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Report Title

Final Report: Distributed Cooperative Control of Multiple Nonlinear Systems with Nonholonomic Constraints and Uncertainty

ABSTRACT

This is the final report of the project. It covers the obtained results, the papers published and students supported. The report also includes the research progress for this project over three years.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

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Paper

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06/26/2014 9.00	Chunyu Chen, Felipe De La Torre, Wenjie Dong. Distributed Exponentially Tracking Control of Multiple Wheeled Mobile Robots , 2014 American Control Conference (ACC). 04-JUN-14, . : ,
06/26/2014 8.00	Wenjie Dong, Felipe De La Torre, Yifan Xing. Distributed Output Tracking Control of Heterogeneous linear Agents, 2014 American Control Conference (ACC). 04-JUN-14, . : ,
06/26/2014 10.00	Wenjie Dong, Felipe De La Torre. Distributed Exponential Formation Control of Multiple Wheeled Mobile Robots, the International Conference of Control, Dynamic Systems, and Robotics. 07-MAY-14, . : ,
06/26/2014 13.00	Chunyu Chen, Wenjie Dong. Coordination of Mechanical Systems with Information and VelocityConstraints , 2014 IEEE 53rd Annual Conference on Decision and Control (CDC). 15-DEC-14, . : ,
06/26/2014 14.00	Yifan Xing, Wenjie Dong. Robut/Adaptive Tracking Control of Wave-Adaptive Modular Vessel withUncertainty , 2014 IEEE 53rd Annual Conference on Decision and Control (CDC). 15-DEC-14, . : ,
06/26/2014 15.00	Chunyu Chen, Wenjie Dong. Distributed Formation Tracking Control of Multiple Mobile RoboticSystems , 2014 IEEE 53rd Annual Conference on Decision and Control (CDC). 15-DEC-14, . : ,
08/01/2013 3.00	Sheng Zhao, , Wenjie Dong, , Jay A. Farrell. Quaternion-based Trajectory Tracking Control of VTOL-UAVs using Command Filtered Backstepping, American control conferences. 18-JUN-13, . : ,
08/01/2013 4.00	Wenjie Dong, , Mounir Ben Ghalia,, Chunyu Chen,, Yifan Xing. Teleoperation of a Cluster of Mobile Robots Subject to Model Uncertainty, American control conference. 18-JUN-13, . : ,
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06/26/2014 16.00	Wenjie Dong. Neural Network Based Distributed Control ofMechanical Systems with/without Constraints , IEEE Transactions on Neural Networks and Learning Systems (01 2014)
08/01/2013 2.00	Wenjie Dong. Distributed tracking control of networked chained systems, International Journal of Control (06 2013)
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Books

<u>Received</u>	<u>Book</u>
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TOTAL:

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Patents Awarded

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Wenjie Dong, 2014 Navy Summer Faculty research fellowship.

Wenjie Dong, Outstanding Faculty award, February 25, 2015.

Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	Discipline
Felipe De La Torre	0.70	
Yifan Xing	0.70	
Edson Gallegos	0.50	
Veera Venkata S Korupu	0.50	
FTE Equivalent:	2.40	
Total Number:	4	

Names of Post Doctorates

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Names of Under Graduate students supported

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FTE Equivalent:	
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Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

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The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:..... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):..... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:..... 0.00

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The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:..... 0.00

Names of Personnel receiving masters degrees

NAME

Yifan Xing

Total Number: 1

Names of personnel receiving PHDs

NAME

Total Number:

Names of other research staff

NAME

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Riverside CA 925210001

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Inventions (DD882)**Scientific Progress**

See Attachment.

Technology Transfer

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Progress Report
of
Project “Distributed Cooperative Control of Multiple
Nonlinear Systems with Nonholonomic Constraints and
Uncertainty”

Dr. Wenjie Dong

April 4, 2015

Department of Electrical Engineering
The University of Texas - Pan American



Contents

1	Problems Considered In This Project	1
1.1	Cooperative Control of Multiple Nonholonomic Dynamic Systems	2
1.2	Cooperative Control of Multiple Nonholonomic Dynamic Systems with Non-ideal Nonholonomic Constraints	3
2	Summary of Achievements	5
2.1	Achievements in Research	5
2.2	Achievements in Education	9
2.3	Publications	9
3	Consensus of Multiple Nonholonomic Mechanical Systems	11
3.1	Distributed Controller Design for Chained Systems	11
3.2	Distributed Controller Design for Driftless Systems	14
3.3	Distributed Controller Design for Dynamical Systems	16
3.4	Distributed Controller Design for General Dynamical Systems	19
3.5	Simulation	21
4	Distributed Tracking Control of Networked Chained Systems	29
4.1	Introduction	29
4.2	Problem Statement and Preliminary Results	31
4.2.1	Problem Statement	31
4.2.2	Preliminary Results	33
4.3	Distributed State Feedback Tracking Control	38
4.4	Distributed output feedback distributed control	42
4.5	Simulations	46
4.6	Conclusion	50
5	Leader-Following Control of Multiple Nonholonomic Systems Over Directed Communication Graphs	51
5.1	Introduction	51
5.2	Problem Statement	53
5.3	Distributed State Feedback Controller Design	56
5.3.1	Variable transformation	56
5.3.2	State Feedback Controller Design	58
5.4	Distributed Output Feedback Controller Design	61

5.5	An Application	64
5.6	Conclusion	65
6	Distributed Exponentially Tracking Control of Multiple Wheeled Mobile Robots	68
6.1	Introduction	68
6.2	Problem Statement	69
6.3	Cooperative Controller Design	70
6.4	Simulations	72
6.5	Conclusion	72
7	Neural Network Based Distributed Control of Mechanical Systems with/without Constraints	75
7.1	Introduction	75
7.1.1	Related Work	76
7.1.2	Our Contributions	76
7.2	Problem Statement and Preliminary Results	77
7.2.1	Problem Statement	77
7.2.2	Some Results on Linear time-varying systems	78
7.2.3	Some Results of Algebraic Graph Theory	80
7.2.4	Neural Network Approximation	83
7.3	Distributed Controller Design	85
7.4	Distributed Tracking Control of Multiple Mechanical Systems	92
7.5	Simulations	95
7.5.1	Formation control of Multiple Mobile Robots	95
7.5.2	Synchronization of Multiple 2-DOF Manipulators	100
7.6	Conclusion	100
8	Distributed Formation Tracking Control of Multiple Mobile Robotic Systems	103
8.1	Introduction	103
8.2	Problem Statement	105
8.3	Main Results	108
8.4	Distributed Controller for Kinematic Systems	109
8.5	Distributed controller for dynamics	113
8.6	distributed controller for time-varying communication topology	115
8.7	Simulation Results	115
9	Distributed Exponentially Tracking Control of Multiple Wheeled Mobile Robots	118
9.1	Introduction	118
9.2	Problem Statement	119
9.3	Distributed Controller Design	121
9.4	Cooperative Control Laws for Time-varying Communication Graph	126
9.5	simulation	126

9.6	Conclusion	127
10	Teleoperation of a Cluster of Mobile Robots Subject to Model Uncertainty	129
10.1	Introduction	129
10.2	Problem Statement	130
10.3	Controller Design	133
10.3.1	Step 1: Controller Design for Kinematic Systems	133
10.3.2	Step 2: Controller Design for Dynamics Systems	136
10.4	Simulation	138
10.5	Conclusion	139
11	Consensus of Multiple Nonholonomic Mechanical Systems with Non-ideal Nonholonomic Constraints	140
11.1	Distributed Controller Design for Kinematic Systems	140
11.2	Distributed Controller Design for Dynamical Systems	143
11.3	Simulation	145
12	Distributed Tracking Control of Multiple Nonholonomic Mechanical Systems with Non-ideal Nonholonomic Constraints	153
12.1	Distributed Controller Design for Kinematic Systems	153
12.2	Distributed Controller Design for Dynamical Systems	156
12.3	Simulation	158
13	Conclusions	167

Abstract

This is the progress report of the project “Distributed Cooperative Control of Multiple Non-linear Systems with Nonholonomic Constraints and Uncertainty”. It includes our achievements in research and education with the support of this project.

Chapter 1

Problems Considered In This Project

Consider a group of m mechanical systems with nonholonomic constraints and uncertainty, with the aid of Lagrangian formulation the motion of system j is defined by [1, 2]

$$M_j(q_{*j}, \alpha_j, \beta_j)\ddot{q}_{*j} + C_j(q_{*j}, \dot{q}_{*j}, \alpha_j, \beta_j)\dot{q}_{*j} + G_j(q_{*j}, \alpha_j, \beta_j) + D_j(q_{*j}) = B_j(q_{*j}, \alpha_j, \beta_j)\tau_j + J_j(q_{*j}, \alpha_j)^\top \lambda_j \quad (1.1)$$

$$J_j(q_{*j}, \alpha_j)\dot{q}_{*j} = 0 \quad (1.2)$$

where $q_{*j} = [q_{1j}, \dots, q_{nj}]^\top$ is the state of system j , α_j is the geometric parameter uncertainty, β_j is the inertia parameter uncertainty, M_j is an $n \times n$ bounded positive-definite symmetric matrix, $C_j\dot{q}_{*j}$ is centripetal and Coriolis force, G_j is gravitational force, D_j includes unmodeled dynamics, disturbance, and noise, B_j is an $n \times r$ input matrix, τ_j is an r -vector of control input, J_j is an $(n-s) \times n$ matrix with $s = n - \text{Rank}(J_j)$, λ_j is an $(n-s)$ -vector of the constraint force on system j , and the superscript $\top$ denotes the transpose. The constraint (1.2) is assumed to be completely nonholonomic [3]. It should be noted that in eqn. (1.1) there are both parameter uncertainty (i.e., α_j and β_j) and non-parameter uncertainty (i.e., D_j).

For the m systems, there is information exchange between systems by sensors or wireless communication. For simple presentation, information obtained by sensors and wireless communications is not distinguished and is considered as the same. For each system, the available information for feedback design is its own information and the information received from its neighbors. If each system is considered as a node, the information exchange between systems can be described by a direct graph (i.e., digraph) $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where $\mathcal{V} = \{1, 2, \dots, m\}$ is a node set and $\mathcal{E}$ is an edge set with element e_{ij} that describes

the information flow from node i to node j . If the information of system i is available to system j , there will be an edge e_{ij} in $\mathcal{E}$. System i is said to be a *neighbor* of system j if the information of system i is available to system j . For system j , the indices of its neighbors form a set which is denoted by $\mathcal{N}_j$. Therefore, the available information to system j for controller design is the information of system j and the information of system i for all $i \in \mathcal{N}_j$. For example, for a group of five systems with information flow as in Figure 1.1,

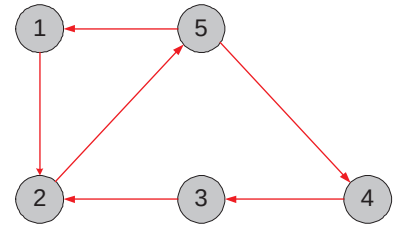


Figure 1.1: Information flow between systems

the information flow can be defined by the digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where $\mathcal{V} = \{1, 2, 3, 4, 5\}$ and $\mathcal{E} = \{e_{12}, e_{25}, e_{32}, e_{43}, e_{54}, e_{51}\}$.

Before proposing the research problems in this project, eqns. (1.1)-(1.2) will be converted into an appropriate form. Let the vector fields $g_{1j}(q_{*j}, \alpha_j), \dots, g_{sj}(q_{*j}, \alpha_j)$ form a basis of the null space of $J_j(q_{*j}, \alpha_j)$, by eqn. (1.2) there exists an s -vector $u_{*j} = [u_{1j}, \dots, u_{sj}]^\top$ such that

$$\dot{q}_{*j} = g_{*j}(q_{*j}, \alpha_j)u_{*j} = g_{1j}(q_{*j}, \alpha_j)u_{1j} + \dots + g_{sj}(q_{*j}, \alpha_j)u_{sj} \quad (1.3)$$

where $g_{*j}(q_{*j}, \alpha_j) = [g_{1j}(q_{*j}, \alpha_j), \dots, g_{sj}(q_{*j}, \alpha_j)] \in R^{n \times s}$. If α_j is assumed to be a constant vector, differentiating both sides of (1.3) and substituting it into (1.1) and multiplying both sides by $g_{*j}(q_{*j}, \alpha_j)^\top$, one has

$$\bar{M}_j(q_{*j}, \alpha_j, \beta_j)\dot{u}_{*j} + \bar{C}_j(q_{*j}, \dot{q}_{*j}, \alpha_j, \beta_j)u_{*j} + \bar{G}_j(q_{*j}, \alpha_j, \beta_j) + \bar{D}_j(q_{*j}, \alpha_j) = \bar{B}_j(q_{*j}, \alpha_j, \beta_j)\tau_j \quad (1.4)$$

where it is applied the fact that $g_j(q_{*j}, \alpha_j)^\top J_j(q_{*j}, \alpha_j)^\top = 0$, and $\bar{M}_j = g_{*j}^\top M_j g_{*j}$, $\bar{C}_j = g_{*j}^\top M_j \dot{g}_{*j} + g_{*j}^\top C_j g_{*j}$, $\bar{G}_j = g_{*j}^\top G_j$, $\bar{D}_j = g_{*j}^\top D_j$, and $\bar{B}_j = g_{*j}^\top B_j$.

System (1.3) is called the *kinematics* of system (1.1)-(1.2). System (1.4) is called the *dynamics* of system (1.1)-(1.2). Eqn. (1.3) is a drift-less nonlinear system and is called a nonholonomic system. System (1.3)-(1.4) describes the motion of system (1.1)-(1.2). The control problem defined for system (1.1)-(1.2) is equivalent to the control problem defined for system (1.3)-(1.4). Hereafter, control problems are defined for system (1.3)-(1.4) instead of system (1.1)-(1.2).

System (1.3) is a general drift-less nonlinear system with uncertainty. For many practical systems, for example wheeled mobile robots and unmanned aerial vehicles, system (1.3) can be locally or globally converted into the well-known chained form by an appropriate state transformation [4]. For example, if $s = 2$ and α_j is known, there exists an appropriate state transformation

$$x_{*j} = T_1(q_{*j}, \alpha_j), \quad u_{*j} = T_2(q_{*j}, \alpha_j)v_{*j} \quad (1.5)$$

for $1 \leq j \leq m$ such that system (1.3)-(1.4) is transformed into the following well-known extended chained form

$$\dot{x}_{1j} = v_{1j}, \quad \dot{x}_{2j} = v_{2j}, \quad \dot{x}_{ij} = v_{1j}x_{i-1,j}, \quad 3 \leq i \leq n \quad (1.6)$$

$$\tilde{M}_j \dot{v}_{*j} + \tilde{C}_j v_{*j} + \tilde{G}_j + \tilde{D}_j = \tilde{B}_j \tau_j \quad (1.7)$$

where $x_{*j} = [x_{1j}, \dots, x_{nj}]^\top$, $v_{*j} = [v_{1j}, v_{2j}]^\top$, $\tilde{M}_j = T_2^\top \bar{M}_j T_2$, $\tilde{C}_j = T_2^\top (\bar{M}_j \dot{T}_2 + \bar{C}_j T_2)$, $\tilde{G}_j = T_2^\top \bar{G}_j$, $\tilde{D}_j = T_2^\top \bar{D}_j$, and $\tilde{B}_j = T_2^\top \bar{B}_j$.

System (1.6) is the well-known chained form system with two inputs. If $s > 2$, in many practical applications, system (1.3) can be converted into the chained form system with multiple inputs and multiple-generators [3] through appropriate state transformations.

1.1 Cooperative Control of Multiple Nonholonomic Dynamic Systems

For multiple systems in (1.3)-(1.4), the following problems are considered.

Consensus Problem of Dynamic Systems: For a group of m systems in (1.3)-(1.4), if there is uncertainty in eqns. (1.3)-(1.4) and the information exchange digraph $\mathcal{G}$ is fixed or time-varying, the problem is how to design a distributed control law τ_j for system j based on its own information and its neighbors' information such that

$$\lim_{t \rightarrow \infty} (q_{*j} - c) = 0, \quad 1 \leq j \leq m, \quad (1.8)$$

where c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

Distributed Tracking Control of Dynamic Systems: For a group of m systems in (1.3)-(1.4) and a desired trajectory q^d which is available to a *portion* of the m systems. If there is uncertainty in eqns. (1.3)-(1.4) and the information exchange digraph $\mathcal{G}$ is fixed or time-varying, the problem is how to design a distributed control law τ_j for system j based on its own information and its neighbors' information such that

$$\lim_{t \rightarrow \infty} (q_{*j} - q^d) = 0, \quad 1 \leq j \leq m. \quad (1.9)$$

1.2 Cooperative Control of Multiple Nonholonomic Dynamic Systems with Non-ideal Nonholonomic Constraints

In eqns. (1.1)-(1.2), the right-side of the constraint (1.2) is assumed to be zero. In practice, since the structure of a mechanical system is not rigid or other reasons, the right-side of the constraint (1.2) may not be zero. To deal with this situation, the constraint can be described by

$$J_j(q_{*j}, \alpha_j) \dot{q}_{*j} = \epsilon_j(q_{*j}) \quad (1.10)$$

where $\epsilon_j(q_{*j})$ is a small perturbation. Let the vector fields $g_{1j}(q_{*j}, \alpha_j), \dots, g_{sj}(q_{*j}, \alpha_j)$ form a basis of the null space of $J_j(q_{*j}, \alpha_j)$, by eqn. (1.10) there exists an s -vector $u_{*j} = [u_{1j}, \dots, u_{sj}]^\top$ such that

$$\begin{aligned} \dot{q}_{*j} &= g_{*j}(q_{*j}, \alpha_j) u_{*j} + J_j^\top(q_{*j}, \alpha_j) [J_j(q_{*j}, \alpha_j) J_j^\top(q_{*j}, \alpha_j)]^{-1} \epsilon_j(q_{*j}) \\ &= g_{*j}(q_{*j}, \alpha_j) u_{*j} + \Delta_j(q_{*j}, \alpha_j) \end{aligned} \quad (1.11)$$

where $g_{*j}(q_{*j}, \alpha_j) = [g_{1j}(q_{*j}, \alpha_j), \dots, g_{sj}(q_{*j}, \alpha_j)] \in R^{n \times s}$. If α_j is assumed to be a constant vector, differentiating both sides of (1.11) and substituting it into (1.1) and multiplying both sides by $g_{*j}(q_{*j}, \alpha_j)^\top$, one has (1.4), where $\bar{D}_j$ is a new matrix and is unknown.

Different from eqn. (1.3), the system in eqn. (1.11) is not a drift-free system. Since ϵ_j is generally small, system (1.11) can be considered as a system in (1.3) with small perturbation. In eqn. (1.11), there are both parameter uncertainty and non-parameter uncertainty.

The dynamics (1.4) are ignored and u_{*j} is assumed to be a virtual control input. It is assumed that the geometric parameter α_j is known and that eqn. (1.11) can be

converted into the following perturbed chained form by appropriate transformations $x_{*j} = T_1(q_{*j}, \alpha_j)$ and $u_{*j} = T_2(q_{*j}, \alpha_j)v_{*j}$.

$$\begin{aligned} \dot{x}_{1j} &= v_{1j} + \phi_{1j}(x_{1j}), & \dot{x}_{2j} &= v_{2j} + \phi_{2j}(\bar{x}_{3j}, v_{1j}), \\ \dot{x}_{ij} &= x_{i-1,j}v_{1j} + v_{1j}\phi_{ij}(\bar{x}_{ij}, v_{1j}), & 3 \leq i \leq n \end{aligned} \quad (1.12)$$

where $\bar{x}_{ij} = [x_{ij}, x_{i+1,j}, \dots, x_{nj}]^\top$, ϕ_{ij} are unknown functions and satisfy some assumptions, and (v_{1j}, v_{2j}) are new inputs.

The following problems are proposed in this project.

Consensus Problem of Dynamic Systems with Non-ideal Nonholonomic Constraints: For a group of m systems in (1.11) and (1.4), if there is uncertainty in eqns. (1.11) and (1.4) and the information exchange digraph $\mathcal{G}$ is fixed or (1.8) is satisfied, where c is unprescribed constants which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

Distributed Tracking Control of Dynamic Systems with Non-ideal Nonholonomic Constraints: For a group of m systems in (1.11) and (1.4) and a desired trajectory q^d which is available to a *portion* of the m systems. If there is uncertainty in eqns. (1.11) and (1.4) and the information exchange digraph $\mathcal{G}$ is fixed or time-varying, the problem is how to design a distributed control law τ_j for system j based on its own information and its neighbors' information such that (1.9) is satisfied.

Chapter 2

Summary of Achievements

In this chapter, we summarize our achievements in research and education.

2.1 Achievements in Research

In this project, we studied the four control problems and have proposed different methods for the proposed problems. Fruitful results have been obtained.

1. For the consensus problem of dynamic system, we proposed a “transverse function” based approach with the aid of backstepping techniques (See Chapter 3). In order to solve the consensus problem of the dynamic systems, the solutions were proposed in two steps. In the first step, we assume that the dynamics of each system can be ignored and consider the consensus problem of kinematic systems. With the aid of the transverse functions, distributed controllers were proposed for kinematic systems. In the second step, we propose distributed controllers for the dynamical systems with the aid of the backstepping techniques and the results in the first step. Considering different uncertainty in the dynamics of each systems, distributed adaptive and distributed robust controllers were proposed, respectively. For details, please see Chapter 3.
2. For the distributed tracking control of dynamic system, the problem is solved in two steps. In the first step, we solve the distributed tracking problem of multiple kinematic systems. In the second step, we solve the distributed tracking control problem of dynamic systems with the aid of the results in the first step.
 - For tracking control of kinematic systems, we have obtained the following results.
 - (a) Distributed tracking control of multiple nonholonomic chained systems is considered by using neighbours’ information. With the aid of the cascade structure of each system and properties of persistently excited signals, distributed state feedback tracking controllers and distributed output feedback tracking controllers are proposed such that the tracking errors exponentially converge to zero. To show applications of the proposed results, formation control of wheeled mobile robots is considered. Distributed controllers are obtained with the aid of the proposed theorems. See Chapter 4 for details.

- (b) The leader-following consensus problem of multiple chained systems with directed communication topology is considered. If the state of each system is measurable, distributed state feedback controllers are proposed using neighbors' state information with the aid of Lyapunov techniques and properties of Laplacian matrix for time-invariant communication graph and time-varying communication graph. It is shown that the states of the systems reach consensus exponentially. If the state of each system is not measurable, distributed observer-based output feedback control laws are proposed. As an application of the proposed results, formation control of wheeled mobile robots is studied. The simulation results show the effectiveness of the proposed results. See Chapter 5 for details.
 - (c) The distributed formation control of multiple nonholonomic wheeled mobile robots with a leader is considered. Distributed tracking control laws are proposed with the aid of results of cascade systems such that the centroid of the states of a group of mobile robot exponentially tracks the leader. See Chapter 6 for details.
- For tracking control of multiple dynamic systems, we have obtained the following results.
 - (a) The leader-following control problem of multiple mechanical systems with/without velocity constraints using neighbors' information is considered with the aid of neural networks. With the aid of the approximation property of neural networks, the cascade structure of each system, and the properties of linear time-varying systems, based distributed robust adaptive state feedback controllers are proposed such that the state of each follower system asymptotically converges to the state of a leader system. To verify the proposed results, formation control of wheeled mobile robots and synchronization of multiple 2-DOF manipulators are considered. Distributed controllers are obtained with the aid of the proposed results. Simulation results show the effectiveness of the proposed results. See Chapter 7 for details.
 - (b) Formation tracking control of multiple wheeled mobile robots is studied. The reference trajectory is considered as a virtual leader vehicle system while the real multiple vehicle systems are considered as follower agents. Chained-form systems and theories of cascaded systems and communication graph are introduced to design control methods for kinematic systems. In addition to distributed control algorithms for kinematic multi-vehicle systems, formation control of vehicles dynamics is addressed with the aid of backstepping method, parametrical uncertainties of vehicles mechanics are estimated by sliding mode control. See Chapter 8 for details.
 - (c) Distributed tracking control is considered for multiple wheeled mobile robots. Laplacian matrix is introduced to characterize the communication topology. Since there are parameter uncertainties for each mechanical system, adaptive control method is applied for controller design of the dynamical systems. Distributed adaptive state feedback control laws are presented with the aid of the agents neighboring information. See Chapter 9 for details.

- (d) It is considered the teleoperation of a cluster of mobile robots that are required to follow a desired trajectory while maintaining a desired rigid formation pattern. The centroid of the robot formation is modeled as a virtual robot. Distributed control laws designed using a backstepping method are proposed for each robot with the aid of neighbors information. It is shown that the motion of the centroid of the cluster of robots is synchronized to that of the virtual robot. The theoretical results are validated by simulations on a cluster of five mobile robots. See Chapter 10 for details.
3. For the consensus problem of dynamic systems with non-ideal nonholonomic constraints, a transverse function based approach was proposed. In this approach, two problems were solved step by step. We first propose distributed controllers for kinematic systems with the aid of graph theory. Then, we propose distributed controllers for multiple dynamic systems. If the uncertainty in the dynamics is parametric, distributed adaptive controllers are proposed. If the uncertainty in the dynamics is non-parametric, distributed robust controllers are proposed. For details, please see Chapter 11,
 4. For the distributed tracking control of dynamic systems with non-ideal nonholonomic constraints, we considered distributed tracking control of kinematic systems and distributed tracking control of dynamic systems. For these two problems, practical distributed tracking controllers are proposed with the aid of robust control and graph theory. The method proposed for the problems can be applied to solve distributed tracking control of multiple systems with more general forms. For detail, please see Chapter 12.
 5. In addition solving the problems proposed in this proposal, we also solve the cooperative of other related problems. The following problems have been studied and the solutions for them have been obtained.
 - (a) Consensus Seeking of Nonlinear Systems: Two consensus problems of multiple nonlinear systems are considered. In the first consensus problem, distributed control laws for multiple nonlinear systems are proposed such that the state of each system converges to a constant agreement vector with the aid of communications between systems. In the second consensus problem, distributed robust/adaptive control laws for multiple nonlinear systems are proposed such that the state of each system converges to the state of a reference system whose state is available to a portion of multiple systems.
 - (b) Distributed Estimated-Based Tracking Control of Multiple Uncertain Nonlinear Systems: It is considered the tracking control of multiple uncertain nonlinear systems with a desired signal which is not available to each system. An estimation-based controller design approach is proposed. Distributed estimation-based adaptive controllers are proposed with the aid of Lyapunov techniques and results from graph theory. Simulation results show the effectiveness of the proposed controllers.

- (c) Distributed Output Tracking Control of Heterogeneous linear Agents: Distributed output tracking control of a leader is considered. If the state of each system is measurable, distributed state feedback controllers are proposed such that the output of each system asymptotically converges to the output of the leader with the aid of the internal model principle. If the state of each system is not measurable, distributed output feedback controllers are proposed with the aid of state estimation such that the output of each system asymptotically converges to the output of the leader. Simulation study validates the proposed results.
 - (d) Consensus of uncertain nonlinear systems: For multiple nonlinear systems the consensus control problem is considered. Consensus algorithms are proposed with the aid of Lyapunov techniques and results from graph theory. To show the effectiveness of the proposed algorithms, simulation results are presented.
 - (e) Consensus seeking of heterogeneous nonlinear agents: It is considered the consensus seeking problem of multiple heterogeneous nonlinear systems with a leader. With the aid of distributed estimation, distributed state feedback laws are proposed such that the output of each system converges to the output of the leader system. Simulation results show effectiveness of the proposed control laws.
 - (f) Robust/Adaptive Tracking Control of Wave-Adaptive Modular Vessel with Uncertainty: It is considered the tracking control of a wave adaptive modular vessel (WAM-V) with unknown inertia parameters and disturbance. To overcome the underactuated nature, practical control laws are proposed. If the inertia parameters are not exactly known, a robust control law is proposed such that the tracking error of the position of the WAM-V exponentially converges to a small neighborhood of the origin. If the inertia parameters are unknown, a robust adaptive control law is proposed such that the tracking error of the position of the WAM-V asymptotically converges to a small neighborhood of the origin. In both cases, the radius of the neighborhood can be as small as possible by choosing a small control parameter. Simulation has been done to show the effectiveness of the proposed control laws.
 - (g) Robust/Adaptive Tracking Control of Wave-Adaptive Modular Vessel with Unknown Inertia Parameters: This paper considers formation control of multiple wave-adaptive-modular vessels (WAM-Vs) with the aid of neighbors' information. Considering the water currents, it is assumed that the dynamics of each WAM-V is not exactly known and is subjected to disturbance. To overcome the underactuated nature of a WAM-V, practical distributed robust tracking controllers are proposed if estimates of inertia parameters are known. If the inertia parameters are unknown, practical distributed adaptive tracking controllers are proposed. Simulation results show the effectiveness of the proposed controllers.
6. In order to test our proposed results, a testbed of cooperative control of four wheeled robots has been built. In this testbed, there are four P3-AT mobile robots. Each robot is equipped with a camera and a laser sensor. Between robots, there is wireless communication with the aid of university wireless infrastructure or an access point. The proposed algorithms can be applied to make the robots move in a desired formation.

7. Implement of control algorithms in the testbed. Some algorithms have been implemented in the testbed in our lab. The effectiveness of the proposed algorithms have been verified.

2.2 Achievements in Education

With the aid of the support of this funds, five graduate students have been financially supported. The students have been involved in this research and received training in their research. Some students have presented their research results in well-known professional automatic control meetings and published their research results in peer-reviewed conference proceedings.

1. Chunyu Chen, Yifan Xing, Vladimir Djapic, and Wenjie Dong, “Distributed Formation Tracking Control of Multiple Mobile Robotic Systems,” Proc. of IEEE Decision and Control Conference, December, 2014.
2. C. Chen F. De La Torre, and W. Dong, “Distributed Exponentially Tracking Control of Multiple Wheeled Mobile Robots,” Proc. of American Control Conference, 2014, pp. 4014-4019.
3. Felipe De La Torre and W. Dong, “Distributed exponential formation control of multiple wheeled mobile robots,” Proc. of Int. Conf. on control, Dynamic Systems, and Robotics, Ottawa, Canada, May, 2014.
4. Z. Sheng, W. Dong, and Jay Farrell, “Quaternion-Based Trajectory Tracking Control of VTOL-UAVs Using Command Filtered Backstepping,” Proc. of American control Conference, pp. 1020-1025, 2013.

Two graduate students have graduated and two master thesis have been produced.

1. Chunyu Chen, Distributed formation tracking control of multiple car-like robots. Master thesis, Department of Electrical Engineering, the University of Texas - Pan American, May, 2014.
2. Yifan Xing, Distributed coordinate tracking control of multiple wheeled mobile robots. Master thesis, Department of Electrical Engineering, the University of Texas - Pan American, March, 2015.

2.3 Publications

For this project, four peer-reviewed journal papers and seven peer-reviewed conference papers have been published.

Peer-reviewed journal papers:

1. W. Dong and V. Djapic, "Leader-following control of multiple nonholonomic systems with over directed communication graphs," *Int. J. of Systems Science*, accepted for publication, 2014
2. W. Dong, C. Chen, and Y. Xing, "Distributed estimation-based tracking control of multiple uncertain non-linear systems", *Int. J. of Systems Science*, vol. 45, no. 10, pp. 2088-2099, 2014.
3. W. Dong, "Distributed Tracking Control of Networked Chained Systems," *Int. J. of Control*, vol. 86, no. 12, pp. 2159-2174, 2013.
4. W. Dong, "Adaptive Consensus Seeking of Multiple Nonlinear Systems," *Int. J. of Adaptive Control and Signal Processing*, vol. 26, no. 5, pp. 419-434, 2012.

Peer-reviewed conference papers:

1. Chunyu Chen, Yifan Xing, Vladimir Djapic, and Wenjie Dong, "Distributed Formation Tracking Control of Multiple Mobile Robotic Systems," *Proc. of IEEE Decision and Control Conference*, December, 2014.
2. W. Dong, F. De La Torre, and Y. Xing, "Distributed Output Tracking Control of Heterogeneous linear Agents," *Proc. of American Control Conference*, 2014, pp. 4677-4682.
3. C. Chen F. De La Torre, and W. Dong, "Distributed Exponentially Tracking Control of Multiple Wheeled Mobile Robots," *Proc. of American Control Conference*, 2014, pp. 4014-4019.
4. Felipe De La Torre and W. Dong , "Distributed exponential formation control of multiple wheeled mobile robots," *Proc. of Int. Conf. on control, Dynamic Systems, and Robotics*, Ottawa, Canada, May, 2014.
5. Z. Sheng, W. Dong, and Jay Farrell, "Quaternion-Based Trajectory Tracking Control of VTOL-UAVs Using Command Filtered Backstepping," *Proc. of American control Conference*, pp. 1020-1025, 2013.
6. W. Dong, M. Ben Ghalia, Y. Xing, and C. Chen, "Teleoperation of a Cluster of Mobile Robots Subject to Model Uncertainty," *Proc. of American control Conference*, pp. 6451-6456, 2013.
7. W. Dong, "On consensus of multiple nonlinear systems," *IEEE 7th Sensor Array, and Multichannel Signal Processing Workshop*, pp. 385-388, 2012.

In the peer-reviewed conference papers, four of them were first-authored by our graduate students. There are also several papers are under preparation for publication.

Chapter 3

Consensus of Multiple Nonholonomic Mechanical Systems

In this chapter, we consider the consensus problem of multiple nonholonomic mechanical systems. In order to solve the problem defined in the proposal, we solve several consensus problems step by step.

3.1 Distributed Controller Design for Chained Systems

Consider m systems in (1.5), i.e.,

$$\dot{x}_{1j} = v_{1j} \quad (3.1)$$

$$\dot{x}_{2j} = v_{2j} \quad (3.2)$$

$$\dot{x}_{ij} = v_{1j}x_{i-1,j}, \quad 3 \leq i \leq n \quad (3.3)$$

The communication between systems is defined by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. We consider the following problem in this section.

Consensus of Multiple Chained Systems: For a group of m systems in (3.1)-(3.3), the problem is how to design a distributed control law (v_{1j}, v_{2j}) for system j based on its own information and its neighbors' information such that

$$\lim_{t \rightarrow \infty} (x_{*j} - c) = 0, \quad 1 \leq j \leq m, \quad (3.4)$$

where $x_{*j} = [x_{1j}, \dots, x_{nj}]^\top$ and c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

The system (3.1)-(3.3) has a cascade structure. (3.1) is a linear system and (3.2)-(3.3) is a linear time-varying system. We propose the following results for the system in (3.1).

Lemma 3.1. *For m systems in (3.1), the distributed control law*

$$v_{1j} = \eta_{1j} \quad (3.5)$$

$$\eta_{1j} = -k_1 x_{1j} + \zeta_{1j} \quad (3.6)$$

$$\dot{\zeta}_{1j} = -k_2\zeta_{1j} + \zeta_{2j} \quad (3.7)$$

$\vdots$

$$\dot{\zeta}_{n-3,j} = -k_{n-2}\zeta_{n-3,j} + \zeta_{n-2,j} \quad (3.8)$$

$$\dot{\zeta}_{n-2,j} = -\sum_{i \in \mathcal{N}_j} a_{ji}(\zeta_{n-2,j} - \zeta_{n-2,i}) - b(\zeta_{n-2,j} - \alpha) + \dot{\alpha} \quad (3.9)$$

ensures that

$$\lim_{t \rightarrow \infty} (x_{1j} - x_{1l}) \stackrel{exp.}{=} 0, \quad 1 \leq j \neq l \leq m \quad (3.10)$$

where a_{ji} are positive constants, $b > 0$, $k_i > 0$ for $1 \leq i \leq n-2$, and α is a variable. Furthermore, if α is a bounded persistently excited signal and $\dot{\alpha}$ is also bounded, then v_{1j} is a bounded PE signal and $\lim_{t \rightarrow \infty} (v_{1j} - v_{1l}) \stackrel{exp.}{=} 0$ for $1 \leq j \neq l \leq m$.

Proof: Let $\tilde{\zeta}_{n-2,j} = \zeta_{n-2,j} - \alpha$ and $\tilde{\zeta}_{n-2,*} = [\tilde{\zeta}_{n-2,1}, \dots, \tilde{\zeta}_{n-2,m}]^\top$, with the control laws we have

$$\dot{\tilde{\zeta}}_{n-2,*} = -\mathcal{L}\tilde{\zeta}_{n-2,*} - b\tilde{\zeta}_{n-2,*} = -(\mathcal{L} + bI)\tilde{\zeta}_{n-2,*}. \quad (3.11)$$

where $\mathcal{L}$ is the Laplacian matrix. Since $b > 0$, $(\mathcal{L} + bI)$ is a Hurwitz matrix. Therefore, $\tilde{\zeta}_{n-2,*}$ exponentially converges to zero, which means that $\zeta_{n-2,j}$ exponentially converges to α .

Let $\tilde{x}_{1,jl} = x_{1j} - x_{1l}$ and $\tilde{\zeta}_{i,jl} = \zeta_{ij} - \zeta_{il}$ for $1 \leq i \leq n-3$ and $1 \leq j \neq l \leq m$, then

$$\begin{aligned} \dot{\tilde{x}}_{1,jl} &= -k_1\tilde{x}_{1,jl} + \tilde{\zeta}_{1,jl} \\ \dot{\tilde{\zeta}}_{1,jl} &= -k_2\tilde{\zeta}_{1,jl} + \tilde{\zeta}_{2,jl} \\ &\vdots \\ \dot{\tilde{\zeta}}_{n-3,jl} &= -k_{n-2}\tilde{\zeta}_{n-3,jl} + \tilde{\zeta}_{n-2,jl} \end{aligned}$$

Since k_{n-2} is positive and $\tilde{\zeta}_{n-2,jl}$ exponentially converges to zero, it is obvious that $\tilde{\zeta}_{n-3,jl}$ exponentially converges to zero. Similarly, it can be proved that $\tilde{\zeta}_{n-3,jl}$ exponentially converges to zero. Repeat this procedure, it can be proved that $\tilde{x}_{1,jl}$ exponentially converges to zero.

Since $\lim_{t \rightarrow \infty} (x_{1j} - x_{1l}) \stackrel{exp.}{=} 0$ and $\lim_{t \rightarrow \infty} (\zeta_{1j} - \zeta_{1l}) \stackrel{exp.}{=} 0$, $\lim_{t \rightarrow \infty} (v_{1j} - v_{1l}) \stackrel{exp.}{=} 0$. By (3.6)-(3.9), $x_{1j} = H(s)\zeta_{n-2,j}$ where $H(s) = \frac{1}{\prod_{r=1}^{n-3}(s+k_r)}$ and $H(s)$ is a stable, minimum phase, proper rational transfer function. So, $v_{1j} = sH(s)\zeta_{n-2,j}$. It is obvious that $sH(s)$ is also a stable, minimum phase, proper rational transfer function. By Lemma 4.8.3 in [5], v_{1j} is a bounded PE signal. ■

Remark 3.1. In Lemma 3.1, α is a control parameter. The reason for introducing it will be clear later.

Remark 3.2. In Lemma 3.1, b is a positive constant. Actually, b can be zero for some systems.

With the aid of the results in Lemma 3.1, we are ready to design controllers for the systems in (3.2)-(3.3). We define the variables

$$z_{ij} = x_{ij} + \beta_{ij}, \quad 2 \leq i \leq n \quad (3.12)$$

where

$$\begin{aligned} \beta_{nj} &= 0 \\ \beta_{n-1,j} &= v_{1j}^{2n-5} z_{nj} \\ \beta_{lj} &= v_{1j}^{2n-5} z_{l+1,j} + \frac{1}{v_{1j}} \dot{\beta}_{l+1,j}, \quad l = n-2, n-3, \dots, 2 \end{aligned}$$

then we have

$$\dot{z}_{2j} = v_{2j} + \dot{\beta}_{2j} \quad (3.13)$$

$$\dot{z}_{ij} = -v_{1j}^{2n-4} z_{ij} + v_{1j} z_{i-1,j}, \quad 3 \leq i \leq n \quad (3.14)$$

It should be noted that the transform defined in (3.4) is global since β_{ij} is well-defined because its special form. The following results can be proved.

Lemma 3.2. *By the transform (3.4), if*

$$\lim_{t \rightarrow \infty} (z_{ij} - z_{il}) = 0, \quad 2 \leq i \leq n; 1 \leq j \neq l \leq m \quad (3.15)$$

then

$$\lim_{t \rightarrow \infty} (x_{ij} - x_{il}) = 0, \quad 2 \leq i \leq n; 1 \leq j \neq l \leq m. \quad (3.16)$$

The system (3.13)-(3.14) has a special structure. We will take advantage of this structure and have the following results.

Lemma 3.3. *If v_{1j} is a bounded PE signal, z_{2j} and z_{2l} are bounded, and*

$$\lim_{t \rightarrow \infty} (z_{2j} - z_{2l}) = 0, \quad 1 \leq j \neq l \leq m \quad (3.17)$$

then (3.15) holds.

Proof: For systems j and l and $j \neq l$, we define $e = z_{3j} - z_{3l}$. Then

$$\dot{e} = -v_{1j}^{2n-4} e + (v_{1l}^{2n-4} - v_{1j}^{2n-4}) z_{2l} + (v_{1j} z_{2j} - v_{2l} z_{2l}) \quad (3.18)$$

Since v_{1j} is a bounded PE signal and $(z_{2j} - z_{2l})$ converges to zero, it is obvious that e converges to zero, which means that z_{3j} converges to z_{3l} . Similarly, we can prove that z_{ij} converges to z_{il} for $i = 4, 5, \dots, n$. ■

Thanks to Lemma 3.3 and Lemma 3.2, it only needs to design v_{2j} such that (3.17) holds. We have the following results.

Lemma 3.4. *If the communication digraph has a spanning tree, the control law*

$$v_{2j} = \eta_{2j} \quad (3.19)$$

$$\eta_{2j} = - \sum_{i \in \mathcal{N}_j} a_{ji}(z_{2j} - z_{2i}) - \dot{\beta}_{2j} \quad (3.20)$$

ensures that (3.16) holds.

Combine the results in Lemma 3.1 and Lemma 3.4, we have the following results.

Theorem 3.1. *For the m systems in (3.1)-(3.2), if the communication digraph has a spanning tree and α is a bounded PE signal and $\dot{\alpha}$ is bounded, the control laws (3.5) and (3.20) for system j ensure that (3.4) holds, where the control parameters are defined in Lemmas 3.1 and 3.4.*

In Theorem 3.1, $\dot{\alpha}$ is required to be a bounded PE signal. There are many choices of $\dot{\alpha}$. For example, we can choose $\alpha(t) = \sin t$.

There are other methods for designing distributed controllers for the kinematic systems in (3.1)-(3.3). For example, the method proposed in our paper [6] can be applied to propose distributed controllers.

3.2 Distributed Controller Design for Driftless Systems

In this section, we considered the following driftless systems

$$\dot{q}_{*j} = g_{1j}v_{1j} \cdots + g_{sj}v_{sj} = g_{*j}v_{*j} \quad (3.21)$$

where $g_{1j}, \dots, g_{sj}$ are smooth functions on R^n such that in a neighborhood of 0 the dimension of the distribution $\Delta_j(q_{*j}) = \text{Span}\{g(q_{*j}) : g \in \text{Lie}\{g_{1j}, \dots, g_{sj}\}\}$ is n .

The problem considered in this section is as follows.

Consensus of Multiple Systems: For a group of m systems in (3.21), the problem is how to design a distributed control law u_{*j} for system j based on its own information and its neighbors' information such that

$$\lim_{t \rightarrow \infty} (q_{*j} - c) = 0, \quad 1 \leq j \leq m, \quad (3.22)$$

where c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

Practical distributed controllers can be proposed with the aid of the transverse function. For simplicity, we assume the communication between systems is bi-directional in this section.

With the aid of the results in [7], we have the following results.

Lemma 3.5. *For the system in (3.21), if the dimension of the distribution $\text{Span}\{g(x) : g \in \text{Lie}\{g_{1j}, \dots, g_{sj}\}\}$ is n , there exists a function $f_j(\beta_{*j}, \epsilon_j) \in R^n$ such that the matrix*

$$H_j(\beta_{*j}) = \left[g_{1j}(f_j), g_{2j}(f_j), \dots, g_{sj}, \frac{\partial f_j}{\partial \beta_{1j}}, \dots, \frac{\partial f_j}{\partial \beta_{n-s,j}} \right]$$

is nonsingular for any β_{*j} and $\epsilon_j > 0$, where $\beta_{*j} = [\beta_{1j}, \dots, \beta_{n-2,j}]^\top$, $\beta \in R^{n-2}$ and the function f_j has the following properties:

1. f_j is bounded for any β_{*j} ;

2. $\lim_{\epsilon_j \rightarrow 0} f_j(\beta_{*j}, \epsilon_j) = 0$.

The proof of Lemma 3.5 can be found in [7, 8]. The construction of f_j can be found also in [7, 8]. The function f_j is called the transverse function.

With the aid of Lemma 3.5, it can be found the function $f_j(\beta_j, \epsilon_j)$ such that G_j is nonsingular. Define

$$z_{*j} = q_{*j} f_j(\beta_{*j}, \epsilon_j)^{-1} \quad (3.23)$$

then, we have

$$\dot{z}_{*j} = dr_{f_j(\beta_{*j})}(q_{*j}) dl_{z_{*j}}(f_j(\beta_{*j})) H_j(\beta_{*j}) \begin{bmatrix} v_{*j} \\ -\dot{\beta}_{*j} \end{bmatrix} \quad (3.24)$$

where the meaning of the notations can be found in [7].

We define the neighbors's difference as

$$e_{*j} = [e_{1j}, \dots, e_{nj}] = \sum_{i \in \mathcal{N}_j} a_{ji} (z_{*j} - z_{*i}). \quad (3.25)$$

If $\dot{\beta}_{*j}$ is considered as an additional input, we propose the following distributed control law.

Theorem 3.2. *For the m systems in (3.21), if the communication graph has a spanning tree, the control law*

$$v_{1j} = \eta_{1j} \quad (3.26)$$

$$\vdots$$

$$v_{sj} = \eta_{sj} \quad (3.27)$$

$$\begin{bmatrix} \eta_{1j} \\ \vdots \\ \eta_{sj} \\ -\dot{\beta}_{*j} \end{bmatrix} = -H_j(\beta_{*j})^{-1} dl_{z_{*j}}^{-1}(q_{*j}) dr_{f_j(\beta_{*j})}(z_{*j}) \left[\sum_{i \in \mathcal{N}_j} a_{ji} (z_{*j} - z_{*i}) \right] \quad (3.28)$$

ensures that

$$\lim_{t \rightarrow \infty} \|q_{*j} - q_{*i}\| \leq \delta(\epsilon, \epsilon_i). \quad (3.29)$$

where $\delta(\epsilon, \epsilon_i)$ is a nonnegative continuous function of ϵ_i and ϵ_j and $\lim_{|\epsilon_j| + |\epsilon_i| \rightarrow 0} \delta(\epsilon, \epsilon_i) = 0$.

Proof: By Lemma 3.5, G_j is nonsingular. So, the control law exists. Substitute the control law into the system, we have

$$\dot{z}_{1j} = -e_{1j} \quad (3.30)$$

$$\vdots \quad (3.31)$$

$$\dot{z}_{nj} = -e_{nj} \quad (3.32)$$

Choose a function

$$V_i = z_{i*}^\top \mathcal{L} z_{i*}$$

where $z_{i*} = [z_{i1}, \dots, z_{in}]^\top$ and $\mathcal{L}$ is the Laplacian matrix, we have

$$\dot{V}_i = -z_{i*}^\top \mathcal{L}^2 z_{i*}$$

By integrating both sides of the above inequality, it can be shown that z_{i*} is bounded and $\mathcal{L} z_{i*}$ converges to zero, which means that $(z_{ij} - z_{il})$ converges to zero for $1 \leq j \neq l \leq m$. Therefore, (3.29) holds. $\blacksquare$

Remark 3.3. If ϵ_j is chosen to be a small constant, $\|f_j - f_i\|$ is a small constant, which means that $\|x_{*j} - x_{*i}\|$ converges to a small neighborhood of the origin. We say (3.22) is achieved practically.

Remark 3.4. In Theorem 3.2, nothing is said about β_{*j} . So, β_{*j} may be bounded or unbounded. Thanks to the properties of the function f_j , the boundedness of β_{*j} plays no role in the consensus problem.

3.3 Distributed Controller Design for Dynamical Systems

We consider m systems. The j -th system is defined as

$$\dot{x}_{1j} = v_{1j}, \quad \dot{x}_{2j} = v_{2j}, \quad \dot{x}_{ij} = v_{1j} x_{i-1,j}, \quad 3 \leq i \leq n \quad (3.33)$$

$$\tilde{M}_j \dot{v}_{*j} + \tilde{C}_j v_{*j} + \tilde{G}_j + \tilde{D}_j = \tilde{B}_j \tau_j \quad (3.34)$$

where $v_{*j} = [v_{1j}, v_{2j}]^\top$ and $x_{*j} = [x_{1j}, \dots, x_{nj}]^\top$. The following properties are satisfied.

Property 3.1. $\tilde{M}_j$ is bounded and $\tilde{M}_j - 2\tilde{C}_j$ is skew-symmetric.

Property 3.2. For any differentiable vector $\xi \in R^2$,

$$\tilde{M}_j \dot{\xi} + \tilde{C}_j \xi + \tilde{G}_j = \tilde{Y}(x_{*j}, \dot{x}_{*j}, \xi, \dot{\xi}) a_j$$

where $\tilde{Y}_j$ is a known function of x_{*j} , $\dot{x}_{*j}$, ξ , and $\dot{\xi}$, and a_j is the inertia parameter vector.

The problem considered in this section is defined as follows.

Consensus of Multiple Systems: For a group of m systems in (3.33)-(3.34), the problem is how to design a distributed control law τ_j for system j based on its own information and its neighbors' information such that

$$\lim_{t \rightarrow \infty} (x_{*j} - c) = 0, \quad 1 \leq j \leq m, \quad (3.35)$$

where $x_{*j} = [x_{1j}, \dots, x_{nj}]^\top$ and c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

In the dynamics (3.34), we first assume that the inertia parameter vector a_j is a constant and is unknown.

In order to design controllers for the systems in (3.33)-(3.34), we let

$$\tilde{v}_{1j} = v_{1j} - \eta_{1j} \quad (3.36)$$

$$\tilde{v}_{2j} = v_{2j} - \eta_{2j}, \quad 1 \leq j \leq m \quad (3.37)$$

where η_{1j} and η_{2j} are defined in (3.5) and (3.20), respectively. Then we have

$$\dot{x}_{1j} = \eta_{1j} + \tilde{v}_{1j} \quad (3.38)$$

$$\dot{x}_{2j} = \eta_{2j} + \tilde{v}_{2j} \quad (3.39)$$

$$\dot{x}_{ij} = (\eta_{1j} + \tilde{v}_{1j})x_{i-1,j}, \quad 3 \leq i \leq n \quad (3.40)$$

$$\tilde{M}_j \dot{\tilde{v}}_{*j} + \tilde{C}_j \tilde{v}_{*j} = \tilde{B}_j \tau_j - (\tilde{M}_j \dot{\eta}_{*j} + \tilde{C}_j \eta_{*j} + \tilde{G}_j + \tilde{D}_j) \quad (3.41)$$

where $v_{*j} = [v_{1j}, v_{2j}]^\top$.

For the systems in (3.38)-(3.40), we have the following results.

