The obtained computer simulation results suggest that this method tends to perform better than the conventional approach to change detection called the energy detector. Unlike the energy detector, which is based on mathematical energy, the optical theorem detector is based on physical wave energy associated to scattering, dissipation in the scatterer, as well as energy storage in the vicinity of the scatterer (reactive energy).

The most logical future direction is the rigorous determination of the conditions (background medium, probing signal, receiver aperture, etc.) under which the optical theorem detector approach is guaranteed to perform better than alternative change detection approaches such as the energy detector. Another direction is the possible comparison with alternative wave based methods such as the time reversal detector. On the other hand, the time reversal detector uses an additional probing experiment that is not used in the optical theorem detector. Thus it is not obvious that these two detectors can be compared directly. Perhaps a normalization of the data can be adopted for this purpose, e.g., via the addition of a second optical theorem experiment, which may be optimized from the prior data.

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Est mat on

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FA9550-12-1-0285

**Principal Investigator Name**
**The full name of the principal investigator on the grant or contract.**

Edw n Marengo

**Program Manager**
**The AFOSR Program Manager currently assigned to the award**

Tr stan Nguyen

**Reporting Period Start Date**

6/1/2012

**Reporting Period End Date**

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**Abstract**

In th s research program we deve oped nove  ana yt ca  and computat ona  methods to est mate the support of rad at ng sources and scatterers from know edge of the correspond ng far f e d rad at on pattern or scatter ng amp  tude.
The focus was the deve opment of methods to est mate the so-ca  ed m n mum source reg on or scatter ng support of a far f e d. The support of any source that generates the g ven far f e d must conta n th s m n mum source reg on. The resu ts were der ved  n the framework of the He mho tz
equat on. The der ved methodo ogy cons sted of two components, one for the est mat on of convex support  nformat on, and a subsequent procedure to est mate the nonconvex support. The convex support  nformat on was
obta ned us ng an exter or  nverse d ffract on framework. Th s a  ows est mat on of the m n mum convex source reg on, wh ch

 s the convex hu   of the m n mum source reg on. Nonconvex support  nformat on  s extracted  n subsequent steps v a a comp ementary  nter or  nverse d ffract on approach. Th s g ves nonconvex bounds for the m n mum source reg on. In many pract ca  prob ems, the  atter co nc des w th the true source or scatterer support.
The der ved source and scatterer support est mat on methods had a pure y ana yt ca  form as we   as a pract ca computat ona  var ant. These methods were va  dated w th ana yt ca  and computat ona  examp es dur ng the course of th s research. App  cat ons of the der ved methods to  mag ng were a so deve oped. Th s work a so  ed to the Cramer-Rao bound quant f cat on of the est mab   ty of target separat on for mu t p e scatter ng systems. It a so  ed to the deve opment of a new change detect on approach  n random and comp ex med a that  s based on the opt ca  theorem of scatter ng theory. The method was found to
be qu te effect ve, perform ng better than a ternat ve methods such as the energy detector.

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E.A. Marengo and J. Tu,   Opt ca  theorem for act ve med a", 2015 IEEE Internat ona  Sympos um on Antennas and Propagat on and North Amer can Rad o Sc ence Meet ng, Vancouver, Canada, Ju y 19-25, 2015 (accepted).

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**Changes in research objectives (if any):**

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**Extensions granted or milestones slipped, if any:**

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**LRIR Title**

**Reporting Period**

**Laboratory Task Manager**

**Program Officer**

**Research Objectives**

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**Funding Summary by Cost Category (by FY, $K)**

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| Equ pment/Fac  t es | | | |
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