Lemma 3.6. *For the m systems in (3.38)-(3.40), if the communication digraph has a spanning tree, then*

1. $x_{ij} - x_{il}$ is bounded for $1 \leq i \leq n$ and $1 \leq j \neq l \leq m$ if $\tilde{v}_{1j}$ and $\tilde{v}_{2j}$ are bounded for $1 \leq j \leq m$.
2. $x_{ij} - x_{il}$ converges to zero for $1 \leq i \leq n$ and $1 \leq j \neq l \leq m$ if $\tilde{v}_{1j}$ and $\tilde{v}_{2j}$ converge to zero for $1 \leq j \leq m$.

Proof: Let $\tilde{x}_{1,jl} = x_{1j} - x_{1l}$, we have

$$\dot{\tilde{x}}_{1,jl} = -k_1 \tilde{x}_{1,jl} + \tilde{\zeta}_{1,jl} + \tilde{v}_{1j} - \tilde{v}_{1l} \quad (3.42)$$

Since $\tilde{\zeta}_{1,jl}$ exponentially converges to zero, the system (3.42) has the input-to-state stability property for input $(\tilde{v}_{1j} - \tilde{v}_{1l})$. This means that $\tilde{x}_{1,jl}$ is bounded if $\tilde{v}_{1j}$ and $\tilde{v}_{1l}$ are bounded and $\tilde{x}_{1,jl}$ converges to zero if $\tilde{v}_{1j}$ and $\tilde{v}_{1l}$ converge to zero.

For the systems in (3.39)-(3.40), with the aid of the transform in (3.4), we have

$$\dot{z}_{2j} = \eta_{2j} + \dot{\beta}_{2j} + \tilde{v}_{2j} \quad (3.43)$$

$$\dot{z}_{ij} = -v_{1j}^{2n-4} z_{ij} + v_{1j} z_{i-1,j}, \quad 3 \leq i \leq n \quad (3.44)$$

It can be proved recursively that z_{ij} is bounded if $z_{i-1,j}$ is bounded and z_{ij} converges to zero if $z_{i-1,j}$ converges to zero. Therefore, $x_{ij} - x_{il}$ converges to zero for $2 \leq i \leq n$ and $1 \leq j \neq l \leq m$ if $\tilde{v}_{1j}$ and $\tilde{v}_{2j}$ converge to zero for $1 \leq j \leq m$. $\blacksquare$

With the aid of the results in Lemma 3.6, we can design controllers such that $\tilde{v}_{1*}$ and $\tilde{v}_{2*}$ are bounded and converge to zero.

For the dynamics of each system, we have

$$\tilde{M}_j \dot{\eta}_{*j} + \tilde{C}_j \eta_{*j} + \tilde{G}_j = \tilde{Y}_j(x_{*j}, \dot{x}_{*j}, \eta_{*j}, \dot{\eta}_{*j})a_j \quad (3.45)$$

where a_j is the inertia parameter. For the disturbance $\tilde{D}_j$, it is assumed that

$$\|\tilde{D}_j\| \leq \rho_j(x_{*j}) \quad (3.46)$$

where ρ_j is a known function of x_{*j} . If the inertia parameter a_j is a constant and is unknown, we have the following results.

Theorem 3.3. *For the m systems in (3.33)-(3.34), if the communication digraph has a spanning tree, the control law*

$$\tau_j = \tilde{B}_j^{-1} \left[-K_j \tilde{v}_{*j} + \tilde{Y}_j \hat{a}_j - \rho_j \text{sign}(\tilde{v}_{*j}) \right] \quad (3.47)$$

$$\dot{\hat{a}}_j = -\Gamma_j \tilde{Y}_j^\top \tilde{v}_{*j} \quad (3.48)$$

ensures that (3.35) holds and $\hat{a}_j$ is bounded, where K_j and Γ_j are positive constant matrices.

Proof: Let

$$V_j = \frac{1}{2} \tilde{v}_{*j}^\top \tilde{M}_j \tilde{v}_{*j} + \frac{1}{2} (\hat{a}_j - a_j) \Gamma_j^{-1} (\hat{a}_j - a_j)$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned} \dot{V}_j &= -\tilde{v}_{*j}^\top K_j \tilde{v}_{*j} - \rho_j \tilde{v}_{*j}^\top \text{sign}(\tilde{v}_{*j}) - \tilde{v}_{*j}^\top \tilde{D}_j \\ &\leq -\tilde{v}_{*j}^\top K_j \tilde{v}_{*j} \leq 0 \end{aligned}$$

Therefore, V_j is bounded, which means that $\tilde{v}_{*j}$ and $\hat{a}_j$ are bounded. By Barbalat's lemma, it can be shown that $\tilde{v}_{*j}$ converges to zero. $\blacksquare$

In Theorem 3.3, the unknown inertia parameter is estimated by an adaptive control law. If an estimate of a_j is $\bar{a}_j$ and

$$\|a_j - \bar{a}_j\| \leq \gamma_j$$

for $1 \leq j \leq m$ and γ_j is a known constant, we propose the following robust control laws.

Theorem 3.4. *For the m systems in (3.33)-(3.34), if the communication digraph has a spanning tree, the control law*

$$\tau_j = \tilde{B}_j^{-1} \left[-K_j \tilde{v}_{*j} + \tilde{Y}_j \bar{a}_j - \gamma_j \tilde{Y}_j \text{sign}(\tilde{Y}_j^\top \tilde{v}_{*j}) - \rho_j \text{sign}(\tilde{v}_{*j}) \right] \quad (3.49)$$

ensures that (3.35) holds, where K_j is a positive constant matrix.

Proof: Let

$$V_j = \frac{1}{2} \tilde{v}_{*j}^\top \tilde{v}_{*j}$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned} \dot{V}_j &= -\tilde{v}_{*j}^\top K_j \tilde{v}_{*j} - \rho_j \tilde{v}_{*j}^\top \text{sign}(\tilde{v}_{*j}) - \tilde{v}_{*j}^\top \tilde{D}_j \\ &\quad + \tilde{v}_{*j}^\top \tilde{Y}_j (\bar{a}_j - a_j) - \gamma_j \tilde{v}_{*j}^\top \tilde{Y}_j \text{sign}(\tilde{Y}_j^\top \tilde{v}_{*j}) \\ &\leq -\tilde{v}_{*j}^\top K_j \tilde{v}_{*j} \leq 0 \end{aligned}$$

Therefore, V_j is bounded, which means that $\tilde{v}_{*j}$ is bounded. By Barbalat's lemma, it can be shown that $\tilde{v}_{*j}$ converges to zero. $\blacksquare$

In Theorem 3.4, the unknown inertia parameter a_j is not required to be a constant. In the control laws, γ_j is required to be known. It is possible to estimate it.

3.4 Distributed Controller Design for General Dynamical Systems

Consider m systems where the j -th system is defined by

$$\dot{q}_{*j} = g_{1j}v_{1j} \cdots + g_{sj}v_{sj} = g_{*j}v_{*j} \quad (3.50)$$

$$\tilde{M}_j \dot{v}_{*j} + \tilde{C}_j v_{*j} + \tilde{G}_j + \tilde{D}_j = \tilde{B}_j \tau_j \quad (3.51)$$

where $v_{*j} = [v_{1j}, v_{2j}]^\top$, $g_{1j}, \dots, g_{sj}$ are smooth functions on R^n such that in a neighborhood of 0 the dimension of the distribution $\Delta_j(q_{*j}) = \text{Span}\{g(q_{*j}) : g \in \text{Lie}\{g_{1j}, \dots, g_{sj}\}\}$ is n .

The following properties are satisfied.

Property 3.3. $\tilde{M}_j$ is bounded and $\dot{\tilde{M}}_j - 2\tilde{C}_j$ is skew-symmetric.

Property 3.4. For any differentiable vector $\xi \in R^2$,

$$\tilde{M}_j \dot{\xi} + \tilde{C}_j \xi + \tilde{G}_j = \tilde{Y}(x_{*j}, \dot{x}_{*j}, \xi, \dot{\xi}) a_j$$

where $\tilde{Y}_j$ is a known function of x_{*j} , $\dot{x}_{*j}$, ξ , and $\dot{\xi}$, and a_j is the inertia parameter vector.

The problem considered in this section is defined as follows.

Consensus of Multiple Systems: For a group of m systems in (3.50)-(3.51), the problem is how to design a distributed control law τ_j for system j based on its own information and its neighbors' information such that

$$\lim_{t \rightarrow \infty} (q_{*j} - c) = 0, \quad 1 \leq j \leq m, \quad (3.52)$$

where c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

In the dynamics (3.51), we first assume that the inertia parameter vector a_j is a constant and is unknown.

In order to design controllers, we let

$$\tilde{v}_{ij} = v_{ij} - \eta_{ij}, \quad 1 \leq j \leq s \quad (3.53)$$

where η_{ij} is defined in (3.26)-(3.27), respectively. Then we have

$$\dot{z}_{*j} = -e_{*j} + g_{1j}\tilde{v}_{1j} + \cdots + g_{sj}\tilde{v}_{sj} \quad (3.54)$$

$$\tilde{M}_j \dot{\tilde{v}}_{*j} + \tilde{C}_j \tilde{v}_{*j} = \tilde{B}_j \tau_j - (\tilde{M}_j \dot{\eta}_{*j} + \tilde{C}_j \eta_{*j} + \tilde{G}_j + \tilde{D}_j) \quad (3.55)$$

For the dynamics of each system, we have

$$\tilde{M}_j \dot{\eta}_{*j} + \tilde{C}_j \eta_{*j} + \tilde{G}_j = \tilde{Y}_j(x_{*j}, \dot{x}_{*j}, \eta_{*j}, \dot{\eta}_{*j}) a_j \quad (3.56)$$

where a_j is the inertia parameter. For the disturbance $\tilde{D}_j$, it is assumed that

$$\|\tilde{D}_j\| \leq \rho_j(x_{*j}) \quad (3.57)$$

where ρ_j is a known function of q_{*j} . If the inertia parameter a_j is a constant and is unknown, we have the following results.

Theorem 3.5. For the m systems in (3.50)-(3.51), if the communication graph has a spanning tree, the control law

$$\tau_j = \tilde{B}_j^{-1} \left[-K_j \tilde{v}_{*j} + \tilde{Y}_j \hat{a}_j - \rho_j \text{sign}(\tilde{v}_{*j}) - \Lambda_j \right] \quad (3.58)$$

$$\dot{\hat{a}}_j = -\Gamma_j \tilde{Y}_j^\top \tilde{v}_{*j} \quad (3.59)$$

ensures that $\hat{a}_j$ is bounded and (3.52) holds, where K_j and Γ_j are positive constant matrices, and

$$\Lambda_j = \begin{bmatrix} \sum_{i=1}^n e_{ij} [g_{1j}]_i \\ \vdots \\ \sum_{i=1}^n e_{ij} [g_{sj}]_i \end{bmatrix} \quad (3.60)$$

where $[\cdot]_i$ denotes the i -th element in $[\cdot]$.

Proof: Let

$$V = \sum_{i=1}^n z_{i*}^\top \mathcal{L} z_{i*} + \sum_{j=1}^m \frac{1}{2} \tilde{v}_{*j}^\top \tilde{M}_j \tilde{v}_{*j} + \sum_{j=1}^m \frac{1}{2} (\hat{a}_j - a_j) \Gamma_j^{-1} (\hat{a}_j - a_j)$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^n z_{i*}^\top \mathcal{L}^2 z_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} - \sum_{j=1}^m \rho_j \tilde{v}_{*j}^\top \text{sign}(\tilde{v}_{*j}) - \sum_{j=1}^m \tilde{v}_{*j}^\top \tilde{D}_j \\ &\leq -\sum_{i=1}^n z_{i*}^\top \mathcal{L}^2 z_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} \end{aligned}$$

By integrating both sides of the above inequality, it can be shown that V is bounded, which means that z_{i*} , $\tilde{v}_{*j}$ and $\hat{a}_j$ are bounded. Furthermore, it can be shown that by Barbalat's lemma that z_{i*} and $\tilde{v}_{*j}$ converge to zero. Therefore, (3.29) holds. $\blacksquare$

In Theorem 3.5, the unknown inertia parameter is estimated by an adaptive control law. If an estimate of a_j is $\bar{a}_j$ and

$$\|a_j - \bar{a}_j\| \leq \gamma_j$$

for $1 \leq j \leq m$ and γ_j is a known constant, we propose the following robust control laws.

Theorem 3.6. For the m systems in (3.50)-(3.51), if the communication graph has a spanning tree, the control law

$$\tau_j = \tilde{B}_j^{-1} \left[-K_j \tilde{v}_{*j} + \tilde{Y}_j \bar{a}_j - \gamma_j \tilde{Y}_j \text{sign}(\tilde{Y}_j^\top \tilde{v}_{*j}) - \rho_j \text{sign}(\tilde{v}_{*j}) - \Lambda_j \right] \quad (3.61)$$

ensures that (3.52) holds, where K_j is a positive constant matrix and Λ_j is defined in (3.60).

Proof: Let

$$V = \sum_{i=1}^n z_{i*}^\top \mathcal{L} z_{i*} + \sum_{j=1}^m \frac{1}{2} \tilde{v}_{*j}^\top \tilde{M}_j \tilde{v}_{*j}$$

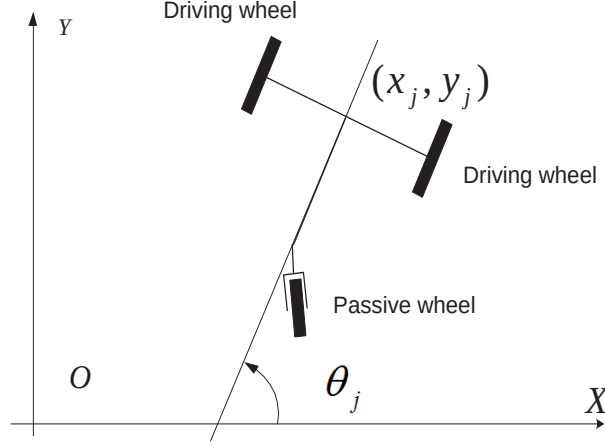


Figure 3.1: Configuration of robot j

Differentiating it along the closed-loop system, we have

$$\begin{aligned}
\dot{V}_j &\leq -\sum_{i=1}^n z_{i*}^\top \mathcal{L}^2 z_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} - \sum_{j=1}^m \rho_j \tilde{v}_{*j}^\top \text{sign}(\tilde{v}_{*j}) - \sum_{j=1}^m \tilde{v}_{*j}^\top \tilde{D}_j \\
&\quad + \sum_{j=1}^m \tilde{v}_{*j}^\top \tilde{Y}_j (\bar{a}_j - a_j) - \sum_{j=1}^m \gamma_j \tilde{v}_{*j}^\top \tilde{Y}_j \text{sign}(\tilde{Y}_j^\top \tilde{v}_{*j}) \\
&\leq -\sum_{i=1}^n z_{i*}^\top \mathcal{L}^2 z_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} \leq 0
\end{aligned}$$

Therefore, V is bounded, which means that z_{i*} and $\tilde{v}_{*j}$ are bounded. By Barbalat's lemma, it can be shown that z_{i*} and $\tilde{v}_{*j}$ converge to zero. $\blacksquare$

In Theorem 3.6, the unknown inertia parameter a_j is not required to be a constant. In the control laws, γ_j is required to be known. It is possible to estimate it.

3.5 Simulation

To verify the proposed results, simulation has been done for three nonholonomic wheeled mobile robots on a horizontal plane. Robot j is constituted by a rigid trolley equipped with 3 nondeformable wheels. The orientation of the 2 wheels with respect to the trolley is fixed, while the orientation of the third wheel is varying. See Fig. 3.1 for details. We assume the plane of each wheel remains vertical and the wheel rotates around its (horizontal) axis. The contact between the wheels and the ground satisfies non slipping condition. The mobile robot is driven by 2 motors which provide torques acting on the rotational axes of the 2 wheels whose orientation is fixed.

The constraint of the non slipping condition can be written as

$$\dot{x}_j \sin \theta_j - \dot{y}_j \cos \theta_j = 0 \quad (3.62)$$

where (x_j, y_j) is the position of robot j and θ_j is the orientation of robot j . The dynamics of robot j are described by the following differential equations

$$\begin{cases} m_j \ddot{x}_j &= \lambda_j \cos \theta_j + \frac{1}{R_j}(\tau_{1j} + \tau_{2j}) \cos \theta_j \\ m_j \ddot{y}_j &= -\lambda_j \sin \theta_j + \frac{1}{R_j}(\tau_{1j} + \tau_{2j}) \sin \theta_j \\ I_j \ddot{\theta}_j &= \frac{L_j}{R_j}(\tau_{1j} - \tau_{2j}) \end{cases} \quad (3.63)$$

where m_j is the mass of robot j , and I_j is its inertia moment around the vertical axis at point Q. R_j is the radius of the wheels and $2L_j$ the length of the axis of the front wheels, and τ_{1j} and τ_{2j} are the torques provided by the motors.

Let $q_{*j} = [x_j, y_j, \theta_j]^\top$,

$$M_j(q_{*j}) = \begin{bmatrix} m_j & 0 & 0 \\ 0 & m_j & 0 \\ 0 & 0 & I_j \end{bmatrix}, C_j(q_{*j}, \dot{q}_{*j}) = 0, G_j(q_{*j}) = 0$$

$$B_j(q_{*j}) = \frac{1}{R_j} \begin{bmatrix} \cos \theta_j & \cos \theta_j \\ \sin \theta_j & \sin \theta_j \\ L_j & -L_j \end{bmatrix}, J_j = [\sin \theta_j, -\cos \theta_j, 0]$$

The system (3.62)-(3.63) is in the form of (3.1)-(3.2).

Let

$$g_{*j} = \begin{bmatrix} \cos \theta_j & 0 \\ \sin \theta_j & 0 \\ 0 & 1 \end{bmatrix}$$

then Equation (3.62) and (3.63) are converted into

$$\begin{cases} \dot{x}_j &= u_{1j} \cos \theta_j \\ \dot{y}_j &= u_{1j} \sin \theta_j \\ \dot{\theta}_j &= u_{2j} \\ m_j \dot{u}_{1j} &= \frac{1}{R_j}(\tau_{1j} + \tau_{2j}) \\ I_j \dot{u}_{2j} &= \frac{L_j}{R_j}(\tau_{1j} - \tau_{2j}) \end{cases} \quad (3.64)$$

With the transformation

$$\begin{cases} x_{1j} &= -\theta_j \\ x_{2j} &= x_j \cos \theta_j + y_j \sin \theta_j \\ x_{3j} &= -x_j \sin \theta_j + y_j \cos \theta_j \\ v_{1j} &= -u_{2j} \\ v_{2j} &= u_{1j} - x_{3j} v_{1j} \end{cases}$$

Equation (3.64) can be converted into the following standard form

$$\begin{cases} \dot{x}_{1j} = v_{1j} \\ \dot{x}_{2j} = v_{2j} \\ \dot{x}_{3j} = x_{2j}v_{1j} \\ \tilde{M}_j\dot{v}_{*j} + \tilde{C}_jv_{*j} = \tilde{B}_j\tau_{*j} \end{cases} \quad (3.65)$$

where

$$\tilde{M}_j = \begin{bmatrix} I_j + m_j x_{3j}^2 & m_j x_{3j} \\ m_j x_{3j} & m_j \end{bmatrix}, \tilde{C}_j = \begin{bmatrix} m_j x_{3j} \dot{x}_{3j} & 0 \\ m_j \dot{x}_{3j} & 0 \end{bmatrix}, \tilde{B}_j = \frac{1}{R_j} \begin{bmatrix} x_{3j} + L_j & x_{3j} - L_j \\ 1 & 1 \end{bmatrix}$$

and

$$\tilde{M}_j(q_{*j})\dot{\xi} + \tilde{C}_j(q_{*j}, \dot{q}_{*j})\xi = \tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})a_j$$

where the inertia parameter vector $a_j = [m_j, I_j]^T$,

$$\tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi}) = \begin{bmatrix} x_{3j}^2 \dot{\xi}_1 + x_{3j} \dot{\xi}_2 + x_{3j} \dot{x}_{3j} \xi_1 & \dot{\xi}_1 \\ x_{3j} \dot{\xi}_1 + \dot{\xi}_2 + \dot{x}_{3j} \xi_1 & 0 \end{bmatrix}$$

The consensus problem of the kinematics in (3.65) can be solved with the aid of the results proposed in Theorem 3.1. The controller is proposed as

$$v_{1j} = \eta_{1j} \quad (3.66)$$

$$\eta_{1j} = -k_1 x_{1j} + \zeta_{1j} \quad (3.67)$$

$$\dot{\zeta}_{1j} = -\sum_{i \in \mathcal{N}_j} a_{ji}(\zeta_{1j} - \zeta_{1i}) - b(\zeta_{1j} - \alpha) + \dot{\alpha} \quad (3.68)$$

$$v_{2j} = \eta_{2j} \quad (3.69)$$

$$\eta_{2j} = -\sum_{i \in \mathcal{N}_j} a_{ji}(z_{2j} - z_{2i}) \quad (3.70)$$

where $k_1 > 0$, $a_{ji} > 0$, $b > 0$, and $\alpha = \sin(14t)$, $z_{2j} = x_{2j} + \beta_{2j}$, $z_{3j} = x_{3j} + \beta_{3j}$, $\beta_{3j} = 0$, and $\beta_{2j} = v_{1j}z_{3j}$. Figs. 3.2-3.4 show the time response of x_{1*} , x_{2*} , and x_{3*} , respectively. It is shown that the state of three systems reach consensus.

The consensus problem of the dynamics in (3.65) can be solved with the aid of the results in Theorem 3.3. The controller is proposed as in (3.6)-(3.48) if the inertia parameter vector a_j is a constant and is unknown. Figs. 3.5-3.7 show the time response of x_{1*} , x_{2*} , and x_{3*} , respectively. It is shown that the state of three systems reach consensus.

If the inertia parameter vector a_j is a constant and is unknown, we can also solve the consensus problem by the robust control algorithms in Theorem 3.4. Figs. 3.8-3.10 show the time response of x_{1*} , x_{2*} , and x_{3*} , respectively. It is shown that the state of three systems reach consensus.

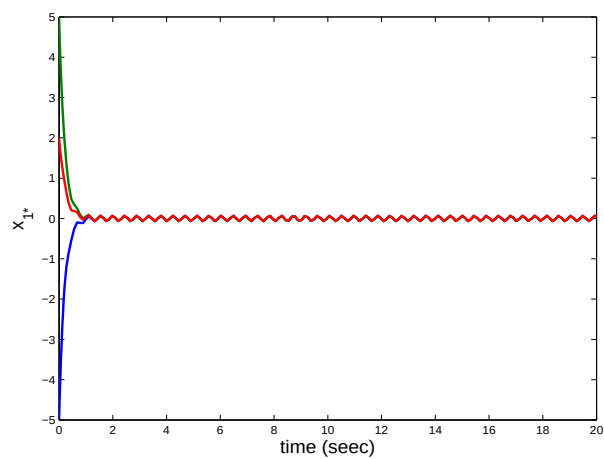


Figure 3.2: Response of x_{1*} .

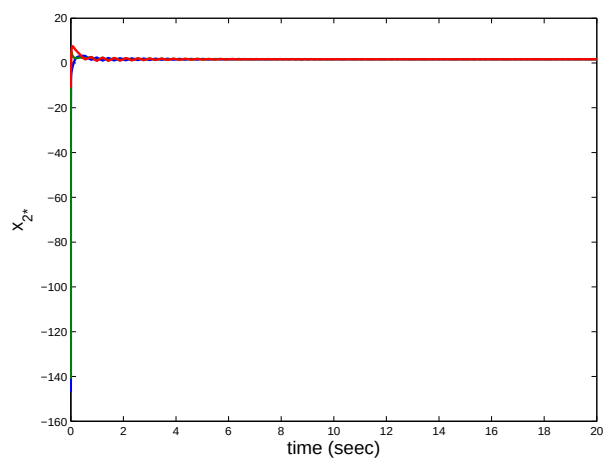


Figure 3.3: Response of x_{2*} .

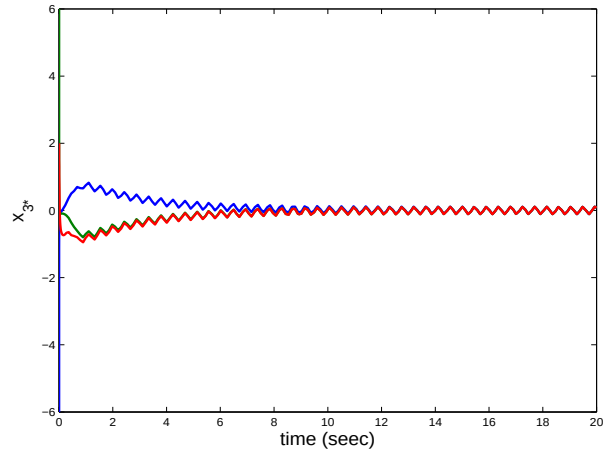


Figure 3.4: Response of x_{2*} .

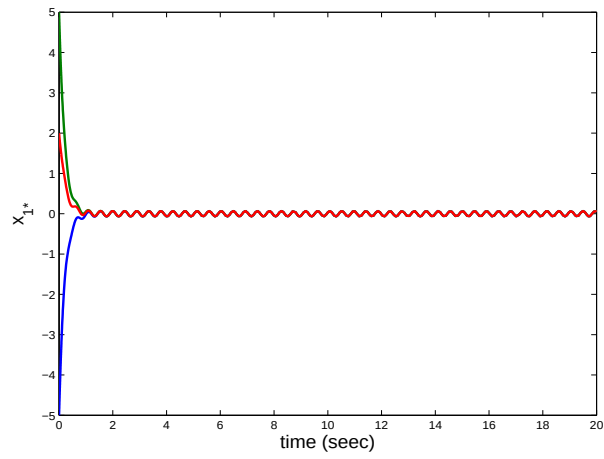


Figure 3.5: Response of x_{1*} .

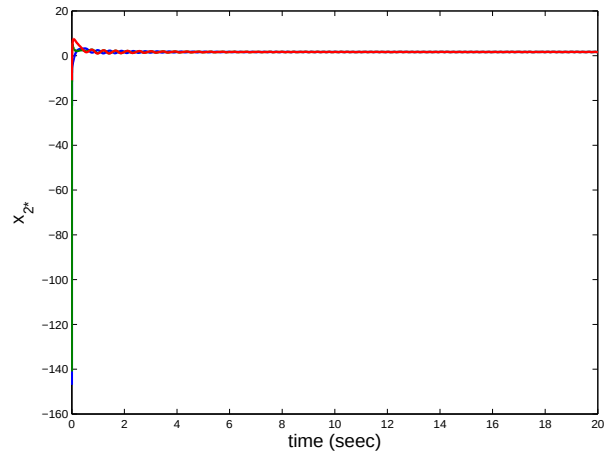


Figure 3.6: Response of x_{2*} .

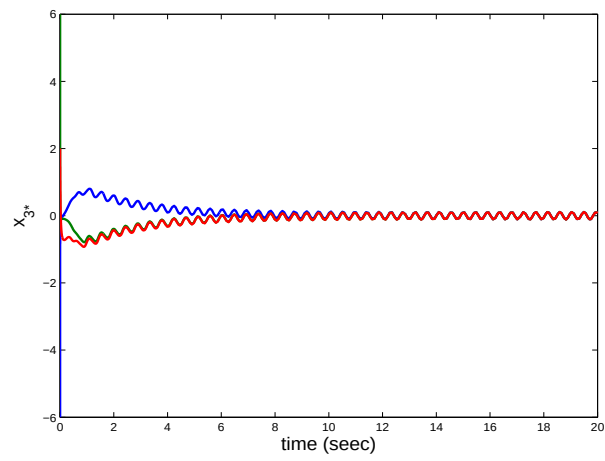


Figure 3.7: Response of x_{2*} .

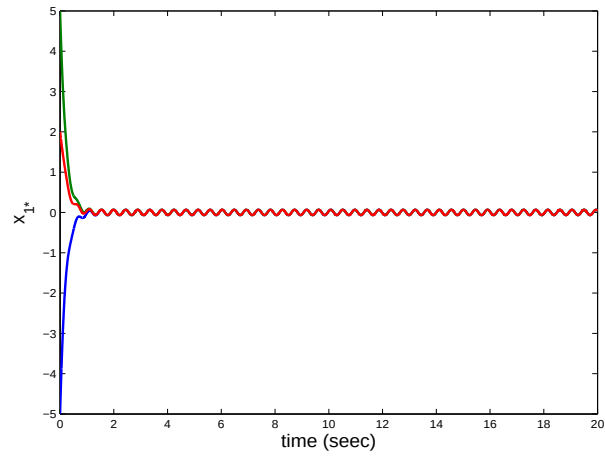


Figure 3.8: Response of x_{1*} .

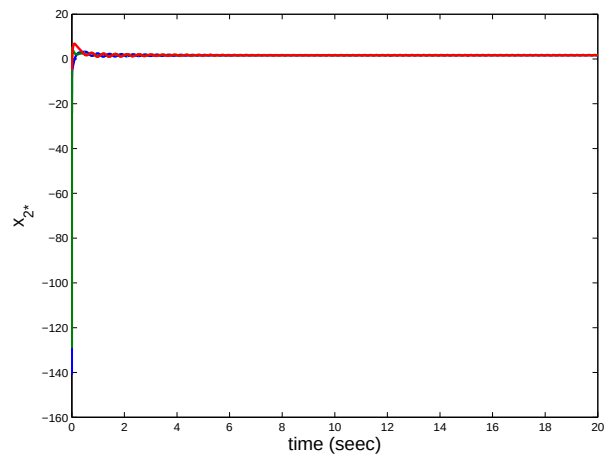


Figure 3.9: Response of x_{2*} .

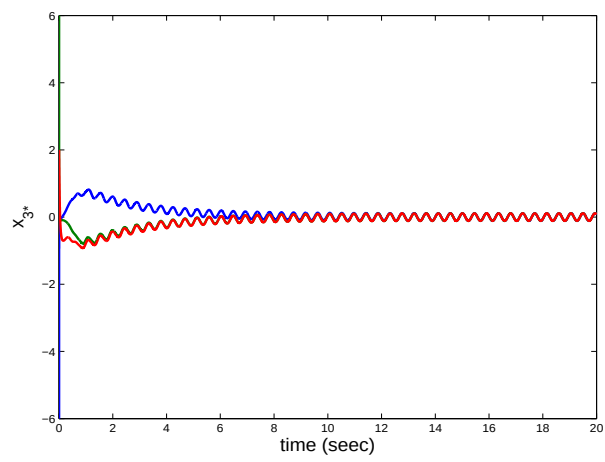


Figure 3.10: Response of x_{2*} .

Chapter 4

Distributed Tracking Control of Networked Chained Systems

4.1 Introduction

In the last decades, control of nonholonomic systems had been an active research area. An important feature of a nonholonomic system is that the number of its inputs is less than the number of its degree of the freedom. This feature makes control problems of a nonholonomic systems challenging. In [9], it has shown that there does not exist a pure-state feedback control law for a nonholonomic system such that its state converges to its equilibrium. However, with the effort of many researchers several types of stabilizing control laws have been proposed, for example, discontinuous control laws in [10] and [11], time-varying control laws in [12] and [13], hybrid control laws in [14] and [15]. Tracking control of a nonholonomic system has also been extensively studied in the last decades due to its applications in tracking control of wheeled mobile robots and unmanned aerial vehicles. With the effort of different researchers, several tracking controllers have been proposed. The first tracking controller was proposed in [16] for a mobile robot. With the aid of backstepping techniques, semi-global tracking controllers are proposed for a chained form system in [17]. In [11], global state and output tracking controllers are proposed for chained form systems with the aid of Lyapunov techniques. With the aid of results on cascade systems, linear tracking controllers are proposed for chained form systems in [18].

Recently, cooperative control of multiple systems has become an active research area and has attracted multi-disciplinary researchers in a wide range of fields, including control system theory, physics, biology, applied mathematics, computer science, and robotics. In the past decade, most of researchers focused their research on cooperative control of multiple identical linear systems or multiple specific mobile robots with simplified identical kinematic models. Various control strategies have been proposed, such as the behavior-based method in [19] and [20], the virtual structure method in [21] and [22], the leader-follower method in [23] and [24], the artificial potentials method in [25] and [26], and the graph theoretical method in [27] and [28], to name a few. In cooperative control, consensus seeking plays an important role (see [29] and [30]). Many cooperative control problems can be solved with the aid of consensus algorithms or techniques developed for solving the consensus seeking problem.

These cooperative control problems include collective behavior of flocks and swarms, sensor fusion, formation control of multiple robot systems, synchronization of coupled oscillators, etc. In [31], alignment of multiple discrete-time agents is discussed and control laws are proposed by using local information. In [32], a theoretical analysis of the consensus property of the Vicsek model is presented with the aid of results from algebraic graph and matrix theories. For networked continuous-time systems, a theoretical framework for consensus control problems is introduced in [33]. In [34], the results obtained in [32] and [33] are extended. In addition to the algebraic graph approach for linear systems, nonlinear analysis tools can also be used to study consensus algorithms. In [35], a set-valued Lyapunov approach is proposed to design consensus algorithms with uni-directional time-dependent communication links. In [36], nonlinear contraction theory is used to study synchronization and schooling applications. In [37], passivity is applied to design consensus algorithms over an undirected communication topology. In [38] and [39], consensus algorithms are proposed for nonlinear systems with the aid of Lyapunov techniques. In [40], consensus algorithms are proposed for high-order nonlinear systems with the aid of backstepping techniques.

Distributed tracking control of multiple systems with a reference system whose state is available to a subset of a group of the systems is an important cooperative control problem. This cooperative control problem is also referred to as a consensus control problem of multiple systems with a reference system. Different from the consensus seeking problem reviewed above the desired value of the state of each system is defined by the state of a reference system. Extensive research on distributed tracking control problem has been conducted for multiple linear systems in the past decade and some results have been obtained. In [41, 42], tracking control of multiple first-order linear systems with a reference system is discussed. Distributed controllers are proposed with the aid of distributed estimators. In [43] and [44], distributed tracking control is considered for multiple first-order and second-order systems. Distributed discontinuous controllers are proposed such that the state of each system converges to a desired trajectory within finite time under the condition that the desired trajectory is available to a portion of the group of systems. Distributed tracking control of multiple Lagrangian mechanical systems is considered in [39] and distributed tracking controllers are proposed.

Flocking and synchronization of multiple systems can be considered as a distributed tracking control problem. In [45], flocking of multiple second-order systems is solved with the aid of potential functions under the assumption that a desired trajectory is available to each system. In [46], flocking of multiple systems is discussed for fixed and switching communication cases such that the velocities of the systems reach an agreement. In [47] and [48], flocking algorithms of multiple second-order linear systems are proposed under the assumption that the information of a virtual leader is available to a portion of systems. In [49] and [50], distributed adaptive control for synchronization of unknown nonlinear networked systems is considered. Distributed adaptive control laws are proposed with the aid of neural network approximation such that the tracking error is uniformly ultimately bounded (UUB). In [51], adaptive leader-following control for multiple systems with uncertainties is considered with the aid of neural-networks. Adaptive control laws are proposed. In [52], formation control of multiple wheeled mobile robots is considered using neighbors' information. Distributed controllers are proposed with the aid of a transformation based on backstepping

techniques.

In this chapter, we consider distributed state feedback tracking control and distributed output feedback tracking control for multiple nonholonomic chained form systems with a reference system whose state is available to a subset of the group of systems. Considering the cascade structure of each system, distributed state feedback tracking controllers are proposed in two steps with the aid of the results of time-varying linear systems and the properties of persistently excited signals. In the first step, a distributed control law is designed for the first input of each system with the aid of distributed estimator design and the sliding mode control. In the second step, a distributed control law is designed for the second input with the aid of linear time-varying theory and distributed estimator design. The proposed distributed tracking controllers ensure that the tracking errors uniformly exponentially converge to the state of the reference system. If the state of each system is not measurable, distributed output feedback tracking controllers are proposed by integrating observer design to the proposed distributed state feedback tracking controller design. The proposed distributed output feedback tracking controllers can ensure that the tracking errors uniformly exponentially converge to zero. In literature, distributed tracking control was considered for multiple linear systems. In this paper, we consider distributed tracking control of multiple chained form systems which represent wheeled mobile robots and unmanned aerial vehicles and propose distributed tracking controllers. The proposed results can be applied to solve the formation control problem of wheeled mobile robots. Simulation results verify the effectiveness of the proposed results.

The remaining parts of this chapter are organized as follows. In Section 4.2, the problems considered in this article are defined and some preliminary results are presented. In Section 4.3, distributed state feedback tracking controllers are proposed. In Section 4.4, distributed output feedback tracking controllers are proposed. In Section 4.5, simulation results are presented. The last section concludes this article.

4.2 Problem Statement and Preliminary Results

4.2.1 Problem Statement

Consider m identical nonholonomic chained form systems. The j -th system is described by

$$\dot{q}_{1j} = u_{1j} \quad (4.1)$$

$$\dot{q}_{2j} = u_{2j} \quad (4.2)$$

$$\dot{q}_{ij} = q_{i-1,j} u_{1j}, \quad 3 \leq i \leq n \quad (4.3)$$

$$y_j = [q_{1j}, q_{nj}]^\top \quad (4.4)$$

where $q_{*j} = [q_{1j}, \dots, q_{nj}]^\top$ and $u_{*j} = [u_{1j}, u_{2j}]^\top$ are the state and input of system j , respectively. y_j is the output of system j .

The communication between the systems can be described by the edge set $\mathcal{E}$ of a directed graph (or digraph for short) $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where the m systems are represented by the m nodes in $\mathcal{V}$. The existence of an edge $(l, j) \in \mathcal{E}$ means that the information (the state or output) of system l is available to system j for control (i.e., unidirectional communication).

Bidirectional communication, if it exists, would be represented by the edge (j, l) also being in the edge set $\mathcal{E}$. The symbol $\mathcal{N}_j$ denotes the neighbors of node j and is a set of indices of the systems whose information is available to system j . A directed path in a digraph is an ordered sequence of vertices such that any ordered pair of vertices appearing consecutively in the sequence is an edge of the digraph. A digraph is called strongly connected if for any two different nodes i and j in $\mathcal{V}$ there exists a directed path from node i to node j .

It is given a desired trajectory $q_{*,m+1} = [q_{1,m+1}, \dots, q_{n,m+1}]^\top$ which is generated by the reference system

$$\dot{q}_{1,m+1} = u_{1,m+1}, \quad \dot{q}_{2,m+1} = u_{2,m+1}, \quad \dot{q}_{i,m+1} = q_{i-1,m+1}u_{1,m+1}, \quad 3 \leq i \leq n, \quad y_{m+1} = q_{*,m+1} \quad (4.5)$$

where $u_{1,m+1}$ and $u_{2,m+1}$ are known time-varying functions. The desired trajectory is available to a portion of the group of m systems. For simplicity, the reference system (4.5) is denoted as system $(m+1)$. The m systems in (4.1)-(4.4) and the reference system in (4.5) can be considered as a group of $(m+1)$ systems and the communication between $(m+1)$ systems is denoted by the digraph $\mathcal{G}^e = \{\mathcal{V}^e, \mathcal{E}^e\}$. The neighbor set of node j is denoted by $\mathcal{N}_j^e$. Since the reference system does not receive information from other systems, the neighbor set of node $(m+1)$ is an empty set, i.e., $\mathcal{N}_{m+1}^e = \emptyset$. Node $(m+1)$ is reachable to node j if there exists a directed path from node $(m+1)$ to node j . Node $(m+1)$ is said to be globally reachable if node $(m+1)$ is reachable to node j for $1 \leq j \leq m$.

The cooperative control problems that will be discussed in this article are defined as follows.

Problem 1: (Distributed state feedback tracking control) For m systems in (4.1)-(4.3) and a reference system in (4.5), the control problem is how to design a control law u_{*j} for system j using its own state q_{*j} and its neighbor's state q_{*l} for $l \in \mathcal{N}_j^e$ such that

$$\lim_{t \rightarrow \infty} (q_{*j}(t) - q_{*,m+1}(t)) = 0 \quad (4.6)$$

for $1 \leq j \leq m$.

Problem 2: (Distributed output feedback tracking control) For m systems in (4.1)-(4.3) and a reference system in (4.5), the control problem is to design a control law u_{*j} for system j using its own output y_j and its neighbor's output y_l for $l \in \mathcal{N}_j^e$ such that (4.6) is satisfied for $1 \leq j \leq m$.

In *Problem 1*, the state of each system is available by itself and is broadcasted to its neighbors. While in *Problem 2* the output of each system is available to itself and is broadcasted to its neighbors. In both problems, the desired state $q_{*,m+1}$ is only available to a subset of a group of the systems, which make the tracking control problems challenging. The tracking controller design procedure for tracking control of a single system in literature cannot solve *Problems 1-2* because the information of the reference system is not available to each system. In this paper, distributed tracking controllers are proposed with the aid of neighbors' information.

For the reference system, the following assumption is made.

Assumption 4.1. *For the reference system (4.5), $\dot{u}_{1,m+1}$, $\dot{u}_{2,m+1}$, and $\dot{q}_{*,m+1}$ are bounded. Moreover, $u_{1,m+1}$ is an absolutely continuous PE signal.*

4.2.2 Preliminary Results

Throughout this article, $\|\cdot\|_2$ stands for the Euclidean norm of vectors and induced norm of matrices. We denote by B_r the open ball $B_r := \{x \in \mathbb{R}^n : \|x\|_2 < r\}$. A function $\alpha : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{K}$ ($\alpha \in \mathcal{K}$), if it is continuous, strictly increasing and zero at zero. If moreover $\alpha(s) \rightarrow \infty$ as $s \rightarrow \infty$ we say that $\alpha \in \mathcal{K}_\infty$. The origin of the system $\dot{x} = f(t, x)$ is globally uniformly stable (GUS) if there exists $\alpha \in \mathcal{K}_\infty$ such that $\|x(t)\|_2 \leq \alpha(\|x_0\|_2)$ for all $t \geq t_0$ and all $t_0 \geq 0$. It is uniformly globally asymptotically stable (UGAS) if in addition to UGS, for each r and $\epsilon > 0$, there exists $T(r, \epsilon) > 0$ such that $(t_0, x_0) \in \mathbb{R}_{\geq 0} \times B_r$ implies that $\|x(t)\|_2 \leq \epsilon$ for all $t \geq t_0 + T$. The origin of the system is said to be globally uniformly exponentially stable (GUES) if there exist two strictly positive constants γ_1 and γ_2 such that

$$\|x(t, t_0, x_0)\|_2 \leq \gamma_1 \|x_0\|_2 e^{-\gamma_2(t-t_0)}$$

for any initial condition $x_0(t_0)$ and $t \geq t_0$. A function $\phi(t) \in \mathbb{R}$ is said to be persistently excited (PE) if there exist $\mu > 0$ and $T > 0$ such that

$$\int_t^{t+T} \phi(\tau)^2 d\tau \geq \mu, \quad \forall t \geq 0.$$

The following lemmas are useful in this paper.

Lemma 4.1. *For a linear time-varying system*

$$\begin{bmatrix} \dot{\chi}_1 \\ \dot{\chi}_2 \\ \dot{\chi}_3 \\ \vdots \\ \dot{\chi}_{l-1} \\ \dot{\chi}_l \end{bmatrix} = \begin{bmatrix} -c_1 & -c_2\phi(t) & 0 & 0 & \cdots & 0 & 0 \\ \phi(t) & 0 & -c_3\phi(t) & 0 & \cdots & 0 & 0 \\ 0 & \phi(t) & 0 & -c_4\phi(t) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & -c_l\phi(t) \\ 0 & 0 & 0 & 0 & \cdots & \phi(t) & 0 \end{bmatrix} \begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \vdots \\ \chi_{l-1} \\ \chi_l \end{bmatrix} \quad (4.7)$$

where c_i ($1 \leq i \leq l$) are positive constants, if $\phi(t)$ is an absolutely continuous PE signal and $\max_{t \in [0, \infty)} \{|\phi(t)|, |\dot{\phi}(t)|\} \leq \phi_M < \infty$ almost everywhere, then the system (4.7) is GUES.

Lemma 4.1 is a modified version of Theorem 2 in [53] and can be proved similarly (see Remark 1 in [54]). So, the proof of Lemma 4.1 is omitted here for space limitation.

Lemma 4.2. *For a linear time-varying system*

$$\begin{bmatrix} \dot{\chi}_1 \\ \dot{\chi}_2 \\ \dot{\chi}_3 \\ \vdots \\ \dot{\chi}_{l-1} \\ \dot{\chi}_l \end{bmatrix} = \begin{bmatrix} -c_1 & -c_2\phi(t) & -c_3 & \cdots & -c_{l-1}\phi(t)^{\text{mod}(l-1,2)} & -c_l\phi(t)^{\text{mod}(l,2)} \\ \phi(t) & 0 & 0 & \cdots & 0 & 0 \\ 0 & \phi(t) & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & \phi(t) & 0 \end{bmatrix} \begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \vdots \\ \chi_{l-1} \\ \chi_l \end{bmatrix} \quad (4.8)$$

where $\text{mod}(l, 2)$ denotes the remainder of the Euclidean division of l by 2, and constants c_i ($1 \leq i \leq l$) are chosen such that the polynomial

$$\lambda^l + c_1 \lambda^{m-1} + \cdots + c_{l-1} \lambda + c_l \quad (4.9)$$

is Hurwitz (i.e. all roots are in the left half of the open complex plane), if $\phi(t)$ is an absolutely continuous PE signal and $\max_{t \in [0, \infty)} \{|\phi(t)|, |\dot{\phi}(t)|\} \leq \phi_M < \infty$ almost everywhere, then the system (4.8) is GUES.

Proof: Let $\chi = [\chi_1, \chi_2, \dots, \chi_l]^\top$, the system can be written as

$$\dot{\chi} = \phi(t) \begin{bmatrix} -c_1 & -c_2 & -c_3 & \cdots & -c_{l-1} & -c_l \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \chi + (\phi(t) - 1) \begin{bmatrix} c_1 \chi_1 + c_3 \chi_3 + \cdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (4.10)$$

Define the Hurwitz determinants

$$\Delta_i = \begin{vmatrix} c_1 & c_3 & c_5 & \cdots & c_{2i-1} \\ 1 & c_2 & c_4 & \cdots & c_{2i-2} \\ 0 & c_1 & c_3 & \cdots & c_{2i-3} \\ 0 & 1 & c_2 & \cdots & c_{2i-4} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & c_i \end{vmatrix}, \quad i = 1, 2, \dots, l \quad (4.11)$$

where if an element c_j appears in Δ_i with $j > i$ it is assumed to be zero. It is well-known that the determinants Δ_i are all positive if and only if (4.9) is Hurwitz ([55]). Define

$$\gamma_1 = \Delta_1 = c_1, \quad \gamma_2 = \frac{\Delta_2}{\Delta_1}, \quad \gamma_3 = \frac{\Delta_3}{\Delta_1 \Delta_2}, \quad \gamma_i = \frac{\Delta_{i-3} \Delta_i}{\Delta_{i-2} \Delta_{i-1}}, \quad i = 4, 5, \dots, l \quad (4.12)$$

it has proved in [56] and [18] that there exists a nonsingular constant matrix $P \in R^{l \times l}$ such that

$$P \begin{bmatrix} -c_1 & -c_2 & -c_3 & \cdots & -c_{l-1} & -c_l \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix} P^{-1} = \begin{bmatrix} -\gamma_1 & -\gamma_2 & 0 & \cdots & 0 & 0 \\ 1 & 0 & -\gamma_3 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -\gamma_l \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}.$$

The state transformation $z = P\chi$ transforms system (4.10) into

$$\dot{z} = \begin{bmatrix} -\gamma_1 & -\gamma_2 \phi & 0 & \cdots & 0 & 0 \\ \phi & 0 & -\gamma_3 \phi & \cdots & 0 & 0 \\ 0 & \phi & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -\gamma_l \phi \\ 0 & 0 & 0 & \cdots & \phi & 0 \end{bmatrix} z \quad (4.13)$$

Since ϕ is a PE signal and $\max_{t \in [0, \infty)} \{|\phi(t)|, |\dot{\phi}(t)|\} \leq \phi_M < \infty$ almost everywhere, by Lemma 4.1 system (4.13) is GUES. Noting the transformation matrix P is a nonsingular matrix, system (4.10) is GUES. ■

Lemma 4.3. *For the system*

$$\dot{x} = (A(t) + F(t))x \quad (4.14)$$

if $A(t)$ and $F(t)$ are bounded, $\dot{x} = A(t)x$ is GUES, and $F(t)$ exponentially converges to zero, then the system (4.14) is GUES.

Proof: Since $\dot{x} = A(t)x$ is GUES and $A(t)$ is bounded, by Theorem 7.8 in [57]

$$Q(t) = \int_t^\infty \Phi_A^\top(\sigma, t) \Phi_A(\sigma, t) d\sigma$$

is a continuously differentiable symmetric matrix for all t and is such that

$$\eta_1 I \leq Q(t) \leq \eta_2 I \quad (4.15)$$

$$A^\top(t)Q(t) + Q(t)A(t) + \dot{Q}(t) = -I \quad (4.16)$$

where η_1 and η_2 are finite positive constants, and $\Phi(\sigma, t)$ is the state transition matrix of the system $\dot{x} = A(t)x$. By (4.16), we have

$$[A(t) + F(t)]^\top Q(t) + Q(t)[A(t) + F(t)] + \dot{Q}(t) = F^\top(t)Q(t) + Q(t)F(t) - I.$$

Since $Q(t)$ and $F(t)$ are bounded and $F(t)$ exponentially converges to zero, there exists a finite time t_1 such that $F^\top(t)Q(t) + Q(t)F(t) \leq I$, which means that

$$[A(t) + F(t)]^\top Q(t) + Q(t)[A(t) + F(t)] + \dot{Q}(t) \leq vI$$

for $t \geq t_1$, where v is a positive constant. For time $t \in [0, t_1)$ the state x is bounded because $A(t)$ and $F(t)$ are bounded. By Theorem 7.4 in [57], the system (4.14) is GUES. ■

For a cascade system

$$\dot{x} = f_1(t, x) + g(t, x, y)y \quad (4.17)$$

$$\dot{y} = f_2(t, y) \quad (4.18)$$

where $x \in R^{n_1}$, $y \in R^{n_2}$, $f_1(t, x)$ is continuously differentiable in (t, x) and $f_2(t, y)$, $g(t, x, y)$ are continuous in their arguments, and locally Lipschitz in y and (x, y) , respectively, the following result has been proved in [58].

Lemma 4.4. *The cascade system (4.17)-(4.18) is GUES if the following conditions hold:*

1. *the system $\dot{x} = f_1(t, x)$ is GUES;*
2. *the function $g(t, x, y)$ satisfies*

$$\|g(t, x, y)\|_2 \leq \theta_1(\|y\|_2) + \theta_2(\|y\|_2)\|x\|_2$$

for all $t \geq t_0$, where θ_1 and θ_2 are continuous nonnegative functions;

3. the system $\dot{y} = f_2(t, y)$ is GUES.

For the digraph $\mathcal{G}^e$ with a weight matrix $\mathcal{A}^e = [a_{ji}]_{(m+1) \times (m+1)}$ ($a_{ji} > 0$), its weighted Laplacian matrix $\mathcal{L}^e = [L_{ji}^e]_{(m+1) \times (m+1)}$ is defined as

$$L_{ji}^e = \begin{cases} -a_{ji}, & \text{if } i \neq j \text{ and } i \in \mathcal{N}_j^e \\ 0, & \text{if } i \neq j \text{ and } i \notin \mathcal{N}_j^e \\ \sum_{k \in \mathcal{N}_j^e} a_{jk}, & \text{if } i = j. \end{cases} \quad (4.19)$$

For the digraph $\mathcal{G}$ with the weight matrix $\mathcal{A}$ which is formed by the first m rows and the first m columns of $\mathcal{A}^e$, its weighted Laplacian matrix $\mathcal{L} = [L_{ji}]_{m \times m}$ is defined as

$$L_{ji} = \begin{cases} -a_{ji}, & \text{if } i \neq j \text{ and } i \in \mathcal{N}_j \\ 0, & \text{if } i \neq j \text{ and } i \notin \mathcal{N}_j \\ \sum_{k \in \mathcal{N}_j} a_{jk}, & \text{if } i = j. \end{cases} \quad (4.20)$$

It is obvious that

$$\mathcal{L}^e = \left[\begin{array}{c|c} \mathcal{L}_{11}^e & -\mathcal{L}_{12}^e \\ \hline \mathbf{0} & 0 \end{array} \right] = \left[\begin{array}{c|c} \mathcal{L} + \text{diag}(\mathcal{L}_{12}^e) & -\mathcal{L}_{12}^e \\ \hline \mathbf{0} & 0 \end{array} \right] \quad (4.21)$$

where $\mathbf{0}$ is a vector with element zero,

$$\mathcal{L}_{11}^e = \mathcal{L} + \text{diag}(\mathcal{L}_{12}^e) \quad (4.22)$$

$$\mathcal{L}_{12}^e = [a_{1,m+1}\mu_1, \dots, a_{m,m+1}\mu_m]^\top \quad (4.23)$$

and

$$\mu_j = \begin{cases} 1, & \text{if node } m+1 \text{ is available to node } j \\ 0, & \text{otherwise} \end{cases} \quad (4.24)$$

for $1 \leq j \leq m$, and $\text{diag}(\mathcal{L}_{12}^e)$ denotes a diagonal matrix with the diagonal being the vector $\mathcal{L}_{12}^e$.

For the weighted Laplacian matrix $\mathcal{L}^e$, the following results will be applied in this paper.

Lemma 4.5. *For the digraph $\mathcal{G}^e$ with weighted matrix $\mathcal{A}^e = [a_{ji}]_{(m+1) \times (m+1)}$ ($a_{ji} > 0$), its weighted Laplacian matrix $\mathcal{L}^e$ is defined in (4.19). If node $(m+1)$ is globally reachable, then*

1. $\mathcal{L}^e \mathbf{1} = \mathbf{0}$ where $\mathbf{1}$ is a vector with element 1;
2. zero is a simple eigenvalue of $\mathcal{L}^e$ and non-zero eigenvalues of $\mathcal{L}^e$ all have positive real parts;
3. $-\mathcal{L}_{11}^e$ is a Hurwitz matrix;
4. there exists a positive definite diagonal matrix P such that $Q = P\mathcal{L}_{11}^e + (\mathcal{L}_{11}^e)^\top P$ is a positive definite matrix.

Proof: The results in (1) is obvious due to the definition of the weighted Laplacian matrix. The results in (2) was proved in [59]. Noting the structure of $\mathcal{L}^e$, The results in (3) is obvious. By Theorem 4.25 in [38], the results in (4) can be derived from the results in (2). $\blacksquare$

Lemma 4.6. *For the digraph $\mathcal{G}^e$ with the weight matrix $\mathcal{A}^e = [a_{ji}]_{(m+1) \times (m+1)}$ ($a_{ji} > 0$), its weighted Laplacian matrix is defined in (4.19). Consider the system*

$$\dot{\xi} = -\mathcal{L}_{11}^e(\xi - \xi_{m+1}\mathbf{1}) - \rho \operatorname{sign}(\mathcal{L}_{11}^e(\xi_j - \xi_{m+1}\mathbf{1})) \quad (4.25)$$

where $\mathcal{L}_{11}^e$ is defined in (4.22), $\xi_{m+1} \in R$ is a differentiable signal and $|\dot{\xi}_{m+1}|$ is bounded, ρ is a control parameter, if node $(m+1)$ is globally reachable, $(\xi - \xi_{m+1}\mathbf{1})$ globally uniformly exponentially converges to zero, where ρ is chosen such that

$$\rho \geq \max_{t \in [0, \infty)} |\dot{\xi}_{m+1}(t)|. \quad (4.26)$$

Proof: Let $\tilde{\xi} = (\xi - \xi_{m+1}\mathbf{1})/\rho$, one has

$$\dot{\tilde{\xi}} = -\mathcal{L}_{11}^e \tilde{\xi} - \operatorname{sign}(\mathcal{L}_{11}^e \tilde{\xi}) - \rho^{-1} \dot{\xi}_{m+1} \quad (4.27)$$

Let $z = \mathcal{L}_{11}^e \tilde{\xi}$, then

$$\dot{z} = -\mathcal{L}_{11}^e z - \mathcal{L}_{11}^e \operatorname{sign}(z) - \mathcal{L}_{11}^e \rho^{-1} \dot{\xi}_{m+1} \quad (4.28)$$

Since node $(m+1)$ is globally reachable, there exists a positive definite matrix $P = \operatorname{diag}(P_1, P_2, \dots, P_m)$ such that $Q = P\mathcal{L}_{11}^e + (\mathcal{L}_{11}^e)^\top P$ is a positive definite matrix. Choose a Lyapunov function $V = z^\top Pz$ and differentiate it along the solutions of (4.28), one has

$$\begin{aligned} \dot{V} &= -z^\top Qz - 2z^\top P\mathcal{L}_{11}^e \operatorname{sign}(z) - 2z^\top P\mathcal{L}_{11}^e \rho^{-1} \dot{\xi}_{m+1} \\ &= -z^\top Qz - 2z^\top P\mathcal{L} \operatorname{sign}(z) - 2z^\top P \operatorname{diag}(\mathcal{L}_{12}^e) \operatorname{sign}(z) - 2z^\top P \operatorname{diag}(\mathcal{L}_{12}^e) \rho^{-1} \dot{\xi}_{m+1} \\ &\leq -z^\top Qz - 2z^\top P \operatorname{diag}(\mathcal{L}_{12}^e) \operatorname{sign}(z) - 2z^\top P \operatorname{diag}(\mathcal{L}_{12}^e) \rho^{-1} \dot{\xi}_{m+1} \end{aligned}$$

where $\mathcal{L}$ is defined in (4.20) and $\mathcal{L}\mathbf{1} = \mathbf{0}$, $\mathcal{L}_{12}^e$ is defined in (4.23). In the above equations, we apply the fact that $z^\top P\mathcal{L} \operatorname{sign}(z) \geq 0$. Since $\dot{\xi}_{m+1}$ is bounded, if ρ is chosen such that (4.26) is satisfied, then

$$z^\top P \operatorname{diag}(\mathcal{L}_{12}^e) \operatorname{sign}(z) + z^\top P \operatorname{diag}(\mathcal{L}_{12}^e) \rho^{-1} \dot{\xi}_{m+1} \geq \sum_{j=1}^m P_j a_{j,m+1} \mu_j |z_j| \rho^{-1} (\rho - |\dot{\xi}_{m+1}|) \geq 0 \quad (4.29)$$

So,

$$\dot{V} \leq -z^\top Qz \leq -\lambda_{\min}(Q) z^\top z \leq -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} V. \quad (4.30)$$

Therefore, V is bounded and exponentially converges to zero. So, $(\xi - \xi_{m+1}\mathbf{1})$ globally uniformly exponentially converges to zero. $\blacksquare$

4.3 Distributed State Feedback Tracking Control

In order to solve *Problem 1*, we consider system (4.1)-(4.3) as a cascade system of (4.1) and (4.2)-(4.3). System (4.1) is a first-order linear system. For distributed tracking control of m first-order linear systems, we have the following results.

Lemma 4.7. *For m systems in (4.1) and the reference system in (4.5) with Assumption 4.1, if node $(m+1)$ is globally reachable in the communication digraph $\mathcal{G}^e$, then the distributed control laws*

$$u_{1j} = - \sum_{i \in \mathcal{N}_j} a_{ji}(q_{1j} - q_{1i}) - a_{j,m+1}\mu_j(q_{1j} - q_{1,m+1}) + \xi_{1j} \quad (4.31)$$

$$\begin{aligned} \dot{\xi}_{1j} = & - \sum_{i \in \mathcal{N}_j} a_{ji}(\xi_{1j} - \xi_{1i}) - a_{j,m+1}\mu_j(\xi_{1j} - \xi_{1,m+1}) \\ & - \rho \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\xi_{1j} - \xi_{1i}) + a_{j,m+1}\mu_j(\xi_{1j} - \xi_{1,m+1}) \right] \end{aligned} \quad (4.32)$$

for $1 \leq j \leq m$ ensure that q_{1j} globally uniformly exponentially converges to $q_{1,m+1}$ and ξ_{1j} globally uniformly exponentially converges to $\xi_{1,m+1}$, where ρ is a sufficiently large number, $\xi_{1,m+1} = u_{1,m+1}$, and $a_{ji} > 0$.

Proof: The m systems in (4.32) can be written as

$$\dot{\xi}_{1*} = -\mathcal{L}_{11}^e(\xi_{1*} - \xi_{1,m+1}\mathbf{1}) - \rho \operatorname{sign}(\mathcal{L}_{11}^e(\xi_{1*} - \xi_{1,m+1}\mathbf{1})) \quad (4.33)$$

where $\xi_{1*} = [\xi_{11}, \xi_{12}, \dots, \xi_{1m}]^\top$. Since node $(m+1)$ is globally reachable in the communication digraph $\mathcal{G}^e$, by Lemma 4.6 $(\xi_{1j} - \xi_{1,m+1})$ uniformly exponentially converges to zero if ρ is chosen such that $\rho \geq \max_{t \in [0, \infty)} |\dot{\xi}_{1,m+1}(t)|$.

Let $\tilde{q}_{1j} = q_{1j} - q_{1,m+1}$ and $\tilde{q}_{1*} = [\tilde{q}_{11}, \dots, \tilde{q}_{1m}]^\top$, with the control law (4.31) we have

$$\dot{\tilde{q}}_{1*} = -\mathcal{L}_{11}^e \tilde{q}_{1*} + \tilde{\xi}_{1*}. \quad (4.34)$$

System (4.34) can be considered as a linear system subjected to disturbance which uniformly exponentially converges to zero. Since node $(m+1)$ is globally reachable in the communication digraph $\mathcal{G}^e$, by Lemma 4.5 $\tilde{q}_{1*}$ uniformly exponentially converges to zero, which means that $(q_{1j} - q_{1,m+1})$ globally uniformly exponentially converges to zero. ■

With the aid of Lemma 4.7, the control law u_{2j} can be proposed and the following results can be obtained.

Theorem 4.1. *For m systems in (4.1)-(4.3) and a reference system (4.5) with Assumption 4.1, if node $(m+1)$ is globally reachable in the communication digraph $\mathcal{G}^e$, then the distributed control laws (4.31)-(4.32) and*

$$u_{2j} = - \sum_{l=2}^n k_{l-1} u_{1j}^{mod(l,2)} q_{lj} + \xi_{2j} \quad (4.35)$$

$$\begin{aligned} \dot{\xi}_{2j} = & - \sum_{i \in \mathcal{N}_j} a_{ji}(\xi_{2j} - \xi_{2i}) - a_{j,m+1}\mu_j(\xi_{2j} - \xi_{2,m+1}) \\ & - \beta \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\xi_{2j} - \xi_{2i}) + a_{j,m+1}\mu_j(\xi_{2j} - \xi_{2,m+1}) \right] \end{aligned} \quad (4.36)$$

for $1 \leq j \leq m$ ensure that q_{*j} globally uniformly exponentially converges to $q_{*,m+1}$ and (ξ_{1j}, ξ_{2j}) globally uniformly exponentially converges to $(\xi_{1,m+1}, \xi_{2,m+1})$, where β is a sufficiently large number,

$$\xi_{2,m+1} = u_{2,m+1} + k_1 q_{2,m+1} + k_2 u_{1,m+1} q_{3,m+1} + k_3 q_{4,m+1} + \cdots + k_{n-1} u_{1,m+1}^{mod(n,2)} q_{n,m+1} \quad (4.37)$$

and k_l ($1 \leq l \leq n-1$) are chosen such that the polynomial

$$\lambda^{n-1} + k_1 \lambda^{n-2} + \cdots + k_{n-2} \lambda + k_{n-1} \quad (4.38)$$

is Hurwitz.

Proof: Define

$$s_{l-1,j} = \sum_{i \in \mathcal{N}_j} a_{ji}(q_{lj} - q_{li}) + a_{j,m+1} \mu_j (q_{lj} - q_{l,m+1}) \quad (4.39)$$

for $2 \leq l \leq n$ and $1 \leq j \leq m$, the closed-loop systems can be written as

$$\begin{cases} \dot{s}_{1j} &= -\sum_{l=2}^n k_{l-1} u_{1,m+1}^{mod(l,2)} s_{l-1,j} + \sum_{i \in \mathcal{N}_j} a_{ji} (\xi_{2j} - \xi_{2i}) + a_{j,m+1} \mu_j (\xi_{2j} - \xi_{2,m+1}) \\ &\quad - \sum_{l=2}^n k_{l-1} \left[\sum_{i \in \mathcal{N}_j} a_{ji} \left((u_{1j}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{lj} - (u_{1i}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{li} \right) \right. \\ &\quad \left. + a_{j,m+1} \mu_j (u_{1j}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{lj} \right] \\ \dot{s}_{2j} &= u_{1,m+1} s_{1j} + \sum_{i \in \mathcal{N}_j} a_{ji} [q_{2j} (u_{1j} - u_{1,m+1}) - q_{2i} (u_{1i} - u_{1,m+1})] \\ &\quad + a_{j,m+1} \mu_j (u_{1j} - u_{1,m+1}) q_{2j} \\ &\quad \vdots \\ \dot{s}_{n-1,j} &= u_{1,m+1} s_{n-2,j} + \sum_{i \in \mathcal{N}_j} a_{ji} [q_{n-1,j} (u_{1j} - u_{1,m+1}) - q_{n-1,i} (u_{1i} - u_{1,m+1})] \\ &\quad + a_{j,m+1} \mu_j (u_{1j} - u_{1,m+1}) q_{n-1,j} \\ \dot{\xi}_{2j} &= - \sum_{i \in \mathcal{N}_j} a_{ji} (\xi_{2j} - \xi_{2i}) - a_{j,m+1} \mu_j (\xi_{2j} - \xi_{2,m+1}) \end{cases} \quad (4.40)$$

$$- \beta \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji} (\xi_{2j} - \xi_{2i}) + a_{j,m+1} \mu_j (\xi_{2j} - \xi_{2,m+1}) \right] \quad (4.41)$$

$$\dot{q}_{1j} = - \sum_{i \in \mathcal{N}_j} a_{ji} (q_{1j} - q_{1i}) - a_{j,m+1} \mu_j (q_{1j} - q_{1,m+1}) + \xi_{1j} \quad (4.42)$$

$$\begin{aligned} \dot{\xi}_{1j} &= - \sum_{i \in \mathcal{N}_j} a_{ji} (\xi_{1j} - \xi_{1i}) - a_{j,m+1} \mu_j (\xi_{1j} - \xi_{1,m+1}) \\ &\quad - \rho \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji} (\xi_{1j} - \xi_{1i}) + a_{j,m+1} \mu_j (\xi_{1j} - \xi_{1,m+1}) \right] \end{aligned} \quad (4.43)$$

Let $\tilde{\xi}_{2j} = \xi_{2j} - \xi_{2,m+1}$, $\tilde{q}_{1j} = q_{1j} - q_{1,m+1}$, and $\tilde{u}_{1j} = u_{1j} - u_{1,m+1}$, then we have

$$\left\{ \begin{array}{l} \dot{s}_{1j} = -\sum_{l=2}^n k_{l-1} u_{1,m+1}^{mod(l,2)} s_{l-1,j} + \sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) + a_{j,m+1} \mu_j \tilde{\xi}_{2j} \\ \quad - \sum_{l=2}^n k_{l-1} \left[\sum_{i \in \mathcal{N}_j} a_{ji} \left((u_{1j}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{lj} - (u_{1i}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{li} \right) \right. \\ \quad \left. + a_{j,m+1} \mu_j (u_{1j}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{lj} \right] \\ \dot{s}_{2j} = u_{1,m+1} s_{1j} + \sum_{i \in \mathcal{N}_j} a_{ji} (q_{2j} \tilde{u}_{1j} - q_{2i} \tilde{u}_{1i}) + a_{j,m+1} \mu_j \tilde{u}_{1j} q_{2j} \\ \vdots \\ \dot{s}_{n-1,j} = u_{1,m+1} s_{n-2,j} + \sum_{i \in \mathcal{N}_j} a_{ji} (q_{n-1,j} \tilde{u}_{1j} - q_{n-1,i} \tilde{u}_{1i}) + a_{j,m+1} \mu_j \tilde{u}_{1j} q_{n-1,j} \end{array} \right. \quad (4.44)$$

$$\begin{aligned} \dot{\tilde{\xi}}_{2j} = & - \sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) - a_{j,m+1} \mu_j \tilde{\xi}_{2j} - \dot{\xi}_{2,m+1} \\ & - \beta \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) + a_{j,m+1} \mu_j \tilde{\xi}_{2j} \right] \end{aligned} \quad (4.45)$$

$$\dot{\tilde{q}}_{1j} = - \sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{q}_{1j} - \tilde{q}_{1i}) - a_{j,m+1} \mu_j \tilde{q}_{1j} + \tilde{\xi}_{1j} \quad (4.46)$$

$$\begin{aligned} \dot{\tilde{\xi}}_{1j} = & - \sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{1j} - \tilde{\xi}_{1i}) - a_{j,m+1} \mu_j \tilde{\xi}_{1j} - \dot{\xi}_{1,m+1} \\ & - \rho \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{1j} - \tilde{\xi}_{1i}) + a_{j,m+1} \mu_j \tilde{\xi}_{1j} \right] \end{aligned} \quad (4.47)$$

The system in eqns. (4.44)-(4.46) for $1 \leq j \leq m$ can be considered as a cascade system (4.17)-(4.18) with $x = [x_{*1}^\top, \dots, x_{*m}^\top]^\top = [[s_{11}, s_{21}, \dots, s_{n-1,1}], \dots, [s_{1m}, s_{2m}, \dots, s_{n-1,m}]]^\top$, $y = [y_{*1}^\top, \dots, y_{*m}^\top]^\top = [[\tilde{\xi}_{21}, \tilde{q}_{11}, \tilde{\xi}_{11}], \dots, [\tilde{\xi}_{2m}, \tilde{q}_{1m}, \tilde{\xi}_{1m}]]^\top$, $s_{*j} = [s_{1j}, \dots, s_{n-1,j}]^\top$,

$$f_{1j} = \begin{bmatrix} -k_1 & -k_2 u_{1,m+1} & \cdots & -k_{n-2} u_{1,m+1}^{mod(n-1,2)} & -k_{n-1} u_{1,m+1}^{mod(n,2)} \\ u_{1,m+1} & 0 & \cdots & 0 & 0 \\ 0 & u_{1,m+1} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & u_{1,m+1} & 0 \end{bmatrix} s_{*j},$$

$$f_1(t, x) = [f_{11}^\top, f_{12}^\top, \dots, f_{1m}^\top]^\top$$

$$f_{2j} = \begin{bmatrix} - \sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) - a_{j,m+1} \mu_j \tilde{\xi}_{2j} - \rho \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) + a_{j,m+1} \mu_j \tilde{\xi}_{2j} \right] - \dot{\xi}_{2,m+1} \\ \quad - \sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{q}_{1j} - \tilde{q}_{1i}) - a_{j,m+1} \mu_j \tilde{q}_{1j} + \tilde{\xi}_{1j} \\ - \sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{1j} - \tilde{\xi}_{1i}) - a_{j,m+1} \mu_j \tilde{\xi}_{1j} - \beta \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{1j} - \tilde{\xi}_{1i}) + a_{j,m+1} \mu_j \tilde{\xi}_{1j} \right] - \dot{\xi}_{1,m+1} \end{bmatrix}$$

$$f_2(t, y) = [f_{21}^\top, f_{22}^\top, \dots, f_{2m}^\top]^\top$$

$$g_{1j} = \begin{bmatrix} -\sum_{l=2}^n k_{l-1} \left[\sum_{i \in \mathcal{N}_j} a_{ji} \left((u_{1j}^{\text{mod}(l,2)} - u_{1,m+1}^{\text{mod}(l,2)}) q_{lj} - (u_{1i}^{\text{mod}(l,2)} - u_{1,m+1}^{\text{mod}(l,2)}) q_{li} \right) \right. \\ \left. + a_{j,m+1} \mu_j (u_{1j}^{\text{mod}(l,2)} - u_{1,m+1}^{\text{mod}(l,2)}) q_{lj} \right] + \sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) + a_{j,m+1} \mu_j \tilde{\xi}_{2j} \\ \dots \dots \dots \\ \sum_{i \in \mathcal{N}_j} a_{ji} (q_{2j} \tilde{u}_{1j} - q_{2i} \tilde{u}_{1i}) + a_{j,m+1} \mu_j \tilde{u}_{1j} q_{2j} \\ \vdots \\ \sum_{i \in \mathcal{N}_j} a_{ji} (q_{n-1,j} \tilde{u}_{1j} - q_{n-1,i} \tilde{u}_{1i}) + a_{j,m+1} \mu_j \tilde{u}_{1j} q_{n-1,j} \end{bmatrix}$$

$$g(t, x, y)y = [g_{11}^\top, g_{12}^\top, \dots, g_{1m}^\top]^\top$$

Since node $(m+1)$ is globally reachable in the communication digraph $\mathcal{G}^e$, by the proof of Lemma 4.7 it can be proved that $\tilde{\xi}_{2j}$ globally uniformly exponentially converges to zero. With the aid of the proof of Lemma 4.7, $\tilde{\xi}_{1*}$ and $\tilde{q}_{1*}$ globally uniformly exponentially converge to zero, respectively. Therefore, the systems in (4.45)-(4.47) are globally uniformly exponentially stable, which means that $\dot{y} = f_2(y)$ is globally uniformly exponentially stable. Furthermore, $\tilde{u}_{1j}$ is GUES.

Since $u_{1,m+1}$ is a PE signal, by Lemma 4.2 the system $\dot{x}_{*j} = f_{1j}$ is GUES. So, $\dot{x} = f_1(x)$ is GUES.

By (4.39), $q_{l*} = (\mathcal{L}_{11}^e)^{-1} s_{l-1,*} + \mathbf{1} q_{l,m+1}$. It can be shown that $\|g(t, x, y)\|_2 \leq \theta_1(\|y\|_2) + \theta_2(\|y\|_2)\|x\|_2$ where θ_1 and θ_2 are nonnegative continuous functions. By Lemma 4.4, the system (4.44)-(4.47) is GUES, which means that s_{lj} , $\tilde{\xi}_{1j}$, and $\tilde{\xi}_{2j}$ globally uniformly exponentially converge to zero for $1 \leq l \leq n-1$ and $1 \leq j \leq m$. By (4.39), $s_{l-1,*} = \mathcal{L}_{11}^e \tilde{q}_{l*}$, where $\tilde{q}_{l*} = q_{l*} - q_{l,m+1} \mathbf{1}$. So, $\tilde{q}_{l*} = (\mathcal{L}_{12}^e)^{-1} s_{l-1,*}$. Therefore, $\tilde{q}_{l*}$ globally uniformly exponentially converges to zero. ■

In the distributed control laws (4.31)-(4.32) and (4.35)-(4.36), ρ and β should be chosen such that

$$\rho \geq \max_{t \in [0, \infty)} |\dot{\xi}_{1,m+1}(t)|, \quad \beta \geq \max_{t \in [0, \infty)} |\dot{\xi}_{2,m+1}(t)|. \quad (4.48)$$

Since $\xi_{1,m+1}$ and $\xi_{2,m+1}$ are unknown to each system, the lower bounds of ρ and β in (4.48) are unknown for each system. Therefore, ρ and β are required to be chosen large enough in the theorem.

In Theorem 4.1, (4.36) is an estimator of $\xi_{2,m+1}$. The estimator (4.36) is distributed. In [52], formation control of wheeled mobile robots is considered and distributed controllers are proposed using backstepping techniques. In this paper, we considered the distributed tracking control problem of multiple chained form systems and proposed distributed tracking controllers with the aid of results for cascade systems. The proposed results in this paper can be applied to solve the formation control problem of wheeled mobile robots (see Section

4.5). In [60], cooperative tracking control of multiple chained systems was considered under the condition that a desired trajectory is known to each system. While in this paper cooperative tracking control problem is solved under the condition that a desired trajectory is only available to a portion of a group of systems. It is obvious that the method proposed in [60] cannot solve the tracking control problem considered in this paper. In [60], the controllers were designed with the aid of backstepping techniques and results of graph theory. In this paper distributed tracking controllers are designed with the aid of the cascade structure of each system, results of time-varying systems, and results of graph theory.

4.4 Distributed output feedback distributed control

In the last section, it is assumed that the state of each system is available for feedback control. In this section, it is assumed that the output of each system is available for feedback control. In order to solve *Problem 2*, we build an estimator for each system to estimate the state of each system and integrate the estimators to the distributed state feedback laws proposed in the last section.

With the aid of Theorem 4.1 and the observer design theory for linear time-varying systems in [61], we have the following results.

Theorem 4.2. *For m systems in (4.1)-(4.4) and the reference system (4.5) with Assumption 4.1, if node $(m+1)$ is globally reachable in the communication digraph $\mathcal{G}^e$, then the distributed control laws (4.31)-(4.32) and*

$$u_{2j} = - \sum_{l=2}^n k_{l-1} u_{1j}^{mod(l,2)} \hat{q}_{lj} + \xi_{2j} \quad (4.49)$$

$$\begin{aligned} \dot{\xi}_{2j} = & - \sum_{i \in \mathcal{N}_j} a_{ji} (\xi_{2j} - \xi_{2i}) - a_{j,m+1} \mu_j (\xi_{2j} - \xi_{2,m+1}) \\ & - \beta \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji} (\xi_{2j} - \xi_{2i}) + a_{j,m+1} \mu_j (\xi_{2j} - \xi_{2,m+1}) \right] \end{aligned} \quad (4.50)$$

for $1 \leq j \leq m$ ensure that $q_{*j}(t)$ globally uniformly exponentially converges to $q_{*,m+1}(t)$ and (ξ_{1j}, ξ_{2j}) uniformly exponentially converges to $(\xi_{1,m+1}, \xi_{2,m+1})$, where β is a sufficiently large number, and $\hat{q}_{lj}$ is generated by the estimator

$$\begin{bmatrix} \dot{\hat{q}}_{2j} \\ \dot{\hat{q}}_{3j} \\ \dot{\hat{q}}_{4j} \\ \vdots \\ \dot{\hat{q}}_{n-1,j} \\ \dot{\hat{q}}_{nj} \end{bmatrix} = \begin{bmatrix} -k_1 & -k_2 u_{1j} & -k_3 & \cdots & -k_{n-2} u_{1j}^{mod(n-1,2)} & -k_{n-1} u_{1j}^{mod(n,2)} \\ u_{1j} & 0 & 0 & \cdots & 0 & 0 \\ 0 & u_{1j} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & u_{1j} & 0 \end{bmatrix} \begin{bmatrix} \hat{q}_{2j} \\ \hat{q}_{3j} \\ \hat{q}_{4j} \\ \vdots \\ \hat{q}_{n-1,j} \\ \hat{q}_{nj} \end{bmatrix}$$

$$+ \begin{bmatrix} l_{n-1} u_{1j}^{mod(n-2,2)} \\ l_{n-2} u_{1j}^{mod(n-3,2)} \\ l_{n-3} u_{1j}^{mod(n-4,2)} \\ \vdots \\ l_2 u_{1j} \\ l_1 \end{bmatrix} (q_{nj} - \hat{q}_{nj}) + \begin{bmatrix} \xi_{2j} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (4.51)$$

constants k_i and l_i ($1 \leq i \leq n-1$) are chosen such that the polynomials

$$\lambda^{n-1} + k_1 \lambda^{n-2} + k_2 \lambda^{n-3} + k_{n-2} \lambda + k_{n-1} \quad (4.52)$$

$$\lambda^{n-1} + l_1 \lambda^{n-2} + l_2 \lambda^{n-3} + l_{n-2} \lambda + l_{n-1} \quad (4.53)$$

are Hurwitz.

Proof: Let $s_{l-1,j}$ be defined in (4.39), $\tilde{q}_{lj} = q_{lj} - \hat{q}_{lj}$, $\tilde{\xi}_{2j} = \xi_{2j} - \xi_{2,m+1}$ and $\tilde{q}_{1j} = q_{1j} - q_{1,m+1}$, for $2 \leq l \leq n$ and $1 \leq j \leq m$, the closed-loop systems can be written as

$$\left\{ \begin{array}{l} \dot{s}_{1j} = -\sum_{l=2}^n k_{l-1} u_{1,m+1}^{mod(l,2)} s_{l-1,j} + \sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) + a_{j,m+1} \mu_j \tilde{\xi}_{2j} \\ \quad - \sum_{l=2}^n k_{l-1} \left[\sum_{i \in \mathcal{N}_j} a_{ji} ((u_{1j}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{lj} - (u_{1i}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{li}) \right. \\ \quad \left. + a_{j,m+1} \mu_j (u_{1j}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{lj} \right] + \sum_{l=2}^n k_{l-1} u_{1,m+1}^{mod(l,2)} \tilde{q}_{lj} \\ \dot{s}_{2j} = u_{1,m+1} s_{1j} + \sum_{i \in \mathcal{N}_j} a_{ji} [q_{2j} \tilde{u}_{1j} - q_{2i} \tilde{u}_{1i}] + a_{j,m+1} \mu_j \tilde{u}_{1j} q_{2j} \\ \vdots \\ \dot{s}_{n-1,j} = u_{1,m+1} s_{n-2,j} + \sum_{i \in \mathcal{N}_j} a_{ji} [q_{n-1,j} \tilde{u}_{1j} - q_{n-1,i} \tilde{u}_{1i}] \\ \quad + a_{j,m+1} \mu_j \tilde{u}_{1j} q_{n-1,j} \end{array} \right. \quad (4.54)$$

$$\dot{\tilde{q}}_{2j} = l_{n-1} u_{1,m+1}^{mod(n-2,2)} \tilde{q}_{nj} + l_{n-1} \tilde{u}_{1j}^{mod(n-2,2)} \tilde{q}_{nj} \quad (4.55)$$

$$\dot{\tilde{q}}_{3j} = u_{1,m+1} \tilde{q}_{2j} + l_{n-2} u_{1,m+1}^{mod(n-3,2)} \tilde{q}_{nj} + \tilde{u}_{1j} \tilde{q}_{2j} + l_{n-2} \tilde{u}_{1j}^{mod(n-3,2)} \tilde{q}_{nj} \quad (4.56)$$

$\vdots$

$$\dot{\tilde{q}}_{n-1,j} = u_{1,m+1} \tilde{q}_{n-2,j} + l_2 u_{1,m+1} \tilde{q}_{nj} + \tilde{u}_{1j} \tilde{q}_{n-2,j} + l_2 \tilde{u}_{1j} \tilde{q}_{nj} \quad (4.57)$$

$$\dot{\tilde{q}}_{nj} = u_{1,m+1} \tilde{q}_{n-1,j} + l_1 \tilde{q}_{nj} + \tilde{u}_{1j} \tilde{q}_{n-1,j} \quad (4.58)$$

$$\dot{\tilde{\xi}}_{2j} = -\sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) - a_{j,m+1} \mu_j \tilde{\xi}_{2j} - \dot{\xi}_{2,m+1}$$

$$- \beta \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) + a_{j,m+1} \mu_j \tilde{\xi}_{2j} \right] \quad (4.59)$$

$$\dot{\tilde{q}}_{1j} = -\sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{q}_{1j} - \tilde{q}_{1i}) - a_{j,m+1} \mu_j \tilde{q}_{1j} + \tilde{\xi}_{1j} \quad (4.60)$$

$$\begin{aligned}
\dot{\tilde{\xi}}_{1j} &= - \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\xi}_{1j} - \tilde{\xi}_{1i}) - a_{j,m+1}\mu_j\tilde{\xi}_{1j} - \dot{\xi}_{1,m+1} \\
&\quad - \rho \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\xi}_{1j} - \tilde{\xi}_{1i}) + a_{j,m+1}\mu_j\tilde{\xi}_{1j} \right]
\end{aligned} \tag{4.61}$$

where $\tilde{u}_{1j} = u_{1j} - u_{1,m+1}$ for $1 \leq j \leq m$.

The system in eqns. (4.54)-(4.60) for $1 \leq j \leq m$ can be considered as a cascade system

$$\dot{x} = f_1(t, x) + g_1(t, x, z, y)[z^\top, y^\top]^\top \tag{4.62}$$

$$\dot{z} = f_3(t, z) + g_2(t, y, z) \tag{4.63}$$

$$\dot{y} = f_2(t, y) \tag{4.64}$$

where $x = [x_{*1}^\top, \dots, x_{*m}^\top]^\top = [[s_{11}, s_{21}, \dots, s_{n-1,1}], \dots, [s_{1m}, s_{2m}, \dots, s_{n-1,m}]]^\top$, $z = [z_{*1}^\top, \dots, z_{*m}^\top]^\top = [[\tilde{q}_{21}, \dots, \tilde{q}_{n1}], \dots, [\tilde{q}_{2m}, \dots, \tilde{q}_{nm}]]^\top$, $y = [y_{*1}^\top, \dots, y_{*m}^\top]^\top = [[\tilde{\xi}_{21}, \tilde{q}_{11}, \tilde{\xi}_{11}], \dots, [\tilde{\xi}_{2m}, \tilde{q}_{1m}, \tilde{\xi}_{1m}]]^\top$,

$$f_{1j} = \begin{bmatrix} -k_1 & -k_2 u_{1,m+1} & \cdots & -k_{n-2} u_{1,m+1}^{\operatorname{mod}(n-1,2)} & -k_{n-1} u_{1,m+1}^{\operatorname{mod}(n,2)} \\ u_{1,m+1} & 0 & \cdots & 0 & 0 \\ 0 & u_{1,m+1} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & u_{1,m+1} & 0 \end{bmatrix} s_{*j},$$

$$f_1(t, x) = \begin{bmatrix} f_{11} \\ f_{12} \\ \vdots \\ f_{1m} \end{bmatrix}$$

$$f_{2j} = \begin{bmatrix} - \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) - a_{j,m+1}\mu_j\tilde{\xi}_{2j} - \beta \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) + a_{j,m+1}\mu_j\tilde{\xi}_{2j} \right] - \dot{\xi}_{2,m+1} \\ - \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{q}_{1j} - \tilde{q}_{1i}) - a_{j,m+1}\mu_j\tilde{q}_{1j} + \tilde{\xi}_{1j} \\ - \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\xi}_{1j} - \tilde{\xi}_{1i}) - a_{j,m+1}\mu_j\tilde{\xi}_{1j} - \rho \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\xi}_{1j} - \tilde{\xi}_{1i}) + a_{j,m+1}\mu_j\tilde{\xi}_{1j} \right] - \dot{\xi}_{1,m+1} \end{bmatrix},$$

$$f_2 = [f_{21}^\top, f_{22}^\top, \dots, f_{2m}^\top]^\top$$

$$f_{3j} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & l_{n-1} u_{1,m+1}^{\operatorname{mod}(n-2,2)} \\ u_{1,m+1} & 0 & \cdots & 0 & 0 & l_{n-2} u_{1,m+1}^{\operatorname{mod}(n-3,2)} \\ 0 & u_{1,m+1} & \cdots & 0 & 0 & l_{n-2} u_{1,m+1}^{\operatorname{mod}(n-4,2)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & u_{1,m+1} & 0 & l_2 u_{1,m+1} \\ 0 & 0 & \cdots & 0 & u_{1,m+1} & l_1 \end{bmatrix} z_{*j},$$

$$f_3(t, z) = [f_{31}^\top, f_{32}^\top, \dots, f_{3m}^\top]^\top$$

$$\begin{aligned}
g_{2j} &= \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & l_{n-1}\tilde{u}_{1j}^{mod(n-2,2)} \\ \tilde{u}_{1,m+1} & 0 & \cdots & 0 & 0 & l_{n-2}\tilde{u}_{1j}^{mod(n-3,2)} \\ 0 & \tilde{u}_{1,m+1} & \cdots & 0 & 0 & l_{n-2}\tilde{u}_{1j}^{mod(n-4,2)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \tilde{u}_{1,m+1} & 0 & l_2\tilde{u}_{1j} \\ 0 & 0 & \cdots & 0 & \tilde{u}_{1,m+1} & l_1 \end{bmatrix} z_{*j}, \\
g_2(t, z) &= [g_{21}^\top, g_{22}^\top, \dots, g_{2m}^\top]^\top \\
g_{1j}(t, x, y) &= \begin{bmatrix} -\sum_{l=2}^n k_{l-1} \left[\sum_{i \in \mathcal{N}_j} a_{ji} \left((u_{1j}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{lj} - (u_{1i}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{li} \right) \right. \\ \left. + a_{j,m+1} \mu_j (u_{1j}^{mod(l,2)} - u_{1,m+1}^{mod(l,2)}) q_{lj} \right] + \sum_{i \in \mathcal{N}_j} a_{ji} (\tilde{\xi}_{2j} - \tilde{\xi}_{2i}) + a_{j,m+1} \mu_j \tilde{\xi}_{2j} + \\ \sum_{l=2}^n k_{l-1} u_{1,m+1}^{mod(l,2)} \tilde{q}_{lj} \\ \dots \dots \dots \\ \sum_{i \in \mathcal{N}_j} a_{ji} (q_{2j} \tilde{u}_{1j} - q_{2i} \tilde{u}_{1i}) + a_{j,m+1} \mu_j \tilde{u}_{1j} q_{2j} \\ \vdots \\ \sum_{i \in \mathcal{N}_j} a_{ji} (q_{n-1,j} \tilde{u}_{1j} - q_{n-1,i} \tilde{u}_{1i}) + a_{j,m+1} \mu_j \tilde{u}_{1j} q_{n-1,j} \end{bmatrix} \\
g(t, x, y) [z^\top, y^\top]^\top &= [g_{11}^\top, g_{12}^\top, \dots, g_{1m}^\top]^\top
\end{aligned}$$

Since node $(m+1)$ is globally reachable in the communication digraph $\mathcal{G}^e$, by the proof of Lemma 4.7 it can be proved that $\tilde{\xi}_{2j}$ globally uniformly exponentially converges to zero. With the aid of the proof of Lemma 4.7, $\tilde{\xi}_{1*}$ and $\tilde{q}_{1*}$ globally uniformly exponentially converge to zero, respectively. Therefore, the systems in (4.59)-(4.61) are globally uniformly exponentially stable, which means that $\dot{y} = f_2(y)$ is GUES. Furthermore, it can be shown that $\tilde{u}_{1j}$ uniformly exponentially converges to zero.

Since $u_{1,m+1}$ is an absolutely continuous PE signal, with the aid of Lemma 4.2 and reordering of the state z of the system (i.e., reverse the order of the state) it can be shown that $\dot{z} = f_3(t, z)$ is GUES. Since $\tilde{u}_{1j}$ exponentially converges to zero, by Lemma 4.3 the state z of the system (4.63) uniformly exponentially converges to zero.

By Lemma 4.2 the system $\dot{x}_{*j} = f_{1j}$ is GUES. So, $\dot{x} = f_1(t, x)$ is GUES. By (4.39), $q_{l*} = (\mathcal{L}_{11}^e)^{-1} s_{l-1,*} + q_{l,m+1}$. It can be shown that $\|g(t, x, z, y)\| \leq \theta_1(\|(y, z)\|) + \theta_2(\|(y, z)\|)\|x\|$ where θ_1 and θ_2 are nonnegative continuous functions. By Lemma 4.4, the system (4.54)-(4.61) is GUES, which means that s_{lj} , $\tilde{\xi}_{1j}$, and $\tilde{\xi}_{2j}$ globally uniformly exponentially converges to zero for $1 \leq l \leq n-1$ and $1 \leq j \leq m$. By (4.39), $s_{l-1,*} = \mathcal{L}_{11}^e \tilde{q}_{l*}$ where $\tilde{q}_{l*} = q_{l*} - q_{l,m+1} \mathbf{1}$. So, $\tilde{q}_{l*} = (\mathcal{L}_{11}^e)^{-1} s_{l-1,*}$. Therefore, $\tilde{q}_{l*}$ globally uniformly exponentially converges to zero. ■

In Theorem 4.2, (4.50) is an estimator of $\xi_{2,m+1}$, which is the same as that in Theorem 4.1. (4.51) is an observer of state $[q_{2j}, q_{3j}, \dots, q_{nj}]^\top$. Observer (4.51) is proposed with the aid of the observer theory for linear time-varying systems in chapter 15 in [61], the duality property of state feedback design and observer design for linear systems, and the state feedback results in Section 4.3. (4.49) is a tracking controller based on the estimated states.

4.5 Simulations

To verify the proposed results, simulation has been done for formation control of five non-holonomic wheeled mobile robots on a horizontal plane. The kinematic of robot j is

$$\dot{x}_j = v_j \cos \theta_j, \quad \dot{y}_j = v_j \sin \theta_j, \quad \dot{\theta}_j = \omega_j \quad (4.65)$$

where (x_j, y_j) is the location of robot j , θ_j is the orientation of robot j , v_j and ω_j are the control inputs. It is given a desired formation defined by a geometric pattern $\mathcal{P}$ whose vertexes are at coordinate: (p_{1x}, p_{1y}) , (p_{2x}, p_{2y}) , (p_{3x}, p_{3y}) , (p_{4x}, p_{4y}) , and (p_{5x}, p_{5y}) . It is also given a desired virtual robot

$$\dot{x}_6 = v_6 \cos \theta_6, \quad \dot{y}_6 = v_6 \sin \theta_6, \quad \dot{\theta}_6 = \omega_6. \quad (4.66)$$

The formation control problem of five mobile robots is to design distributed control laws such that

$$\lim_{t \rightarrow \infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix}, \quad 1 \leq i \neq j \leq 5 \quad (4.67)$$

$$\lim_{t \rightarrow \infty} (\theta_j - \theta_6) = 0, \quad 1 \leq j \leq 5 \quad (4.68)$$

$$\lim_{t \rightarrow \infty} \left(\sum_{j=1}^5 \frac{x_j}{5} - x_6 \right) = 0, \quad \lim_{t \rightarrow \infty} \left(\sum_{j=1}^5 \frac{y_j}{5} - y_6 \right) = 0. \quad (4.69)$$

The formation control problem can be solved with the aid of the results proposed in the last sections. To this end, we define

$$\begin{cases} q_{1j} &= \theta_j \\ q_{2j} &= (x_j - p_{jx}) \cos \theta_j + (y_j - p_{jy}) \sin \theta_j \\ q_{3j} &= (x_j - p_{jx}) \sin \theta_j - (y_j - p_{jy}) \cos \theta_j \\ u_{1j} &= \omega_j \\ u_{2j} &= v_j - \omega_j q_{3j} \end{cases} \quad (4.70)$$

for $1 \leq j \leq 6$, where $p_{6x} = p_{6y} = 0$. Then, we have

$$\dot{q}_{1j} = u_{1j}, \quad \dot{q}_{2j} = u_{2j}, \quad \dot{q}_{3j} = u_{1j} q_{2j} \quad (4.71)$$

which is a special case of system (4.1)-(4.3). It can be shown that the formation control problem is solved if (4.6) is satisfied for $1 \leq j \leq 5$. Therefore, the proposed distributed tracking control laws in the last sections can solve the formation control problem.

In the simulation, it is assumed that the desired formation pattern is shown as in Fig. 4.1 and the state of the reference system is $(x_6, y_6, \theta_6) = (10 \sin(0.5t), -10 \cos(0.5t), 0.5t)$. If the state of each system is available for feedback control, the distributed tracking control laws in Theorem 4.1 solve the formation control problem. Fig. 4.2 shows the communication digraph between systems. Figs. 4.3-4.5 show the response of $(q_{1j} - q_{16})$, $(q_{2j} - q_{26})$, and $(q_{3j} - q_{36})$ for $1 \leq j \leq 5$, respectively. The simulation results show that (4.6) is satisfied.

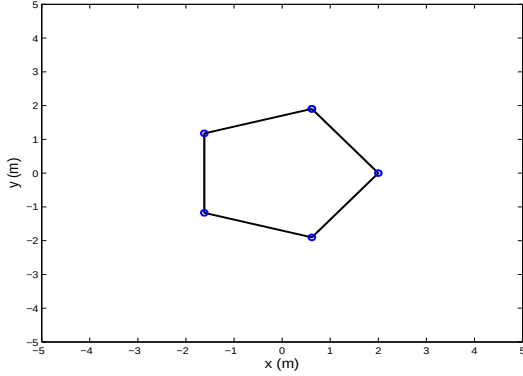


Figure 4.1: Geometric pattern of the formation

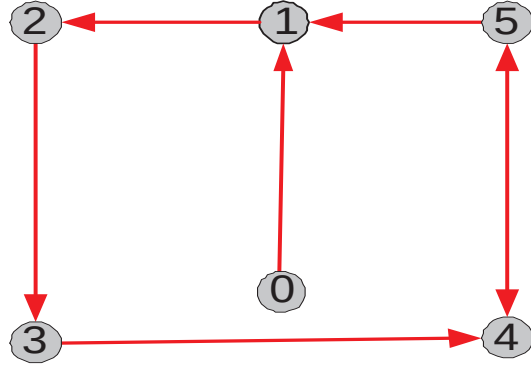


Figure 4.2: Communication digraph $\mathcal{G}$

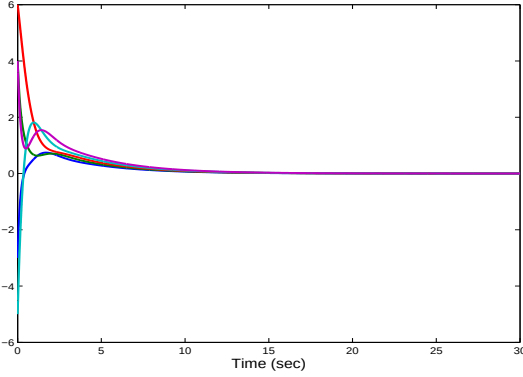


Figure 4.3: Response of $(q_{1j} - q_{16})$ for $1 \leq i \leq 5$.

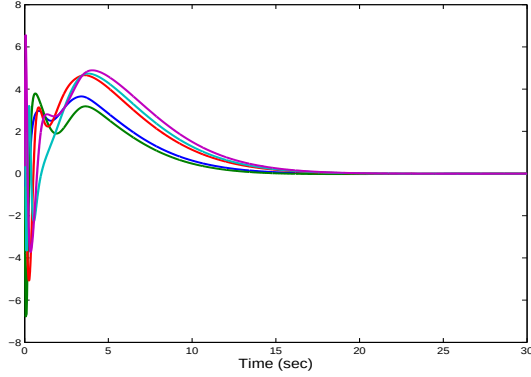


Figure 4.4: Response of $(q_{2j} - q_{26})$ for $1 \leq i \leq 5$.

Fig. 4.6 shows the centroid of x_i ($1 \leq i \leq 5$) (i.e., $\sum_{j=1}^5 x_j/5$) and x_6 . Fig. 4.7 shows the centroid of y_i ($1 \leq i \leq 5$) (i.e., $\sum_{j=1}^5 y_j/5$) and y_6 . Fig. 4.8 shows the path of the centroid of the five robots and its desired path. The simulation results verify that (4.68)-(4.69) are satisfied. Eqn. (4.67) is also verified and the response of them is omitted here.

If the output q_{1j} and q_{3j} is available and can be broadcasted to its neighbors, the distributed output tracking laws can be obtained with the aid of Theorem 4.2. It is assumed that the communication digraph and the state of the virtual system are the same as before. Figs. 4.9-4.10 show the response of $(q_{2j} - \hat{q}_{2j})$ and $(q_{3j} - \hat{q}_{3j})$ for $1 \leq j \leq 5$, respectively. The simulation results show that the estimation errors converge to zero. Figs. 4.11-4.13 show the response of $(q_{1j} - q_{16})$, $(q_{2j} - q_{26})$, and $(q_{3j} - q_{36})$ for $1 \leq j \leq 5$, respectively. The simulation results show that (4.6) is satisfied. Fig. 4.14 shows the centroid of x_i ($1 \leq i \leq 5$) (i.e., $\sum_{j=1}^5 x_j/5$) and x_6 . Fig. 4.15 shows the centroid of y_i ($1 \leq i \leq 5$) (i.e., $\sum_{j=1}^5 y_j/5$) and y_6 . Fig. 4.16 shows the path of the centroid of the five robots and its desired path. Simulation results verify that (4.68)-(4.69) are satisfied. Eqn. (4.67) is also verified and the response of them is omitted here.

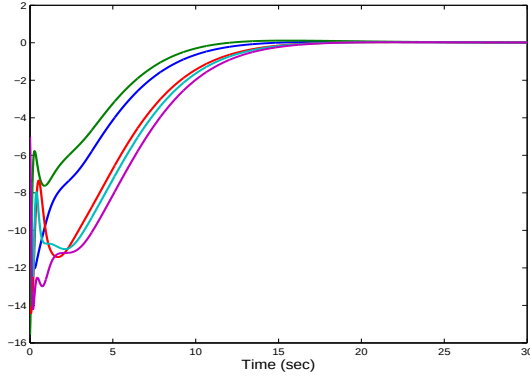


Figure 4.5: Response of $(q_{3j} - q_{36})$ for $1 \leq i \leq 5$.

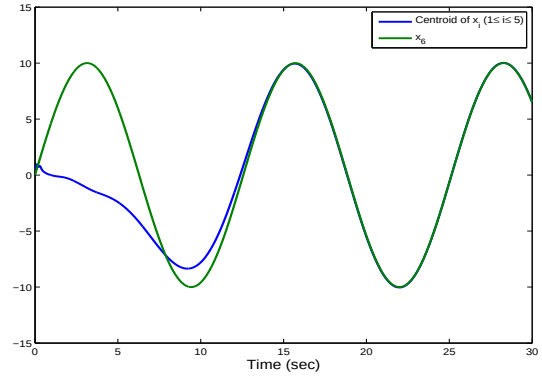


Figure 4.6: Response of the centroid of x_i (solid) for $1 \leq i \leq 5$ and x_6 (dashed).

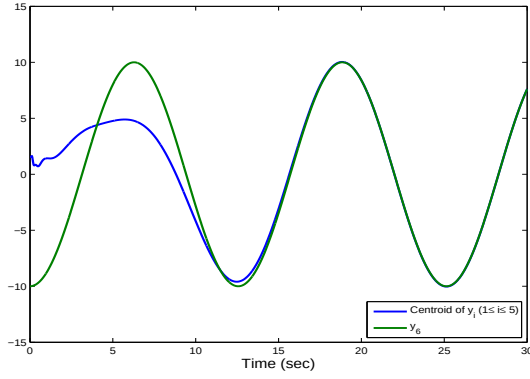


Figure 4.7: Response of the centroid of y_i (solid) for $1 \leq i \leq 5$ and y_6 (dashed).

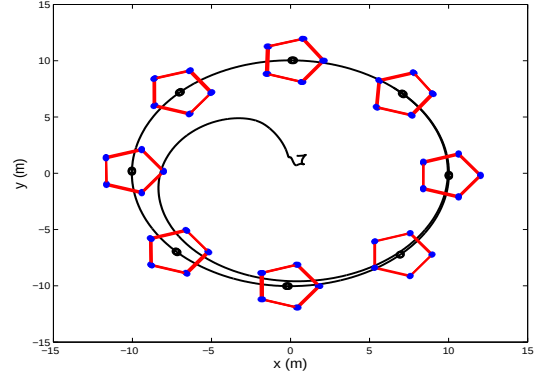


Figure 4.8: The path of the centroid of the five robots (dashed line), the desired path (solid line) of the centroid of robots, and formation of the five robots at several moments (red pentagons).

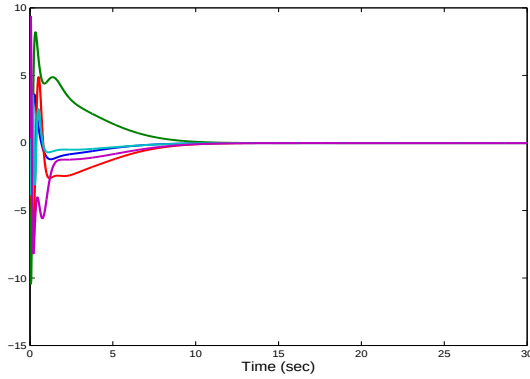


Figure 4.9: Response of $(q_{2j} - \hat{q}_{2j})$ for $1 \leq i \leq 5$.

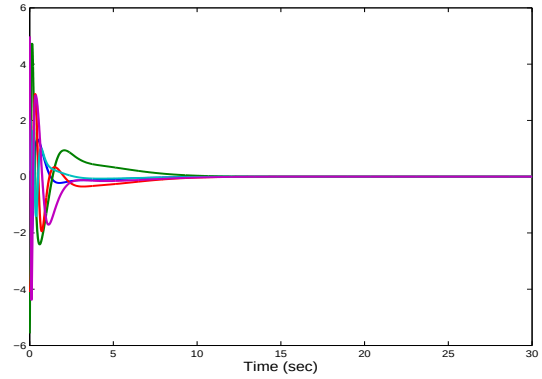


Figure 4.10: Response of $(q_{3j} - \hat{q}_{3j})$ for $1 \leq i \leq 5$.

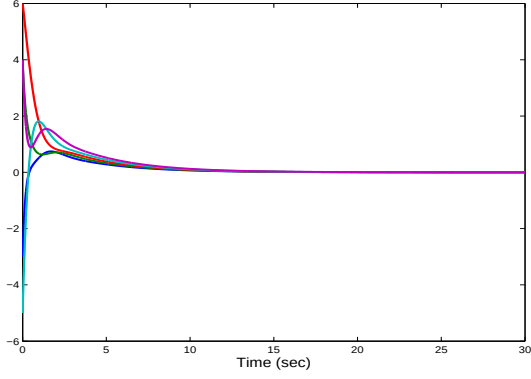


Figure 4.11: Response of $(q_{1j} - q_{16})$ for $1 \leq i \leq 5$.

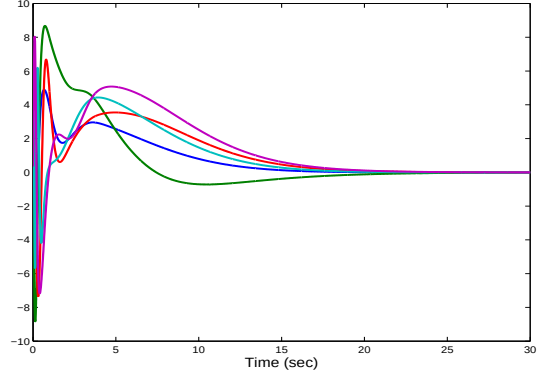


Figure 4.12: Response of $(q_{2j} - q_{26})$ for $1 \leq i \leq 5$.

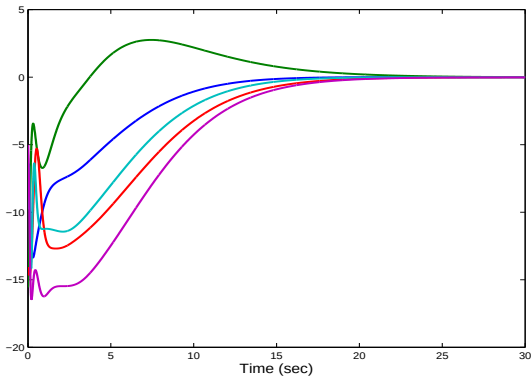


Figure 4.13: Response of $(q_{3j} - q_{36})$ for $1 \leq i \leq 5$.

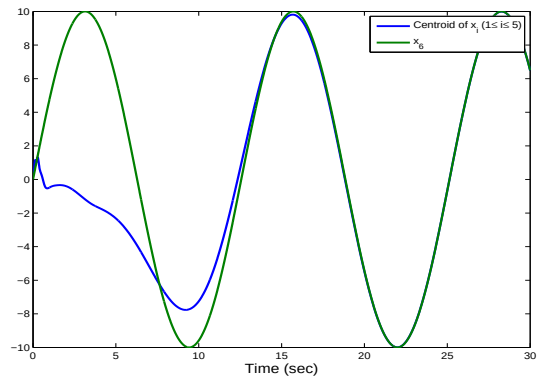


Figure 4.14: Response of the centroid of x_i (solid) for $1 \leq i \leq 5$ and x_6 (dashed).

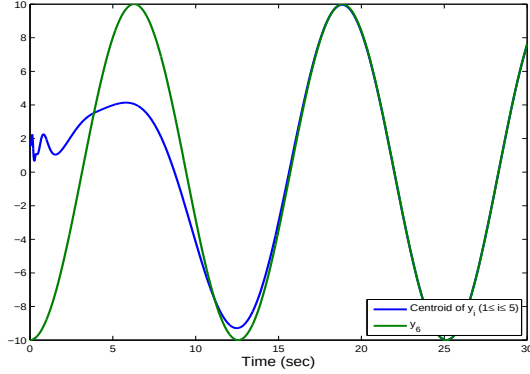


Figure 4.15: Response of the centroid of y_i (solid) for $1 \leq i \leq 5$ and y_6 (dashed).

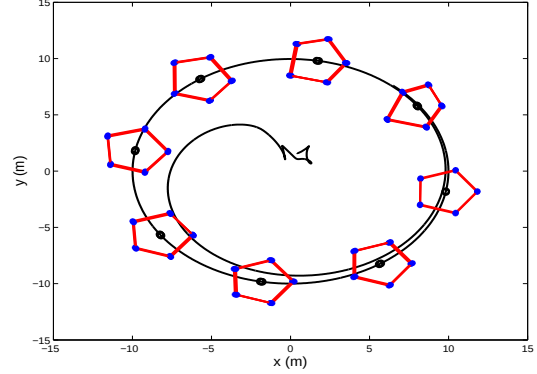


Figure 4.16: The path of the centroid of the five robots (dashed line), the desired path (solid line) of the centroid of robots, and formation of the five robots at several moments (red pentagons).

4.6 Conclusion

This chapter discusses the distributed tracking control of multiple nonholonomic systems. Distributed tracking control laws are proposed with the aid of results for cascade systems and results from graph theory. Simulation results verify the proposed results. In this paper, we assume that the communications between systems are fixed. If the communication graph is time-varying, the proposed distributed tracking control laws also solve the defined control problem if the communication graph is bidirectional and strongly connected at each instant.

Chapter 5

Leader-Following Control of Multiple Nonholonomic Systems Over Directed Communication Graphs

5.1 Introduction

Distributed cooperative control of multiple systems has been extensively studied in the past decade. The focus of this control problem is on the cooperation between multiple systems by using neighbors' information. The interaction among systems and the integrity of systems are important. One of fundamental approaches to achieving group behavior through local information is to make a function of the state of each system agree on the same value (i.e., reach consensus). The problem on how to make multiple systems come into a consensus on some function of states is the so-called consensus problem. The consensus problem has many engineering applications in practice, such as formation control in [62], flocking behavior analysis in [45], attitude alignment in [63], etc.

In literature, both the leaderless consensus problem and the leader-following consensus problem have been studied based on whether or not there is a leader specifying the agreement value. In the early research work, the leaderless consensus problem had been studied for multiple first-order linear systems in [32] and [33]. Extensions of the results in [32] and [33] were presented in [34]. In [64] and [65], the leaderless consensus problem of multiple identical linear systems was considered and observer-based output feedback controllers were proposed for time-invariant communication topology. For the leaderless consensus problem of multiple linear systems with different dynamics paper [66] proposed observer-based distributed controllers by extending the results in [65]. For the leaderless consensus problem of multiple nonlinear systems, papers [35] and [67] proposed distributed controllers with the aid of the set-valued Lyapunov theory for multiple discrete-time systems and continuous time systems. In [40], the leaderless consensus problem of multiple nonlinear systems in feed-forward form was considered. Distributed controllers were proposed with the aid of backstepping techniques.

The leader-following consensus problem has been extensively studied in recent years. For multiple first-order linear systems with a leader, consensus algorithms were proposed with

the aid of distributed estimators in [41] and [42]. For multiple second-order linear systems with a leader, consensus algorithms were proposed in [43, 44, 68, 69] and [70] such that the tracking errors between systems converge to zero within finite time. For multiple high-order linear systems, the leader-following consensus problem was studied in [71–73] and [74]. The leader-following consensus problem can be considered as a distributed output regulation problem where a leader is considered as an exosystem. In [75], it was shown that the internal model principle is necessary and sufficient for the distributed output regulation problem of multiple linear systems. With the aid of internal model principle several dynamic control laws have been proposed in [76, 77] and [78].

Flocking and synchronization of multiple systems are closely related to the consensus problem. In [45], flocking of multiple second-order systems was solved with the aid of potential functions under the assumption that a desired trajectory is available to each system. In [47, 48, 79] and [80], flocking algorithms of multiple second-order linear systems were proposed under the assumption that the information of a virtual leader is available to a portion of systems. In [81], distributed algorithms were proposed for flocking of multiple robots with the aid of backstepping. In [49] and [50], synchronization of unknown nonlinear networked systems was considered. Distributed adaptive control laws were proposed with the aid of neural network approximation such that the tracking error is uniformly ultimately bounded (UUB). In [82], the synchronization problem of general dynamical networks was considered for directed and weakly connected network topology. Exponential control laws were proposed by using the Lyapunov functional method and the Kronecker product technique. Some other papers which are closed to this paper are [83–85].

Though extensive research has been carried out for the leader-following consensus problem of multiple linear and nonlinear systems, there is few research on the leader-following control problem of multiple nonholonomic systems. However, in practice many systems are nonholonomic systems and have underactuated nature. For example, wheeled mobile robots (WMRs) in [4], unmanned aerial vehicles (UAVs) in [86], etc. It is necessary and important to study the leader-following control problem of nonholonomic systems. In [87], the leader-following consensus problem of multiple chained system has been studied. Distributed controllers are proposed with the aid of the results on cascade systems. In this paper, we still consider this problem. A new approach is proposed with the aid of backstepping techniques. If the state of each system is measurable, distributed adaptive state feedback controllers are proposed with the aid of backstepping techniques and results from graph theory by using neighbors' local information. If the state of each system is not measurable, distributed adaptive output feedback controllers are proposed with the aid of observer design for each system. Simulation validates the proposed results. Compared to the results in [87], the contributions of our results are as follows:

- A new approach is proposed for distributed tracking control of nonlinear systems with the aid of backstepping techniques and the properties of Laplacian matrix of directed graphs.
- The proposed distributed adaptive controllers can estimate the unknown information on-line by local information. In [87] some information of the leader system should be known in advance for each follower system. Therefore, the controllers proposed in

this paper are intrinsically decentralized and are independent of the information of the leader system.

The remaining parts of this chapter are organized as follows. In Section 5.2, the problem considered in this paper is defined. In Section 5.3, distributed state feedback controllers are proposed. In Section 5.4, distributed output feedback controllers are proposed. In Section 5.5, an application of the proposed results is presented. The last section concludes this paper.

5.2 Problem Statement

Consider a group of m nonholonomic systems in the chained form. The motion of system j is described by

$$\dot{x}_{1j} = v_{1j}, \quad \dot{x}_{2j} = v_{2j}, \quad \dot{x}_{ij} = v_{1j}x_{i-1,j}, \quad 3 \leq i \leq n, \quad y_j = [x_{1j}, x_{nj}]^\top \quad (5.1)$$

where $x_{*j} = [x_{1j}, x_{2j}, \dots, x_{nj}]^\top$ is the state, v_{1j} and v_{2j} are control inputs, and y_j is the output. For each system the available information for feedback is its own information and its neighbors' information measured by sensors or received from wireless networks. If each system is considered as a node, the communication between systems can be described by a directed graph (digraph for short) $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \dots, m\}$ is a node set, and $\mathcal{E}$ is an edge set with ordered pair (i, j) which describes the communication from node i to node j . If the information of node i is available to node j , node i is called a neighbor of node j . The set of all neighbors of node j is denoted by $\mathcal{N}_j$. Node i is said to be reachable for node j if there exists a set of edges which connect node i to node j with respecting to their directions. Node i is said to be globally reachable if it is reachable from every other node in the graph $\mathcal{G}$.

Remark 5.1. *The model in (5.1) is one type of canonical forms of nonholonomic systems. Wheeled mobile robots with two control inputs can be described by (5.1) after state and input transformation in [3] and [4]. Some unmanned aerial vehicles can also be described by (5.1) (see [86]). In (5.1), the output y_j only has two states. The availability of these two states are the least requirement for the output feedback control problem considered in this article.*

It is given a leader (labeled as system 0) defined by

$$\dot{x}_{10} = v_{10}, \quad \dot{x}_{20} = v_{20}, \quad \dot{x}_{i0} = v_{10}x_{i-1,0}, \quad 3 \leq i \leq n, \quad y_0 = [x_{10}, x_{20}, \dots, x_{n0}]^\top \quad (5.2)$$

where v_{10} and v_{20} are known time-varying functions, $x_{*0} = [x_{10}, x_{20}, \dots, x_{n0}]^\top$ is the state, and y_0 is the output. We assume that the state of the leader is available to a portion of the m systems in (5.1). Consider system 0 in (5.2) and the m systems in (5.1) as a group of systems. The communication graph of the $m + 1$ systems is denoted by $\mathcal{G}^e$. The neighbor set of node j is denoted by $\mathcal{N}_j^e$ for $1 \leq j \leq m$. Since system 0 is the leader, $\mathcal{N}_0^e = \emptyset$.

The following two distributed control problems are considered in this paper.

Distributed State Feedback Control Problem: Design distributed control laws v_{1j} and v_{2j} for system j using its own state x_{*j} and its neighbors' states x_{*i} for $i \in \mathcal{N}_j^e$ such that

$$\lim_{t \rightarrow \infty} (x_{*j} - x_{*0}) = 0 \quad (5.3)$$

for $1 \leq j \leq m$.

Distributed Output Feedback Control Problem: Design distributed control laws v_{1j} and v_{2j} for system j using its own output y_j and its neighbors' outputs y_i for $i \in \mathcal{N}_j^e$ such that (5.3) holds for $1 \leq j \leq m$.

In order to solve the defined control problems, the following assumptions are made on the leader.

Assumption 5.1. Variable v_{10} is differentiable up to $(n-2)$ -th order. The $\frac{d^i v_{10}}{dt^i}$ ($0 \leq i \leq 2$) are bounded and

$$\int_t^{t+T_1} v_{10}^{2n-4}(\tau) d\tau \geq \alpha_1, \quad \int_t^{t+T_2} v_{10}^2(\tau) d\tau \geq \alpha_2, \quad \forall t \geq 0$$

where α_i and T_i ($1 \leq i \leq 2$) are positive constants.

Assumption 5.2. The vector $\bar{x}_{30} = [x_{30}, x_{40}, \dots, x_{n0}]^\top$ is bounded.

Assumption 5.1 means that the signal v_{10}^{n-2} and v_{10} are persistently excited signals (PE signals) (see [5] for the definition of a PE signal). This assumption is due to the fact that the considered systems are nonholonomic.

For a PE signal, the following lemma is useful.

Lemma 5.1. (Lemma 4.8.3 in [5]) Signals $\xi_1(t) \in R$ and $\xi_2(t) \in R$ are bounded. If ξ_1 is a PE signal and $(\xi_1 - \xi_2)$ converges to zero, then ξ_2 is a PE signal.

Lemma 5.2. For the system

$$\dot{\zeta} = -\eta_1^2(t)\zeta + \eta_2(t) \quad (5.4)$$

where $\eta_1(t) \in R$ is a bounded PE signal, if $\eta_2(t)$ is bounded and $\eta_2(t)$ converges to zero, then ζ converges to zero.

Proof: The solution of (5.4) is

$$\begin{aligned} \zeta(t) &= e^{\int_0^t -\eta_1^2(\tau) d\tau} \zeta(0) + \int_0^t e^{\int_\tau^t -\eta_1^2(\nu) d\nu} \eta_2(\tau) d\tau \leq e^{-\delta t + b_1} \zeta(0) + \int_0^t e^{-\delta(t-\tau) + b_2} \eta_2(\tau) d\tau \\ &= e^{b_1} e^{-\delta t} \zeta(0) + e^{b_2} \int_0^t e^{-\delta(t-\tau)} \eta_2(\tau) d\tau \end{aligned}$$

where $\delta > 0$, $b_1 > 0$, and $b_2 > 0$. Since $\eta_2(t)$ is bounded and converges to zero, $\int_0^t e^{-\delta(t-\tau)} \eta_2(\tau) d\tau$ converges to zero with the aid of the theorem in Section 4.1 in [88]. Therefore, ζ converges to zero. ■

For a digraph $\mathcal{G}$ with weight matrix $\mathcal{A} = [a_{ji}]_{m \times m}$ ($a_{ji} > 0$), its weighted Laplacian matrix $\mathcal{L} = [L_{ji}]$ is defined by

$$L_{ji} = \begin{cases} -a_{ji}, & \text{if } i \neq j \text{ and } i \in \mathcal{N}_j \\ \sum_{l \in \mathcal{N}_j} a_{jl}, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases} \quad (5.5)$$

On the weighted Laplacian matrix, the following results are useful.

Lemma 5.3. *For the digraph $\mathcal{G}$ with weight matrix $\mathcal{A} = [a_{ji}]$ ($a_{ji} > 0$) and a nonzero nonnegative vector $\xi \in \mathbb{R}^m$, if the information of ξ is globally reachable to the nodes in the digraph $\mathcal{G}$, then we have the following results:*

1. *the real part of the eigenvalues of the matrix $(\mathcal{L} + \text{diag}(\xi))$ are all positive;*
2. *there exists a diagonal positive definite matrix P such that $Q = P(\mathcal{L} + \text{diag}(\xi)) + (\mathcal{L} + \text{diag}(\xi))^\top P$ is a symmetric positive definite matrix;*
3. *for the system*

$$\dot{\chi} = -\mathcal{L}\chi - \text{diag}(\xi)(\chi - d\mathbf{1}) - M \text{sign}(\mathcal{L}\chi + \text{diag}(\xi)(\chi - d\mathbf{1})) \quad (5.6)$$

where $\mathbf{1} = [1, 1, \dots, 1]^\top$, if $d(t) \in R$ is a differentiable signal and $\max_{t \in [0, \infty)} |\dot{d}(t)| \leq M < \infty$, then $(\chi - d\mathbf{1})$ exponentially converges to zero.

Proof: 1. Item 1 has been proved in Lemma 1.6 in [89].

2. For item 2, with the aid of the result in Item 1 and Theorem 4.25 in [38] $(\mathcal{L} + \text{diag}(\xi))$ is a non-singular M -matrix and there exists a diagonal positive matrix $P = \text{diag}(P_1, P_2, \dots, P_m)$ such that $(P(\mathcal{L} + \text{diag}(\xi)) + (\mathcal{L} + \text{diag}(\xi))^\top P)$ is positive definite.

3. It is shown in Lemma 2.7 in [87]. ■

If the digraph $\mathcal{G}$ is bidirectional, the following results can be obtained.

Lemma 5.4. *For the digraph $\mathcal{G}$ with weight matrix $\mathcal{A} = [a_{ji}]$ ($a_{ji} = a_{ij} > 0$), if the graph $\mathcal{G}$ is bidirectional and connected and $\xi \in \mathbb{R}^m$ is a nonzero nonnegative vector, then we have the following results:*

1. *the matrix $(\mathcal{L} + \text{diag}(\xi))$ is a symmetric positive definite matrix;*
2. *for the system*

$$\begin{aligned} \dot{\chi} &= -\mathcal{L}\chi - \text{diag}(\xi)(\chi - d\mathbf{1}) - \text{diag}(\hat{M}_1, \dots, \hat{M}_m) \text{sign}(\mathcal{L}\chi + \text{diag}(\xi)(\chi - d\mathbf{1})) \\ \dot{\hat{M}}_j &= \gamma_j \left| \sum_{i \in \mathcal{N}_j} a_{ji}(\chi_j - \chi_i) + \xi_j(\chi_j - d) \right|, \quad 1 \leq j \leq m \end{aligned} \quad (5.8)$$

where $\gamma_j > 0$ ($1 \leq j \leq m$) and $\chi = [\chi_1, \chi_2, \dots, \chi_m]^\top$, if $d(t) \in R$ is a differentiable signal and $\max_{t \in [0, \infty)} |\dot{d}(t)| \leq M < \infty$, then $(\chi - d\mathbf{1})$ asymptotically converges to zero and $\hat{M}_j$ is bounded.

Proof: 1. Item 1 has been proved in [38].

2. Let $\tilde{\chi} = \chi - d\mathbf{1}$ and $q = [q_1, q_2, \dots, q_m]^\top = (\mathcal{L} + \text{diag}(\xi))\tilde{\chi}$, we choose a Lyapunov function candidate

$$V = \tilde{\chi}^\top (\mathcal{L} + \text{diag}(\xi))\tilde{\chi} + \sum_{j=1}^m \gamma_j^{-1} (M - \hat{M}_j)^2 \quad (5.9)$$

and differentiate it along the solutions of (5.7)-(5.8), one has

$$\begin{aligned} \dot{V} &= -2\tilde{\chi}^\top (\mathcal{L} + \text{diag}(\xi))^2 \tilde{\chi} - 2\tilde{\chi}^\top (\mathcal{L} + \text{diag}(\xi)) \text{diag}(\hat{M}_1, \dots, \hat{M}_m) \text{sign}(q) \\ &\quad - 2\tilde{\chi}^\top (\mathcal{L} + \text{diag}(\xi)) \mathbf{1} \dot{d} - 2 \sum_{j=1}^m \gamma_j^{-1} (M - \hat{M}_j) \dot{\hat{M}}_j \\ &= -2\tilde{\chi}^\top (\mathcal{L} + \text{diag}(\xi))^2 \tilde{\chi} - 2Mq^\top \text{sign}(q) - 2q^\top \mathbf{1} \dot{d} - 2 \sum_{j=1}^m \gamma_j^{-1} (M - \hat{M}_j) \dot{\hat{M}}_j \\ &\quad + 2q^\top \text{diag}(M - \hat{M}_1, \dots, M - \hat{M}_m) \text{sign}(q) \\ &\leq -2\tilde{\chi}^\top (\mathcal{L} + \text{diag}(\xi))^2 \tilde{\chi} - 2M \sum_{j=1}^m |q_j| + 2 \sum_{j=1}^m M |q_j| - 2 \sum_{j=1}^m \gamma_j^{-1} (M - \hat{M}_j) \dot{\hat{M}}_j \\ &\quad + 2 \sum_{j=1}^m (M - \hat{M}_j) |q_j| \\ &\leq -2\tilde{\chi}^\top (\mathcal{L} + \text{diag}(\xi))^2 \tilde{\chi} \leq 0 \end{aligned} \quad (5.10)$$

which means that V is bounded. So, $\tilde{\chi}$ and $\hat{M}_j$ are bounded. By integrating both sides of (5.10), it can be shown that $\tilde{\chi}$ is square-integrable. With the aid of Barbalat's lemma, $\tilde{\chi}$ converges to zero. So, $(\chi - d\mathbf{1})$ converges to zero. $\blacksquare$

5.3 Distributed State Feedback Controller Design

5.3.1 Variable transformation

In order to design distributed controllers, we first introduce a variable transform such that system (5.1) is transformed to a special form. For system j , we define the variables

$$z_{ij} = x_{ij} - \alpha_{ij}, \quad 1 \leq i \leq n \quad (5.11)$$

with the aid of the backstepping techniques, where

$$\alpha_{1j} = 0, \quad \alpha_{nj} = 0, \quad \alpha_{n-1,j} = -k_n v_{1j}^{2n-5} z_{nj}, \quad (5.12)$$

$$\alpha_{n-2,j} = -k_{n-1} v_{1j}^{2n-5} z_{n-1,j} - z_{nj} - (2n-5) k_n v_{1j}^{2n-7} z_{nj} \dot{v}_{1j} - k_n v_{1j}^{2n-6} \dot{z}_{nj} \quad (5.13)$$

$$\alpha_{n-i,j} = -k_{n-i+1} v_{1j}^{2n-5} z_{n-i+1,j} - z_{n-i+2,j} + \sum_{l=0}^{i-2} \frac{1}{v_{1j}} \frac{\partial \alpha_{n-i+1}}{\partial v_{1j}^{[l]}} v_{1j}^{[l+1]} + \sum_{l=0}^{i-2} \frac{\partial \alpha_{n-i+1}}{\partial z_{n-l,j}} \frac{\dot{z}_{n-l,j}}{v_{1j}} \quad (5.14)$$

$$i = 3, \dots, n-2$$

$k_i > 0$ and $v_{1j}^{[l]}$ denotes the l -th order derivative of v_{1j} with respect to time, then

$$\dot{z}_{1j} = v_{1j} \quad (5.15)$$

$$\dot{z}_{2j} = v_{2j} - \frac{d\alpha_{2j}}{dt} \quad (5.16)$$

$$\dot{z}_{3j} = -k_3 v_{1j}^{2n-4} z_{3j} + v_{1j}(z_{2j} - z_{4j}) \quad (5.17)$$

$\vdots$

$$\dot{z}_{n-1,j} = -k_{n-1} v_{1j}^{2n-4} z_{n-1,j} + v_{1j}(z_{n-2,j} - z_{nj}) \quad (5.18)$$

$$\dot{z}_{nj} = -k_n v_{1j}^{2n-4} z_{nj} + v_{1j} z_{n-1,j} \quad (5.19)$$

for $j = 0, 1, 2, \dots, m$.

Remark 5.2. Noting the special structure of α_{ij} , it can be proved that α_{ij} does not contain the term $\frac{1}{v_{1j}}$ after expanding each term in the transformation. Therefore, α_{ij} is well defined even if v_{1j} is zero at some time.

For the new variables, the following lemma can be proved and its proof is omitted.

Lemma 5.5. For the $(m+1)$ systems in (5.15)-(5.19), if

$$\lim_{t \rightarrow \infty} (z_{ij} - z_{i0}) = 0 \text{ and } \lim_{t \rightarrow \infty} (v_{1j}^{[l]} - v_{10}^{[l]}) = 0 \quad (5.20)$$

for $1 \leq i \leq n$, $0 \leq l \leq n-3$, and $1 \leq j \leq m$, then (5.3) hold.

With the aid of Lemma 5.5, design of controllers for the distributed state feedback control problem is equivalent to design of distributed controllers v_{1j} and v_{2j} for the m systems in (5.15)-(5.19) ($1 \leq j \leq m$) such that (5.20) is satisfied.

Noting the special structure of the systems in (5.15)-(5.19), the conditions in (5.20) can be refined in the following lemma.

Lemma 5.6. For the $(m+1)$ systems in (5.15)-(5.19), under Assumptions 5.1-5.2, if

$$\lim_{t \rightarrow \infty} (v_{1j}^{[l]} - v_{10}^{[l]}) = 0 \quad (5.21)$$

$$\lim_{t \rightarrow \infty} (z_{2j} - z_{20}) = 0 \quad (5.22)$$

for $1 \leq j \leq m$ and $0 \leq l \leq n-3$, then

$$\lim_{t \rightarrow \infty} (z_{ij} - z_{i0}) = 0 \quad (5.23)$$

for $3 \leq i \leq n$ and $1 \leq j \leq m$.

Proof: Let $\tilde{z}_{ij} = z_{ij} - z_{i0}$ for $2 \leq i \leq n$, then

$$\begin{aligned} \dot{\tilde{z}}_{3j} &= -k_3 v_{1j}^{2n-4} \tilde{z}_{3j} + v_{1j}(\tilde{z}_{2j} - \tilde{z}_{4j}) + k_3(v_{10}^{2n-4} - v_{1j}^{2n-4})z_{30} \\ &\quad + (v_{1j} - v_{10})(z_{20} - z_{40}) \end{aligned} \quad (5.24)$$

$\vdots$

$$\begin{aligned} \dot{\tilde{z}}_{n-1,j} &= -k_{n-1} v_{1j}^{2n-4} \tilde{z}_{n-1,j} + v_{1j}(\tilde{z}_{n-2,j} - \tilde{z}_{n0}) + k_{n-1}(v_{10}^{2n-4} - v_{1j}^{2n-4})z_{n-1,0} \\ &\quad + (v_{1j} - v_{10})(z_{n-2,0} - z_{n0}) \end{aligned} \quad (5.25)$$

$$\dot{\tilde{z}}_{nj} = -k_n v_{1j}^{2n-4} \tilde{z}_{nj} + v_{1j} \tilde{z}_{n-1,j} + k_n(v_{10}^{2n-4} - v_{1j}^{2n-4})z_{n0} + (v_{1j} - v_{10})z_{n-1,0} \quad (5.26)$$

Let a nonnegative function

$$V = \frac{1}{2} \sum_{i=3}^n \tilde{z}_{ij}^2$$

and differentiate it along the solution of the systems in (5.24)-(5.26), we have

$$\begin{aligned} \dot{V} = & - \sum_{l=3}^n k_l \tilde{z}_{lj}^2 v_{1j}^{2n-4} + \sum_{l=3}^n k_l \tilde{z}_{lj} z_{l0} (v_{10}^{2n-4} - v_{1j}^{2n-4}) + \sum_{l=3}^{n-1} \tilde{z}_{lj} (z_{l-1,0} - z_{l+1,0}) (v_{1j} - v_{10}) \\ & + \tilde{z}_{nj} z_{n-1,0} (v_{1j} - v_{10}) + \tilde{z}_{3j} \tilde{z}_{2j} v_{1j}. \end{aligned}$$

Noting the facts: (1). $\tilde{z}_{30} = [z_{30}, z_{40}, \dots, z_{n0}]^\top$ is bounded due to Assumption 5.2; (2). v_{1j} and v_{10} are bounded; and (3). eqns. (5.21)-(5.22) hold, we have

$$\dot{V} \leq - \sum_{l=3}^n \underline{k} \tilde{z}_{lj}^2 v_{1j}^{2n-4} + \sum_{l=3}^n |\tilde{z}_{lj}| f_1(t) = -2\underline{k} v_{1j}^{2n-4} V + \sum_{l=3}^n |\tilde{z}_{lj}| f_1(t) \leq -2\underline{k} v_{1j}^{2n-4} V + \sqrt{2V} f_1(t)$$

where $\underline{k} = \min_{3 \leq l \leq n} \{k_l\}$ and $f_1(t)$ is nonnegative and converges to zero. Let $V_1 = \sqrt{V}$, it can be proved that

$$\dot{V}_1 \leq -\underline{k} v_{1j}^{2n-4} V_1 + \frac{f_1(t)}{\sqrt{2}}.$$

Since $(v_{1j} - v_{10})$ satisfies (5.21) and v_{10} satisfies Assumption 5.1, v_{1j}^{2n-4} is a PE signal by Lemma 5.1. With the aid of the comparison lemma in [90] and Lemma 5.2, V_1 converges to zero. Therefore, V converges to zero. So, (5.23) holds. $\blacksquare$

Lemma 5.6 means that the convergence of z_{2j} to z_{20} and v_{1j} to v_{10} guarantees the convergence of z_{ij} to z_{i0} for $3 \leq j \leq m$. Therefore, with the aid of Lemmas 5.5-5.6 the distributed state feedback control problem is solved if distributed control laws v_{1j} and v_{2j} can be designed such that (5.21)-(5.22) hold and

$$\lim_{t \rightarrow \infty} (z_{1j} - z_{10}) = 0 \quad (5.27)$$

for $1 \leq j \leq m$.

5.3.2 State Feedback Controller Design

We design control law v_{1j} first. Noting that $v_{1j}^{[l]}$ for $0 \leq l \leq n-3$ are required in α_{ij} , v_{1j} should be differentiable up to $(n-2)$ -th order. To this end, we propose dynamic controllers as follows.

Lemma 5.7. *For the $(m+1)$ systems in (5.15), if the state of system 0 is globally reachable to other systems, then the distributed control laws*

$$v_{1j} = -\beta_1 z_{1j} + \xi_{1j} \quad (5.28)$$

$$\dot{\xi}_{1j} = -\beta_2 \xi_{1j} - z_{1j} + \xi_{2j} \quad (5.29)$$

$\vdots$

$$\dot{\xi}_{n-3,j} = -\beta_{n-2} \xi_{n-3,j} - \xi_{n-4,j} + \xi_{n-2,j} \quad (5.30)$$

$$\dot{\xi}_{n-2,j} = -s_{1j} - \rho_1 \operatorname{sign}(s_{1j}) \quad (5.31)$$

for $1 \leq j \leq m$ ensure that (5.21) and (5.27) hold, where β_i ($1 \leq i \leq n-2$) are positive constants,

$$s_{1j} = \sum_{i \in \mathcal{N}_j} a_{ji}(\xi_{n-2,j} - \xi_{n-2,i}) + b_j \mu_j (\xi_{n-2,j} - \xi_{n-2,0}) \quad (5.32)$$

$$\mu_j = \begin{cases} 1, & \text{if the state of system 0 is available to system } j; \\ 0, & \text{otherwise} \end{cases} \quad (5.33)$$

$a_{ji} > 0$, $b_j > 0$, and ρ_1 is a sufficiently large number.

Proof: Let $\xi_{n-2,*} = [\xi_{n-2,1}, \xi_{n-2,2}, \dots, \xi_{n-2,m}]^\top$, one has

$$\dot{\xi}_{n-2,*} = -\mathcal{L}\xi_{n-2,*} - B(\xi_{n-2,*} - \xi_{n-2,0}\mathbf{1}) - \rho_1 \text{sign}(\mathcal{L}\xi_{n-2,*} + B(\xi_{n-2,*} - \xi_{n-2,0}\mathbf{1})) \quad (5.34)$$

where $B = \text{diag}(b_1\mu_1, b_2\mu_2, \dots, b_m\mu_m)$. If ρ_1 is chosen such that

$$\rho_1 \geq \max_{t \in [0, \infty)} |\dot{\xi}_{n-2,0}(t)|, \quad (5.35)$$

by Lemma 5.3 $\xi_{n-2,j}$ exponentially converges to $\xi_{n-2,0}$ for $1 \leq j \leq m$.

Next, we show that (5.21) and (5.27) hold for $1 \leq j \leq m$. Define $\tilde{z}_{1j} = z_{1j} - z_{10}$ and $\tilde{\xi}_{ij} = \xi_{ij} - \xi_{i0}$ for $1 \leq i \leq n-3$, one has

$$\dot{\tilde{z}}_{1j} = -\beta_1 \tilde{z}_{1j} + \tilde{\xi}_{1j} \quad (5.36)$$

$$\dot{\tilde{\xi}}_{1j} = -\beta_2 \tilde{\xi}_{1j} - \tilde{z}_{1j} + \tilde{\xi}_{2j} \quad (5.37)$$

$$\begin{aligned} & \vdots \\ \dot{\tilde{\xi}}_{n-3,j} &= -\beta_{n-2} \tilde{\xi}_{n-3,j} - \tilde{\xi}_{n-4,j} + \tilde{\xi}_{n-2,j} \end{aligned} \quad (5.38)$$

Let

$$V_2 = \frac{1}{2} \sum_{i=1}^{n-3} \tilde{\xi}_{ij}^2 + \frac{1}{2} \tilde{z}_{1j}^2 \quad (5.39)$$

differentiate V_2 along the solution of (5.36)-(5.38), one has

$$\dot{V}_2 = -\beta_1 \tilde{z}_{1j}^2 - \sum_{i=1}^{n-3} \beta_{i+1} \tilde{\xi}_{ij}^2 + \tilde{\xi}_{n-3,j} \tilde{\xi}_{n-2,j} \leq -2\underline{\beta} V_2 + |\tilde{\xi}_{n-2,j}| \sqrt{2V_2} \quad (5.40)$$

where $\underline{\beta} = \min_{1 \leq i \leq n-2} \{\beta_i\}$. Let $V_3 = \sqrt{V_2}$, then

$$\dot{V}_3 \leq -\underline{\beta} V_3 + \frac{|\tilde{\xi}_{n-2,j}|}{\sqrt{2}}.$$

With the aid of the comparison lemma [90] and Lemma 5.2, V_3 exponentially converges to zero. Therefore, V_2 exponentially converges to zero. Hence, (5.21) and (5.27) hold for $1 \leq j \leq m$. ■

For the control input v_{2j} , the following distributed control laws are proposed.

Lemma 5.8. *For $(m + 1)$ systems in (5.16), if the state of system 0 is globally reachable to other systems, then the distributed control laws*

$$v_{2j} = -s_{2j} - \rho_2 \operatorname{sign}(s_{2j}) + \dot{\alpha}_{2j} \quad (5.41)$$

for $1 \leq j \leq m$ ensure that (5.22) holds, where

$$s_{2j} = \sum_{i \in \mathcal{N}_j} a_{ji}(z_{2j} - z_{2i}) + b_j \mu_j(z_{2j} - z_{20}) \quad (5.42)$$

and ρ_2 is a sufficiently large number.

Proof: Let $z_{2*} = [z_{21}, z_{22}, \dots, z_{2m}]^\top$, one has

$$\dot{z}_{2*} = -\mathcal{L}z_{2*} - B(z_{2*} - \dot{z}_{20}\mathbf{1}) - \rho_2 \operatorname{sign}(\mathcal{L}z_{2*} - B(z_{2*} + \dot{z}_{20}\mathbf{1})) \quad (5.43)$$

If ρ_2 is chosen such that

$$\rho_2 \geq \max_{t \in [0, \infty)} |\dot{z}_{20}(t)|, \quad (5.44)$$

by Lemma 5.3 z_{2j} exponentially converges to z_{20} for $1 \leq j \leq m$. ■

With the aid of Lemmas 5.5-5.8, we have the following results.

Theorem 5.1. *For m systems in (5.1) and a leader in (5.2) with Assumptions 5.1-5.2, if the state of the leader is globally reachable to other systems, then the distributed state feedback control laws v_{1j} in (5.28)-(5.31) and v_{2j} in (5.41) ensure that (5.3) holds.*

Proof: By Lemma 5.7, the control laws v_{1j} in (5.28)-(5.31) for $1 \leq j \leq m$ ensure that (5.21) and (5.27) hold. By Lemma 5.8, the control laws v_{2j} in (5.41) ensure that (5.22) holds. By Lemmas 5.5-5.6, (5.3) holds. ■

In Theorem 5.1, ρ_1 and ρ_2 should be chosen such that (5.35) and (5.44) are satisfied. Since $\dot{\xi}_{n-2,0}$ and $\dot{z}_{20}$ are not available to each system, the lower bounds in (5.35) and (5.44) are not available to each system. Therefore, ρ_1 and ρ_2 should be chosen large enough. It is possible to estimate ρ_1 and ρ_2 online by each system with the aid of neighbors' states as in the following theorem.

Theorem 5.2. *For m systems in (5.1) and a leader in (5.2) with Assumptions 5.1-5.2, if the communication graph $\mathcal{G}$ is bidirectional and connected and the state of the leader is available to at least one of the m systems, then the distributed state feedback control laws*

$$v_{1j} = -\beta_1 z_{1j} + \xi_{1j}. \quad (5.45)$$

$$v_{2j} = -s_{2j} - \rho_{2j} \operatorname{sign}(s_{2j}) + \dot{\alpha}_{2j} \quad (5.46)$$

$$\dot{\xi}_{1j} = -\beta_2 \xi_{1j} - z_{1j} + \xi_{2j} \quad (5.47)$$

$\vdots$

$$\dot{\xi}_{n-3,j} = -\beta_{n-2} \xi_{n-3,j} - \xi_{n-4,j} + \xi_{n-2,j} \quad (5.48)$$

$$\dot{\xi}_{n-2,j} = -s_{1j} - \rho_{1j} \operatorname{sign}(s_{1j}) \quad (5.49)$$

$$\dot{\rho}_{1j} = \gamma_{1j} |s_{1j}| \quad (5.50)$$

$$\dot{\rho}_{2j} = \gamma_{2j} |s_{2j}| \quad (5.51)$$

ensure that (5.3) holds and ρ_{1j} and ρ_{2j} are bounded, where $\beta_i > 0$, $\gamma_{1j} > 0$, $\gamma_{2j} > 0$, s_{1j} and s_{2j} are defined in (5.32) and (5.42), respectively, and $a_{ji} = a_{ij} > 0$.

Proof: Let $\xi_{n-2,*} = [\xi_{n-2,1}, \xi_{n-2,2}, \dots, \xi_{n-2,m}]^\top$, one has

$$\begin{aligned} \dot{\xi}_{n-2,*} &= -\mathcal{L}\xi_{n-2,*} - B(\xi_{n-2,*} - \xi_{n-2,0}\mathbf{1}) \\ &\quad - \text{diag}(\rho_{11}, \dots, \rho_{1m})\text{sign}(\mathcal{L}\xi_{n-2,*} + B(\xi_{n-2,*} - \xi_{n-2,0}\mathbf{1})) \end{aligned} \quad (5.52)$$

For the systems in (5.52) and (5.50), by Lemma 5.4 $\xi_{n-2,j}$ converges to $\xi_{n-2,0}$ and ρ_{1j} is bounded for $1 \leq j \leq m$.

By following the proof of Lemma 5.7, it can be shown that (5.21) and (5.27) hold for $1 \leq j \leq m$. Similarly, we can show that (5.22) holds. By Lemmas 5.5-5.6, (5.3) holds. ■

In Theorem 5.1, the communication graph is assumed to be fixed. If the communication topology is switching, we have the following results.

Theorem 5.3. *For m systems in (5.1) and a leader in (5.2) with Assumptions 5.1-5.2, if the communication graph $\mathcal{G}$ is bidirectional and connected and the state of the leader is available to at least one of the m systems at each instant, then the distributed state feedback control laws v_{1j} in (5.28)-(5.31) and v_{2j} in (5.41) ensure that (5.3) holds.*

Theorem 5.3 can be proved by following the proofs of Lemmas 5.7-5.8 and Theorem 5.1. However, since the communication topology is switching the proof should be revised with the aid of the fact that $\mathcal{L}$ is symmetric. For brevity of the paper, the proof is omitted.

In the above theorems, in the control laws for system j the required information are the state of system j and the state of system i for $i \in \mathcal{N}_j$. The state of the leader is not required to be known to each system. In contrast to the leader-following approach in [23], the leader-following communication pattern has been pre-defined before the controller design. In Theorem 5.1, there is no pre-defined patterns for the communication between systems. Instead, the requirement is that the m systems are strongly connected and the state of the leader is available to at least one of the m systems. Moreover, the requirement on the communication digraph can be relaxed to the the state of the leader system is accessible to each system.

In [87], the tracking control of multiple chained systems with a leader was considered. Distributed tracking controllers were proposed with the aid of the results of linear-time varying systems. In this paper, we consider the same control problem and propose new distributed tracking controllers with the aid of a new special structure of the transformed systems and adaptive control theory. In this paper, the proposed distributed tracking controllers are adaptive ones and can estimate the bound of unknown information of the leader system online by local information. While in [87] the bound of of some information of the leader system should be known in advance.

5.4 Distributed Output Feedback Controller Design

If states x_{ij} for $2 \leq i \leq n-1$ are not measurable, we design an estimator for system j . The following observer can be proposed with the aid of the results in [18].

Lemma 5.9. *For system j , the observer is proposed as*

$$\dot{\hat{x}}_{2j} = v_{2j} + l_{m-1}v_{1j}^{mod(m-2,2)}(x_{2n} - \hat{x}_{2n}) \quad (5.53)$$

$$\dot{\hat{x}}_{3j} = v_{1j}\hat{x}_{2j} + l_{m-2}v_{1j}^{mod(m-3,2)}(x_{2n} - \hat{x}_{2n}) \quad (5.54)$$

$\vdots$

$$\dot{\hat{x}}_{n-1,j} = v_{1j}\hat{x}_{n-2,j} + l_2v_{1j}(x_{2n} - \hat{x}_{2n}) \quad (5.55)$$

$$\dot{\hat{x}}_{nj} = v_{1j}\hat{x}_{n-1,j} + l_1(x_{2n} - \hat{x}_{2n}) \quad (5.56)$$

where l_i ($1 \leq i \leq m-1$) are chosen such that the polynomial

$$\lambda^{m-1} + l_1\lambda^{m-2} + \dots + l_{m-2}\lambda + l_{m-1}$$

is Hurwitz (i.e. have their roots in the left half of the open complex plane) and $mod(k, 2)$ denotes the fractional part of $\frac{k}{2}$. If v_{1j} is a PE signal, then $(x_{ij} - \hat{x}_{ij})$ for $2 \leq i \leq n$ exponentially converge to zero.

With the aid of the observer in (5.53)-(5.56), we design distributed controllers by following the controller design procedure in the last section. For simplicity of notation, we let $\hat{x}_{i0} = x_{i0}$ for $2 \leq i \leq n$. Define the variables

$$z_{1j} = x_{1j} - \alpha_{1j}, \quad z_{2j} = \hat{x}_{2j} - \alpha_{2j}, \dots, \quad z_{nj} = \hat{x}_{nj} - \alpha_{nj} \quad (5.57)$$

where α_{ij} ($1 \leq i \leq n$, $0 \leq j \leq m$) are defined in (5.12)-(5.14), we have the following equations

$$\dot{z}_{1j} = v_{1j} \quad (5.58)$$

$$\dot{z}_{2j} = v_{2j} - \frac{d\alpha_{2j}}{dt} + l_{m-1}v_{1j}^{mod(m-2,2)}(x_{nj} - \hat{x}_{nj}) \quad (5.59)$$

$$\dot{z}_{3j} = -k_3v_{1j}^{2n-4}z_{3j} + v_{1j}(z_{2j} - z_{4j}) + l_{m-2}v_{1j}^{mod(m-3,2)}(x_{nj} - \hat{x}_{nj}) \quad (5.60)$$

$\vdots$

$$\dot{z}_{n-1,j} = -k_{n-1}v_{1j}^{2n-4}z_{n-1,j} + v_{1j}(z_{n-2,j} - z_{nj}) + l_2v_{1j}(x_{nj} - \hat{x}_{nj}) \quad (5.61)$$

$$\dot{z}_{nj} = -k_nv_{1j}^{2n-4}z_{nj} + v_{1j}z_{n-1,j} + l_1(x_{nj} - \hat{x}_{nj}) \quad (5.62)$$

for $j = 0, 1, 2, \dots, m$.

For the new variables in (5.57), it can be proved that Lemmas 5.5-5.6 hold. Since x_{1j} is available for controller design, the controllers v_{1j} ($1 \leq j \leq m$) can be designed as these in Lemma 5.7. Next, we propose distributed controllers v_{2j} ($1 \leq j \leq m$).

Lemma 5.10. *For $(m+1)$ systems in (5.59)-(5.62), if the state of system 0 is globally reachable to other systems, then the distributed output feedback control laws v_{2j} defined in (5.41) ensure that (5.22) holds, where in (5.41) ρ_2 is a sufficiently large number and s_{2j} is defined in (5.42) with z_{2j} defined in (5.57).*

Proof: With the control laws v_{1j} in Lemma 5.5, v_{1j} exponentially converges to v_{10} . By Lemma 5.1 and Assumption 5.1, v_{1j} is a PE signal. By Lemma 5.9, $(x_{nj} - \hat{x}_{nj})$ exponentially converges to zero. By the control law v_{2j} , one has

$$\dot{z}_{2j} = -s_{2j} - \rho_2 \operatorname{sign}(s_{2j}) + l_{m-1} v_{1j}^{\operatorname{mod}(m-2,2)} (x_{nj} - \hat{x}_{nj}) \quad (5.63)$$

If $(x_{nj} - \hat{x}_{nj}) = 0$ and ρ_2 is chosen such that

$$\rho_2 \geq \max_{t \in [0, \infty)} |\dot{z}_{20}(t)|$$

by Lemma 5.3 z_{2j} exponentially converges to z_{20} . With the aid of the inverse Lyapunov theorem, it can be proved that z_{2j} exponentially converges to zero. ■

With the aid of Lemmas 5.7 and 5.10, we have the following results.

Theorem 5.4. *For m systems in (5.1) and a leader in (5.2) with Assumptions 5.1-5.2, if the state of the leader is globally reachable to other systems, then the distributed output feedback control laws v_{1j} in (5.28)-(5.31) and v_{2j} in (5.41) ensure that (5.3) holds, where ρ_1 and ρ_2 are sufficiently large numbers and s_{2j} is defined in (5.42) with z_{2j} defined in (5.57).*

Proof: By Lemma 5.7, the control laws v_{1j} in (5.28) for $1 \leq j \leq m$ ensure that (5.21) and (5.27) hold. By Lemma 5.8, the control laws v_{2j} in (5.41) ensure that (5.22) holds. By Lemmas 5.5 and 5.6, (5.3) holds. ■

In Theorem 5.4, ρ_1 and ρ_2 can be estimated on-line by each system with the aid of neighbors' outputs as in the following theorem.

Theorem 5.5. *For m systems in (5.1) and a leader in (5.2) with Assumptions 5.1-5.2, if the communication graph $\mathcal{G}$ is bidirectional and connected and the state of the leader is available to at least one of the m systems, then the distributed output feedback control laws v_{1j} in (5.45) and v_{2j} in (5.46)-(5.49) with the observer in (5.53)-(5.56) and the updated laws ρ_{1j} and ρ_{2j} in (5.50)-(5.51) ensure that (5.3) holds and ρ_{1j} and ρ_{2j} are bounded, where $\beta_i > 0$, $\gamma_{1j} > 0$, $\gamma_{2j} > 0$, s_{1j} and s_{2j} are defined in (5.32) and (5.42) where z_{2j} defined in (5.57), respectively, and $a_{ji} = a_{ij} > 0$.*

If the communication topology is switching, we have the following results.

Theorem 5.6. *For m systems in (5.1) and a leader in (5.2) with Assumptions 5.1-5.2, if the communication graph $\mathcal{G}$ is bidirectional and connected and the state of the leader is available to at least one of the m systems at each instant, then the distributed output feedback control laws v_{1j} in (5.28) and v_{2j} in (5.41) ensure that (5.3) holds, where ρ_1 and ρ_2 are sufficiently large numbers and s_{2j} is defined in (5.42) with z_{2j} defined in (5.57).*

Theorems 5.5 and 5.6 can be proved similarly as the proofs of Theorems 5.2 and 5.4. For brevity of the paper, the proof is omitted.

5.5 An Application

To show the effectiveness of the proposed results, an application of the proposed results to formation control is considered. It is assumed that there are five identical nonholonomic car-like robots moving on a horizontal plane. The structure and configuration of each robot is shown in Fig. 5.1. The kinematics of robot j is

$$\dot{x}_j = v_j \cos(\theta_j + \varphi_j), \dot{y}_j = v_j \sin(\theta_j + \varphi_j), \dot{\theta}_j = v_j \sin \varphi_j / l_j, \dot{\varphi}_j = \omega_j \quad (5.64)$$

where v_j and ω_j are the driving and steering velocity inputs, respectively. The communication between five systems is defined by a digraph $\mathcal{G}$. It is given a desired formation $\mathcal{F}$ defined by coordinates (p_{xj}, p_{yj}) ($1 \leq j \leq 5$) which satisfy $\sum_{j=1}^5 p_{xj} = 0$ and $\sum_{j=1}^5 p_{yj} = 0$ and a reference system defined by

$$\dot{x}_0 = v_0 \cos(\theta_0 + \varphi_0), \dot{y}_0 = v_0 \sin(\theta_0 + \varphi_0), \dot{\theta}_0 = v_0 \sin \varphi_0 / l_0, \dot{\varphi}_0 = \omega_0 \quad (5.65)$$

where v_0 and ω_0 are known time-varying functions. The state of the reference system is available to a portion of the five robots. We consider the following formation control problem.

Formation Control Problem: Design distributed control laws v_j and ω_j for system j using its own state information and its neighbor's state information such that

$$\lim_{t \rightarrow \infty} (x_j - x_i) = p_{xj} - p_{xi}, \quad \lim_{t \rightarrow \infty} (y_j - y_i) = p_{yj} - p_{yi}, \quad 1 \leq j \neq i \leq 5 \quad (5.66)$$

$$\lim_{t \rightarrow \infty} \left(\frac{1}{5} \sum_{j=1}^5 x_j - x_0 \right) = 0, \quad \lim_{t \rightarrow \infty} \left(\frac{1}{5} \sum_{j=1}^5 y_j - y_0 \right) = 0 \quad (5.67)$$

In the formation control problem, (5.66) means that the five robots come into formation and (5.67) means that the centroid of the five robots asymptotically tracks the position of the reference system.

In order to solve the formation problem with the aid of the proposed results, we transform the formation control problem to the control problem considered in this paper. To this end, we define

$$\begin{cases} x_{1j} &= \theta_j + \varphi_j \\ x_{2j} &= (x_j - p_{xj}) \cos(\theta_j + \varphi_j) + (y_j - p_{yj}) \sin(\theta_j + \varphi_j) \\ x_{3j} &= (x_j - p_{xj}) \sin(\theta_j + \varphi_j) - (y_j - p_{yj}) \cos(\theta_j + \varphi_j) \\ v_{1j} &= \sin \varphi_j v_j / l_j + \omega_j \\ v_{2j} &= v_j - \omega_j x_{3j} \end{cases} \quad (5.68)$$

for $0 \leq j \leq m$. Then, we have

$$\dot{x}_{1j} = v_{1j}, \quad \dot{x}_{2j} = v_{2j}, \quad \dot{x}_{3j} = v_{1j} x_{2j}, \quad 0 \leq j \leq 5. \quad (5.69)$$

It can be shown that for the systems in (5.69) if (5.3) holds the formation control problem is solved.

In the simulation, it is assumed that in the reference system (i.e., $j = 0$ in (5.69)) $v_{10} = 0.5$, $v_{20} = -0.25 \sin(0.5t)$, and the initial condition $x_{*0} = [0, 0.5, 0]^T$, which means that the state of the reference system is $(x_{10}, x_{20}, x_{30}) = (0.5t, 0.5 \cos(0.5t), 0.5 \sin(0.5t))$. So,

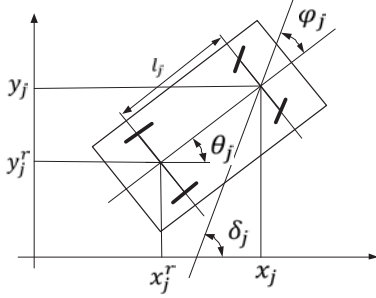


Figure 5.1: Configuration of a car-like robot

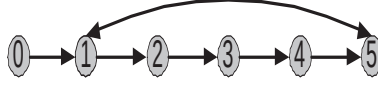


Figure 5.2: Communication graph $\mathcal{G}$

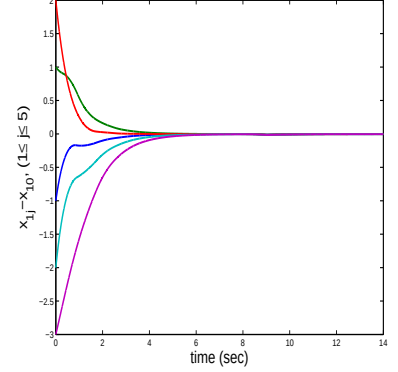


Figure 5.3: Response of $(x_{1j} - x_{10})$ for $1 \leq j \leq 5$.

Assumptions 5.1-5.2 are satisfied. We assume that the communication digraph is shown in Fig. 5.2. The cooperative controllers can be obtained with the aid of Theorem 5.1. We choose the control parameters $a_{ji} = 2$, $k_3 = 2$, $b_j = 2$, $\rho_1 = 2$, and $\rho_2 = 2$. Fig. 5.3 shows the response of $(x_{j1} - x_{10})$ ($1 \leq j \leq 5$). Fig. 5.4 shows the response of $(x_{2j} - x_{20})$ ($1 \leq j \leq 5$). Fig. 5.5 shows the response of $(x_{3j} - x_{30})$ ($1 \leq j \leq 5$). Figs 5.6 and 5.7 show the response of $(\frac{1}{5} \sum_{j=1}^5 x_j - x_0)$ and $(\frac{1}{5} \sum_{j=1}^5 y_j - y_0)$. The simulation results show that the formation control problem can be solved with the aid of the the proposed results in Theorem 5.1.

If the communication graph is time-varying and switches according to the following logic

$$\mathcal{G} = \begin{cases} \mathcal{G} \text{ in Fig. 5.8,} & \text{if } t - \text{round}(t) \geq 0 \\ \mathcal{G} \text{ in Fig. 5.9,} & \text{if } t - \text{round}(t) < 0 \end{cases}$$

the control laws in Theorem 5.3 solve the formation control problem. Fig. 5.10 shows the response of $(x_{j1} - x_{10})$ ($1 \leq j \leq 5$). Fig. 5.11 shows the response of $(x_{2j} - x_{20})$ ($1 \leq j \leq 5$). Fig. 5.12 shows the response of $(x_{3j} - x_{30})$ ($1 \leq j \leq 5$). The response of $(\sum_{j=1}^5 \frac{x_j}{5} - x_0)$ is shown in Fig. 5.13. The response of $(\sum_{j=1}^5 \frac{y_j}{5} - y_0)$ is shown in Fig. 5.14. The simulation results show that the formation control problem can be solved with the aid of the proposed distributed control laws in Theorem 5.3.

5.6 Conclusion

This chapter has discussed the leader-following control problem of multiple nonholonomic systems for time-invariant and time-varying communication graphs. With the aid of the properties of persistently excited signals and results from graph theory distributed state feedback control laws and output feedback control laws were proposed. An application of the proposed results to formation control of wheeled mobile robots was considered. Simulation results show the effectiveness of the proposed control laws.

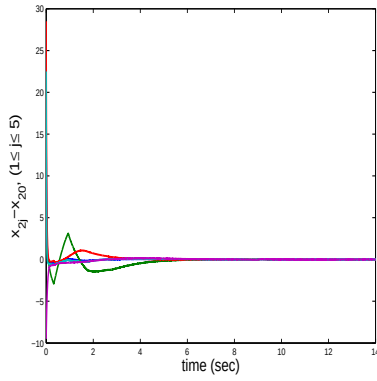


Figure 5.4: Response of $(x_{2j} - x_{20})$ for $1 \leq i \leq 5$.

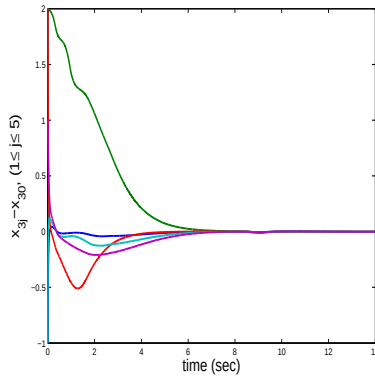


Figure 5.5: Response of $(x_{3j} - x_{30})$ for $1 \leq i \leq 5$.

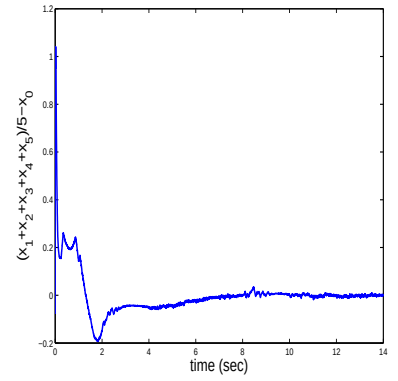


Figure 5.6: Response of $(\frac{1}{5} \sum_{j=1}^5 x_j - x_0)$.

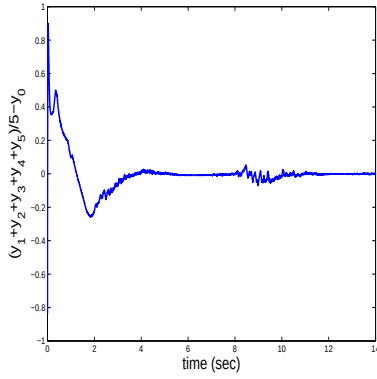


Figure 5.7: Response of $(\frac{1}{5} \sum_{j=1}^5 y_j - y_0)$.

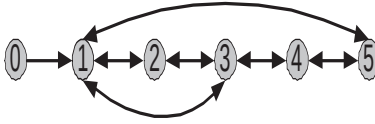


Figure 5.8: Communication graph $\mathcal{G}$

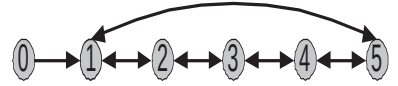


Figure 5.9: Communication graph $\mathcal{G}$

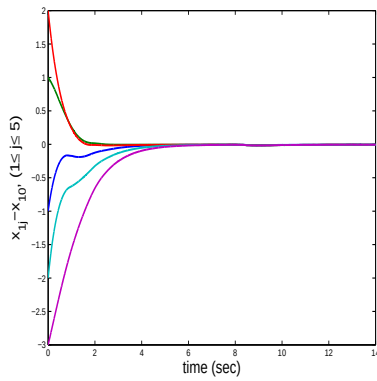


Figure 5.10: Response of $(x_{1j} - x_{10})$ for $1 \leq i \leq 5$.

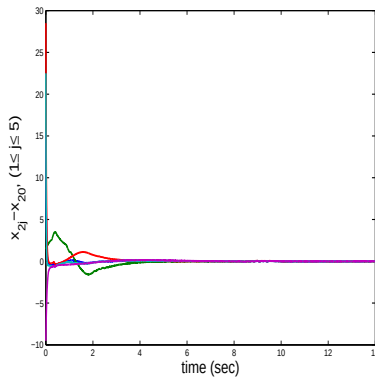


Figure 5.11: Response of $(x_{2j} - x_{20})$ for $1 \leq i \leq 5$.

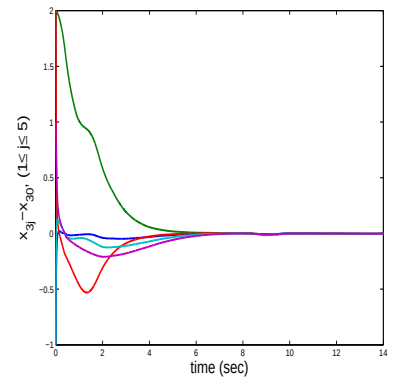


Figure 5.12: Response of $(x_{3j} - x_{30})$ for $1 \leq i \leq 5$.

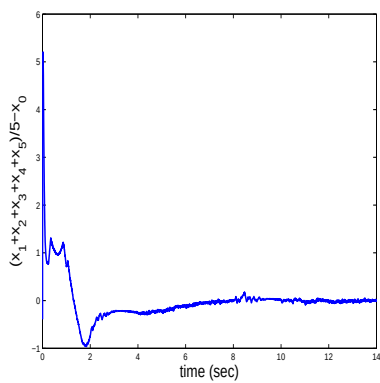


Figure 5.13: Response of $(\sum_{j=1}^5 \frac{x_j}{5} - x_0)$.

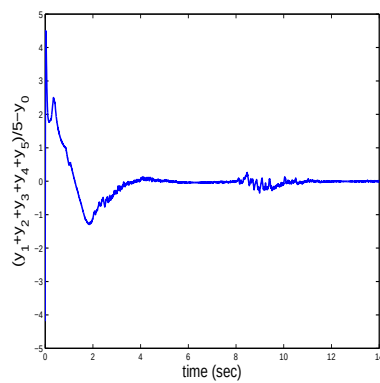


Figure 5.14: Response of $(\sum_{j=1}^5 \frac{y_j}{5} - y_0)$.

Chapter 6

Distributed Exponentially Tracking Control of Multiple Wheeled Mobile Robots

6.1 Introduction

There are lots of applications of wheeled mobile robots in practice and control strategies have been developed for decades. Wheeled mobile robots with nonslip constraints are typical nonholonomic systems. Stabilization of a single nonholonomic system is challenging due to the fact that no smooth pure state feedback control law exists for stabilization. Moreover, no dynamic continuous time-invariant feedback controller is available to render the closed loop system locally asymptotically stable [91]. With the efforts of researchers, several approaches have been proposed for stabilizing nonholonomic systems, which can be classified into the following three aspects, namely, discontinuous time-invariant feedback, the time-varying feedback, and the hybrid feedback. For details, see [92] and the references therein.

It is challenging to design tracking controllers for wheeled mobile robots because of its nonlinear feature. Samson and Ait-Abderrahim proposed the first tracking controller for a mobile robot in [16]. Then a tracking controller was designed through linear approximation for nonholonomic systems in [93]. In [94], the tracking task of wheeled mobile robots was fulfilled by linearizing both static and dynamic feedback. Fliess et al solved the tracking problem utilizing results of “differentially flat” nonlinear systems [95]. With the aid of backstepping techniques, semi-global tracking controllers were proposed for a chained-form system in [17]. In [96], global state and output tracking controllers were proposed for chained-form systems with the aid of Lyapunov techniques. Based on the results of cascade systems, linear tracking controllers were proposed for chained-form systems in [18, 97].

Due to the practical requirement of specific tasks, the consensus problem without a leader has been extensively studied in the past decades. In [32], matrix theory was applied to propose local control laws for first-order linear discrete-time systems such that the states of multiple systems converge to a constant value. In [33], Laplacian matrix of a communication graph was exploited to propose local control laws for the consensus problem of multiple first-order linear continuous-time systems. In [34], consensus algorithms were proposed with

relaxed assumption on the communication graphs in [33]. Consensus problem with a leader has also been studied systematically and several control laws have been proposed. In [43], consensus problems of first-order and second-order linear systems were considered. Distributed controllers were proposed such that the state of each system converge to a desired trajectory within finite time under the condition that the desired trajectory is available to a portion of the group of systems. In [72], leader-following consensus of high-order linear systems was considered over a switching communication topology. Distributed controllers were proposed with the aid of Riccati-inequality-based approach. In [98], consensus control of multi-agent systems was considered. Output feedback controllers were proposed with the aid of H_∞ theory. In [64], consensus of high-order linear systems was considered for time-varying and directed communication topologies. Distributed controllers were proposed with the aid of the observer design approach. In [65, 66], consensus of multiple linear systems was considered in a unified viewpoint and a notation of consensus region was introduced. In [73], the leader-following consensus problem for a group of agents with identical linear systems subject to control input saturation was considered. Linear feedback laws were proposed for fixed and switching communication topology. In [68, 69], consensus of first-order and second-order nonlinear systems was considered. Finite-time control laws were proposed with the aid of a comparison lemma. In [99], cooperative control of multiple mobile robots was considered. Distributed control laws were proposed with the aid of consensus approach.

In this chapter, we study distributed formation control of multiple nonholonomic wheeled mobile robots with a leader whose state is not available to each system such that the group of robots converges to a desired geometric pattern whose centroid follows the leader. New distributed control laws are proposed based on the results of cascade systems and the properties of persistently exciting signals. Compared to the results in [64–66, 72, 73, 98], in this paper cooperative tracking control is solved for multiple nonlinear systems. Compared to the results in [99], a new approach is proposed for cooperative tracking control problem of multiple wheeled mobile robots and the proposed control laws can make the tracking errors uniformly exponentially converge to zero, which is much more applicable in practice.

The remaining parts of this chapter are organized as follows. In Section 2, the considered problem is formulated and some preliminary results are presented. In Section 3, distributed tracking controllers are proposed. In Section 4, controllers are proposed for switching communication graphs. In Section 5, simulation results are presented. The last section concludes this paper.

6.2 Problem Statement

It is considered a group of m wheeled mobile robots which move on a horizontal plane. The motion of robot j is described by

$$\dot{x}_j = v_j \cos \theta_j, \quad \dot{y}_j = v_j \sin \theta_j, \quad \dot{\theta}_j = \omega_j \quad (6.1)$$

where (x_j, y_j) is the position of robot j in a coordinate system, θ_j is the orientation of robot j , v_j is the speed of robot j , and ω_j is angular speed of robot j . The control inputs are v_j and ω_j .

For m system, each system knows its own state and its neighbors' states by communication and/or sensors. For simplicity, it is assumed that the communications between the systems are bidirectional. If we consider each system as a node, the communication between the systems can be described by a (undirected) graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \dots, m\}$ is a node set, and $\mathcal{E}$ is an edge set with unordered pair (i, j) which describes the communication between node i and node j . If the state of node i is available to node j , node i is called a neighbor of node j . The set of all neighbors of node j is denoted by $\mathcal{N}_j$. A graph is called connected if for any two different nodes there exists a set of edges which connect the two nodes.

A formation of m robots is defined by a geometric pattern $\mathcal{P}$. The pattern $\mathcal{P}$ can be described by orthogonal coordinates (p_{jx}, p_{jy}) ($1 \leq j \leq m$). Without loss of generality, we assume that $\sum_{j=1}^m p_{jx} = 0$ and $\sum_{j=1}^m p_{jy} = 0$, i.e., the center of the geometric pattern $\mathcal{P}$ is at the origin of a local orthogonal coordinate system. It is given a reference trajectory $q_0(t) = (x_0(t), y_0(t), \theta_0(t))$ which satisfies

$$\dot{x}_0 = v_0 \cos \theta_0, \quad \dot{y}_0 = v_0 \sin \theta_0, \quad \dot{\theta}_0 = \omega_0 \quad (6.2)$$

where v_0 and ω_0 are known time-varying functions. The state q_0 is assumed to be available to a portion of the m wheeled mobile robots.

Let $q_j = [x_j, y_j, \theta_j]^\top$, the control problem considered in this article is defined as follows.

Control Problem: Design a control laws v_j and ω_j for system j using its own state q_j , its neighbor's state q_l , the relative position with its neighbor (p_{lx}, p_{ly}) for $l \in \mathcal{N}_j$, and the desired trajectory q_0 if it is available to the system such that

$$\lim_{t \rightarrow \infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix} \quad (6.3)$$

$$\lim_{t \rightarrow \infty} (\theta_i - \theta_0) = 0 \quad (6.4)$$

$$\lim_{t \rightarrow \infty} \left[\sum_{l=1}^m \frac{x_l}{m} - x_0 \right] = 0, \quad \lim_{t \rightarrow \infty} \left[\sum_{l=1}^m \frac{y_l}{m} - y_0 \right] = 0 \quad (6.5)$$

for $1 \leq i \neq j \leq m$.

In order to solve the defined problem, the following assumption is made on the desired trajectory.

Assumption 6.1. The $\frac{d^i \theta_0}{dt^i}$ ($0 \leq i \leq 2$) are bounded and $\int_t^{t+T} \dot{\theta}_0^2(\tau) d\tau > \mu$ for some $\mu > 0$ and $T > 0$.

6.3 Cooperative Controller Design

Define the change of variables

$$z_{1j} = \theta_j, z_{2j} = (x_j - p_{jx}) \cos \theta_j + (y_j - p_{jy}) \sin \theta_j, z_{3j} = (x_j - p_{jx}) \sin \theta_j - (y_j - p_{jy}) \cos \theta_j \quad (6.6)$$

for $0 \leq j \leq m$, where $k_3 > 0$ and $p_{0x} = p_{0y} = 0$. The transformed state space model is

$$\dot{z}_{1j} = \omega_j \quad (6.7)$$

$$\dot{z}_{2j} = v_j - \omega_j z_{3j} \quad (6.8)$$

$$\dot{z}_{3j} = \omega_j z_{2j}. \quad (6.9)$$

We have the following results.

Lemma 6.1. *If $\lim_{t \rightarrow \infty} (z_{1j} - z_{10}) = 0$, $\lim_{t \rightarrow \infty} (z_{2j} - z_{20}) = 0$, and $\lim_{t \rightarrow \infty} (z_{3j} - z_{30}) = 0$ for $1 \leq j \leq m$, then (6.3)-(6.5) hold.*

System (6.7)-(6.9) can be considered as a cascade system of (6.7) and (6.8)-(6.9). We first design control law ω_j for system (6.7) such that $\lim_{t \rightarrow \infty} (z_{1j} - z_{10}) = 0$. For a group of m robots ($1 \leq j \leq m$), the communication between robots is described by a graph $\mathcal{G} = \{\mathcal{A}, \mathcal{E}\}$. Given an $m \times m$ constant matrix $A = [a_{ji}]$ with $a_{ji} = a_{ij} > 0$, the Laplacian matrix $\mathcal{L} = [L_{ji}]$ of the graph $\mathcal{G}$ with weight matrix A is defined by

$$L_{ji} = \begin{cases} -a_{ji}, & \text{if } i \in \mathcal{N}_j \text{ and } i \neq j \\ 0, & \text{if } i \notin \mathcal{N}_j \text{ and } i \neq j \\ \sum_{l \neq j, l \in \mathcal{N}_j} a_{jl}, & \text{if } j = i. \end{cases}$$

For the Laplacian matrix, the following result is useful in this paper.

Lemma 6.2 ([99]). *If the communication graph $\mathcal{G}$ is connected, then $(L + \text{diag}(\mu))$ is a positive definite symmetric matrix, where constant vector $\mu = [\mu_1, \mu_2, \dots, \mu_m]^\top$, $\mu_i \geq 0$ ($1 \leq i \leq m$) and at least one of the elements of μ is nonzero.*

With the aid of Lemma 6.2, we have the following lemma.

Lemma 6.3. *For the $(m+1)$ systems in eqn. (6.7) ($0 \leq j \leq m$), if the communication graph $\mathcal{G}$ is connected and the system 0 is available to at least one of the m systems, the control laws*

$$\omega_j = \xi_{1j} - \sum_{i \in \mathcal{N}_j} a_{ji}(z_{1j} - z_{1i}) - b_j \mu_j (z_{1j} - z_{10}) \quad (6.10)$$

$$\begin{aligned} \dot{\xi}_{1j} = & - \sum_{i \in \mathcal{N}_j} a_{ji}(\xi_{1j} - \xi_{1i}) - b_j \mu_j (\xi_{1j} - \xi_{10}) \\ & - \rho_{1j} \text{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\xi_{1j} - \xi_{1i}) - b_j \mu_j (\xi_{1j} - \xi_{10}) \right] \end{aligned} \quad (6.11)$$

for $1 \leq j \leq m$ guarantee that $\lim_{t \rightarrow \infty} (z_{1j} - z_{10}) \stackrel{exp}{=} 0$ and $\lim_{t \rightarrow \infty} (\omega_j - \omega_0) \stackrel{exp}{=} 0$, where $\xi_{10} = \dot{\theta}_0$, $b_j > 0$, ρ_{1j} is sufficiently large, the parameter $\mu_j = 1$ if system 0 is available to system j and $\mu_j = 0$ if system 0 is not available to system j .

The proof of Lemma 6.3 is the same as the proof of Lemma 4 in [99] and is omitted here for space limitation.

With the aid of Lemma 6.3 and the results of cascade system, we have the following results.

Theorem 6.1. *For the $(m + 1)$ systems in eqn. (6.8) ($0 \leq j \leq m$), under Assumption 6.1, if the communication graph $\mathcal{G}$ is connected, then the distributed control laws (6.10)-(6.11) and*

$$\begin{aligned} v_j &= -k_1 z_{2j} - k_2 \omega_j z_{3j} + \omega_j z_{3j} + \xi_{2j} \\ \dot{\xi}_{2j} &= - \sum_{i \in \mathcal{N}_j} a_{ji} (\xi_{2j} - \xi_{2i}) - b_j \mu_j (\xi_{2j} - \xi_{20}) \end{aligned} \quad (6.12)$$

$$- \beta_j \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji} (\xi_{2j} - \xi_{2i}) + b_j \mu_j (\xi_{2j} - \xi_{20}) \right] \quad (6.13)$$

for $1 \leq j \leq m$ ensure that (z_{1j}, z_{2j}, z_{3j}) uniformly exponentially converges to (z_{10}, z_{20}, z_{30}) and (ξ_{1j}, ξ_{2j}) exponentially converges to (ξ_{10}, ξ_{20}) , where β_j is a sufficiently large positive constant, $k_1 > 0$, $k_2 > 0$, and

$$\xi_{20} = v_0 - \omega_0 z_{30} + k_1 z_{20} + k_2 \omega_0 z_{30}. \quad (6.14)$$

In [99], distributed control laws for multiple wheeled mobile robots were proposed such that the state of each system asymptotically converges to a desired state. In this paper, new distributed tracking control laws are proposed with the aid of the results of cascade systems. Moreover, the proposed control laws ensure that the state of each system globally uniformly exponentially converges to a desired state.

6.4 Simulations

To show the effectiveness of the proposed results, simulation has been done for three robots. The desired geometric pattern $\mathcal{P}$ is shown in Fig. 6.1. The pattern $\mathcal{P}$ can be described by orthogonal coordinates $(p_{1x}, p_{1y}) = (-1, 1.7)$, $(p_{2x}, p_{2y}) = (-1, -1.7)$, and $(p_{3x}, p_{3y}) = (2, 0)$. Assume the reference trajectory is $(x_0, y_0, \theta_0) = (10 \sin(0.5t), -10 \cos(0.5t), 0.5t)$, by (6.2) $v_0 = 5$ and $\omega_0 = 0.5$. So, Assumption 6.1 is satisfied.

Assume the communication graph is shown in Fig. 6.2. The cooperative controllers can be obtained by Theorem 6.1. We choose the control parameters $a_{ji} = 2$, $k_3 = 2$, $b_1 = 2$, $\rho_1 = 2$, and $\rho_2 = 2$. Fig. 6.3 shows the centroid of x_i ($1 \leq i \leq 3$) (i.e., $\sum_{j=1}^3 x_j/3$) and x_0 . Fig. 6.4 shows the centroid of y_i ($1 \leq i \leq 3$) (i.e., $\sum_{j=1}^3 y_j/3$) and y_0 . Fig. 6.5 shows $(\theta_i - \theta_0)$ ($1 \leq i \leq 3$). Fig. 6.6 shows the path of the centroid of the three robots and its desired path. From the simulation (6.4)-(6.5) are satisfied. Eqn. (6.3) is also verified and the response of them is omitted here.

6.5 Conclusion

This chapter has discussed the formation control of multiple wheeled mobile robots under the condition that a desired trajectory is available to only a portion of the systems. Distributed control laws were proposed with the aid of Lyapunov techniques and results from graph

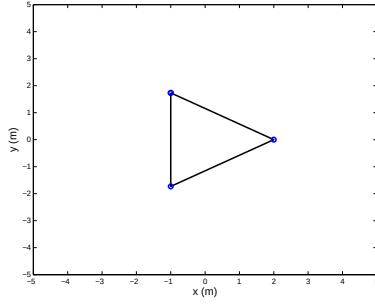


Figure 6.1: Desired geometric pattern.

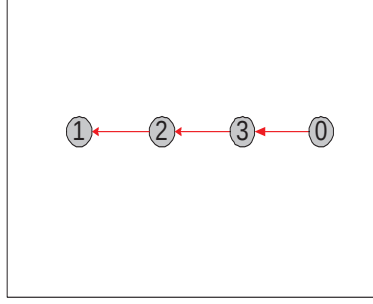


Figure 6.2: Information exchange graph $\mathcal{G}$

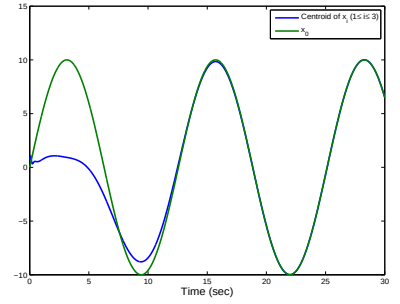


Figure 6.3: Response of the centroid of x_i (solid) for $1 \leq i \leq 3$ and x_0 (dashed).

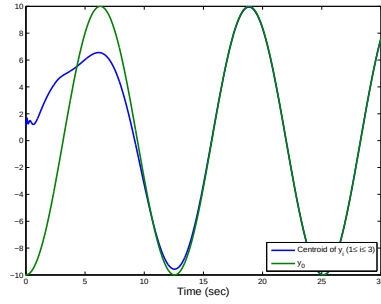


Figure 6.4: Response of the centroid of y_i (solid) for $1 \leq i \leq 3$ and y_0 (dashed).

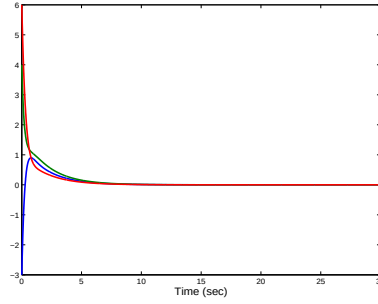


Figure 6.5: Response of $(\theta_i - \theta_0)$ for $1 \leq i \leq 3$.

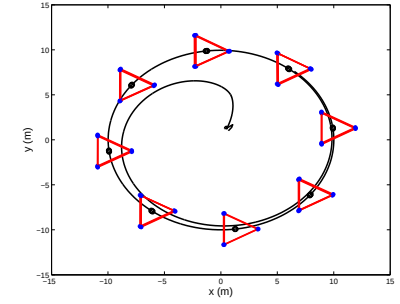


Figure 6.6: The path of the centroid of the three robots (dashed line), the desired path (solid line) of the centroid of robots, and formation of the three robots at several moments (red triangles).

theory. Simulation results show the effectiveness of the proposed control laws. In this paper, the information exchange graph is assumed to be bidirectional. The future work is to extend our results to more general information exchange graphs.

Chapter 7

Neural Network Based Distributed Control of Mechanical Systems with/without Constraints

7.1 Introduction

Wheeled mobile robots (WMRs) have wide spectrum of applications in civil and military areas. Control of a single WMR has been extensively studied in the past decades and many control algorithms have been proposed [92]. As the complex of tasks increases, some tasks are required to be accomplished by multiple WMRs cooperatively. Moreover, compared to a single WMR system a system consisting of multiple WMRs have the features such as (a) reliable, (b) flexible, (c) scalable, etc. Considering these, cooperative control of multiple WMRs has been studied by researchers in the past decades. Early research on cooperative control of multiple WMRs is based on the assumption that all information of each WMR is available to a central station and each WMR is controlled by the commands from the central station. This type of cooperative control is called the centralized cooperative control. In centralized cooperative control, control algorithms can be designed by considering multiple WMRs as a single system and the performance of the whole system can be optimized. However, in centralized cooperative control the control law for each WMR depends on the information of each WMR, which means that the centralized cooperative control system are not scalable and are not robust to communication failure.

To overcome the shortcomings of the centralized cooperative control, distributed cooperative control has been proposed. In distributed cooperative control, there is not a central station and the control law for each WMR only depends on its neighbors' information. There are many distributed cooperative control problems in practice. An important problem is the leader-following control problem where there are multiple WMRs and a leader WMR whose information is only available to a portion of the group of WMRs. The leader-following control problem is to design a distributed control law for each WMR using its neighbors' information such that each WMR follows the leader WMR. In this paper, we consider the leader-following control of multiple WMRs. In order to design control laws, the dynamics of each WMR should be known. However, in practice it is hard to obtain the dynamics

of a WMR exactly. Since neural networks have the ability of approximating a unknown function, we apply neural networks to learn the unknown parts in the dynamics and then design controllers with the aid of the approximation of the neural networks. Furthermore, we take the advantage of adaptive control into our controller design and propose on-line tuning algorithms for the neural networks in the controllers.

7.1.1 Related Work

In the last decades, control of a single WMR had been an active research area. An important feature of a WMR system is that the number of its inputs is less than the number of its degree of the freedom. This feature makes control problems of a WMR challenging. Tracking control of a WMR has been extensively studied and various control laws has been proposed with different methods in [16–18]. However, these control laws cannot be applied to the leader-following control multiple WMRs.

Cooperative control of multiple systems has been studied in the last decades. Various control methods have been proposed in its early research, for example the behavior-based method in [19, 20], the virtual structure method in [21, 22], the artificial potentials method in [25, 26], and the graph theoretical method in [27, 28]. As one of cooperative control problems, leader-following control of multiple linear systems has been extensively studied. For multiple first-order linear systems, distributed controllers were proposed in [41–44] with the aid of sliding mode control and adaptive control. For multiple second-order systems, distributed controllers were proposed in [43, 44, 100]. For multiple high-order linear systems, the leader-following control problem was studied in [71–74]. The leader-following control of multiple Lagrangian mechanical systems was considered in [101] with the aid of graph theory. The leader-following control of nonlinear systems has also been studied. For multiple WMRs, distributed controllers were proposed in [99] with the aid of a transformation based on backstepping techniques. For multiple chained form systems with a leader, distributed tracking controllers were proposed in [87] with the aid of the cascade structure of each system. For multiple first-order and second-order systems with uncertainty, distributed controllers were proposed in [102, 103]. For multiple linear systems with uncertainty, distributed finite-time tracking controllers were proposed in [104] with the aid of Lyapunov techniques. For multiple uncertain mechanical systems, distributed tracking controllers are proposed in [105].

7.1.2 Our Contributions

In this chapter, we consider the leader-following control problem of multiple uncertain mechanical systems with/without velocity constraints. In order to solve the problem, the cascade structure of each system is explored and the property of the input-to-state stability (ISS) is identified for each system. With the aid of the ISS property and the passivity property of each system, distributed neural network based tracking controllers are proposed with the aid of neighbors' information. In the proposed controllers, the unknown dynamics of each system is approximated by neural networks and the state of the leader system is estimated with the aid of sliding mode control and adaptive control. The contributions of this article are as follows: 1) the leader-following control problem is solved for uncertain mechanical

systems with/without velocity constraints, which receives less attention in literature; 2) a novel ISS based approach is proposed for distributed tracking control of uncertain dynamical systems with the aid of neural network approximation and adaptive distributed estimation. The proposed approach can be applied solve the distributed tracking control of uncertain mechanical systems as well as uncertain mechanical systems with constraints; and 3) an application of neural networks in distributed cooperative control of multiple uncertain WMRs is presented and it is shown that neural networks have potential applications of distributed control of uncertain nonlinear systems.

The remaining parts of this chapter are organized as follows. In Section 7.2, the first problem considered in this article is defined and some preliminary results are presented. In Section 7.3, distributed tracking controllers of multiple uncertain mechanical systems with velocity constraints are proposed. In Section 7.4, the proposed controller design method in Section 7.3 is applied to solve the leader-following control of uncertain mechanical systems. In Section 7.5, two applications of the proposed results are considered. Simulation results are presented. The last section concludes this article.

7.2 Problem Statement and Preliminary Results

7.2.1 Problem Statement

Consider m mechanical systems with velocity constraints. The j -th system is defined by

$$M_j(q_{*j})\ddot{q}_{*j} + C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j} + G_j(q_{*j}) = B_j(q_{*j})\tau_j + J(q_{*j})^\top \lambda_j, \quad (7.1)$$

$$J(q_{*j})\dot{q}_{*j} = 0 \quad (7.2)$$

where $q_{*j} = [q_{1j}, \dots, q_{nj}]^\top$ is the state of system j , $M_j(q_{*j}) \in R^{n \times n}$ is a bounded positive-definite symmetric matrix, $C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j}$ is centripetal and Coriolis torque, $G_j(q_{*j})$ is a gravitational torque, $B_j(q_{*j}) \in R^{n \times r}$ is an input transformation matrix, $\tau_j \in R^r$ is the control input, $J(q_{*j}) \in R^{(n-s-1) \times n}$ is a full row rank matrix with $s = n - 1 - \text{Rank}(J(q_{*j}))$, $2 \leq s + 1 < n$, $r \geq s + 1$, $\lambda_j \in R^{n-s-1}$ is the Lagrange multiplier which expresses the constraint force on system j , and the superscript $\top$ denotes the transpose. In system (7.1)-(7.2), the constraint (7.2) is assumed to be completely nonholonomic for each system [3]. In the dynamics, M_j , C_j , and G_j are assumed to be unknown in this paper. However, dynamics (7.1) has the following properties [106]:

Property 7.1. M_j is a bounded symmetric matrix.

Property 7.2. Matrix $(\dot{M}_j - 2C_j)$ is skew-symmetric for a proper definition of C_j . This property is called the passivity property of the dynamics (7.1).

The communication between the systems can be described by the edge set $\mathcal{E}$ of a directed graph (or digraph for short) $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where the m systems are represented by the m nodes in $\mathcal{V}$. The existence of an edge $(l, j) \in \mathcal{E}$ means that the information (the state) of system l is available to system j for control (i.e., unidirectional communication). Bidirectional communication, if it exists, would be represented by the edge (j, l) also being in the edge set

$\mathcal{E}$. The symbol $\mathcal{N}_j$ denotes the neighbors of node j and is a set of indices of the systems whose information is available to system j . A directed path in a digraph is an ordered sequence of vertices such that any ordered pair of vertices appearing consecutively in the sequence is an edge of the digraph.

It is given a desired trajectory $q_{*,m+1} = [q_{1,m+1}, \dots, q_{n,m+1}]^\top$ which is generated by a leader system

$$M_{m+1}(q_{*,m+1})\ddot{q}_{*,m+1} + C_{m+1}(q_{*,m+1}, \dot{q}_{*,m+1})\dot{q}_{*,m+1} + G_{m+1}(q_{*,m+1}) = B_{m+1}(q_{*,m+1})\tau_{m+1} + J(q_{*,m+1})^\top \lambda_{m+1}, \quad (7.3)$$

$$J(q_{*,m+1})\dot{q}_{*,m+1} = 0 \quad (7.4)$$

where $M_{m+1} \in R^{n \times n}$, $C_{m+1} \in R^{n \times n}$, $G_{m+1} \in R^n$, and $B_{m+1} \in R^{n \times r}$ are known matrices. The input τ_{m+1} is a known time-varying function. For simplicity, the leader system (7.3)-(7.4) is denoted as system $(m+1)$. The m systems in (7.1)-(7.2) and the leader system in (7.3)-(7.4) can be considered as a group of $(m+1)$ systems and the communication between $(m+1)$ systems is denoted by the digraph $\mathcal{G}^e = \{\mathcal{V}^e, \mathcal{E}^e\}$. The neighbor set of node j is denoted by $\mathcal{N}_j^e$. Since the leader system does not receive information from other systems, the neighbor set of node $(m+1)$ is an empty set, i.e., $\mathcal{N}_{m+1}^e = \emptyset$. Node $(m+1)$ is reachable to node j ($j \neq m+1$) if there exists a directed path from node $(m+1)$ to node j . Node $(m+1)$ is said to be globally reachable if node $(m+1)$ is reachable to node j for $1 \leq j \leq m$. In this paper, the following assumption is made on the communication digraph $\mathcal{G}^e$.

Assumption 7.1. *In the communication digraph $\mathcal{G}^e$, node $(m+1)$ is globally reachable.*

The control problem that will be discussed first is defined as follows.

Leader-following control: For m systems in (7.1)-(7.2) and a leader system in (7.3)-(7.4), the control problem is how to design a control law τ_j for system j using its own state $(q_{*j}, \dot{q}_{*j})$ and its neighbor's state $(q_{*i}, \dot{q}_{*i})$ for $i \in \mathcal{N}_j^e$ such that

$$\lim_{t \rightarrow \infty} (q_{*j}(t) - q_{*,m+1}(t)) = 0 \quad (7.5)$$

for $1 \leq j \leq m$.

In the leader-following control problem, the state of each system is available to itself and its neighbors. The desired state $q_{*,m+1}$ is only available to a subset of a group of the systems, which make the leader-following control problem challenging.

7.2.2 Some Results on Linear time-varying systems

Throughout this article, $\|\cdot\|$ stands for the Euclidean norm of vectors and induced norm of matrices. For $x = [x_1, \dots, x_n]^\top$, $|x| = [|x_1|, \dots, |x_n|]^\top$, $\text{sign}(x) = [\text{sign}(x_1), \dots, \text{sign}(x_n)]^\top$, and $\text{diag}(x)$ is a diagonal matrix with the element in the main diagonal being x . $\mathbf{0}$ is a vector or matrix with each element being zero and with an appropriate dimension. $\mathbf{1}$ is a vector with each element being one and with an appropriate dimension. For a symmetric matrix $B \in R^{n \times n}$, $\lambda_{\min}(B)$ denotes the smallest eigenvalue of B and $\lambda_{\max}(B)$ denotes the largest eigenvalue of B . We denote by B_r the open ball $B_r := \{x \in \mathbb{R}^n : \|x\| < r\}$. A function

$\alpha(s) : \mathfrak{R}_{\geq 0} \rightarrow \mathfrak{R}_{\geq 0}$ is said to be of class $\mathcal{K}$ ($\alpha \in \mathcal{K}$), if it is continuous, strictly increasing and zero at $s = 0$. Moreover, if $\alpha(s) \rightarrow \infty$ as $s \rightarrow \infty$ we say that $\alpha \in \mathcal{K}_{\infty}$. The origin of the system $\dot{x} = f(t, x)$ is globally uniformly stable (GUS) if there exists $\alpha \in \mathcal{K}_{\infty}$ such that $\|x(t)\| \leq \alpha(\|x_0\|)$ for all $t \geq t_0$ and all $t_0 \geq 0$. It is uniformly globally asymptotically stable (UGAS) if in addition to GUS, for each r and $\epsilon > 0$, there exists $T(r, \epsilon) > 0$ such that $(t_0, x_0) \in \mathfrak{R}_{\geq 0} \times B_r$ implies that $\|x(t)\| \leq \epsilon$ for all $t \geq t_0 + T$. The origin of the system is said to be globally uniformly exponentially stable (GUES) if there exist two strictly positive constants γ_1 and γ_2 such that

$$\|x(t, t_0, x_0)\| \leq \gamma_1 \|x_0\| e^{-\gamma_2(t-t_0)}$$

for any initial condition $x_0(t_0)$ and $t \geq t_0$. A function $\phi(t) \in \mathfrak{R}$ is said to be persistently excited (PE) if there exist $\mu > 0$ and $T > 0$ such that

$$\int_t^{t+T} \phi(\tau)^2 d\tau \geq \mu, \quad \forall t \geq 0.$$

For a PE signal, the following lemma will be applied in this paper.

Lemma 7.1. (*Lemma 4.8.3 in [5]*) *Signal $\xi_1(t)$ is a PE signal, if $\xi_2(t)$ is bounded and converges to zero, then $\xi_1 + \xi_2$ is a PE signal.*

For a linear time-varying system, we have the following results.

Lemma 7.2. ([87]) *For a linear time-varying system*

$$\begin{cases} \dot{\chi}_1 = -c_1\chi_1 - c_2\phi(t)\chi_2 - c_3\chi_3 + \cdots \\ \quad - c_{l-1}\phi(t)^{\text{mod}(l-1,2)}\chi_{l-1} - c_l\phi(t)^{\text{mod}(l,2)}\chi_l \\ \dot{\chi}_2 = \chi_1, \dots, \dot{\chi}_l = \chi_{l-1} \end{cases} \quad (7.6)$$

where $\text{mod}(l, 2)$ denotes the remainder of the Euclidean division of l by 2, and constants c_i ($1 \leq i \leq l$) are chosen such that the polynomial

$$\lambda^l + c_1\lambda^{l-1} + \cdots + c_{l-1}\lambda + c_l \quad (7.7)$$

is Hurwitz (i.e. all roots are in the left half of the open complex plane), if $\phi(t)$ is an absolutely continuous PE signal and $\max_{t \in [0, \infty)} \{|\phi(t)|, |\dot{\phi}(t)|\} \leq \phi_M < \infty$ almost everywhere, then the system (7.6) is GUES.

Lemma 7.3. *For the system*

$$\dot{x} = A(t)x + F(t) \quad (7.8)$$

where $A(t)$ is bounded and $\dot{x} = A(t)x$ is GUES. If $F(t)$ is bounded and converges to zero, then the system (7.8) is GAS.

Proof: Since $\dot{x} = A(t)x$ is GUES and $A(t)$ is bounded, by Theorem 7.8 in [61]

$$Q(t) = \int_t^\infty \Phi_A^\top(\sigma, t) \Phi_A(\sigma, t) d\sigma$$

is a continuously differentiable symmetric matrix for all t and is such that

$$\eta_1 I \leq Q(t) \leq \eta_2 I \quad (7.9)$$

$$A^\top(t)Q(t) + Q(t)A(t) + \dot{Q}(t) \leq -\eta_3 I \quad (7.10)$$

where η_1 , η_2 , and η_3 are finite positive constants, and $\Phi(\sigma, t)$ is the state transition matrix of the system $\dot{x} = A(t)x$. Let $V(t) = x^\top Q(t)x$ and differentiate it along the solution of the system, we have

$$\begin{aligned} \dot{V} &= x^\top (QA + A^\top Q + \dot{Q})x + 2x^\top QF \\ &\leq -\eta_3 x^\top x + 2x^\top QF \leq -\frac{\eta_3}{\eta_1} V + \eta_4 \sqrt{V} \|F\| \end{aligned}$$

where η_4 is a positive constant. By the comparison lemma [90], it can be shown that V converges to zero if F converges to zero. Therefore, x converges to zero. $\blacksquare$

7.2.3 Some Results of Algebraic Graph Theory

For the digraph $\mathcal{G}^e$ with a weight matrix $\mathcal{A}^e = [a_{ji}]_{(m+1) \times (m+1)}$ ($a_{ji} > 0$), its weighted Laplacian matrix $\mathcal{L}^e = [L_{ji}^e]_{(m+1) \times (m+1)}$ is defined as

$$L_{ji}^e = \begin{cases} -a_{ji}, & \text{if } i \neq j \text{ and } i \in \mathcal{N}_j^e \\ 0, & \text{if } i \neq j \text{ and } i \notin \mathcal{N}_j^e \\ \sum_{k \in \mathcal{N}_j^e} a_{jk}, & \text{if } i = j. \end{cases} \quad (7.11)$$

For the digraph $\mathcal{G}$ with the weight matrix $\mathcal{A}$ which is formed by the first m rows and the first m columns of $\mathcal{A}^e$, its weighted Laplacian matrix $\mathcal{L} = [L_{ji}]_{m \times m}$ is defined as

$$L_{ji} = \begin{cases} -a_{ji}, & \text{if } i \neq j \text{ and } i \in \mathcal{N}_j \\ 0, & \text{if } i \neq j \text{ and } i \notin \mathcal{N}_j \\ \sum_{k \in \mathcal{N}_j} a_{jk}, & \text{if } i = j. \end{cases} \quad (7.12)$$

By the definition,

$$\mathcal{L}^e = \left[\begin{array}{c|c} \mathcal{L}_{11}^e & -\mathcal{L}_{12}^e \\ \hline \mathbf{0} & 0 \end{array} \right] = \left[\begin{array}{c|c} \mathcal{L} + \text{diag}(\mathcal{L}_{12}^e) & -\mathcal{L}_{12}^e \\ \hline \mathbf{0} & 0 \end{array} \right] \quad (7.13)$$

where

$$\mathcal{L}_{11}^e = \mathcal{L} + \text{diag}(\mathcal{L}_{12}^e) \quad (7.14)$$

$$\mathcal{L}_{12}^e = [a_{1,m+1}\mu_1, \dots, a_{m,m+1}\mu_m]^\top \quad (7.15)$$

and

$$\mu_j = \begin{cases} 1, & \text{if node } (m+1) \text{ is available to node } j \\ 0, & \text{otherwise} \end{cases} \quad (7.16)$$

for $1 \leq j \leq m$.

For the weighted Laplacian matrix $\mathcal{L}^e$, the following results will be applied in this paper.

Lemma 7.4. ([38, 87]) *For the digraph $\mathcal{G}^e$ with weighted matrix $\mathcal{A}^e = [a_{ji}]_{(m+1) \times (m+1)}$ ($a_{ji} > 0$), its weighted Laplacian matrix $\mathcal{L}^e$ is defined in (7.11). If node $(m+1)$ is globally reachable, then*

1. $\mathcal{L}^e \mathbf{1} = \mathbf{0}$;
2. zero is a simple eigenvalue of $\mathcal{L}^e$ and non-zero eigenvalues of $\mathcal{L}^e$ all have positive real parts;
3. $-\mathcal{L}_{11}^e$ is a Hurwitz matrix;
4. Let

$$P = \left(\text{diag}((\mathcal{L}_{11}^e)^{-1} \mathbf{1}) \right)^{-1} = \text{diag}([P_1, \dots, P_m]),$$

then P and

$$Q = P\mathcal{L}_{11}^e + (\mathcal{L}_{11}^e)^\top P$$

are positive definite matrices.

With the aid of Lemma 7.4, the following results can be obtained.

Lemma 7.5. *For the digraph $\mathcal{G}^e$ with the weight matrix $\mathcal{A}^e = [a_{ji}]_{(m+1) \times (m+1)}$ ($a_{ji} > 0$), consider the system*

$$\dot{\xi}_j = - \sum_{i \in \mathcal{N}^e} a_{ji} (\xi_j - \xi_i) + d_j, \quad 1 \leq j \leq m \quad (7.17)$$

where $\xi_j \in \mathbb{R}$, d_j is bounded and $(d_j - \dot{\xi}_{m+1})$ exponentially converges to zero. If node $(m+1)$ is globally reachable, $(\xi_j - \xi_{m+1})$ globally uniformly exponentially converges to zero for $1 \leq j \leq m$.

Proof: Let $\tilde{\xi}_j = \xi_j - \xi_{m+1}$ for $1 \leq j \leq m$ and $\tilde{\xi} = [\tilde{\xi}_1, \dots, \tilde{\xi}_m]^\top$, then (7.17) can be written in a compact form as

$$\dot{\tilde{\xi}} = -\mathcal{L}_{11}^e \tilde{\xi} + [d_1, \dots, d_m]^\top - \dot{\xi}_{m+1} \mathbf{1}. \quad (7.18)$$

Since $-\mathcal{L}_{11}^e$ is a Hurwitz matrix by Lemma 7.4, system (7.18) is a perturbation of a stable linear system with an exponentially decaying disturbance $([d_1, \dots, d_m]^\top - \dot{\xi}_{m+1} \mathbf{1})$. Therefore, the system (7.18) globally uniformly exponentially converges to zero. $\blacksquare$

Lemma 7.6. For the digraph $\mathcal{G}^e$ with the weight matrix $\mathcal{A}^e = [a_{ji}]_{(m+1) \times (m+1)}$ ($a_{ji} > 0$), consider the system

$$\dot{\xi}_j = - \sum_{i \in \mathcal{N}^e} a_{ji}(\xi_j - \xi_i) - \rho_j \operatorname{sign} \left(\sum_{i \in \mathcal{N}^e} a_{ji}(\xi_j - \xi_i) \right) \quad (7.19)$$

where $\xi_j \in R$ for $1 \leq j \leq m+1$.

1. If node $(m+1)$ is globally reachable and

$$\rho_j(t) \geq |\dot{\xi}_{m+1}(t)|. \quad (7.20)$$

then $(\xi_j - \xi_{m+1})$ globally uniformly exponentially converges to zero for $1 \leq j \leq m$.

2. If the communication between node j and i for $1 \leq j \neq i \leq m$ are bidirectional, node $(m+1)$ is globally reachable, $|\dot{\xi}_{m+1}|$ is bounded, and

$$\dot{\rho} = \operatorname{diag}(\gamma) |\mathcal{L}_{11}^e \tilde{\xi}|. \quad (7.21)$$

where $\rho = [\rho_1, \dots, \rho_m]^\top$, $\gamma = [\gamma_1, \dots, \gamma_m]^\top$, $\gamma_i > 0$ for $1 \leq i \leq m$, and $\tilde{\xi} = [\xi_1 - \xi_{m+1}, \dots, \xi_m - \xi_{m+1}]^\top$, then $(\xi_j - \xi_{m+1})$ asymptotically converges to zero for $1 \leq j \leq m$ and ρ is bounded.

Proof: 1. Let $s = \mathcal{L}_{11}^e \tilde{\xi}$, then

$$\dot{s} = -\mathcal{L}_{11}^e s - \mathcal{L}_{11}^e \operatorname{diag}(\rho) \operatorname{sign}(s) - \operatorname{diag}(\mathcal{L}_{12}^e) \dot{\xi}_{m+1} \mathbf{1}$$

where we apply the fact that $\mathcal{L} \mathbf{1} = \mathbf{0}$. Choose a Lyapunov function candidate $V = s^\top P s$ where P is defined in Lemma 7.4, we have

$$\begin{aligned} \dot{V} &= -s^\top Q x - 2s^\top P \mathcal{L}_{11}^e \operatorname{diag}(\rho) \operatorname{sign}(s) - 2s^\top P \operatorname{diag}(\mathcal{L}_{12}^e) \dot{\xi}_{m+1} \mathbf{1} \\ &= -s^\top Q x - 2s^\top P \mathcal{L} \operatorname{diag}(\rho) \operatorname{sign}(s) - 2s^\top P \mathcal{L}_{12}^e \operatorname{diag}(\rho) \operatorname{sign}(s) - 2s^\top P \operatorname{diag}(\mathcal{L}_{12}^e) \dot{\xi}_{m+1} \mathbf{1} \\ &\leq -s^\top Q x - 2s^\top P \mathcal{L} \operatorname{diag}(\rho) \operatorname{sign}(s) - 2 \sum_{j=1}^m P_j a_{j,m+1} \mu_j |s_j| (\rho_j - |\dot{\xi}_{m+1}|) \\ &\leq -s^\top Q x - 2s^\top P \mathcal{L} \operatorname{diag}(\rho) \operatorname{sign}(s) \\ &\leq -s^\top Q x \leq -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} V \end{aligned}$$

Therefore, V exponentially converges to zero. So, $(\xi_j - \xi_{m+1})$ globally uniformly exponentially converges to zero for $1 \leq j \leq m$.

2. Equation (7.19) can be written as

$$\dot{\tilde{\xi}} = -\mathcal{L}_{11}^e \tilde{\xi} - \operatorname{diag}(\rho) \operatorname{sign}(s) - \dot{\xi}_{m+1} \mathbf{1}$$

Since the communication between node j and i for $1 \leq j \neq i \leq m$ are bidirectional and node $(m+1)$ is globally reachable, $\mathcal{L}_{11}^e$ is a symmetric positive definite matrix. Let a constant $M \geq \dot{\xi}_{m+1}(t)$ for any t , we choose a Lyapunov function candidate

$$V = \tilde{\xi}^\top \mathcal{L}_{11}^e \tilde{\xi} + \sum_{j=1}^m \gamma_j^{-1} (\rho_j - M)^2.$$

Differentiate V along the solution of the system, we have

$$\begin{aligned}
\dot{V} &= -2\tilde{\xi}^\top (\mathcal{L}_{11}^e)^2 \tilde{\xi} - 2\tilde{\xi}^\top \mathcal{L}_{11}^e \text{diag}(\rho) \text{sign}(s) \\
&\quad - 2\tilde{\xi}^\top \mathcal{L}_{11}^e \dot{\xi}_{m+1} \mathbf{1} + 2 \sum_{j=1}^m \gamma_j^{-1} (\rho_j - M) \dot{\rho}_j \\
&= -2\tilde{\xi}^\top (\mathcal{L}_{11}^e)^2 \tilde{\xi} - 2 \sum_{j=1}^m \rho_j |s_j| - \sum_{j=1}^m 2|\dot{\xi}_{m+1}| |s_j| \\
&\quad + 2 \sum_{j=1}^m \gamma_j^{-1} (\rho_j - M) \dot{\rho}_j \\
&\leq -2\tilde{\xi}^\top (\mathcal{L}_{11}^e)^2 \tilde{\xi} - 2 \sum_{j=1}^m \rho_j |s_j| + \sum_{j=1}^m 2|\dot{\xi}_{m+1}| |s_j| \\
&\quad + 2 \sum_{j=1}^m \gamma_j^{-1} (\rho_j - M) \dot{\rho}_j \\
&= -2\tilde{\xi}^\top (\mathcal{L}_{11}^e)^2 \tilde{\xi} - 2 \sum_{j=1}^m (\rho_j - M) |s_j| \\
&\quad - \sum_{j=1}^m 2(M - |\dot{\xi}_{m+1}|) |s_j| + 2 \sum_{j=1}^m \gamma_j^{-1} (\rho_j - M) \dot{\rho}_j \\
&\leq -2\tilde{\xi}^\top (\mathcal{L}_{11}^e)^2 \tilde{\xi} - 2 \sum_{j=1}^m (\rho_j - M) |s_j| \\
&\quad + 2 \sum_{j=1}^m \gamma_j^{-1} (\rho_j - M) \dot{\rho}_j \\
&= -2\tilde{\xi}^\top (\mathcal{L}_{11}^e)^2 \tilde{\xi} \leq 0
\end{aligned} \tag{7.22}$$

which means that V is bounded. Therefore, $\tilde{\xi}$ and ρ_j are bounded. By integrating both sides of (7.22), $\tilde{\xi}$ is square-integrable. Since $\tilde{\xi}$ is bounded, by Barbalat's lemma $\tilde{\xi}$ converges to zero. $\blacksquare$

7.2.4 Neural Network Approximation

The multilayer neural network (NN) is modeled with the aid of the structure of biological nervous systems [107]. It is a nonlinear function which can map an input space R^p into an output space R^q . The NNs have the abilities of function approximation, learning, generalization, classification, etc. A two-layer neural network is shown in Fig. 7.1. It has a hidden layer with l neurons and an output layer with q neurons. The input vector x has p components. Input 1 is for considering thresholds. The output of the neural network can be written as

$$y = [y_1, \dots, y_q]^\top = W^\top \left[\phi \left(V^\top \begin{bmatrix} 1 \\ x \end{bmatrix} \right) \right]$$

where matrices V and W are the hidden layer and the output layer weights and thresholds and ϕ is the activation function of neurons. In W , the elements in the first row are thresholds of the output layer and the elements in other rows are weights of the output layer. In V , the elements in the first row are thresholds of the hidden layer and the elements in other rows are weights of the hidden layer. For the sake of simplicity, the above equation can be written as

$$y = W^\top \phi(V^\top x)$$

by redefining the activation function ϕ and x . For the activation function there are different choices ([108, 109]). Typical choices are

1. sigmoid function: $\phi(s) = \frac{1}{1+e^{\alpha s}}$,
2. hyperbolic tangent function: $\phi(s) = \frac{1-e^{-\alpha s}}{1+e^{-\alpha s}}$,
3. radial basis function: $\phi(s) = \frac{e^{-(s-\alpha)^2}}{\beta}$,
4. etc.

A multilayer neural network can be applied to approximate a nonlinear function. For a continuous function $f(x) : \Omega \rightarrow R^q$ and a small positive number ϵ where Ω is a compact subset of R^p , it has been proven in [110] that there exists a sufficiently large number l of neurons in the hidden layer with suitably selected activation function ϕ and weights and thresholds such that one has

$$\|f(x) - W^\top \phi(V^\top x)\| \leq \epsilon \quad (7.23)$$

for any $x \in \Omega$. Let

$$E(x) = f(x) - W^\top \phi(V^\top x)$$

$E(x)$ is called the NN functional reconstruction error vector. In order to choose a suitable NN to approximate $f(x)$, one needs to suitably select the activation function ϕ , the number of neurons in the hidden layer, and weights and thresholds. The NN is a nonlinear function of the parameter V , which means that the selection of V is hard. The NN is a linear function of parameter W . It can be shown that if the weight matrix V are suitably selected and is fixed it is possible to select weight matrix W such that (7.23) holds ([111, 112]). Actually, this fact can be easily proved with the aid of the first-order Taylor approximation formula at sufficient dense points in Ω .

In this paper, we will assume that the activation function ϕ , the number of neurons in the hidden layer, and the weight matrix V have been chosen and are known. While the weight matrix W is unknown. For convenience, $\phi(V^\top x)$ will be denoted by $\phi(x)$. The optimal weight matrix $\bar{W}$ of W for approximation is defined as

$$\bar{W} = \arg \min_W \left\{ \sup_{x \in \Omega} \|f(x) - W^\top \phi(x)\| \right\}.$$

If $f(x)$ is a known function, the optimal weight matrix $\bar{W}$ can be obtained explicitly as a function of $f(x)$ and $\phi(x)$ with the aid of different optimization methods. If $f(x)$ is an

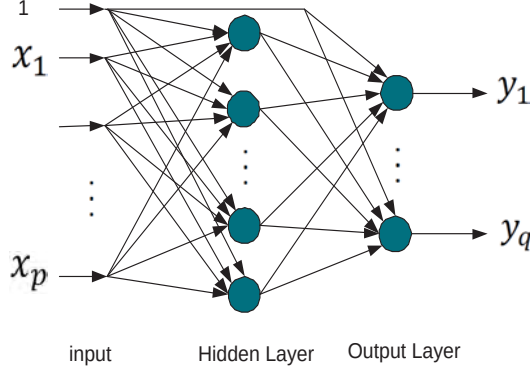


Figure 7.1: Architecture of a two-layer neural network [107]

unknown function, one cannot obtain $\bar{W}$ as a function of $f(x)$ and $\phi(x)$. In order to obtain an estimate of W , one way is to train the NN by the training algorithms as in [111, 113]. Another way is to design online tuning algorithms as in [107, 114] with the aid of Lyapunov stability theory. In this paper, we will propose an online tuning algorithm for W .

7.3 Distributed Controller Design

To solve the cooperative control problem, we convert the systems in (7.1)-(7.2) and (7.3)-(7.4) into a suitable form. For $1 \leq j \leq m+1$, let the vector fields $g_1(q_{*j}), \dots, g_{s+1}(q_{*j})$ form a basis for the null space of $J(q_{*j})$. Then, by (7.2), there exists a vector $u_{*j} = [u_{1j}, \dots, u_{s+1,j}]^\top$ such that

$$\dot{q}_{*j} = g(q_{*j})u_{*j} = g_1(q_{*j})u_{1j} + \dots + g_{s+1}(q_{*j})u_{s+1,j} \quad (7.24)$$

where $g(q_{*j}) = [g_1(q_{*j}), \dots, g_{s+1}(q_{*j})] \in R^{n \times (s+1)}$. Differentiating both sides of (7.24) and substituting it into (7.1) and multiplying on the left by $g^\top(q_{*j})$, we have

$$\bar{M}_j(q_{*j})\dot{u}_{*j} + \bar{C}_j(q_{*j}, \dot{q}_{*j})u_{*j} + \bar{G}_j(q_{*j}) = \bar{B}_j(q_{*j})\tau_j \quad (7.25)$$

where we have used the fact that $g^\top(q_{*j})J^\top(q_{*j}) = 0$, and

$$\begin{aligned} \bar{M}_j(q_{*j}) &= g^\top(q_{*j})M_j(q_{*j})g(q_{*j}) \\ \bar{C}_j(q_{*j}, \dot{q}_{*j}) &= g^\top(q_{*j})M_j(q_{*j})\dot{g}(q_{*j}) + g^\top(q_{*j})C_j(q_{*j}, \dot{q}_{*j})g(q_{*j}) \\ \bar{G}_j(q_{*j}) &= g^\top(q_{*j})G_j(q_{*j}) \\ \bar{B}_j(q_{*j}) &= g^\top(q_{*j})B_j(q_{*j}). \end{aligned}$$

Based on Property 7.2 of dynamics (7.1), the following property can be easily proved:

Property 7.3. *Matrix $\dot{\bar{M}}_j - 2\bar{C}_j$ is skew-symmetric.*

The reduced system (7.24)-(7.25) describes the motion of the system (7.1)-(7.2). Therefore, the leader-following control problem can be considered based on the systems in (7.24)-(7.25). In order to completely actuate each nonholonomic system, $\bar{B}_j(q)$ is assumed to be

a full rank matrix. (7.24) is called the kinematics of system j , while (7.25) is called the dynamics of system j .

To simplify the leader-following control problem, we assume that eqn. (7.24) has two inputs and is in the following chained form after suitable state and input transformations

$$\dot{q}_{1j} = u_{1j}, \quad (7.26)$$

$$\dot{q}_{2j} = u_{2j}, \quad \dot{q}_{ij} = u_{1j}q_{i-1,j}, \quad 3 \leq i \leq n. \quad (7.27)$$

For the leader system, the following assumption is made.

Assumption 7.2. *For the system $(m+1)$ in (7.25)-(7.27), $u_{1,m+1}$ is an absolutely continuous PE signal and $\max_{t \in [0, \infty)} \{|u_{1,m+1}(t)|, |\dot{u}_{1,m+1}(t)|\} < \infty$ almost everywhere. $q_{i,m+1}$ and $\dot{q}_{i,m+1}$ are bounded for $2 \leq i \leq n-1$.*

We design distributed control laws based on the backstepping techniques and the approximation property of neural networks. In the first step, it is assumed that (u_{1j}, u_{2j}) is a control input for system j in (7.26)-(7.27) and we design a distributed control for it such that (7.5) holds. In the second step, we design distributed controller τ_j such that (7.5) holds with the aid of the results in step 1.

• **Step 1:** The system in (7.26)-(7.27) is a cascaded system. The subsystem in (7.26) is a first-order linear system and the subsystem in (7.27) is a linear time-varying system. We take this advantage in controller design. For $(m+1)$ linear systems in (7.26), we propose the distributed controller for system j as

$$u_{1j} = \eta_{1j} \quad (7.28)$$

where

$$\eta_{1j} = - \sum_{i \in \mathcal{N}_j^e} a_{ji}(q_{1j} - q_{1i}) + \xi_{1j} \quad (7.29)$$

ξ_{1j} is an estimate of $\dot{q}_{1,m+1}$ and will be designed later such that

$$\lim_{t \rightarrow \infty} (\xi_{1j} - \dot{q}_{1,m+1}) \stackrel{exp}{=} 0 \quad (7.30)$$

where $\stackrel{exp}{=}$ means "exponentially converges to". Let

$$\tilde{q}_{1j} = q_{1j} - q_{1,m+1},$$

with the control law (7.28), we have

$$\dot{\tilde{q}}_{1j} = - \sum_{i \in \mathcal{N}_j^e} a_{ji}(\tilde{q}_{1j} - \tilde{q}_{1i}) + \xi_{1j} - \dot{q}_{1,m+1}. \quad (7.31)$$

Under Assumption 7.1, by Lemma 7.5 $(q_j - q_{m+1})$ uniformly exponentially converges to zero for $1 \leq j \leq m$. Therefore, u_{1j} exponentially converges to $u_{1,m+1}$ and is a PE signal.

For $(m + 1)$ linear time-varying systems in (7.27), we propose a distributed control law for system j as

$$u_{2j} = \eta_{2j} \quad (7.32)$$

where

$$\eta_{2j} = -k_2 q_{2j} - k_3 u_{1j} q_{3j} - \cdots - k_n u_{1j}^{\text{mod}(n,2)} q_{nj} + \chi_{1j} \quad (7.33)$$

χ_{1j} is an estimate of

$$\begin{aligned} \Lambda = & \dot{q}_{2,m+1} + k_2 q_{2,m+1} + k_3 u_{1,m+1} q_{3,m+1} + \cdots \\ & + k_n u_{1,m+1}^{\text{mod}(n,2)} q_{n,m+1} \end{aligned} \quad (7.34)$$

and will be designed later such that

$$\lim_{t \rightarrow \infty} (\chi_{1j} - \Lambda) \stackrel{exp}{=} 0 \quad (7.35)$$

and $k_2, \dots, k_n$ are chosen such that the polynomial

$$\lambda^{n-1} + k_2 \lambda^{n-2} + \cdots + k_{n-1} \lambda + k_n \quad (7.36)$$

is Hurwitz. Let $\tilde{q}_{ij} = q_{ij} - q_{i,m+1}$ for $2 \leq i \leq n$, with the control law (7.32), we have

$$\begin{aligned} \dot{\tilde{q}}_{2j} = & -k_2 \tilde{q}_{2j} - k_3 u_{1j} \tilde{q}_{3j} - \cdots - k_n u_{1j}^{\text{mod}(n,2)} \tilde{q}_{nj} + \chi_{1j} \\ & - \Lambda - \sum_{\text{odd } l=3}^n k_l (u_{1l} - u_{1,m+1}) q_{l,m+1} \end{aligned} \quad (7.37)$$

$$\begin{aligned} \dot{\tilde{q}}_{ij} = & u_{1j} \tilde{q}_{i-1,j} + (u_{1j} - u_{1,m+1}) q_{i-1,m+1}, \\ & 3 \leq i \leq n. \end{aligned} \quad (7.38)$$

The system in (7.37)-(7.38) can be considered as a perturbation of the following linear time-varying system

$$\dot{\tilde{q}}_{2j} = -k_2 \tilde{q}_{2j} - k_3 u_{1j} \tilde{q}_{3j} - \cdots - k_n u_{1j}^{\text{mod}(n,2)} \tilde{q}_{nj} \quad (7.39)$$

$$\dot{\tilde{q}}_{ij} = u_{1j} \tilde{q}_{i-1,j}, \quad 3 \leq i \leq n. \quad (7.40)$$

with the perturbation

$$\Delta = \begin{bmatrix} \chi_{1j} - \Lambda - \sum_{\text{odd } l=3}^n k_l (u_{1l} - u_{1,m+1}) q_{l,m+1} \\ (u_{1j} - u_{1,m+1}) q_{2,m+1} \\ \vdots \\ (u_{1j} - u_{1,m+1}) q_{n-1,m+1} \end{bmatrix} \quad (7.41)$$

Since u_{1j} is a PE signal, by Lemma 7.2 the system (7.39)-(7.40) is GUES. Since Δ is bounded and converges to zero, by Lemma 7.3 the system (7.37)-(7.38) is GAS. Therefore, $\tilde{q}_{ij}$ converges to zero for $2 \leq i \leq n$.

To summarize, for system (7.26)-(7.27) the controllers (7.28) and (7.32) with η_{1j} defined in (7.29) and η_{2j} defined in (7.33) ensure that (7.5) hold.

• **Step 2:** u_{1j} and u_{2j} were not the real control input. They cannot be chosen to be η_{1j} and η_{2j} . Let $\tilde{u}_{1j} = u_{1j} - \eta_{1j}$ and $\tilde{u}_{2j} = u_{2j} - \eta_{2j}$, the systems in (7.31), (7.37)-(7.38), and (7.25) can be written as

$$\dot{\tilde{q}}_{1j} = - \sum_{i \in \mathcal{N}_j^e} a_{ji}(\tilde{q}_{1j} - \tilde{q}_{1i}) + \xi_{1j} - \dot{q}_{1,m+1} + \tilde{u}_{1j} \quad (7.42)$$

$$\begin{aligned} \dot{\tilde{q}}_{2j} = & -k_2\tilde{q}_{2j} - k_3u_{1j}\tilde{q}_{3j} - \dots - k_nu_{1j}^{\text{mod}(n,2)}\tilde{q}_{nj} + \tilde{u}_{2j} \\ & + \chi_{1j} - \Lambda - \sum_{\text{odd } l=3}^n k_l(u_{1l} - u_{1,m+1})q_{l,m+1} \end{aligned} \quad (7.43)$$

$$\dot{\tilde{q}}_{ij} = u_{1j}\tilde{q}_{i-1,j} + (u_{1j} - u_{1,m+1})q_{i-1,m+1}, 3 \leq i \leq n. \quad (7.44)$$

$$\begin{aligned} \bar{M}_j(q_{*j})\dot{\tilde{u}}_{*j} = & \bar{B}_j(q_{*j})\tau_j - \bar{C}_j(q_{*j}, \dot{q}_{*j})\tilde{u}_{*j} \\ & - (\bar{M}_j\dot{\eta}_{*j} + \bar{C}_j\eta_{*j} + \bar{G}_j) \end{aligned} \quad (7.45)$$

where $u_{*j} = [u_{1j}, u_{2j}]^\top$ and $\eta_{*j} = [\eta_{1j}, \eta_{2j}]^\top$.

For the system in (7.42)-(7.44), the following ISS property can be shown.

Lemma 7.7. *For the system in (7.42)-(7.44), under Assumptions 7.1-7.2, if*

1. ξ_{1j} is bounded and (7.30) is satisfied,
2. χ_{1j} is bounded and (7.35) is satisfied,
3. $\tilde{u}_{1j}$ and $\tilde{u}_{2j}$ are bounded and converge to zero,

then $\tilde{q}_{*j}$ converges to zeros.

Proof: If (7.30) is satisfied, (7.42) is a perturbation of the system (7.31) with the perturbation $\tilde{u}_{1j}$. Since the system (7.31) is GUES, $\tilde{q}_{1j}$ converges to zero if $\tilde{u}_{1j}$ is bounded and converge to zero by Lemma 7.3. Furthermore, u_{1j} is a PE signal by Lemma 7.1.

If (7.35) is satisfied, (7.43)-(7.44) is a perturbation of the system (7.39)-(7.40) with the perturbation $\tilde{u}_{2j}$ and Δ_j . Since u_{1j} is a PE signal, the system (7.39)-(7.40) is GUES by Lemma 7.2. By Lemma 7.3 $\tilde{q}_{ij}$ converges to zero if $\tilde{u}_{2j}$ is bounded and converge to zero. ■

With the aid of Lemma 7.7, we design a distributed controller τ_j for (7.45), ξ_{1j} , and χ_j such that the conditions required in Lemma 7.7 are satisfied. In (7.45),

$$f_j(x_j) = \bar{M}_j(q_{*j})\dot{\eta}_{*j} + \bar{C}_j(q_{*j}, \dot{q}_{*j})\eta_{*j} + \bar{G}_j(q_{*j}) \quad (7.46)$$

is unknown, where $x_j = [q_{*j}^\top, \dot{q}_{*j}^\top, \eta_{*j}^\top, \dot{\eta}_{*j}^\top]^\top$. $f_j(x_j)$ is a static function of x_j . A neural network can be applied to approximate it. We choose a suitable activation function ϕ_j and suitable weights and thresholds in the hidden layer for a NN. For an optimal weight and threshold matrix $\bar{W}_{*j}$ of the NN,

$$f_j(x_j) = \bar{W}_{*j}^\top \phi_j(x_j) + \epsilon_j. \quad (7.47)$$

where the optimal parameter matrix is defined by

$$\bar{W}_{*j} = \arg \min_{W_{*j}} \{ \sup_{x_j \in \Omega} \|f_j(x_j) - W_{*j}^\top \phi_j(x_j)\| \}$$

and $\bar{\epsilon}_j$ is a positive constant such that

$$\|f_j(x_j) - \bar{W}_{*j}^\top \phi_j(x_j)\| \leq \bar{\epsilon}_j. \quad (7.48)$$

It should be noted that $\bar{\epsilon}_j$ is small if the number of the neurons in the hidden layer is large and $\bar{W}_{*j}$ is unknown.

With the approximation of the neural network, (7.45) can be written as

$$\begin{aligned} \bar{M}_j(q_{*j})\dot{\tilde{u}}_{*j} &= \bar{B}_j(q_{*j})\tau_j - \bar{C}_j(q_{*j}, \dot{q}_{*j})\tilde{u}_{*j} \\ &\quad - \bar{W}_{*j}^\top \phi_j(x_j) - \epsilon_j. \end{aligned} \quad (7.49)$$

We choose the control law

$$\tau_j = \bar{B}_j^{-1}(q_{*j}) \left[-K_p \tilde{u}_{*j} + \hat{W}_{*j}^\top \phi_j(x_j) - \rho_{3j} \text{sign}(\tilde{u}_{*j}) \right] \quad (7.50)$$

where K_p is a positive definite symmetric matrix, $\hat{W}_{*j}$ is an estimate of $\bar{W}_{*j}$ and will be designed later, and ρ_{3j} is a positive constant such that

$$\rho_{3j} \geq \bar{\epsilon}_j. \quad (7.51)$$

With the control law (7.50), we have

$$\begin{aligned} \bar{M}_j(q_{*j})\dot{\tilde{u}}_{*j} &= -K_p \tilde{u}_{*j} - \bar{C}_j(q_{*j}, \dot{q}_{*j})\tilde{u}_{*j} \\ &\quad + (\hat{W}_{*j} - \bar{W}_{*j})^\top \phi_j - \rho_{3j} \text{sign}(\tilde{u}_{*j}) - \epsilon_j. \end{aligned} \quad (7.52)$$

The update law is chosen to be

$$\dot{\hat{W}}_{*j} = -\Gamma_j \phi_j \tilde{u}_{*j}^\top \quad (7.53)$$

where Γ_j is a positive definite symmetric matrix. To prove the stability of the closed-loop system with the proposed control law and update law, we choose a Lyapunov function candidate

$$V = \frac{1}{2} \tilde{u}_{*j}^\top \bar{M}_j \tilde{u}_{*j} + \frac{1}{2} \text{tr}((\hat{W}_{*j} - \bar{W}_{*j})^\top \Gamma_j^{-1} (\hat{W}_{*j} - \bar{W}_{*j})) \quad (7.54)$$

where $\text{tr}(\cdot)$ denotes the trace of its variable. Differentiate it along the solution of the closed-loop system, we have

$$\begin{aligned} \dot{V} &= -\tilde{u}_{*j}^\top K_p \tilde{u}_{*j} + \frac{1}{2} \tilde{u}_{*j}^\top (\bar{M}_j - 2\bar{C}_j) \tilde{u}_{*j} \\ &\quad + \tilde{u}_{*j}^\top (\hat{W}_{*j} - \bar{W}_{*j})^\top \phi_j - \tilde{u}_{*j}^\top \rho_{3j} \text{sign}(\tilde{u}_{*j}) - \tilde{u}_{*j}^\top \epsilon_j \\ &\quad + \text{tr}((\hat{W}_{*j} - \bar{W}_{*j})^\top \Gamma_j^{-1} \dot{\hat{W}}_{*j}) \\ &= -\tilde{u}_{*j}^\top K_p \tilde{u}_{*j} - \tilde{u}_{*j}^\top \rho_{3j} \text{sign}(\tilde{u}_{*j}) - \tilde{u}_{*j}^\top \epsilon_j \\ &\leq -\tilde{u}_{*j}^\top K_p \tilde{u}_{*j} - \rho_{3j} (|\tilde{u}_{1j}| + |\tilde{u}_{2j}|) + \bar{\epsilon}_j (|\tilde{u}_{1j}| + |\tilde{u}_{2j}|) \\ &\leq -\tilde{u}_{*j}^\top K_p \tilde{u}_{*j} \leq 0 \end{aligned}$$

which means that V is bounded. So, $\tilde{u}_{*j}$ and $\hat{W}_{*j}$ are bounded. By integrating both sides of the above inequality, it can be shown that $\tilde{u}_{*j}$ is square-integrable. Noting that $\dot{\tilde{u}}_{*j}$ is bounded, by Barbalat's lemma $\tilde{u}_{*j}$ converges to zero.

• **Step 3:** We design ξ_{1j} and χ_{1j} such that ξ_{1j} and χ_{1j} are bounded and (7.30) and (7.35) are satisfied. In the control law (7.50), $\dot{\eta}_{*j}$ is required, which means that ξ_{1j} and χ_{2j} are required to be differentiable. With the aid of the results in [40], we design ξ_{1j} and χ_{1j} ($1 \leq j \leq m$) as follows.

$$\dot{\xi}_{1j} = -\beta_1 \xi_{1j} + \xi_{2j} \quad (7.55)$$

$$\dot{\xi}_{2j} = -\sum_{i \in \mathcal{N}_j^e} a_{ji}(\xi_{2j} - \xi_{2i}) - \rho_{1j} \operatorname{sign} \left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(\xi_{2j} - \xi_{2i}) \right) \quad (7.56)$$

$$\dot{\chi}_{1j} = -\beta_2 \chi_{1j} + \chi_{2j} \quad (7.57)$$

$$\dot{\chi}_{2j} = -\sum_{i \in \mathcal{N}_j^e} a_{ji}(\chi_{2j} - \chi_{2i}) - \rho_{2j} \operatorname{sign} \left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(\chi_{2j} - \chi_{2i}) \right) \quad (7.58)$$

where β_1 and β_2 are positive constants,

$$\xi_{2,m+1} = \ddot{q}_{1,m+1} + \beta_1 \dot{q}_{1,m+1} \quad (7.59)$$

$$\chi_{2,m+1} = \dot{\Lambda} + \beta_2 \Lambda \quad (7.60)$$

and ρ_{1j} and ρ_{2j} are chosen such that

$$\rho_{1j}(t) \geq |\dot{\xi}_{2,m+1}(t)| \quad (7.61)$$

$$\rho_{2j}(t) \geq |\dot{\chi}_{2,m+1}(t)|. \quad (7.62)$$

For the systems in (7.56), ξ_{2j} exponentially converges to $\xi_{2,m+1}$ by Lemma 7.6. Equation (7.55) can be written as

$$\frac{d}{dt}(\xi_{1j} - \dot{q}_{1,m+1}) = -\beta_1(\xi_{1j} - \dot{q}_{1,m+1}) + \xi_{2j} - \xi_{2,m+1}.$$

Since $\beta_1 > 0$, ξ_{1j} exponentially converges to $\dot{q}_{1,m+1}$. Similarly, it can be shown that χ_{1j} exponentially converges to Λ . Therefore, conditions 1 and 2 in Lemma 7.7 are satisfied. By Lemma 7.7, $\tilde{q}_{*j}$ converges to zero.

With the above controller design procedure, we have the following results.

Theorem 7.1. *For m systems in (7.1)-(7.2) and a leader system in (7.3)-(7.4), under Assumptions 7.1-7.2, the distributed control law (7.50) and the update law (7.53) for $1 \leq j \leq m$ ensure that (7.5) holds and $\hat{W}_{*j}$ is bounded, where η_{1j} and η_{2j} are defined in (7.29) and (7.33), ξ_{1j} and χ_{1j} are generated by (7.55)-(7.58), k_l ($2 \leq l \leq n$) are chosen such that the polynomial in (7.36) is Hurwitz, ρ_{1j} , ρ_{2j} , and ρ_{3j} are chosen such that (7.61)-(7.62) and (7.51) are satisfied, $a_{ji} > 0$, $\beta_1 > 0$, $\beta_2 > 0$, K_p and Γ_j are positive definite matrices.*

Remark 7.1. In the proposed control laws, the neural network can be a RBF neural network or other types of neural networks. In short, any neural network works in this paper if it has the approximation property in (7.47). For a specified control system, one can choose specified neural networks based on the information of the dynamics of the system for convenience [107]. In the simulation in Section 7.5, we choose a RBF neural network to approximate the unknown function f_j for demonstration.

Remark 7.2. In the proposed control law (7.50), the parameter matrix $\hat{W}_{*j}$ in the neural network is tuned online by the algorithm in (7.53). Therefore, the neural network does not need to be trained off-line in advance. This is one of the advantages of the proposed control laws. The learning algorithm in (7.53) is derived with the aid of the Lyapunov function candidate (7.54). The learning algorithm (7.53) guarantees that $\hat{W}_{*j}$ is bounded but does not guarantee that $\hat{W}_{*j}$ converges to its optimal value $\bar{W}_{*j}$. For the initial value of $\hat{W}_{*j}$, one may choose them to be any bounded values.

Remark 7.3. In Theorem 7.1, the control laws are distributed. Only neighbors' information is required for control. To deal with uncertainty in each system, neural network based control laws are proposed in Theorem 7.1. It is possible to propose other types of controllers to deal with uncertainty, for example, adaptive control laws, robust control laws, robust adaptive control laws, etc. For space limit, we omit them here.

In Theorem 7.1, it is required that ρ_{3j} satisfies (7.51). However, $\bar{\epsilon}_j$ is generally unknown. To meet this requirement, ρ_{3j} can be chosen to be a large number or ρ_{3j} is estimated on-line as follows.

$$\dot{\rho}_{3j} = \beta_{3j}(|\tilde{u}_{1j}| + |\tilde{u}_{2j}|). \quad (7.63)$$

To verify the rightness of this update law, we choose a Lyapunov function candidate

$$V = \frac{1}{2}\tilde{u}_{*j}^\top \bar{M}_j \tilde{u}_{*j} + \frac{1}{2}\text{tr}((\hat{W}_{*j} - \bar{W}_{*j})^\top \Gamma_j^{-1}(\hat{W}_{*j} - \bar{W}_{*j})) + \frac{1}{2}\beta_{3j}^{-1}(\rho_{3j} - \bar{\epsilon}_j)^2.$$

Differentiate it along the solution of the closed-loop system, we have

$$\begin{aligned} \dot{V} &= -\tilde{u}_{*j}^\top K_p \tilde{u}_{*j} + \frac{1}{2}\tilde{u}_{*j}^\top (\bar{M}_j - 2\bar{C}_j) \tilde{u}_{*j} \\ &\quad + \tilde{u}_{*j}^\top (\hat{W}_{*j} - \bar{W}_{*j})^\top \phi_j - \tilde{u}_{*j}^\top \rho_{3j} \text{sign}(\tilde{u}_{*j}) - \tilde{u}_{*j}^\top \epsilon_j \\ &\quad + \text{tr}((\hat{W}_{*j} - \bar{W}_{*j})^\top \Gamma_j^{-1} \dot{\hat{W}}_{*j}) + \beta_{3j}^{-1}(\rho_{3j} - \bar{\epsilon}_j) \dot{\rho}_{3j} \\ &= -\tilde{u}_{*j}^\top K_p \tilde{u}_{*j} - \tilde{u}_{*j}^\top \rho_{3j} \text{sign}(\tilde{u}_{*j}) - \tilde{u}_{*j}^\top \epsilon_j \\ &\quad + \beta_{3j}^{-1}(\rho_{3j} - \bar{\epsilon}_j) \dot{\rho}_{3j} \\ &\leq -\tilde{u}_{*j}^\top K_p \tilde{u}_{*j} - \bar{\epsilon}_j(|\tilde{u}_{1j}| + |\tilde{u}_{2j}|) - \tilde{u}_{*j}^\top \epsilon_j \\ &\quad - (\rho_{3j} - \bar{\epsilon}_j)(|\tilde{u}_{1j}| + |\tilde{u}_{2j}|) + \beta_{3j}^{-1}(\rho_{3j} - \bar{\epsilon}_j) \dot{\rho}_{3j} \\ &\leq -\tilde{u}_{*j}^\top K_p \tilde{u}_{*j} \leq 0 \end{aligned}$$

which means that V is bounded. So, $\tilde{u}_{*j}$, β_{3j} , and $\hat{W}_{*j}$ are bounded. By integrating both sides of the above inequality, it can be shown that $\tilde{u}_{*j}$ is square-integrable. Noting that $\dot{\tilde{u}}_{*j}$ is bounded, by Barbalat's lemma $\tilde{u}_{*j}$ converges to zero.

With the above analysis, we have the following results.

Theorem 7.2. For m systems in (7.1)-(7.2) and a leader system in (7.3)-(7.4), under Assumptions 7.1-7.2, the distributed control law (7.50) and the update law (7.53) for $1 \leq j \leq m$ ensure that (7.5) holds and $\hat{W}_{*j}$ and ρ_{3j} are bounded, where η_{1j} and η_{2j} are defined in (7.29) and (7.33), ξ_{1j} and χ_{1j} are generated by (7.55)-(7.58), k_l ($2 \leq l \leq n$) are chosen such that the polynomial in (7.36) is Hurwitz, ρ_{1j} and ρ_{2j} are chosen such that (7.61)-(7.62) are satisfied, ρ_{3j} is updated by (7.63), $a_{ji} > 0$, $\beta_1 > 0$, $\beta_2 > 0$, $\beta_{3j} > 0$, K_p and Γ_j are positive definite matrices.

In Theorem 7.2, ρ_{1j} and ρ_{2j} are required to satisfy the inequalities in (7.61)-(7.62). It is possible to estimate them if the following assumptions are satisfied.

Assumption 7.3. The communication graph $\mathcal{G}$ is bidirectional and node $(m+1)$ is globally reachable.

Assumption 7.4. $\dot{\xi}_{2,m+1}$ and $\dot{\chi}_{2,m+1}$ are bounded

Under Assumptions 7.3-7.4, with the aid of Lemma 7.6 the update laws for ρ_{1j} and ρ_{2j} can be chosen as

$$\dot{\rho}_{1j} = \gamma_j \left| \sum_{i \in \mathcal{N}_j^e} a_{ji} (\xi_{2j} - \xi_{2i}) \right| \quad (7.64)$$

$$\dot{\rho}_{2j} = \gamma_j \left| \sum_{i \in \mathcal{N}_j^e} a_{ji} (\chi_{2j} - \chi_{2i}) \right| \quad (7.65)$$

where $\gamma_j > 0$ and $1 \leq j \leq m$. These results are summarized as follows.

Theorem 7.3. For m systems in (7.1)-(7.2) and a leader system in (7.3)-(7.4), under Assumptions 7.2-7.4, the distributed control law (7.50) and the update law (7.53) for $1 \leq j \leq m$ ensure that (7.5) holds and $\hat{W}_{*j}$ and ρ_{3j} are bounded, where η_{1j} and η_{2j} are defined in (7.29) and (7.33), ξ_{1j} and χ_{1j} are generated by (7.55)-(7.58), k_l ($2 \leq l \leq n$) are chosen such that the polynomial in (7.36) is Hurwitz, ρ_{1j} and ρ_{2j} are updated by (7.64)-(7.65), ρ_{3j} is updated by (7.63), $a_{ji} > 0$, $\beta_1 > 0$, $\beta_2 > 0$, $\beta_{3j} > 0$, $\gamma_j > 0$, K_p and Γ_j are positive definite matrices.

7.4 Distributed Tracking Control of Multiple Mechanical Systems

In last section, we solved the distributed tracking control of mechanical systems with velocity constraints. If there is no velocity constraints, the method proposed in the last section also works.

For m mechanical systems, the dynamics of system j without constraint can be written as

$$M_j(q_{*j})\ddot{q}_{*j} + C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j} + G_j(q_{*j}) = \tau_j \quad (7.66)$$

where $q_{*j} = [q_{1j}, \dots, q_{nj}]^\top$ is the state of system j , $M_j(q_{*j}) \in R^{n \times n}$ is a bounded positive-definite symmetric matrix, $C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j}$ is centripetal and Coriolis torque, $G_j(q_{*j})$ is a gravitational torque, and $\tau_j \in R^r$ is the control input. In this paper, it is assumed that the dynamics of (7.66) is unknown and Property 7.2 is satisfied. The communication between systems is described by a communication digraph $\mathcal{G}$.

It is given a leader system whose state is $q_{m+1}(t)$ and is only available to a portion of m systems. The leader system is labeled as system $m + 1$. The communication between $m + 1$ systems is described by the communication digraph $\mathcal{G}^e$. The distributed tracking control problem is to design a distributed tracking control law for each system using its local information such that (7.5) holds for $1 \leq j \leq m$.

The control laws can be designed by following the procedure in the last section. To this end, (7.66) is written as

$$\dot{q}_{*j} = u_{*j} \quad (7.67)$$

$$\bar{M}_j(q_{*j})\dot{u}_{*j} + \bar{C}_j(q_{*j}, \dot{q}_{*j})u_{*j} + \bar{G}_j(q_{*j}) = \tau_j \quad (7.68)$$

where $\bar{M}_j(q_{*j}) = M_j(q_{*j})$, $\bar{C}_j(q_{*j}, \dot{q}_{*j}) = C_j(q_{*j}, \dot{q}_{*j})$, $\bar{G}_j(q_{*j}) = G_j(q_{*j})$. The system in (7.67)-(7.68) has a cascade structure. Controllers are designed in three steps.

• **Step 1:** For system (7.67), the distributed controller is proposed as

$$u_{*j} = \eta_{*j} \quad (7.69)$$

where

$$\eta_{*j} = - \sum_{i \in \mathcal{N}_j^e} a_{ji}(q_{*j} - q_{*i}) + \xi_{*j} \quad (7.70)$$

and ξ_{*j} is an estimate of $\dot{q}_{*,m+1}$ and will be designed later such that

$$\lim_{t \rightarrow \infty} (\xi_{*j} - \dot{q}_{*,m+1}) \stackrel{exp.}{=} 0. \quad (7.71)$$

By Lemma 7.6, it can be shown that (7.5) holds with the control input (7.69).

• **Step 2:** Since u_{*j} is not the real control input, u_{*j} cannot be η_{*j} . Let $\tilde{u}_{*j} = u_{*j} - \eta_{*j}$ and $\tilde{q}_{*j} = q_{*j} - q_{*,m+1}$, then

$$\dot{\tilde{q}}_{*j} = - \sum_{i \in \mathcal{N}_j^e} a_{ji}(\tilde{q}_{*j} - \tilde{q}_{*i}) + \tilde{u}_{*j} + \xi_{*j} - \dot{q}_{*,m+1} \quad (7.72)$$

$$\bar{M}_j(q_{*j})\dot{\tilde{u}}_{*j} + \bar{C}_j(q_{*j}, \dot{q}_{*j})\tilde{u}_{*j} = \tau_j - (\bar{M}_j(q_{*j})\dot{\eta}_{*j} + \bar{C}_j(q_{*j}, \dot{q}_{*j})\eta_{*j} + \bar{G}_j(q_{*j})). \quad (7.73)$$

It can be shown that system (7.72) has the ISS property with input $\tilde{u}_{*j} + \xi_{*j} - \dot{q}_{*,m+1}$. Next, we design τ_j such that $\tilde{u}_{*j}$ converges to zero. Since dynamics (7.66) is unknown, $f_j(x_j)$ defined in (7.46) is unknown. We choose a suitable activation function and suitable weights and thresholds in the hidden layer, then $f_j(x_j)$ can be written as (7.47). We propose the control law as in (7.50) and the update law as in (7.53). Similarly, it can be proved that $\tilde{u}_{*j}$ converges to zero and $\hat{W}_{*j}$ is bounded.

• **Step 3:** In this step, we design ξ_{*j} such that ξ_{*j} is bounded and (7.71) is satisfied. This step is similar as Step 3 in the last section. The control law for ξ_{*j} is proposed as

$$\dot{\xi}_{*j} = -\beta_1 \xi_{*j} + \zeta_{*j} \quad (7.74)$$

$$\dot{\zeta}_{*j} = -\sum_{i \in \mathcal{N}_j^e} a_{ji}(\zeta_{*j} - \zeta_{*i}) - \rho_{1j} \operatorname{sign} \left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(\zeta_{*j} - \zeta_{*i}) \right) \quad (7.75)$$

where

$$\zeta_{*,m+1} = \ddot{q}_{*,m+1} + \beta_1 \dot{q}_{*,m+1} \quad (7.76)$$

and

$$\rho_{1j} \geq \sqrt{\dot{\zeta}_{*,m+1}^\top \dot{\zeta}_{*,m+1}}. \quad (7.77)$$

With the above controller design procedure, we have the following results.

Theorem 7.4. *For m systems in (7.66) and a leader system with state $q_{*,m+1}$, under Assumption 7.1, the distributed control law (7.50) and the update law (7.53) for $1 \leq j \leq m$ ensure that (7.5) holds and $\hat{W}_{*j}$ is bounded, where η_{*j} is defined in (7.70), ξ_{*j} is generated by (7.74)-(7.75), ρ_{1j} and ρ_{3j} are chosen such that (7.77) and (7.51) are satisfied, $a_{ji} > 0$, $\beta_1 > 0$, $\beta_2 > 0$, K_p and Γ_j are positive definite matrices.*

In Theorem 7.4, it is required that ρ_{3j} satisfies (7.51). It is possible to estimate ρ_{3j} as in the following theorem.

Theorem 7.5. *For m systems in (7.66) and a leader system with the state $q_{*,m+1}$, under Assumption 7.1, the distributed control law (7.50) and the update law (7.53) for $1 \leq j \leq m$ ensure that (7.5) holds and $\hat{W}_{*j}$ and ρ_{3j} are bounded, where η_{*j} is defined in (7.70), ξ_{*j} is generated by (7.74)-(7.75), ρ_{1j} is chosen such that (7.77) is satisfied, ρ_{3j} is updated by*

$$\dot{\rho}_{3j} = \beta_{3j} \sum_{i=1}^n |\tilde{u}_{ij}| \quad (7.78)$$

$a_{ji} > 0$, $\beta_1 > 0$, $\beta_{3j} > 0$, K_p and Γ_j are positive definite matrices.

In Theorem 7.5, ρ_{1j} is required to satisfy the inequalities in (7.77). It is possible to estimate them under the following assumption.

Assumption 7.5. *$\frac{d^3}{dt^3} q_{*,m+1}$ and $\ddot{q}_{*,m+1}$ are bounded.*

Theorem 7.6. *For m systems in (7.66) and a leader system with the state $q_{*,m+1}$, under Assumptions 7.3 and 7.5, the distributed control law (7.50) and the update law (7.53) for $1 \leq j \leq m$ ensure that (7.5) holds and $\hat{W}_{*j}$ and ρ_{3j} are bounded, where η_{*j} is defined in (7.70), ξ_{*j} is generated by (7.74)-(7.75), ρ_{1j} is updated by*

$$\dot{\rho}_{1j} = \gamma_j \sum_{l=1}^n \left| \sum_{i \in \mathcal{N}_j^e} a_{ji}(\zeta_{lj} - \zeta_{li}) \right| \quad (7.79)$$

ρ_{3j} is updated by (7.78), $a_{ji} > 0$, $\beta_1 > 0$, $\beta_{3j} > 0$, $\gamma_j > 0$, K_p and Γ_j are positive definite matrices.

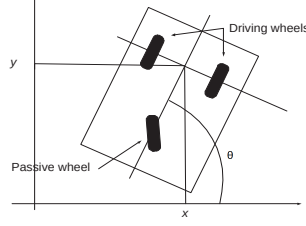


Figure 7.2: Configuration of a wheeled mobile robot

7.5 Simulations

7.5.1 Formation control of Multiple Mobile Robots

To verify the proposed results in Section 7.3, simulation has been done for formation control of five nonholonomic wheeled mobile robots on a horizontal plane (Fig. 7.2). For robot j , its dynamics and the constraints are shown in (7.1)-(7.2), where $q_{*j} = [x_j, y_j, \theta_j]^\top$, $J(q_{*j}) = [\sin \theta_j, \cos \theta_j, 0]$, $M_j = \text{diag}[m_j, m_j, I_j]$, $C_j = \mathbf{0}$, $G_j = \mathbf{0}$,

$$B_j = \frac{1}{R_j} \begin{bmatrix} \cos \theta_j & \cos \theta_j \\ \sin \theta_j & \sin \theta_j \\ L_j & -L_j \end{bmatrix}$$

(x_j, y_j) is the position of robot j , θ_j is the orientation of robot j , R_j is the radius of the driving wheels, $2L_j$ is the distance between the two driving wheels, m_j is the mass of robot j , and I_j is the inertia moment of robot j around the vertical axis. In the controller design, the dynamics of each robot is assumed to be unknown for controller design.

It is given a leader system (7.3)-(7.4) and a desired formation defined by a geometric pattern $\mathcal{P}$ whose vertexes are at coordinate: (p_{x1}, p_{y1}) , (p_{x2}, p_{y2}) , (p_{x3}, p_{y3}) , (p_{x4}, p_{y4}) , and (p_{x5}, p_{y5}) . The leader system is labeled as system 6. The formation control problem of five mobile robots is to design distributed control laws τ_j using neighbors' information such that

$$\lim_{t \rightarrow \infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{xj} \\ p_{iy} - p_{yj} \end{bmatrix}, i \neq j \quad (7.80)$$

$$\lim_{t \rightarrow \infty} (\theta_j - \theta_6) = 0, 1 \leq j \leq 5 \quad (7.81)$$

$$\lim_{t \rightarrow \infty} \left(\sum_{j=1}^5 \frac{x_j}{5} - x_6 \right) = 0, \quad (7.82)$$

$$\lim_{t \rightarrow \infty} \left(\sum_{j=1}^5 \frac{y_j}{5} - y_6 \right) = 0. \quad (7.83)$$

The velocity constraint (7.2) is equivalent to

$$\dot{x}_j = w_{1j} \cos \theta_j, \quad \dot{y}_j = w_{1j} \sin \theta_j, \quad \dot{\theta}_j = w_{2j} \quad (7.84)$$

$$\psi_j = \left[e^{-\frac{q_{1j}^2}{2}} \quad e^{-\frac{q_{2j}^2}{2}} \quad e^{-\frac{q_{3j}^2}{2}} \quad e^{-\frac{q_{1j}^2}{2}} \quad e^{-\frac{q_{2j}^2}{2}} \quad e^{-\frac{q_{3j}^2}{2}} \quad e^{-\frac{\eta_{1j}^2}{2}} \quad e^{-\frac{\eta_{2j}^2}{2}} \quad e^{-\frac{\eta_{1j}^2}{2}} \quad e^{-\frac{\eta_{2j}^2}{2}} \right]^\top \quad (7.88)$$

where w_{1j} and w_{2j} are the speed and angular speed of robot j . The formation control problem can be solved with the aid of the results proposed in the last section. To this end, we define

$$\begin{cases} q_{1j} &= \theta_j \\ q_{2j} &= (x_j - p_{xj}) \cos \theta_j + (y_j - p_{yj}) \sin \theta_j \\ q_{3j} &= (x_j - p_{xj}) \sin \theta_j - (y_j - p_{yj}) \cos \theta_j \\ u_{1j} &= w_{2j} \\ u_{2j} &= w_{1j} - w_{2j} q_{3j} \end{cases} \quad (7.85)$$

for $1 \leq j \leq 6$, where $p_{x6} = p_{y6} = 0$. Then,

$$\dot{q}_{1j} = u_{1j}, \quad \dot{q}_{2j} = u_{2j}, \quad \dot{q}_{3j} = u_{1j} q_{2j} \quad (7.86)$$

which is a special case of system (7.27). In dynamics (7.25),

$$\bar{M}_j = \begin{bmatrix} m_j q_{3j}^2 + I_j & m_j q_{3j} \\ m_j q_{3j} & m_j \end{bmatrix}, \quad \bar{C}_j = \begin{bmatrix} m_j q_{3j} \dot{q}_{3j} & 0 \\ m_j \dot{q}_{3j} & 0 \end{bmatrix},$$

$$\bar{B}_j = \begin{bmatrix} \frac{q_{3j} + L_j}{R_j} & \frac{q_{3j} - L_j}{R_j} \\ 1 & 1 \end{bmatrix}, \quad \bar{G}_j = \mathbf{0}.$$

In (7.85), the notation q_{ij} is reused in conformity to the notation in Section 7.3. It can be shown that the formation control problem is solved if (7.5) is satisfied for $1 \leq j \leq 5$. Therefore, the proposed distributed control laws in Section 7.3 can solve the formation control problem.

In the simulation, it is assumed that the desired formation pattern is shown in Fig. 7.3 and the state of the leader system is $(x_6, y_6, \theta_6) = (10 \sin(0.5t), -10 \cos(0.5t), 0.5t)$. The distributed control laws in Theorem 7.1 solve the formation control problem. In the control laws, we choose the activation function

$$\phi_j(z) = e^{-z^2/2} \quad (7.87)$$

and ψ_j is defined in (7.88). ϕ_j can be chosen as other forms which are omitted here for space limit. Fig. 7.4 shows the communication digraph between systems. Figs. 7.5-7.7 show the response of $(x_j - x_6 - p_{xj})$, $(y_j - y_6 - p_{yj})$, and $(\theta_j - \theta_6)$ for $1 \leq j \leq 5$, respectively. The simulation results show that (7.5) is satisfied. Fig. 7.8 shows the centroid of x_i ($1 \leq i \leq 5$) (i.e., $\sum_{j=1}^5 x_j/5$) and x_6 . Fig. 7.9 shows the centroid of y_i ($1 \leq i \leq 5$) (i.e., $\sum_{j=1}^5 y_j/5$) and y_6 . Fig. 7.10 shows the path of the centroid of the five robots and its desired path. Fig. 7.11 shows the estimates of the parameter W_{*j} in the neural networks. The simulation results verify that (7.81)-(7.83) are satisfied. Eqn. (7.80) is also verified and the response of them is omitted here. The results in Theorem 7.2-7.3 were also validated by simulations. Due to space limitation, we omit them.

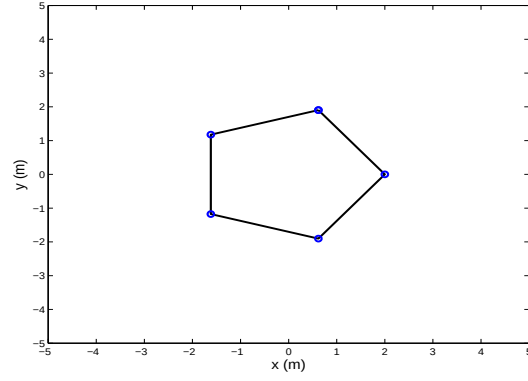


Figure 7.3: Geometric pattern of the formation

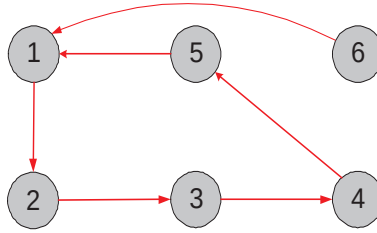


Figure 7.4: Communication digraph $\mathcal{G}$

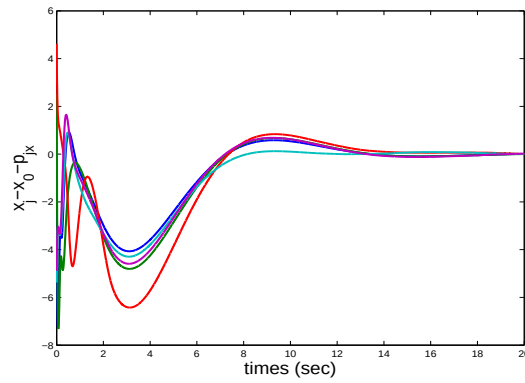


Figure 7.5: Response of $(x_j - x_6 - p_{xj})$ for $1 \leq i \leq 5$.

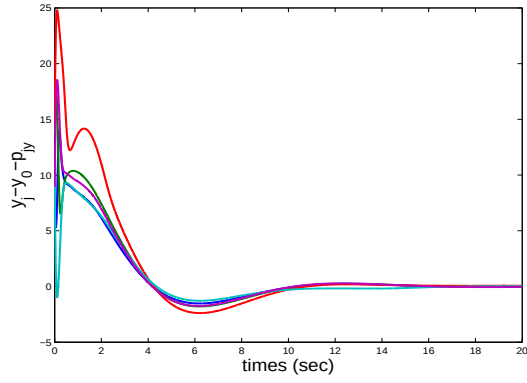


Figure 7.6: Response of $(y_j - y_0 - p_{yj})$ for $1 \leq i \leq 5$.

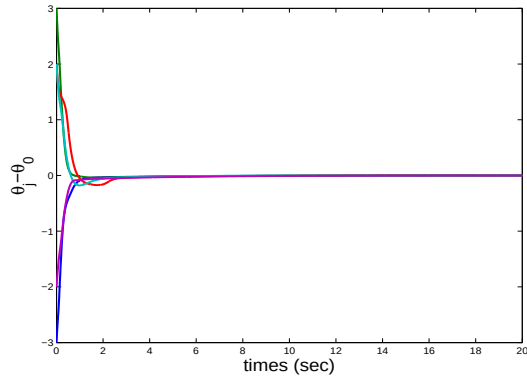


Figure 7.7: Response of $(\theta_j - \theta_0)$ for $1 \leq i \leq 5$.

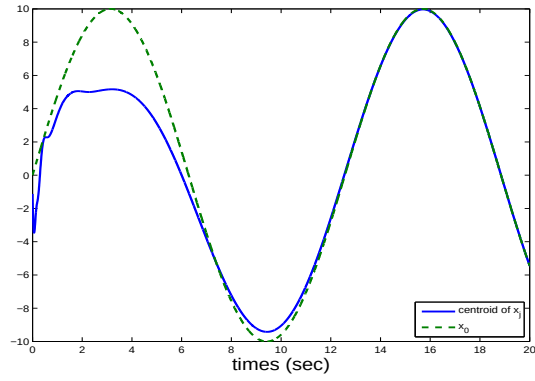


Figure 7.8: Response of the centroid of x_i (solid) for $1 \leq i \leq 5$ and x_6 (dashed).

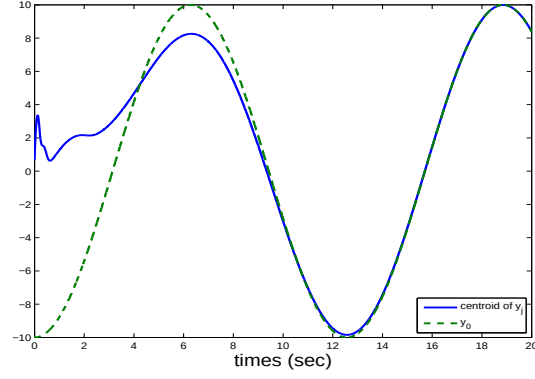


Figure 7.9: Response of the centroid of y_i (solid) for $1 \leq i \leq 5$ and y_6 (dashed).

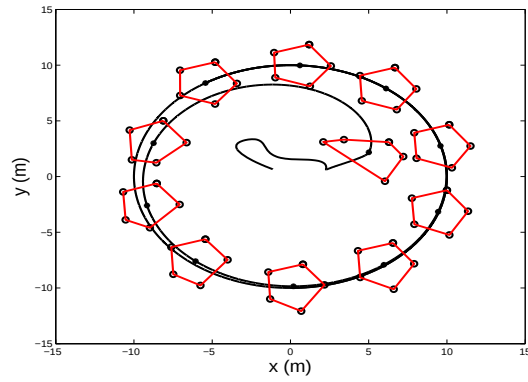


Figure 7.10: The path of the centroid of the five robots (dashed line), the desired path (solid line) of the centroid of robots, and formation of the five robots at several moments (red pentagons).

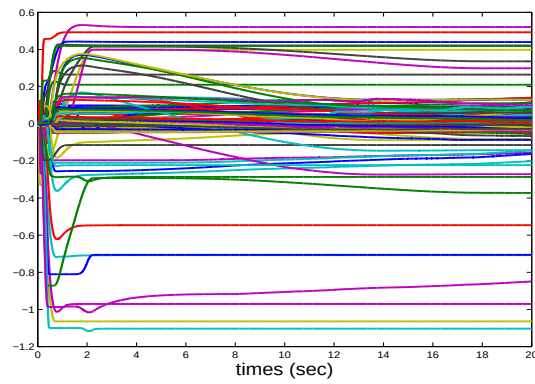


Figure 7.11: Estimates of parameter W_{*j} in neural networks.

$$\psi_j = \left[e^{-\frac{q_{1j}^2}{2}} \quad e^{-\frac{q_{2j}^2}{2}} \quad e^{-\frac{\dot{q}_{1j}^2}{2}} \quad e^{-\frac{\dot{q}_{2j}^2}{2}} \quad e^{-\frac{\eta_{1j}^2}{2}} \quad e^{-\frac{\eta_{2j}^2}{2}} \quad e^{-\frac{\dot{\eta}_{1j}^2}{2}} \quad e^{-\frac{\dot{\eta}_{2j}^2}{2}} \right]^\top \quad (7.89)$$

7.5.2 Synchronization of Multiple 2-DOF Manipulators

To verify the proposed results, simulation has been done for three follower systems and one leader system. For the three follower systems, each system is planar manipulator with two revolute joints (see [115]). The dynamics of each system can be written as (7.66), where $q_j \in R^2$ is the joint variable, τ_j is the control input,

$$\begin{aligned} M_j &= \begin{bmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{bmatrix} \\ C_j &= \begin{bmatrix} h\dot{q}_{2j} & h(\dot{q}_{1j} + \dot{q}_{2j}) \\ -h\dot{q}_{1j} & 0 \end{bmatrix} \\ G_j &= \begin{bmatrix} (m_1 l_{c1} + m_2 l_1)g \cos q_{1j} + m_2 l_{c2} g \cos(q_{1j} + q_{2j}) \\ m_2 l_{c2} \cos(q_{1j} + q_{2j}) \end{bmatrix} \\ d_{11} &= m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos q_{2j}) + I_1 + I_2 \\ d_{12} &= m_2 (l_{c2}^2 + l_1 l_{c2} \cos q_{2j}) + I_2 \\ d_{22} &= m_2 l_{c2}^2 + I_2 \\ h &= -m_2 l_1 l_{c2} \sin q_{2j} \end{aligned}$$

where the physical meaning of each variable above can be found in [115]. For simplicity, three follower systems are assumed to be identical. The communication between systems is defined by the digraph in Fig. 7.12. The state of the leader system is $q_{*4} = [2 \cos t, 3 \sin 2t]^\top$ and is available to system 1. The distributed control problem is to design distributed control law for each system such that $(q_{*j} - q_{*4})$ converges to zero.

In the simulation, the dynamics is unknown. By Theorem 7.5, the control law is (7.50) and the update law is (7.53). In the control law, the activation function is chosen as in (7.87) where ψ_j is defined in (7.89). Fig. 7.13 shows the time response of $(q_{1j} - q_{14})$ for $1 \leq j \leq 3$. Fig. 7.14 shows the time response of $(q_{2j} - q_{24})$ for $1 \leq j \leq 3$. It is shown that $(q_{*j} - q_{*4})$ converges to zero, which means that the proposed results in Theorem 7.5 are right. The simulation of the control laws in Theorem 7.6 has also been done. The simulation results verify the results in Theorem 7.6. Due to space limitation, we omit them here.

7.6 Conclusion

This chapter considers distributed leader-following cooperative control of uncertain mechanical systems with/without velocity constraints. Neural network based distributed control laws are proposed with the aid of distributed estimation, robust control, and neural network

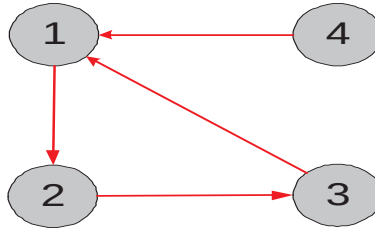


Figure 7.12: Communication digraph.

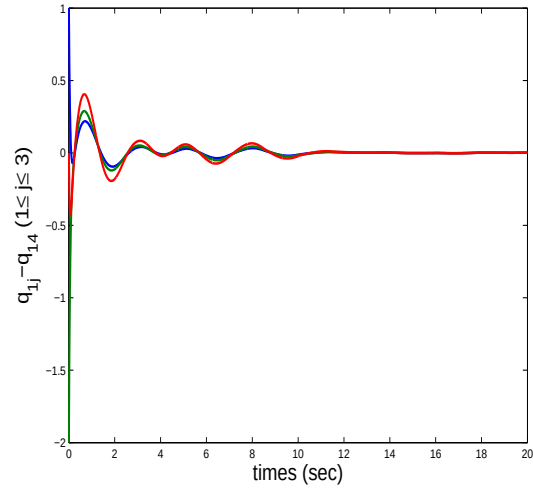


Figure 7.13: Response of $(q_{1j} - q_{14})$ for $1 \leq j \leq 3$

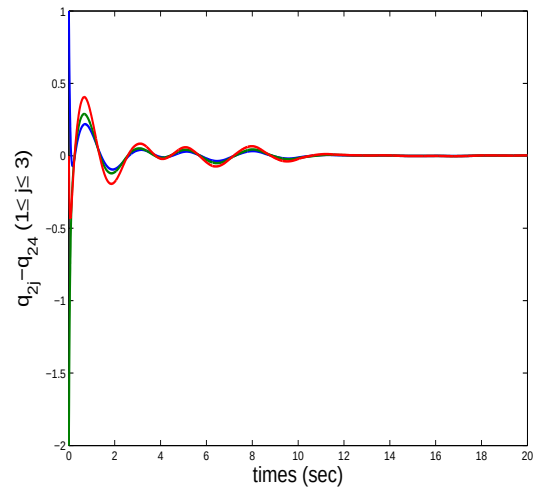


Figure 7.14: Response of $(q_{2j} - q_{24})$ for $1 \leq j \leq 3$

approximation techniques. Applications of the proposed results are presented. Simulation results show the effectiveness of the proposed results.

Chapter 8

Distributed Formation Tracking Control of Multiple Mobile Robotic Systems

8.1 Introduction

Consensus problem for multi-agent systems has been intensively studied due to developments of distributed computing and graph theory,

Robotic mobile vehicle systems are applied into variety of environments including military and industry with advantages of high flexibility and various capabilities, when equipped with accessories such as hand grippers and infrared detector, mobile vehicles can fulfil complex tasks of rescue, security and navigation. Cooperation of multiple vehicles receive more attentions in recent years as group performances guarantee higher efficiency and robustness, compared with monolithic system, multivehicle systems are more applicable to spatially distributed environments where mobile vehicles are required to function in different locations, disability of a portion of vehicles won't affect the overall performance by simply being cut off from the rest sound ones. Before developing capabilities for multivehicle systems, it is important to study the coordination problem since individual vehicle in the group is expected to move in specific manners. various control methods are applied to formation control of multivehicle systems and rendezvous problem. Coordination of multivehicle systems can be transformed into consensus problem as all vehicle systems are supposed to converge to a reference state, from analysis of vehicle models, it is known kinematic of vehicle systems is a first-order nonlinear system, by linearization from state transform or other nonlinear control methods, vehicle kinematic system can be stablized and converge to consensus values. Consensus on multiple single-integrator systems is addressed in [29, 33, 34, 45]. In [34], Consensus control for both continuous-time and discrete single-integrator is addressed, the authors consider switching communication graphs and the conception of union of digraph is introduced to solve the consensus for dynamically changing topology. Consensus control algorithms for multiple second-order systems are studied in [116][117]. In [117], Tracking with a time-varying second-order leader is addressed, leader of both bounded acceleration and linear form are considered. In [118], Consensus problem for multiple high-order linear

systems with time delays in both the communication network and inputs is considered, observer based output feedback protocols are proposed for arbitrarily large yet bounded delays. In [119], The authors use decentralized discrete-time block control scheme to achieve formation and trajectory tracking, both first-order and second-order systems are discussed, each agent is provided with discrete-time state observers and formation tracking can be achieved applying the proposed block control with a consensus scheme. In [120], distributed consensus tracking algorithms without velocity measurements under both fixed and switching network topologies for first-order and second-order kinematics are proposed for a leader-follower communication topology. A mild connectivity requirement is adopted for distributed consensus tracking and swarm tracking.

Formation tracking is one common problem of multiple vehicle coordination, a time-varying reference signal is regarded as the desired trajectory and all the vehicle agents are supposed to track it. Unlike centralized formation tracking control, in which individual follower can utilize information from reference signals directly along with its own states, distributed formation tracking control utilizes information of vehicle's own states and that from its neighbors from which it can receive information from wireless communication. In general communication topologies, only a portion of vehicles can communicate with the reference signal, control of all the other vehicles depends on control information from their neighbors. Distributed formation control is addressed in [121][122][123][6], in [121], a new kinematics model for leader-follower system is addressed and globally stable controller is designed with the aid of backstepping methods. In [122][123], Formation stabilization for multiple nonholonomic mobile robots is addressed and distributed control methods are studied for vehicles kinematic systems with the aid of cascaded system, a leader-follower graph is applied to describe the communication topology of multivehicle systems. In [6], Distributed formation tracking methods are addressed with the aid of σ process by suitable variable transformation. Delayed communication is considered for the proposed controllers. In [124], a novel formation control technique of multiple wheeled mobile robots employing artificial potential field based navigation is addressed, the communication graph is considered as a leader-follower topology, the leader motion by artificial field and followers motion by control. In [125], Instead of measuring the poses directly from sensors, the authors apply continuous-time extended Kalman filter to estimate the poses under the assumption each robot has only coarse positioning sensors. In [126], the leader-waypoint-follower formation is constructed based on relative motion states, the followers are designed to move to their desired waypoints, both stable tracking and receding horizon tracking control methods are applied to guarantee the convergence errors stable and tend toward zero efficiently. In [127], a formation controller is designed to make the formation control system robust against the unmeasured velocity of the leader robot with the Lyapunov redesign technique.

In real-life operations, control of kinematics is not practical due to the unavailability of velocities, engine-generated torques actually control the motion of vehicles. From analysis of vehicle dynamics. It is learned parametrical uncertainties exist in the dynamic systems. Uncertainties are either estimated by adaptive control methods [128][129] or through neural networks' on-line training process [130][131]. In [128][129], Parametrical uncertainties are estimated by adaptive control methods and a dynamics-based controller is designed with the aid of kinematic-based controller and backstepping methods. In [130][131], Neural networks

are applied to estimate the dynamics of the robots and computed-torque controller is designed with the aid of kinematic-based formation controller and backstepping methods. In [132], A dynamics-based controller consisting of a feedback linearization part and a sliding mode compensator, is designed for leader-tracking.

In this chapter, distributed formation tracking of multiple wheeled unicycles is considered, variable transformations are utilized to change vehicle' kinematic system into chained-form system, exponential stability of cascaded system is introduced for controlling distributed formation tracking algorithms for transformed kinematic systems and exponential stability of the chained-form systems is proved to guarantee all vehicles track the desired reference trajectory with fixed formation. Distributed controller for vehicles' dynamics is designed with the aid of backstepping methods and kinematics-based controller, sliding mode control method is applied to estimate the parametrical uncertainties. Compared with the distributed algorithms in [133], the control methods proposed in this paper is more practical and implementable since the derivatives in kinematic control part is removed, besides, the variable transformation and control laws are conciser compared with that in [133], which also simplifies the control in real-life operations. Simulations have been done to prove the proposed control algorithms.

8.2 Problem Statement

With the aid of Lagrange-D'Alembert principle, m unicycle systems with three generalized states are defined as

$$M_j(q_{*j})\ddot{q}_{*j} + C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j} + G_j(q_{*j}) = B_j(q_{*j})\tau_j + J^\top(q_{*j})\lambda_j \quad (8.1)$$

$$J(q_{*j})\dot{q}_{*j} = 0 \quad (8.2)$$

where $q_{*j} = [q_{1j}, q_{2j}, q_{3j}]^\top = [x_j, y_j, \theta_j]^\top$ are the states of system j , $M_j(q_{*j})$ is a 3×3 positive-definite symmetric matrix, $C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j}$ represents centripetal and Coriolis force, $G_j(q_{*j})$ is the gravitational force, $B_j(q_{*j})$ is a 3×2 input transformation matrix, $J(q_{*j})$ is the two-dimensional control inputs vector, λ_j is the constraint force imposed on system j . Then derivatives of generalized states $\dot{q}_{*j}$ can be defined as

$$\dot{q}_{*j} = g(q_{*j})V_{*j} = \begin{bmatrix} \cos \theta_j & 0 \\ \sin \theta_j & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_j \\ \omega_j \end{bmatrix} \quad (8.3)$$

In the right hand of (8.1) there are nonholonomic constraints $J^\top(q_{*j})\lambda_j$ which have nothing to do with controlling torques, notice $g(q_{*j}) * J^\top(q_{*j}) = \mathbf{0}$, then by replacing q_{*j} in (8.1) with V_{*j} , nonholonomic constraints are removed.

$$\bar{M}_j(q_{*j})\dot{V}_{*j} + \bar{C}_j(q_{*j}, \dot{q}_{*j})V_{*j} + \bar{G}_j(q_{*j}) = \bar{B}_j(q_{*j})\tau_j \quad (8.4)$$

where

$$\bar{M}_j(q_{*j}) = g^\top(q_{*j})M_j(q_{*j})g(q_{*j})$$

$$\begin{aligned}
\bar{C}_j(q_{*j}, \dot{q}_{*j}) &= g^\top(q_{*j})M_j(q_{*j})\dot{g}(q_{*j}) \\
&\quad + g^\top(q_{*j})C_j(q_{*j}, \dot{q}_{*j})g(q_{*j}) \\
\bar{B}_j(q_{*j}) &= g^\top(q_{*j})B_j(q_{*j})
\end{aligned}$$

The communication between vehicles is characterized by a directed tree $G = (\mathcal{V}, \mathcal{E})$ through this paper, where $\mathcal{V}$ is the set of vehicle vertices with $\mathcal{V} = \{v_1, v_2, \dots, v_m\}$ and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$. $e_{ij} = (v_i, v_j)$ is an element of $\mathcal{E}$ if the state information flow goes from v_i to v_j , v_i is called a neighbor of v_j and $\mathcal{N}_j$ represents the sets of all neighbors of v_j . For the m mobile vehicle systems, adjacency matrix $A = [a_{ji}]$ is given to characterize edge set $\mathcal{E}$ with $a_{ji} = a_{ij} > 0$ and $a_{ii} = 0$ since individual vehicle doesn't need communication to receive its own states. Laplacian matrix $\mathcal{L} = [L_{ji}]$ of the graph $\mathcal{G}$ with adjacency matrix A is defined as

$$L_{ji} = \begin{cases} -a_{ji}, & \text{if } i \in \mathcal{N}_j \text{ and } i \neq j \\ 0, & \text{if } i \notin \mathcal{N}_j \text{ and } i \neq j \\ \sum_{l \neq j, l \in \mathcal{N}_j} a_{jl}, & \text{if } j = i. \end{cases}$$

The time-varying reference signal is defined by a sinusoidal trajectory

$$\begin{aligned}
\dot{x}_0 &= v_0 \cos \theta_0 \\
\dot{y}_0 &= v_0 \sin \theta_0 \\
\dot{\theta}_0 &= \omega_0
\end{aligned} \tag{8.5}$$

where the subscript 0 means the reference signal is regarded as a virtual vehicle agent, the desired formation of multivehicle systems is denoted by $\mathcal{P}$ with a group of orthogonal coordinates $p_j = (p_{jx}, p_{jy})$ for $(1 \leq j \leq m)$, p_j satisfy $\sum_{j=1}^m p_{jx} = 0$ and $\sum_{j=1}^m p_{jy} = 0$. $p_0 = (p_{0x}, p_{0y})$ for the virtual leader is supposed to be $(0, 0)$.

Distributed formation tracking control is defined as designing control laws $V_{*j} = [v_j, \omega_j]$ for system j by using its own state information and its neighbors' state information such that

$$\lim_{t \rightarrow \infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix} \tag{8.6}$$

$$\lim_{t \rightarrow \infty} (\theta_i - \theta_0) = 0 \tag{8.7}$$

$$\lim_{t \rightarrow \infty} \left[\sum_{i=1}^m \frac{x_i}{m} - x_0 \right] = 0, \lim_{t \rightarrow \infty} \left[\sum_{l=1}^m \frac{y_l}{m} - y_0 \right] = 0 \tag{8.8}$$

for $1 \leq i \neq j \leq m$.

It is known that Eqn. (8.6) guarantees vehicles keep formation $\mathcal{P}$ during the motion. Eqn. (8.7) vehicles reach the consensus orientation. Eqn. (8.8) ensures the centroid of multivehicle system converges to the virtual leader.

Define the states transformation equations as

$$\begin{cases} q_{1j} = \theta_j \\ q_{2j} = (x_j - p_{jx}) \cos \theta_j + (y_j - p_{jy}) \sin \theta_j \\ q_{3j} = (x_j - p_{jx}) \sin \theta_j - (y_j - p_{jy}) \cos \theta_j \\ v_{1j} = \omega_j \\ v_{2j} = v_j - v_{1j} q_{3j} \end{cases} \tag{8.9}$$

Then the transformed state space is

$$\dot{q}_{1j} = v_{1j} \quad (8.10)$$

$$\dot{q}_{2j} = v_{2j} \quad (8.11)$$

$$\dot{q}_{3j} = v_{1j}q_{2j} \quad (8.12)$$

The virtual leader in (8.5) is also transformed to the chained-form system in (8.10)-(8.12) by (8.9) as

$$\dot{q}_{10} = v_{10}$$

$$\dot{q}_{20} = v_{20}$$

$$\dot{q}_{30} = v_{10}q_{20}$$

Lemma 8.1. *If $\lim_{t \rightarrow \infty}(q_{1j} - q_{10}) = 0$, $\lim_{t \rightarrow \infty}(q_{2j} - q_{20}) = 0$, and $\lim_{t \rightarrow \infty}(q_{3j} - q_{30}) = 0$ for $1 \leq j \leq m$, then (8.6)-(8.8) hold.*

Proof: by $q_{1j} = \theta_j$ in (8.9), (8.7) holds, by definitions of q_{2j} and q_{3j} in (8.9) it follows that

$$\begin{bmatrix} x_j - p_{jx} \\ y_j - p_{jy} \end{bmatrix} = \begin{bmatrix} \cos \theta_j & \sin \theta_j \\ \sin \theta_j & -\cos \theta_j \end{bmatrix} \begin{bmatrix} q_{2j} \\ q_{3j} \end{bmatrix}$$

therefore,

$$\begin{aligned} \lim_{t \rightarrow \infty} \begin{bmatrix} x_j - p_{jx} \\ y_j - p_{jy} \end{bmatrix} &= \lim_{t \rightarrow \infty} \begin{bmatrix} \cos \theta_j & \sin \theta_j \\ \sin \theta_j & -\cos \theta_j \end{bmatrix} \begin{bmatrix} q_{2j} \\ q_{3j} \end{bmatrix} \\ &= \begin{bmatrix} \cos \theta_0 & \sin \theta_0 \\ \sin \theta_0 & -\cos \theta_0 \end{bmatrix} \begin{bmatrix} q_{20} \\ q_{30} \end{bmatrix} \\ &= \begin{bmatrix} x_0 - p_{0x} \\ y_0 - p_{0y} \end{bmatrix} \end{aligned}$$

$\lim_{t \rightarrow \infty}[x_j - p_{jx} - x_0, y_j - p_{jy} - y_0] = [0, 0]$, then (8.6)-(8.8) hold. ■

Define $q_{*j} = [q_{1j}, q_{2j}, q_{3j}]$, by Lemma 8.1, the control problem can be transformed to designing control laws $v_{*j} = v_{1j}, v_{2j}$ such that

$$\lim_{t \rightarrow \infty}(q_{*j} - q_{*0}) = 0 \quad (8.13)$$

with the its own states information and that from its neighbors.

Notice

$$V_{*j} = \mathcal{B}_j v_{*j} \quad (8.14)$$

where $\mathcal{B}_j = \begin{bmatrix} 1 & 0 \\ z_{3j} & 1 \end{bmatrix}$.

Substitute V_{*j} in (8.14) into (8.4), we have

$$\hat{M}_j(q_{*j})\dot{v}_{*j} + \hat{C}_j(q_{*j}, \dot{q}_{*j})v_{*j} + \hat{G}_j(q_{*j}) = \hat{B}_j(q_{*j})\tau_j \quad (8.15)$$

where

$$\begin{aligned}\hat{M}_j(q_{*j}) &= \mathcal{B}_j^\top \bar{M}_j(q_{*j}) \mathcal{B}_j \\ \hat{C}_j(q_{*j}, \dot{q}_{*j}) &= \mathcal{B}_j^\top \bar{M}_j(q_{*j}) \dot{\mathcal{B}}_j + \mathcal{B}_j^\top \bar{C}_j(q_{*j}, \dot{q}_{*j}) \mathcal{B}_j \\ \bar{B}_j(q_{*j}) &= \mathcal{B}_j^\top \bar{B}_j(q_{*j})\end{aligned}$$

Eqn. (8.15) has the following properties

Property 1: Matrix $\hat{M}_j - 2\hat{C}_j$ is skew-symmetric.

Property 2: For any differentiable vector $v \in R^3$, we have

$$\hat{M}_j(q_{*j})\dot{v}_{*j} + \hat{C}_j(q_{*j}, \dot{q}_{*j})v_{*j} + \hat{G}_j(q_{*j}) = \hat{Y}_j(q_{*j}, \dot{q}_{*j}, v_{*j}, \dot{v}_{*j})a_j$$

where a_j is an inertia parameter vector of mass and moment of inertia, $\hat{Y}_j(q_{*j}, \dot{q}_{*j}, v_{*j}, \dot{v}_{*j})$ is a known function of q_{*j} , $\dot{q}_{*j}$, v_{*j} and $\dot{v}_{*j}$. a_j is supposed to be unknown.

by (8.15) it is known τ_j are the real control inputs, we first propose a control law of v_{*j} for (8.10), then we design control laws of τ_j with the knowledge of backstepping method.

8.3 Main Results

Before designing distributed formation tracking control of multivehicle systems, we introduce a theorem of exponential stability of cascaded system, consider the cascaded system

$$\begin{aligned}\dot{x}_1 &= f_1(t, x_1) + g(t, x_1, x_2)x_2 \\ \dot{x}_2 &= f_2(t, x_2)\end{aligned}\tag{8.16}$$

where $x_1 \in \mathcal{R}^n$, $x_2 \in \mathcal{R}^n$, $f_1(t, x_1)$ is continuously differentiable in (t, x_1) , $g(t, x_1, x_2)$ and $f_2(t, x_2)$ are locally Lipschitz in (x_1, x_2) and x_2 .

Theorem 8.1. *(8.16) is globally exponential stable if it satisfies the following three assumptions [134][135].*

Assumption 8.1. $\dot{x}_1 = f_1(t, x_1)$ is globally exponential stable and there exists a continuously differentiable function $V(t, x_1) : \mathcal{R}_+ \times \mathcal{R}^n$ which satisfies

$$\begin{aligned}k_1||x_1||^\alpha &\leq V(t, x_1) \leq k_2||x_1||^\alpha \\ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_1}f_1(t, x_1) &\leq -k_3||x_1||^\alpha\end{aligned}$$

where k_1, k_2, k_3 and α are positive numbers.

Assumption 8.2. the interconnection function $g(t, x_1, x_2)$ satisfies for all $t_0 \geq 0$

$$||g(t, x_1, x_2)|| \leq \theta_1(||x_2||) + \theta_2(||x_2||)x_1$$

where θ_1 and θ_2 are continuous functions.

Assumption 8.3. $\dot{x}_2 = f_2(t, x_2)$ is globally exponentially stable and for all $t_0 \geq 0$, it follows that

$$\int_{t_0}^{\infty} \|f_2(t_0, t, x_2(t_0))\| \leq k(\|x_2(t_0)\|)$$

where k is a class $\mathcal{K}$ function.

Consider a linear time-varying system

$$\dot{x} = \begin{bmatrix} 0 & \cdots & \cdots & \cdots & 0 \\ \phi(t) & 0 & \cdots & \cdots & 0 \\ 0 & \vdots & \vdots & \vdots & 0 \\ 0 & \vdots & \vdots & \vdots & 0 \\ 0 & \vdots & \vdots & \phi(t) & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} u \quad (8.17)$$

where $\phi(t)$ is a bounded continuously differentiable Lipschitz function.

Lemma 8.2. *The control algorithm*

$$u = -k_2 x_1 - k_3 \phi(t) x_2 - k_4 x_3 - k_5 \phi(t) x_4 - \dots \quad (8.18)$$

ensures system (8.17) is globally exponentially stable if k_i are such that

$$\lambda^n + k_1 \lambda^{n-1} + \dots + k_n$$

is Hurwitz [134].

8.4 Distributed Controller for Kinematic Systems

The following assumption is made on the leader agent.

Assumption 8.4. The $\frac{d^i v_{*0}}{dt^i}$ ($0 \leq i \leq 2$) are bounded and $\int_t^{t+T} v_{*0}^2(\tau) d\tau > \alpha$ for some $\alpha > 0$ and $T > 0$.

Assumption 8.4 means signal v_{*0} is persistently excited signal (PE signal).

Theorem 8.2. *For m systems in (8.10)-(8.12), if a directed spanning tree exists in the directed communication graph with the virtual leader the root of the tree, then the distributed control laws*

$$v_{1j} = u_{1j} = - \sum_{i \in \mathcal{N}_j} a_{ji} (q_{1j} - q_{1i}) - a_{j,m+1} (q_{1j} - q_{10}) + \delta_{1j} \quad (8.19)$$

$$\begin{aligned} \dot{\delta}_{1j} = & - \sum_{i \in \mathcal{N}_j} a_{ji} (\delta_{1j} - \delta_{1i}) - a_{j,m+1} (\delta_{1j} - \delta_{10}) \\ & - \rho_1 \text{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji} (\delta_{1j} - \delta_{1i}) - a_{j,m+1} (\delta_{1j} - \delta_{10}) \right] \end{aligned} \quad (8.20)$$

for $1 \leq j \leq m$, $\delta_{10} = u_{10}$, guarantee that q_{1j} globally exponentially converge to q_{10} .

Proof: Define $\tilde{\delta}_{1j} = \delta_{1j} - \delta_{10}$, (8.20) can be written as

$$\begin{aligned} \dot{\tilde{\delta}}_{1j} = & - \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}) - a_{j,m+1}\tilde{\delta}_{1j} \\ & - \rho_1 \text{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}) - a_{j,m+1}\tilde{\delta}_{1j} \right] - \dot{\delta}_{10} \end{aligned} \quad (8.21)$$

Define $\delta_{1*} = [\delta_{1j}, \delta_{2j}, \dots, \delta_{mj}]$, with the aid of Laplacian matrix (8.21) can be written as

$$\dot{\tilde{\delta}}_{1*} = -(\mathcal{L} + B)\tilde{\delta}_{1*} - \rho_1 \text{sign} [(\mathcal{L} + B)\tilde{\delta}_{1*}] - \dot{\delta}_{10} \mathbf{1} \quad (8.22)$$

where $\mathcal{L}$ is Laplacian matrix of the communication diagraph, $B = \text{diag}(a_{1,m+1}, a_{2,m+1}, \dots, a_{m,m+1})$ is the diagonal matrix representing communication with the virtual leader, from the characteristics of $\mathcal{L}$ it is known the directed spanning tree guarantees $\mathcal{L} + B$ is positive symmetric with eigenvalues in the right half of the complex plane [136]. Choose Lyapunov function $V_1 = \frac{1}{2}\tilde{\delta}_{1*}^T \tilde{\delta}_{1*}$ where $\delta_{1*} = [\delta_{11}, \dots, \delta_{1m}]$, and differentiate it along (8.22), it follows that

$$\begin{aligned} \dot{V}_1 &= -\tilde{\delta}_{1*}^T (\mathcal{L} + B) \tilde{\delta}_{1*} - \tilde{\delta}_{1*}^T \rho_1 \text{sign} [(\mathcal{L} + B)\tilde{\delta}_{1*}] - \tilde{\delta}_{1*}^T \dot{\delta}_{10} \mathbf{1} \\ &= -\tilde{\delta}_{1*}^T (\mathcal{L} + B) \tilde{\delta}_{1*} - \tilde{\delta}_{1*}^T \rho_1 \text{sign} [(\mathcal{L} + B)\tilde{\delta}_{1*}] \\ &\quad - ((\mathcal{L} + B)\tilde{\delta}_{1*})^T (L + B)^{-1} \dot{\delta}_{10} \mathbf{1} \end{aligned}$$

Let σ be the minimum eigenvalue of $(\mathcal{L} + B)^{-1}$, then

$$\dot{V}_1 \leq -\tilde{\delta}_{1*}^T (\mathcal{L} + B) \tilde{\delta}_{1*} - \sigma \rho_1 \Phi^T \text{sign} [(\mathcal{L} + B)\tilde{\delta}_{1*}] - \Phi^T (L + B)^{-1} \dot{\delta}_{10} \mathbf{1}$$

where $\Phi = ((\mathcal{L} + B)\tilde{\delta}_{1*})$. It can be proved if ρ_1 satisfies

$$\rho_1 \geq \frac{\|(\mathcal{L} + B)^{-1}\| \|\dot{\delta}_{10} \mathbf{1}\|}{\sigma}$$

then $V_1 \leq -\tilde{\delta}_{1*}^T (\mathcal{L} + B) \tilde{\delta}_{1*}$, let γ be the smallest eigenvalue of $\mathcal{L} + B$, we have

$$\begin{aligned} \dot{V}_1 &\leq -\tilde{\delta}_{1*}^T (\mathcal{L} + B) \tilde{\delta}_{1*} \leq -\gamma \delta_1^2 = \frac{\gamma}{0.5} \times V_1 \\ \dot{V}_1 &\leq -2\gamma V_1 \end{aligned}$$

which means V_1 is globally exponentially stable then δ_1 globally exponentially converge to δ_{10} .

Substitute v_{1j} in (8.19) into (8.10) we have

$$\dot{q}_{1j} = - \sum_{i \in \mathcal{N}_j} a_{ji}(q_{1j} - q_{1i}) - a_{j,m+1}(q_{1j} - q_{10}) + \delta_{1j} \quad (8.23)$$

define $\tilde{q}_{1j} = q_{1j} - q_{10}$, (8.23) can be written as

$$\dot{\tilde{q}}_{1j} = - \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{q}_{1j} - \tilde{q}_{1i}) - a_{j,m+1}\tilde{q}_{1j} + \tilde{\delta}_{1j} \quad (8.24)$$

similarly like (8.22), (8.24) can be written as

$$\dot{\tilde{q}}_{1*} = -(\mathcal{L} + B)\tilde{q}_{1*} + \tilde{\delta}_{1*} \quad (8.25)$$

since $\tilde{\delta}_1$ is globally exponentially stable, it can be proved q_{1j} globally exponentially converge to q_{10} . $\blacksquare$

Theorem 8.3. *For m systems in (8.10)-(8.12), if a directed spanning tree exists in the directed communication graph with the virtual leader the root of the tree, then the distributed control laws*

$$v_{2j} = u_{2j} = -k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j} \quad (8.26)$$

$$\begin{aligned} \dot{\delta}_{2j} = & - \sum_{i \in \mathcal{N}_j} a_{ji}(\delta_{2j} - \delta_{2i}) - a_{j,m+1}(\delta_{2j} - \delta_{20}) \\ & - \rho_2 \text{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\delta_{2j} - \delta_{2i}) - a_{j,m+1}(\delta_{2j} - \delta_{20}) \right] \end{aligned} \quad (8.27)$$

for $1 \leq j \leq m$, $\delta_{20} = u_{20} + k_2 q_{20} + k_3 u_{10} q_{30}$, guarantee that q_{2j} globally exponentially converge to q_{20} and q_{3j} globally exponentially converge to q_{30} .

Proof: substitute v_{2j} in (8.26) into (8.11), it follows that

$$\dot{q}_{2j} = -k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j} \quad (8.28)$$

Define $\zeta_{*j} = [\zeta_{1j}, \zeta_{2j}]$ as

$$\begin{aligned} \zeta_{1j} &= \sum_{i \in \mathcal{N}_j} a_{ji}(q_{2j} - q_{2i}) - a_{j,m+1}(q_{2j} - q_{20}) \\ \zeta_{2j} &= \sum_{i \in \mathcal{N}_j} a_{ji}(q_{3j} - q_{3i}) - a_{j,m+1}(q_{3j} - q_{30}) \end{aligned}$$

differentiate ζ_{*j} along (8.28) and (8.12) we have

$$\begin{aligned} \dot{\zeta}_{1j} &= \sum_{i \in \mathcal{N}_j} a_{ji}[(-k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j}) - (-k_2 q_{2i} - k_3 u_{1i} q_{3i} + \delta_{2i})] \\ &\quad - a_{j,m+1}[(-k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j}) - (-k_2 q_{20} - k_3 u_{10} q_{30} + \delta_{20})] \\ &= -k_2 \zeta_{1j} - k_3 u_{10} \zeta_{2j} + \sum_{i \in \mathcal{N}_j} a_{ji}(\delta_{2j} - \delta_{2i}) - a_{j,m+1}(\delta_{2j} - \delta_{20}) \\ &\quad - k_3 \left[\sum_{i \in \mathcal{N}_j} a_{ji}((u_{1j} - u_{10})q_{3j} - (u_{1i} - u_{10})q_{3i}) - a_{j,m+1}(u_{1j} - u_{10})q_{3j} \right] \end{aligned} \quad (8.29)$$

$$\begin{aligned} \dot{\zeta}_{2j} &= \sum_{i \in \mathcal{N}_j} a_{ji}(u_{1j} q_{2j} - u_{1i} q_{2i}) - a_{j,m+1}(u_{1j} q_{2j} - u_{10} q_{20}) \\ &= u_{10} \zeta_{1j} + \left[\sum_{i \in \mathcal{N}_j} a_{ji}((u_{1j} - u_{10})q_{2j} - (u_{1i} - u_{10})q_{2i}) - a_{j,m+1}(u_{1j} - u_{10})q_{2j} \right] \end{aligned} \quad (8.30)$$

define $\tilde{\delta}_{2j} = \delta_{2j} - \delta_{20}$, $\tilde{u}_{1j} = u_{1j} - u_{10}$, then (8.29) and (8.30) can be written as

$$\begin{aligned}\dot{\zeta}_{1j} &= -k_2\zeta_{1j} - k_3u_{10}\zeta_{2j} + \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j} \\ &\quad - k_3 \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{u}_{1j}q_{3j} - \tilde{u}_{1j}q_{3i}) - a_{j,m+1}\tilde{u}_{1j}q_{3j} \right]\end{aligned}\quad (8.31)$$

$$\dot{\zeta}_{2j} = u_{10}\zeta_{1j} + \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{u}_{1j}q_{2j} - \tilde{u}_{1j}q_{2i}) - a_{j,m+1}\tilde{u}_{1j}q_{2j} \right] \quad (8.32)$$

notice $\dot{\delta}_{2j}$ and $\dot{\delta}_{1j}$ have the same structure, by (8.21) we have

$$\begin{aligned}\dot{\delta}_{2j} &= - \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j} \\ &\quad - \rho_2 \text{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j} \right] - \dot{\delta}_{20}\end{aligned}\quad (8.33)$$

Define

$$\begin{aligned}x_1 &= [[\zeta_{11}, \zeta_{21}], [\zeta_{12}, \zeta_{22}], \dots, [\zeta_{1m}, \zeta_{2m}]]^\top \\ x_2 &= [[\tilde{\delta}_{21}, \tilde{q}_{11}, \tilde{\delta}_{11}], \dots, [\tilde{\delta}_{2m}, \tilde{q}_{1m}, \tilde{\delta}_{1m}]]^\top\end{aligned}$$

by (8.31) and (8.32) we have

$$\begin{aligned}f_{1j} &= \begin{bmatrix} -k_2 & -k_3u_{10} \\ u_{10} & 0 \end{bmatrix}, f_1(t, x_1) = [f_{11}, \dots, f_{1m}] \\ g_j &= \begin{bmatrix} \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j} \\ -k_3[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{u}_{1j}q_{3j} - \tilde{u}_{1j}q_{3i}) - a_{j,m+1}\tilde{u}_{1j}q_{3j}] \\ \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{u}_{1j}q_{2j} - \tilde{u}_{1j}q_{2i}) - a_{j,m+1}\tilde{u}_{1j}q_{2j} \end{bmatrix} \\ g(t, x_1, x_2)x_2 &= [g_1, g_2, \dots, g_m]\end{aligned}$$

by (8.33), (8.24) and (8.21) we have

$$\begin{aligned}f_{2j} &= \begin{bmatrix} - \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j} \\ -\rho_2 \text{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j} \right] \\ -\dot{\delta}_{20} \\ - \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{q}_{1j} - \tilde{q}_{1i}) - a_{j,m+1}\tilde{q}_{1j} + \tilde{\delta}_{1j} \\ - \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}) - a_{j,m+1}\tilde{\delta}_{1j} \\ -\rho_1 \text{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}) - a_{j,m+1}\tilde{\delta}_{1j} \right] \\ -\dot{\delta}_{10} \end{bmatrix} \\ f_2(t, x_2) &= [f_{21}, f_{22}, \dots, f_{2m}]\end{aligned}$$

Consider (8.17), the closed-loop states equation by substituting u in (8.18) is

$$\dot{x} = \begin{bmatrix} -k_2 & -k_3\phi(t) & -k_4 & \cdots & 0 \\ \phi(t) & 0 & \cdots & \cdots & 0 \\ 0 & \vdots & \vdots & \vdots & 0 \\ 0 & \vdots & \vdots & \vdots & 0 \\ 0 & \vdots & \vdots & \phi(t) & 0 \end{bmatrix} x \quad (8.34)$$

notice the second order matrix f_{1j} is special case of (8.34), since u_{10} is continuous PE signal, then by Lemma 8.2, $\dot{x}_1 = f_1(t, x_1)$ is globally exponentially stable, then the Assumption 1 in Theorem 8.1 holds, g_j can be proved to satisfy Assumption 2 in Theorem 8.1 and from the expression of f_{2j} it can be proved $\dot{x}_2 = f_2(t, x_2)$ is globally exponentially stable and Assumption 3 holds, therefore, $x_1 = [[\zeta_{11}, \zeta_{21}], [\zeta_{12}, \zeta_{22}], \dots, [\zeta_{1m}, \zeta_{2m}]]$ is globally exponentially stable. From the definition of ζ_{1j} and ζ_{2j} ,

$$\begin{aligned} \zeta_{1j} &= \sum_{i \in \mathcal{N}_j} a_{ji}(q_{2j} - q_{2i}) - a_{j,m+1}(q_{2j} - q_{20}) \\ &= \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{q}_{2j} - \tilde{q}_{2i}) - a_{j,m+1}\tilde{q}_{2j} \\ \zeta_{1*} &= (\mathcal{L} + B)\tilde{q}_{2*} \end{aligned}$$

since ζ_{1*} is globally exponentially stable, then $\tilde{q}_{2*} = (\mathcal{L} + B)^{-1}\zeta_{1*}$ is exponentially stable and q_{2j} globally exponentially converge to q_{20} , similarly, we can prove q_{3j} globally exponentially converge to q_{30} . ■

Theorem 8.4. *For m systems in (8.10)-(8.12), if a directed spanning tree exists in the directed communication graph with the virtual leader the root of the tree, then the distributed control laws (8.19)-(8.20) and (8.26)-(8.27) guarantee (8.6)-(8.8) hold.*

Proof: By Theorem 8.2 q_{1j} globally exponentially converge to q_{10} . By Theorem 8.3 q_{2j} globally exponentially converge to q_{20} , q_{3j} globally exponentially converge to q_{30} . By Lemma 8.1 (8.6)-(8.8) hold. ■

8.5 Distributed controller for dynamics

In previous section, distributed formation tracking laws are proposed for vehicles' kinematics, however, from the dynamics in (8.15) it is known the real control inputs are torques generated by vehicle engines, in this section, distributed control algorithms are proposed with the aid of backstepping methods and sliding mode control.

Define $\check{v}_{*j} = v_{*j} - u_{*j}$, where $v_{*j} = [v_{1j}, v_{2j}]$, $u_{*j} = [u_{1j}, u_{2j}]$, we have

Theorem 8.5. *For m systems in (8.10)-(8.12), if a directed spanning tree exists in the directed communication graph with the virtual leader the root of the tree, then the distributed control laws*

$$\tau_j = \hat{B}_j^{-1}(\tilde{M}_j\dot{u}_{*j} + \tilde{C}_ju_{*j} + \tilde{G}_j - k\check{v}_{*j} - Y_j\beta\text{sign}(Y_j^T\check{v}_{*j})) \quad (8.35)$$

where k satisfies $|\tilde{a} - \hat{a}| < k$, and (8.19)-(8.20), (8.26)-(8.27) guarantee (8.6)-(8.8) hold.

Proof: Denote the inertia parameter error vector $\check{a} = \tilde{a} - \hat{a}$, choose Lyapunov function $V_2 = \frac{1}{2}\check{v}_{*j}^T \hat{M}_j \check{v}_{*j}$, differentiate V_2 it follows that

$$\dot{V}_2 = \frac{1}{2}\check{v}_{*j}^T \dot{\hat{M}}_j \check{v}_{*j} + \check{v}_{*j}^T \hat{M}_j \dot{\check{v}}_{*j} \quad (8.36)$$

substitute (8.35) into (8.15) we have

$$\begin{aligned} \hat{M}_j \dot{\check{v}}_{*j} + \hat{C}_j \check{v}_{*j} + \hat{G}_j &= \tilde{M}_j \dot{u}_{*j} + \tilde{C}_j u_{*j} + \tilde{G}_j - k\check{v}_{*j} - Y_j \beta \text{sign}(Y_j^T \check{v}_{*j}) \\ \hat{M}_j (\dot{\check{v}}_{*j} - \dot{u}_{*j}) + \hat{C}_j (\check{v}_{*j} - u_{*j}) &= -k\check{v}_{*j} + Y_j \check{a} + Y_j \beta \text{sign}(Y_j^T \check{v}_{*j}) \\ \hat{M}_j \dot{\check{v}}_{*j} &= -\hat{C}_j \check{v}_{*j} - k\check{v}_{*j} + Y_j \check{a} + Y_j \beta \text{sign}(Y_j^T \check{v}_{*j}) \end{aligned} \quad (8.37)$$

substitute $\hat{M}_j \dot{\check{v}}_{*j}$ into (8.36) it follows that

$$\begin{aligned} \dot{V}_2 &= \frac{1}{2}\check{v}_{*j}^T \dot{\hat{M}}_j \check{v}_{*j} + \check{v}_{*j}^T (-\hat{C}_j \check{v}_{*j} - k\check{v}_{*j} + Y_j \check{a} + Y_j \beta \text{sign}(Y_j^T \check{v}_{*j})) \\ &= \frac{1}{2}\check{v}_{*j}^T (\dot{\hat{M}}_j - \hat{C}_j) \check{v}_{*j} - \check{v}_{*j}^T k\check{v}_{*j} + \check{v}_{*j}^T Y_j \check{a} - \check{v}_{*j}^T Y_j \beta \text{sign}(Y_j^T \check{v}_{*j}) \\ &\leq \frac{1}{2}\check{v}_{*j}^T (\dot{\hat{M}}_j - \hat{C}_j) \check{v}_{*j} - \check{v}_{*j}^T k\check{v}_{*j} \end{aligned}$$

since $\dot{\hat{M}}_j - \hat{C}_j$ is skew-symmetric, $\dot{V}_2 < -\check{v}_{*j}^T k\check{v}_{*j}$, let λ_m be the maximum eigenvalue of $\hat{M}_j$ then $\dot{V}_2 < -\frac{2k}{\lambda_m} V_2$, then $\check{v}_{*j}$ are exponentially stable and $v_{*j} - u_{*j}$ globally exponentially converge to zero.

(8.19) and (8.26) are rewritten as

$$\begin{aligned} v_{1j} &= - \sum_{i \in \mathcal{N}_j} a_{ji}(q_{1j} - q_{1i}) - a_{j,m+1}(q_{1j} - q_{10}) \\ &\quad + \delta_{1j} + \check{v}_{1j} \end{aligned} \quad (8.38)$$

$$v_{2j} = -k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j} + \check{v}_{2j} \quad (8.39)$$

$\zeta_{*j} = [\zeta_{1j}, \zeta_{2j}]$ are the same as defined in Section 8.3, it follows that

$$\begin{aligned} \dot{\zeta}_{1j} &= -k_2 \zeta_{1j} - k_3 u_{1,m+1} \zeta_{2j} \\ &\quad + \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1} \tilde{\delta}_{2j} \\ &\quad + \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{u}_{2j} - \tilde{u}_{2i}) - a_{j,m+1} \tilde{u}_{2j} \\ &\quad - k_3 \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{u}_{1j} q_{3j} - \tilde{u}_{1j} q_{3i}) - a_{j,m+1} \tilde{u}_{1j} q_{3j} \right] \end{aligned} \quad (8.40)$$

$$\dot{\tilde{q}}_{1j} = - \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{q}_{1j} - \tilde{q}_{1i}) - a_{j,m+1} \tilde{q}_{1j} + \tilde{\delta}_{1j} + \tilde{u}_{2j} \quad (8.41)$$

$\dot{\zeta}_{2j}$, $\tilde{\delta}_{1j}$ and $\tilde{\delta}_{2j}$ are unchanged as expressed in (8.32), (8.21) and (8.33).

$$g_j = \begin{bmatrix} \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1} \tilde{\delta}_{2j} \\ \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{u}_{2j} - \tilde{u}_{2i}) - a_{j,m+1} \tilde{u}_{2j} \\ -k_3 [\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{u}_{1j} q_{3j} - \tilde{u}_{1j} q_{3i}) - a_{j,m+1} \tilde{u}_{1j} q_{3j}] \\ \sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{u}_{1j} q_{2j} - \tilde{u}_{1j} q_{2i}) - a_{j,m+1} \tilde{u}_{1j} q_{2j} \end{bmatrix}$$

$$g(t, x_1, x_2)x_2 = [g_1, g_2, \dots, g_m]$$

$$f_{2j} = \begin{bmatrix} -\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j} \\ -\rho_2 \text{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j} \right] \\ -\dot{\delta}_{20} \\ -\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{q}_{1j} - \tilde{q}_{1i}) - a_{j,m+1}\tilde{q}_{1j} + \tilde{\delta}_{1j} + \tilde{u}_{1j} \\ -\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}) - a_{j,m+1}\tilde{\delta}_{1j} \\ -\rho_1 \text{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}) - a_{j,m+1}\tilde{\delta}_{1j} \right] \\ -\dot{\delta}_{10} \end{bmatrix}$$

f_{1j} are the same as that in Section 8.3, it can be proved that the modified cascaded system

$$\begin{aligned} \dot{x}_1 &= f_1(t, x_1) + g(t, x_1, x_2)x_2 \\ \dot{x}_2 &= f_2(t, x_2) \end{aligned}$$

still satisfies the Assumptions in Theorem 8.1 and $q_{*j} = [q_{1j}, q_{2j}, q_{3j}]$ exponentially converge to $q_{*0} = [q_{10}, q_{20}, q_{30}]$, by Lemma 8.1, (8.6)-(8.8) hold. $\blacksquare$

8.6 distributed controller for time-varying communication topology

In real-life operations, due to disconnection or creation of links and nodes failures, the communication topology is time-varying, it has been proved through a infinite sequence of nonoverlapping, uniformly bounded time interval, if the union of the graphs across each interval has a directed spanning tree, then multi-agent systems reach consensus with the aid of theories of SIA matrices [34].

Theorem 8.6. *For m systems in (8.10)-(8.12), if at any nonoverlapping, uniformly bounded time interval, a directed spanning tree exists for union of the graphs across the interval, with the virtual leader the root of the spanning tree, then the distributed control laws (8.19)-(8.20), (8.26)-(8.27) and (8.35) guarantee (8.6)-(8.8) hold.*

8.7 Simulation Results

To show the effectiveness of the proposed results, simulation has been done for four robots. The desired geometric pattern $\mathcal{P}$ is shown in Fig. 8.1, assume the format of the robot systems is in square shape. The pattern $\mathcal{P}$ can be described by orthogonal coordinates $(p_{1x}, p_{1y}) = (0, 1)$, $(p_{2x}, p_{2y}) = (-1, 0)$, $(p_{3x}, p_{3y}) = (0, -1)$ and $(p_{4x}, p_{4y}) = (1, 0)$. For the leading agent, assume the reference trajectory is $(x_5, y_5, \theta_5) = (10 \sin(t), -10 \cos(t), t)$ and $(p_{5x}, p_{5y}) = (0, 0)$, $v_5 = 10$ and $\omega_5 = 1$.

Fig. 8.2 represents the communication graph for the multivehicle systems, ρ_1 , ρ_2 , k_2 and k_3 are assigned 2, k in dynamic control laws is 10. Fig. 8.4 shows the centroid of x_j ($1 \leq j \leq 4$) (i.e., $\sum_{j=1}^4 x_j/4$) and x_0 . Fig. 8.5 shows the centroid of y_j ($1 \leq j \leq 4$) (i.e.,

$\sum_{j=1}^4 y_j/4$) and y_0 . Fig. 8.6 shows $(\theta_j - \theta_0)$ ($1 \leq j \leq 4$). Fig. 8.7 shows the formation tracking of the 4 followers, the blue spots represent each follower robot, the black spot represent the trajectory of centroid of the robots.

Assume the information communication graph switches according to the following logic.

$$\mathcal{G} = \begin{cases} \mathcal{G} \text{ in Fig. 8.2,} & \text{if } t - \text{round}(t) \geq 0 \\ \mathcal{G} \text{ in Fig. 8.3,} & \text{if } t - \text{round}(t) < 0 \end{cases}$$

For the switching topologies defined above, Fig. 8.8 shows the centroid of x_j ($1 \leq i \leq 4$) and x_0 . Fig. 8.9 shows the centroid of y_j ($1 \leq j \leq 4$) and y_0 . Fig. 8.10 shows $(\theta_j - \theta_0)$ ($1 \leq j \leq 4$). Fig. 8.11 shows the formation tracking of the 4 followers.

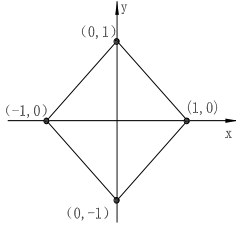


Figure 8.1: Desired geometric formation.

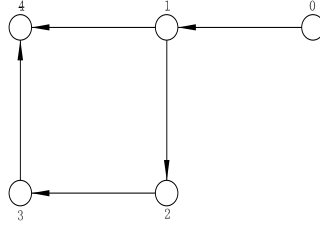


Figure 8.2: Information exchange graph $\mathcal{G}$.

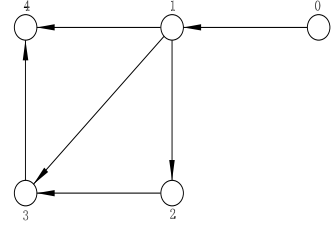


Figure 8.3: Information exchange graph $\mathcal{G}$.

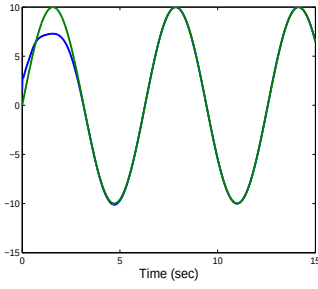


Figure 8.4: Response of the centroid of x_j for $1 \leq j \leq 4$ and x_0 .

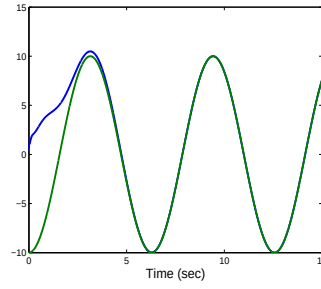


Figure 8.5: Response of the centroid of y_j for $1 \leq j \leq 4$ and y_0 .

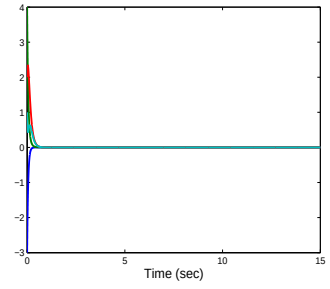


Figure 8.6: Response of $(\theta_j - \theta_0)$ for $1 \leq j \leq 4$.

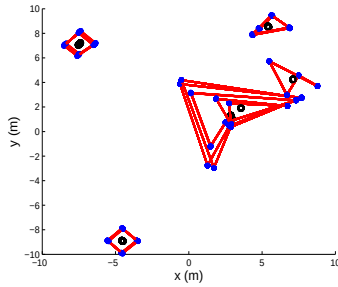


Figure 8.7: formation tracking of 4 followers.

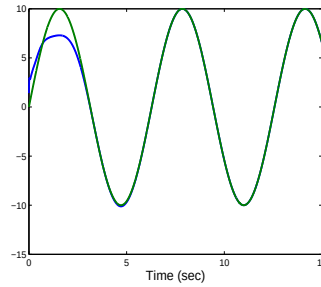


Figure 8.8: Response of the centroid of x_j for $1 \leq j \leq 4$ and x_0 .

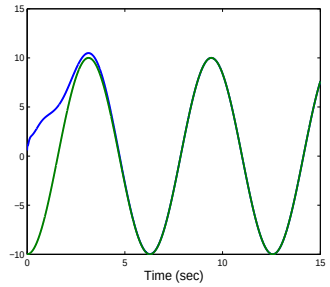


Figure 8.9: Response of the centroid of y_j for $1 \leq j \leq 4$ and y_0 .

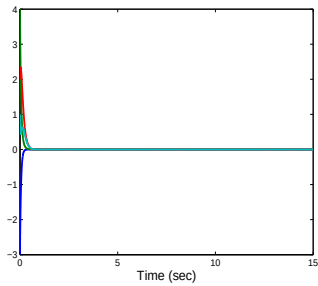


Figure 8.10: Response of $(\theta_j - \theta_0)$ for $1 \leq j \leq 4$.

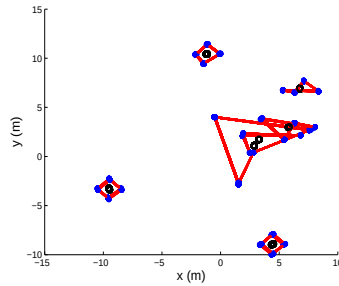


Figure 8.11: formation tracking of 4 followers.

Chapter 9

Distributed Exponentially Tracking Control of Multiple Wheeled Mobile Robots

9.1 Introduction

The coordination of multi-agent systems has been studied intensively in recent years because of its wide use in large object moving, cooperative target pursuit, rescue mission, etc. The configurations of multi-agent systems determine it can perform with less time and achieve higher accuracy in mapping [137, 138]. For multi-agent systems, a leader/follower architecture is usually defined with the follower agents following the trajectory of the leader agent [139–141]. Different control schemes have been proposed to reach the consensus. Co-operative control schemes have received considerable attention due to the development of network consensus theory. It removes the unstableness of communication networks with centralized controlling leader agent, which can be disturbed with connection breakage, with more connections requiring more network bandwidth capacities. Distributed control schemes can reduce the cost of broadband connection by offering connection with the leader agent to only a small part of follower systems. In [142–144], consensus algorithms are proposed for linearized systems with the aids of graph theory, where each agent is considered a vertex in the communication graph, the communication pattern is characterized with a Laplacian matrix, and the consensus goal is achieved with the aid of the characteristics of communication graph. In [142], the authors consider directed communication networks with fixed and switching topologies and also undirected networks. Consensus protocols are proposed. In [145], distributed flocking algorithms are proposed for free-space and obstacle avoidance cases. In [146], the authors consider a weaker condition of spanning tree structure. In [147], the Vicsek model is studied and convergence results are derived. In [148], a distributed control strategy for connectivity preserving swarm aggregation with collision avoidance is presented. In [149], a feedback control strategy is proposed for convergence of a multi-agent system to a desired formation configuration. The cases of agents with single integrator and nonholonomic unicycle-type kinematics are both studied. Formation infeasibility is considered and related with the flocking behavior by proving the convergence of velocity values. In

[150], the authors consider consensus algorithms for double-integrator dynamics for bounded control inputs with connected communication graph and other cases. States consensus is proved under different communication patterns. In [151], the consensus problem of a group of autonomous agents with an active leader is studied. A distributed feedback law along with a distributed state-estimation rule is proposed for each continuous-time dynamical agent. The Lyapunov-based convergence analysis is given for the multi-agent system considered with a varying interconnection topology.

In realities, the state information of the leader agent is not available to all the follower systems, especially when the networks are very large. In [152], the Lyapunov techniques are combined with the graph theory and distributed control laws are proposed for convergence of a desired pattern. In [153], the authors analyze consensus algorithms with a time-varying reference state by using theoretical graph tools. The information flow in [153] is not what is usually considered from the leader to the followers and thus increase the robustness of the whole systems. In [154], an adaptive controller design method is proposed such that each agent reconstructs the reference velocity and recovers the desired formation. For the time-varying reference velocity case, a controller redesign is presented and the parameter convergence is guaranteed. In [155], decentralized cooperative controllers are proposed with backstepping techniques. In [156], the distributed formation control problem for multiple nonholonomic robots is considered and consensus algorithms are proposed with the aid of graph and Lyapunov theories.

In this chapter, we consider multiple wheeled mobile robots with nonholonomic constraints and analyze the distributed control schemes such that the robots converge to the desired formation whose centroid moves along the trajectory of a leader robot. In order to propose distributed controllers, first a variable transformation is introduced to change state models into chained systems. Then we propose a new state model and design the control laws for the new states in order to reach the control goals. In this paper, the dynamic models of wheel robots is taken into consideration. We directly design control laws for the torques imposed on wheels in reality instead of the velocities of robots. The simulation results show the effectiveness of our distributed cooperative control strategies for the dynamics models of leader-follower systems.

9.2 Problem Statement

Considering m simplified car-like robots with three states. We can define the m robot systems with dynamical models by Lagrange-D'Alembert principle

$$M_j(q_{*j})\ddot{q}_{*j} + C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j} + G_j(q_{*j}) = B_j(q_{*j})\tau_j + J^\top(q_{*j})\lambda_j \quad (9.1)$$

$$J(q_{*j})\dot{q}_{*j} = 0 \quad (9.2)$$

where $q_{*j} = [q_{1j}, q_{2j}, q_{3j}]^\top = [x_j, y_j, \theta_j]^\top$ are the states of system j , $M_j(q_{*j})$ is a 3×3 positive-definite symmetric matrix, $C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j}$ represents centripetal and Coriolis force, $G_j(q_{*j})$ is the gravitational force, $B_j(q_{*j})$ is a 3×2 input transformation matrix, $J(q_{*j})$ is the two-dimensional control inputs vector, λ_j is the constraint force imposed on system j . By (9.2),

$\dot{q}_{*j}$ can be defined as

$$\dot{q}_{*j} = g(q_{*j})V_{*j} = \begin{bmatrix} \cos \theta_j & 0 \\ \sin \theta_j & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_j \\ \omega_j \end{bmatrix} \quad (9.3)$$

Eqn. (9.1) has the following two properties for the dynamic models.

Property 1: Matrix $\dot{M}_j - 2C_j$ is skew-symmetric.

Property 2: For any differentiable vector $\delta \in R^3$, we have

$$M_j(q_{*j})\dot{\delta} + C_j(q_{*j}, \dot{q}_{*j})\delta + G_j(q_{*j}) = Y_j(q_{*j}, \dot{q}_{*j}, \delta, \dot{\delta})a_j$$

where a_j is an inertia parameter vector which is related with the mass and moment of inertia, $Y_j(q_{*j}, \dot{q}_{*j}, \delta, \dot{\delta})$ is a known function of q_{*j} , $\dot{q}_{*j}$, δ and $\dot{\delta}$. a_j is supposed to be unknown.

Suppose each robot can acquire state information from some of the other systems. This information communication can be defined in a communication digraph $G = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V}$ is the set of nodes for the directed graph with $\mathcal{V} = \{v_1, v_2, \dots, v_m\}$ and $\mathcal{E}$ is the subset of $\mathcal{V} \times \mathcal{V}$. For system i and j , $e_{ij} = (v_i, v_j)$ belongs to $\mathcal{E}$ if the state information of system i can be received by j and i is called a neighbor of robot j . $\mathcal{N}_j$ represents the sets of all neighbors of robot j . In this paper, we assume that the communication graph is directed.

For the m robot systems, the communication graph is $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. It is given an $m \times m$ constant matrix $A = [a_{ji}]$ with $a_{ji} = a_{ij} > 0$, the Laplacian matrix $\mathcal{L} = [L_{ji}]$ of the graph $\mathcal{G}$ with weight matrix A can be defined by

$$L_{ji} = \begin{cases} -a_{ji}, & \text{if } i \in \mathcal{N}_j \text{ and } i \neq j \\ 0, & \text{if } i \notin \mathcal{N}_j \text{ and } i \neq j \\ \sum_{l \neq j, l \in \mathcal{N}_j} a_{jl}, & \text{if } j = i. \end{cases}$$

$\mathcal{P}$ is defined as formation of the m robots, this geometric pattern can be described by a group of orthogonal coordinates (p_{jx}, p_{jy}) ($1 \leq j \leq m$) which satisfy $\sum_{j=1}^m p_{jx} = 0$ and $\sum_{j=1}^m p_{jy} = 0$. The trajectory of the leader agent is described by $q_0 = (x_0, y_0, \theta_0)$ which satisfies

$$\dot{x}_0 = v_0 \cos \theta_0, \quad \dot{y}_0 = v_0 \sin \theta_0, \quad \dot{\theta}_0 = \omega_0 \quad (9.4)$$

where v_0 and ω_0 are known functions and state q_0 is assumed to be available to only one robot in this paper. The control problem can be described as designing control laws v_j and ω_j for system j using its own state information and its neighbors' state information such that

$$\lim_{t \rightarrow \infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix} \quad (9.5)$$

$$\lim_{t \rightarrow \infty} (\theta_i - \theta_0) = 0 \quad (9.6)$$

$$\lim_{t \rightarrow \infty} \left[\sum_{i=1}^m \frac{x_i}{m} - x_0 \right] = 0, \quad \lim_{t \rightarrow \infty} \left[\sum_{l=1}^m \frac{y_l}{m} - y_0 \right] = 0 \quad (9.7)$$

for $1 \leq i \neq j \leq m$.

In the control problem, Eqn. (9.5) guarantees all the agents converge to the formation $\mathcal{P}$. Eqn. (9.6) ensures that the orientation of each robot converges to the orientation of the leader robot. Eqn. (9.7) means that the centroid of the group of robots converges to the trajectory of the leader robot.

Define the change of variables

$$\begin{cases} z_{1j} = \theta_j \\ z_{2j} = (x_j - p_{jx}) \cos \theta_j + (y_j - p_{jy}) \sin \theta_j \\ z_{3j} = (x_j - p_{jx}) \sin \theta_j - (y_j - p_{jy}) \cos \theta_j \\ v_{1j} = \omega_j \\ v_{2j} = v_j - v_{1j} z_{3j} \end{cases} \quad (9.8)$$

Then the transformed state space is

$$\dot{z}_{1j} = v_{1j}, \quad \dot{z}_{2j} = v_{2j}, \quad \dot{z}_{3j} = v_{1j} z_{2j} \quad (9.9)$$

The leader agent in (9.4) can also be transformed to the model above by (9.8) as

$$\dot{z}_{10} = v_{10}, \dot{z}_{20} = v_{20}, \dot{z}_{30} = v_{10} z_{20}.$$

Lemma 9.1. *If $\lim_{t \rightarrow \infty} (z_{1j} - z_{10}) = 0$, $\lim_{t \rightarrow \infty} (z_{2j} - z_{20}) = 0$, and $\lim_{t \rightarrow \infty} (z_{3j} - z_{30}) = 0$ for $1 \leq j \leq m$, then (9.5)-(9.7) hold.*

The proof is omitted due to limitation of space.

By Lemma 9.1, the control problem is to design control laws v_{1j} and v_{2j} such that

$$\lim_{t \rightarrow \infty} (z_{*j} - z_{*0}) = 0 \quad (9.10)$$

where $z_{*j} = [z_{1j}, z_{2j}, z_{3j}]$ and $z_{*0} = [z_{10}, z_{20}, z_{30}]$, with the its own and neighbors' state information.

In order to remove the constraint component $J^\top(q_{*j})\lambda_j$ and transform the states in (9.1) into $v_{*j} = [v_{1j}, v_{2j}]$, similar transform is implemented as in [157], it follows that

$$\hat{M}_j(q_{*j})\dot{v}_{*j} + \hat{C}_j(q_{*j}, \dot{q}_{*j})v_{*j} + \hat{G}_j(q_{*j}) = \hat{B}_j(q_{*j})\tau_j \quad (9.11)$$

Notice τ_j are the real control inputs, we first propose a control law of v_{*j} for (9.9), then we design control laws of τ_j with the knowledge of backstepping method.

9.3 Distributed Controller Design

The following assumption is made on the leader agent.

Assumption 9.1. *The $\frac{d^i v_{10}}{dt^i}$ ($0 \leq i \leq 2$) are bounded and $\int_t^{t+T} v_{10}^2(\tau) d\tau > \alpha$ for some $\alpha > 0$ and $T > 0$.*

Assumption 9.1 means that signal v_{10} is persistently excited signal (PE signal). For a PE signal, we have the following result.

Lemma 9.2. *For the system*

$$\dot{\delta} = -\psi_1(t)^2 \delta + \psi_2(t) \quad (9.12)$$

where $\psi_1(t)$ is a PE signal, if $\psi_2(t)$ is bounded and converges to zero, then δ converges to zero.

In order to design the controllers, variable transformation is introduced as

$$s_{*j} = z_{*j} - \sigma_{*j} \quad (9.13)$$

where $*$ means 1 to 3., $\sigma_{1j} = \sigma_{3j} = 0$, $\sigma_{2j} = -k_3 v_{1j} s_{3j}$, and $k_3 > 0$. Then Eqn. (9.9) can be transformed into

$$\dot{s}_{1j} = v_{1j} \quad (9.14)$$

$$\dot{s}_{2j} = v_{2j} + k_3(\dot{v}_{1j} s_{3j} + \dot{s}_{3j} v_{1j}) \quad (9.15)$$

$$\dot{s}_{3j} = -k_3 v_{1j}^2 s_{3j} + v_{1j} s_{2j}. \quad (9.16)$$

Then we can have the following lemma.

Lemma 9.3. *For the transformed systems in (9.14)-(9.16) if*

$$\lim_{t \rightarrow \infty} (s_{ij} - s_{i0}) = 0, \lim_{t \rightarrow \infty} (v_{1j} - v_{10}) = 0 \quad (9.17)$$

for $1 \leq i \leq 3$, then (9.10) hold. The proof is omitted here due to space limits.

Lemma 9.4. *For the m systems in (9.14)-(9.16), under Assumption 1, if*

$$\lim_{t \rightarrow \infty} (s_{2j} - s_{20}) = 0, \lim_{t \rightarrow \infty} (v_{1j} - v_{10}) = 0 \quad (9.18)$$

then

$$\lim_{t \rightarrow \infty} (s_{3j} - s_{30}) = 0 \quad (9.19)$$

Proof: Let $\tilde{s}_{3j} = s_{3j} - s_{30}$, then

$$\dot{\tilde{s}}_{3j} = -k_3 v_{1j}^2 s_{3j} + k_3 (v_{10}^2 - v_{1j}^2) s_{30} + v_{1j} s_{2j} - v_{10} s_{20} \quad (9.20)$$

Since $\lim_{t \rightarrow \infty} (v_{1j} - v_{10}) = 0$, v_{1j} is a bounded PE signal. Since $\lim_{t \rightarrow \infty} (s_{2j} - s_{20}) = 0$, $v_{1j} s_{2j} - v_{10} s_{20}$ converges to zero. By Lemma 9.2, (9.19) holds. ■

By Lemma 9.4, if we design control laws to make $\lim_{t \rightarrow \infty} (s_{1j} - s_{10}) = 0$, $\lim_{t \rightarrow \infty} (v_{1j} - v_{10}) = 0$, and $\lim_{t \rightarrow \infty} (s_{2j} - s_{20}) = 0$, then (9.10) holds.

Lemma 9.5. *For the m systems in (9.14)-(9.16), if the communication graph is connected and the state of the leader is available to one of the m follower systems, the control laws*

$$v_{1j} = \varsigma_{1j} = -\alpha_1 s_{1j} + \zeta_{1j} \quad (9.21)$$

$$\dot{\zeta}_{1j} = - \sum_{i \in \mathcal{N}_j} a_{ji} (\zeta_{1j} - \zeta_{1i}) - b_j \mu_j (\zeta_{1j} - \zeta_{10}) -$$

$$\rho_1 \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji} (\zeta_{1j} - \zeta_{1i}) + b_j \mu_j (\zeta_{1j} - \zeta_{10}) \right] \quad (9.22)$$

for $1 \leq j \leq m$ guarantee that $\lim_{t \rightarrow \infty} (s_{1j} - s_{10}) = 0$ and $\lim_{t \rightarrow \infty} (v_{1j} - v_{10}) = 0$, where

$$\mu_j = \begin{cases} 1, & \text{if leader agent state is available to system } j \\ 0, & \text{otherwise} \end{cases}$$

$\alpha_1 > 0$, $b_j > 0$, $\zeta_{10} = v_{10} + \alpha_1 s_{10}$, and ρ_1 is a sufficiently large constant.

Proof: Eqn. (9.22) can be redefined with the knowledge of Laplacian matrix as

$$\dot{\zeta}_{1*} = -\mathcal{L}\zeta_{1*} - \rho_1 \text{sign}(\mathcal{L}\zeta_{1*} + B(\zeta_{1*} - \mathbf{1}\zeta_{10})) - B(\zeta_{1*} - \mathbf{1}\zeta_{10}) \quad (9.23)$$

where $\zeta_{1*} = [\zeta_{11}, \dots, \zeta_{1m}]^\top$ and $B = \text{diag}[b_1\mu_1, \dots, b_m\mu_m]$. Let $\tilde{\zeta}_{1j} = \zeta_{1j} - \zeta_{10}$, then Eqn. (9.23) can be transformed into

$$\dot{\tilde{\zeta}}_{1*} = -(\mathcal{L} + B)\tilde{\zeta}_{1*} - \rho_1 \text{sign}((\mathcal{L} + B)\tilde{\zeta}_{1*}) - \dot{\zeta}_{10}\mathbf{1}. \quad (9.24)$$

where $\tilde{\zeta}_{1*} = [\tilde{\zeta}_{11}, \dots, \tilde{\zeta}_{1m}]^\top$. We choose Lypapunov function $F = \sum_{j=1}^m \frac{1}{2} \tilde{\zeta}_{1j}^2$, and differentiate F along (9.24), then we have

$$\begin{aligned} \dot{F} &= -\tilde{\zeta}_{1*}^\top (\mathcal{L} + B) \tilde{\zeta}_{1*} - \tilde{\zeta}_{1*}^\top \rho_1 \text{sign}((\mathcal{L} + B)\tilde{\zeta}_{1*}) - \tilde{\zeta}_{1*}^\top \dot{\zeta}_{10}\mathbf{1} \\ &= -\tilde{\zeta}_{1*}^\top (\mathcal{L} + B) \tilde{\zeta}_{1*} - \tilde{\zeta}_{1j}^\top \rho_1 \text{sign}((\mathcal{L} + B)\tilde{\zeta}_{1*}) \\ &\quad - ((\mathcal{L} + B)\tilde{\zeta}_{1*})^\top ((\mathcal{L} + B)^\top)^{-1} \dot{\zeta}_{10}\mathbf{1} \end{aligned}$$

Let ξ_1 be minimum eigenvalue of $((\mathcal{L} + B)^\top)^{-1}$, then

$$\begin{aligned} \dot{F} &\leq -\tilde{\zeta}_{1*}^\top (\mathcal{L} + B) \tilde{\zeta}_{1*} - \xi_1 \rho_1 ((\mathcal{L} + B)\tilde{\zeta}_{1*})^\top \text{sign}((\mathcal{L} + B)\tilde{\zeta}_{1*}) \\ &\quad - ((\mathcal{L} + B)\tilde{\zeta}_{1*})^\top ((\mathcal{L} + B)^\top)^{-1} \dot{\zeta}_{10}\mathbf{1} \end{aligned}$$

Since $\mathcal{L} + B$ is symmetric and positive definite, if

$$\rho_1 \geq \frac{\|((\mathcal{L} + B)^\top)^{-1}\| \|\dot{\zeta}_{10}\|}{\xi_1},$$

it can be verified $\dot{F} \leq -\tilde{\zeta}_{1*}^\top (\mathcal{L} + B) \tilde{\zeta}_{1*}$. Therefore, $\lim_{t \rightarrow \infty} \tilde{\zeta}_{1j} = 0$ which means that ζ_{1j} converges to ζ_{10} . Let $V_1 = \frac{1}{2} \tilde{s}_{1j}^2$ where $\tilde{s}_{1j} = s_{1j} - s_{10}$, differentiate V_1 along (9.14), we have

$$\dot{V}_1 = -\alpha_1 \tilde{s}_{1j}^2 + \tilde{s}_{1j} \tilde{\zeta}_{1j} \leq -2\alpha_1 V_1 + |\tilde{\zeta}_{1j}| \sqrt{2V_1}$$

Define $V_2 = \sqrt{V_1}$, then $\dot{V}_2 \leq -\alpha_1 V_2 + \frac{1}{\sqrt{2}} |\tilde{\zeta}_{1j}|$. It can be proved that $\lim_{t \rightarrow \infty} (s_{1j} - s_{10}) = 0$. By (9.21),

$$\begin{aligned} \lim_{t \rightarrow \infty} (v_{1j} - v_{10}) &= \lim_{t \rightarrow \infty} (-\alpha_1 s_{1j} + \zeta_{1j} - v_{10}) \\ &= \lim_{t \rightarrow \infty} (-\alpha_1 s_{1j} + v_{10} + \alpha_1 s_{10} - v_{10}) \\ &= -\lim_{t \rightarrow \infty} \alpha_1 (s_{1j} - s_{10}) = 0. \end{aligned}$$

■

Lemma 9.6. *For the m systems in (9.14)-(9.16), if the communication graph is connected and the state of the leader is available to one of the m follower systems, control inputs*

$$\begin{aligned} v_{2j} = \varsigma_{2j} = & - \sum_{i \in \mathcal{N}_j} a_{ji}(s_{2j} - s_{2i}) - b_j \mu_j(s_{2j} - s_{20}) - \\ & \rho_2 \operatorname{sign} \left[\sum_{i \in \mathcal{N}_j} a_{ji}(s_{2j} - s_{2i}) + b_j \mu_j(s_{2j} - s_{20}) \right] \\ & - k_3(\dot{v}_{1j} s_{3j} + \dot{s}_{3j} v_{1j}) \end{aligned} \quad (9.25)$$

for $1 \leq j \leq m$ guarantee that $\lim_{t \rightarrow \infty} (s_{2j} - s_{20}) = 0$, where ρ_2 is a sufficiently large constant.

Proof: Substitute v_{2j} into (9.15)

$$\dot{s}_{2*} = -\mathcal{L}s_{2*} - B(s_{2*} - \dot{s}_{20}\mathbf{1}) - \rho_2 \operatorname{sign}(\mathcal{L}s_{2*} + B(s_{2*} - \dot{s}_{20}\mathbf{1})) \quad (9.26)$$

where $s_{2*} = [s_{21}, \dots, s_{2m}]^\top$. Let $\tilde{s}_{2*} = s_{2*} - s_{20}\mathbf{1}$, then Eqn. (9.26) can be transformed into

$$\dot{\tilde{s}}_{2*} = -(\mathcal{L} + B)\tilde{s}_{2*} - \rho_2 \operatorname{sign}((\mathcal{L} + B)\tilde{s}_{2*}) - \dot{s}_{20}\mathbf{1} \quad (9.27)$$

Choose Lypapunov function $E = \frac{1}{2} \sum_{j=1}^m \tilde{s}_{2j}^2$, differentiate E along (9.27), we have

$$\begin{aligned} \dot{E} &= -\tilde{s}_{2*}^\top (\mathcal{L} + B) \tilde{s}_{2*} - \tilde{s}_{2*}^\top \rho_2 \operatorname{sign}((\mathcal{L} + B)\tilde{s}_{2*}) - \tilde{s}_{2*}^\top \dot{s}_{20}\mathbf{1} \\ &= -\tilde{s}_{2*}^\top (\mathcal{L} + B) \tilde{s}_{2*} - \tilde{s}_{2*}^\top \rho_2 \operatorname{sign}((\mathcal{L} + B)\tilde{s}_{2*}) \\ &\quad - ((\mathcal{L} + B)\tilde{s}_{2*})^\top ((\mathcal{L} + B)^\top)^{-1} \dot{s}_{20}\mathbf{1} \end{aligned}$$

Let ξ_1 be minimum eigenvalue of $((\mathcal{L} + B)^\top)^{-1}$, if

$$\rho_2 \geq \frac{\|(\mathcal{L} + B)^\top\|^{-1} \|\dot{s}_{20}\|}{\xi_1},$$

$$\dot{E} \leq -\tilde{s}_{2*}^\top (\mathcal{L} + B) \tilde{s}_{2*}$$

It can be proved that E converges to zero. Therefore, s_{2j} converges s_{20} . ■

In Lemmas 5-6 we have proved that $\lim_{t \rightarrow \infty} (z_{1j} - z_{10}) = 0$, $\lim_{t \rightarrow \infty} (z_{2j} - z_{20}) = 0$, and $\lim_{t \rightarrow \infty} (v_{1j} - v_{10}) = 0$. With the aid of Lemma 4, $\lim_{t \rightarrow \infty} (z_{3j} - z_{30}) = 0$. By Lemma 3, (9.10) holds. Next the control laws for real input τ_j are proposed with the knowledge of $v_{*j} = [v_{1j}, v_{2j}]$ designed in (9.21) and (9.25). Define

$$\tilde{v}_{*j} = [\tilde{v}_{1j}, \tilde{v}_{2j}] = [v_{1j} - \varsigma_{1j}, v_{2j} - \varsigma_{2j}],$$

we have the following results.

Lemma 9.7. *For m systems in (9.11), under Assumption 1, if the communication graph is connected and the state of the leader agent is available to one of the m follower systems, the control laws for τ_j*

$$\tau_j = \hat{B}_j^{-1}(-K_j \tilde{u}_{*j} + \hat{Y}_j(\varsigma_{*j}, \dot{\varsigma}_{*j}, z_{*j}, \dot{z}_{*j}) \hat{a}_j) \quad (9.28)$$

and the update laws for $\hat{a}_j$

$$\dot{\hat{a}}_j = -\Psi_j \hat{Y}_j^\top (q_{*j}, \dot{q}_{*j}, \varsigma_{*j}, \dot{\varsigma}_{*j}) \tilde{v}_{*j} \quad (9.29)$$

for $1 \leq j \leq m$ guarantee that $\lim_{t \rightarrow \infty} (s_{ij} - s_{i0}) = 0$ for $1 \leq i \leq 2$ and $1 \leq j \leq m$ and $\hat{a}_j$ is bounded.

Proof: Substitute τ_j in (9.28) into (9.11), we have

$$\hat{M}_j(q_{*j})\dot{v}_{*j} + \hat{C}_j(q_{*j}, \dot{q}_{*j})v_{*j} + \hat{G}_j(q_{*j}) = -K_j \tilde{u}_{*j} + \hat{Y}_j(q_{*j}, \dot{q}_{*j}, \varsigma_{*j}, \dot{\varsigma}_{*j})\hat{a}_j$$

The equation above can be simplified as

$$\hat{M}_j \dot{\tilde{v}}_{*j} + \hat{C}_j \tilde{v}_{*j} = -K_j \tilde{v}_{*j} + \hat{Y}_j \tilde{a}_j \quad (9.30)$$

where $\tilde{a}_j = \hat{a}_j - a_j$. Let the Lyapunov function $V_v = \tilde{v}_{*j}^\top \hat{M}_j \tilde{v}_{*j} + \tilde{a}_j^\top \Psi_j^{-1} \tilde{a}_j$, differentiate V_v with the aid of Property 1 and (9.30), we have

$$\begin{aligned} \dot{V}_v &= \tilde{v}_{*j}^\top \dot{\hat{M}}_j \tilde{v}_{*j} + 2\tilde{v}_{*j}^\top \hat{M}_j \dot{\tilde{v}}_{*j} + 2\dot{\tilde{a}}_j^\top \Psi_j^{-1} \tilde{a}_j \\ &= -2\tilde{v}_{*j}^\top K_j \tilde{v}_{*j} \leq 0 \end{aligned}$$

Therefore, V_v is bounded, which means that $\tilde{v}_{*j}$ and $\hat{a}_j$ are bounded. Furthermore, it can be proved that $\tilde{v}_{*j}$ converges to zero.

For the systems in (9.14)-(9.16), $v_{1j} = \varsigma_{1j} + \tilde{v}_{1j}$ and $v_{2j} = \varsigma_{2j} + \tilde{v}_{2j}$. Next, we prove that $\lim_{t \rightarrow \infty} (s_{1j} - s_{10}) = 0$ and $\lim_{t \rightarrow \infty} (s_{2j} - s_{20}) = 0$. (9.14) can be transformed into

$$\dot{\tilde{s}}_{1j} = -\alpha_1 \tilde{s}_{1j} + \tilde{\zeta}_{1j} + \tilde{v}_{1j}. \quad (9.31)$$

Define $V_3 = \frac{1}{2} \tilde{s}_{1j}^2$ and differentiate it along (9.31), we have

$$\begin{aligned} \dot{V}_3 &= -\alpha_1 \tilde{s}_{1j}^2 + (\tilde{\zeta}_{1j} + \tilde{v}_{1j}) \tilde{s}_{1j} \\ &\leq -2\alpha_1 V_3 + |\tilde{\zeta}_{1j} + \tilde{v}_{1j}| \sqrt{2V_3} \end{aligned}$$

Define $V_4 = \sqrt{V_3}$, then

$$\dot{V}_4 \leq -\alpha_1 V_4 + \frac{1}{\sqrt{2}} |\tilde{\zeta}_{1j} + \tilde{v}_{1j}|.$$

Since $\tilde{\zeta}_{1j}$ and $\tilde{v}_{1j}$ converge to zero, it can be proved that V_4 converges to zero. Therefore, $\lim_{t \rightarrow \infty} (s_{1j} - s_{10}) = 0$. (9.15) can be transformed into

$$\dot{\tilde{s}}_{2*} = -(\mathcal{L} + B) \tilde{s}_{2*} - \rho_2 \text{sign}((\mathcal{L} + B) \tilde{s}_{2*}) - \dot{s}_{20} \mathbf{1} + \tilde{v}_{2*}. \quad (9.32)$$

where $\tilde{s}_{2*} = [\tilde{s}_{21}, \dots, \tilde{s}_{2m}]^\top$. Choose a nonnegative function $E_2 = \sum_{j=1}^m \frac{1}{2} \tilde{s}_{2j}^2$ and differentiate it along (9.32), we have

$$\begin{aligned} \dot{E}_2 &= -\tilde{s}_{2*}^\top (\mathcal{L} + B) \tilde{s}_{2*} - \tilde{s}_{2*}^\top \rho_2 \text{sign}((\mathcal{L} + B) \tilde{s}_{2*}) - \tilde{s}_{2*}^\top (\dot{s}_{20} \mathbf{1} - \tilde{v}_{2*}) \\ &\leq -\tilde{s}_{2*}^\top (\mathcal{L} + B) \tilde{s}_{2*} + \tilde{s}_{2*}^\top \tilde{v}_{2*} \end{aligned}$$

It can be proved that E_2 converges to zero. Therefore, $\tilde{s}_{2j}$ converges to zero and s_{2j} converges to s_{20} . ■

Theorem 9.1. *For the m robot systems ($1 \leq j \leq m$), if the communication graph is connected and the state of the leader agent is available to one of the m follower systems, the control laws in (9.28)-(9.29) guarantee that (9.5)-(9.7) holds under Assumption 1.*

Proof: By Lemma 5-7 we can prove that $\lim_{t \rightarrow \infty} (s_{1j} - s_{10}) = 0$, $\lim_{t \rightarrow \infty} (v_{1j} - v_{10}) = 0$ and $\lim_{t \rightarrow \infty} (s_{2j} - s_{20}) = 0$. By Lemma 4, $\lim_{t \rightarrow \infty} (s_{3j} - s_{30}) = 0$ holds, which means $\lim_{t \rightarrow \infty} (s_{ij} - s_{i0}) = 0$ and $\lim_{t \rightarrow \infty} (v_{1j} - v_{10}) = 0$. By Lemma 3 (9.10) holds. Finally we can prove (9.5)-(9.7) hold by Lemma 1. $\blacksquare$

In literature, similar problems have been considered in [152, 158]. However, in [158] the dynamic model is not considered and in [152] the dynamic system is considered without the nonholonomic constraints. In this paper, we considered the distributed tracking problem for uncertain mechanical systems with nonholonomic constraints and proposed distributed control laws with fast convergence rate.

9.4 Cooperative Control Laws for Time-varying Communication Graph

In the previous section, the communication graph is assumed to be fixed, in reality due to node and link disconnections and creations, the communication graph is time-varying or switching.

Theorem 9.2. *For the m robot systems defined in the previous section, under Assumption 1, if the communication graph $\mathcal{G}$ is time-varying and connected at any finite time interval and the state of the leader agent is available to one of the m follower systems, the control laws (9.28)-(9.29) guarantee that (9.5)-(9.7) holds.*

9.5 simulation

To show the effectiveness of the proposed results, simulation has been done for four robots. The desired geometric pattern $\mathcal{P}$ is shown in Fig. 9.2, assume the format of the robot systems is in square shape. The pattern $\mathcal{P}$ can be described by orthogonal coordinates $(p_{1x}, p_{1y}) = (0, 0.5)$, $(p_{2x}, p_{2y}) = (-0.5, 0)$, $(p_{3x}, p_{3y}) = (0, -0.5)$ and $(p_{4x}, p_{4y}) = (0.5, 0)$. For the leading agent, assume the reference trajectory is $(x_0, y_0, \theta_0) = (5 \sin(t), -5 \cos(t), t)$ and $(p_{0x}, p_{0y}) = (0, 0)$, by (9.4) $v_0 = 5$ and $\omega_0 = 1$.

Fig. 9.3 represents the communication graph, The cooperative controllers can be obtained by Lemma 5-6. We choose the control parameters $a_{ji} = 2$, $k_3 = 2$, $b_1 = 2$, $\rho_1 = 2$, and $\rho_2 = 2$. Fig. 9.5 shows the centroid of x_j ($1 \leq j \leq 4$) (i.e., $\sum_{j=1}^4 x_j/4$) and x_0 . Fig. 9.6 shows the centroid of y_j ($1 \leq j \leq 4$) (i.e., $\sum_{j=1}^4 y_j/4$) and y_0 . Fig. 9.7 shows $(\theta_j - \theta_0)$ ($1 \leq j \leq 4$).

If the information communication graph is time-varying, the control laws in Lemma 5-6 also solve the defined control problem. Assume the information communication graph switches according to the following logic.

$$\mathcal{G} = \begin{cases} \mathcal{G} \text{ in Fig. 9.3,} & \text{if } t - \text{round}(t) \geq 0 \\ \mathcal{G} \text{ in Fig. 9.4,} & \text{if } t - \text{round}(t) < 0 \end{cases}$$

Fig. 9.8 shows the centroid of x_j ($1 \leq i \leq 4$) and x_0 . Fig. 9.9 shows the centroid of y_j ($1 \leq j \leq 4$) and y_0 . Fig. 9.10 shows $(\theta_j - \theta_0)$ ($1 \leq j \leq 4$).

9.6 Conclusion

In this paper, distributed control of multiple follower systems has been studied and control algorithms were proposed for tracking trajectory of the leader agent. In the controller design, dynamics is considered adaptive controllers are designed for estimation of the unknown model parameters. Simulation results have proved the effectiveness of our proposed control laws.

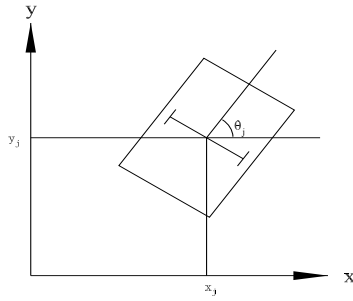


Figure 9.1: Simplified model of wheeled mobile robot.

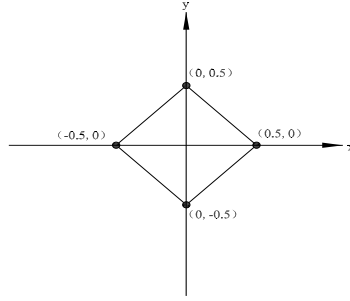


Figure 9.2: Desired geometric formation.

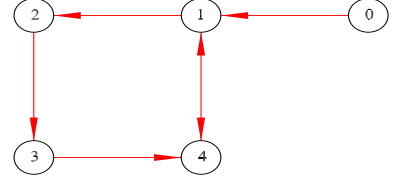


Figure 9.3: Information exchange graph $\mathcal{G}$.

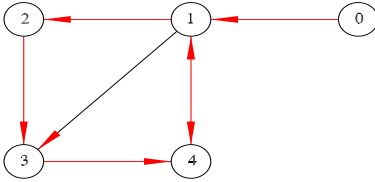


Figure 9.4: Information exchange graph $\mathcal{G}$ (after topology change).

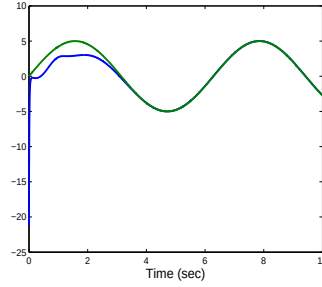


Figure 9.5: Response of the centroid of x_j for $1 \leq j \leq 4$ and x_0 .

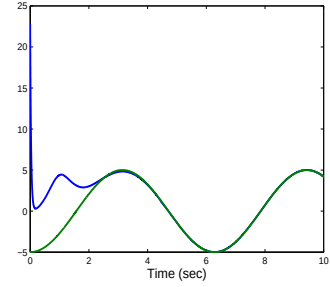


Figure 9.6: Response of the centroid of y_j for $1 \leq j \leq 4$ and y_0 .

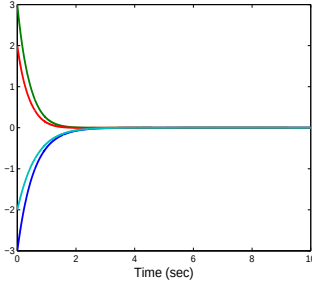


Figure 9.7: Response of $(\theta_j - \theta_0)$ for $1 \leq j \leq 4$.

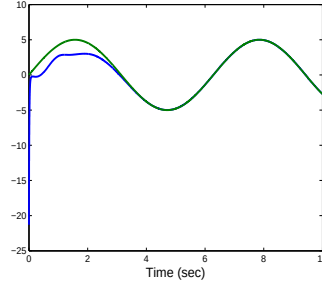


Figure 9.8: Response of the centroid of x_j for $1 \leq j \leq 4$ and x_0 .

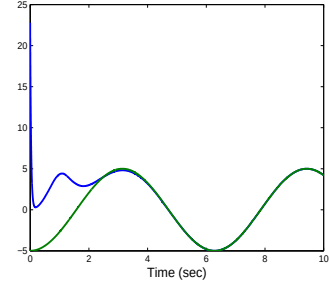


Figure 9.9: Response of the centroid of y_j for $1 \leq j \leq 4$ and y_0 .

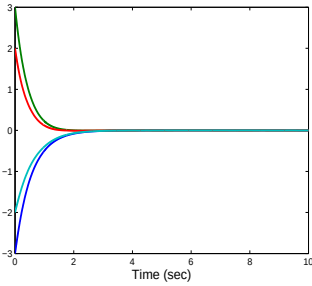


Figure 9.10: Response of $(\theta_j - \theta_0)$ for $1 \leq j \leq 4$.

Chapter 10

Teleoperation of a Cluster of Mobile Robots Subject to Model Uncertainty

10.1 Introduction

In practice, many tasks are required to be accomplished by multiple autonomous robots cooperatively, such as operations in hazardous environments, manipulation in nuclear processing plants, search and rescue missions, and exploration and survey. Due to high requirements of tasks and dynamic environment, many tasks cannot be accomplished alone by autonomous robots. Therefore, it is necessary that a human operator works cooperatively with multiple autonomous robots. Bilateral teleoperation is one type of cooperation between a human operator and a group of autonomous robots. The bilateral teleoperation of a team of robots enables human operators to extend their actions and intelligence to remote locations by allowing them to concentrate on high-level reasoning and decision-making (e.g. strategic path to be navigated by the robot formation).

Bilateral Teleoperation of a single master system and a single slave system has been studied for over four decades. In [159], experiments on the effects of communication delay in teleoperation of a single master and single slave systems was reported. In [160], supervisory control was developed to address the problem of communication delays [160]. In [161], a Lyapunov-based analysis was proposed for teleoperation. In [162, 163], the passivity-based approach was proposed. With the aid of the passivity-based approach, different subsequent schemes have been proposed in the literature to provide performance improvement [164–166].

With the aid of the development of cooperative control theory in the past decade [19, 23–28, 52, 67, 167–174], research in bilateral teleoperation of a single master and multiple slave systems has been reported in several papers.

In [175], coordination of a group of Lagrangian systems was considered with bidirectional communication constraints. With the aid of input-to-state stability, PD-type control laws were proposed such that the state of a slave system converges to a bounded region around the state of a master system. In [176], bilateral teleoperation of multiple mobile vehicles was considered when each slave vehicle can communicate its position and velocity to the master vehicle and vice versa. PD controllers were proposed such that the motion tracking and the formation control are achieved. In [177], bilateral teleoperation of multiple mobile robots

was considered under the assumption that the master system has communication with each slave system. Control laws were proposed with the aid of partial feedback linearization. In [178], bilateral operation of multiple cooperative robots with delayed communication was considered under the condition that the communication and the computation are centralized. Control laws were proposed with the aid of passive decomposition.

In bilateral teleoperation of a single master and multiple slave systems research, most of the results were achieved based on one or more of the following assumptions: (1) each system is not subject to nonholonomic constraints [175, 176, 178]; (2) there is no dynamic model uncertainty [177]; or (3) the master system can communicate with each slave system [176]. This paper considers the problem of the teleoperation of a cluster of mobile robots that appear as one single virtual mobile robot to a remote human operator. The proposed control design assumes the following: (1) each system is subject to nonholonomic constraints; (2) there is model uncertainty for each system; (3) the state information of the virtual system is available only to a subset of the mobile robots; and (4) the state information of each robot is not available to the virtual system.

With the aid of backstepping techniques and the results from graph theory, distributed adaptive control laws are proposed for each robot such that the group of robots come into a desired formation and the centroid of the robot cluster moves along the desired trajectory that is remotely specified by a human operator. Compared to existing results, the advantages of the proposed study are: 1) starting from an initial formation, a group of robots come into a desired formation the centroid of which converges to the desired position of a virtual robot; 2) The desired formation is achieved even though the information about the virtual robot is available to only a subset of the robots in the cluster.

10.2 Problem Statement

This paper considers the problem of the tele-operation of a cluster of m car-like wheeled mobile robots that is controlled to move along a pre-defined path while maintaining a desired rigid formation. The configuration of each robot is shown in Fig. 10.1. It is assumed that each robot is rigid and its wheels do not slip along its axis. For simplicity, it is also assumed that the two wheels on each axle (front and rear) collapse into a single wheel located at the midpoint of the axle (car-like model). The front wheel can be steered while the rear wheel orientation is fixed. The generalized coordinates of robot j are $q_{*j} = [x_j, y_j, \theta_j, \theta_j + \varphi_j]^\top$, where (x_j, y_j) are the cartesian coordinates of the front wheel, θ_j is the orientation of the robot body with respect to the x -axis, and φ_j is the steering angle.

Robot j is subject to two nonholonomic constraints (one for each wheel):

$$\dot{x}_j \sin(\theta_j + \varphi_j) - \dot{y}_j \cos(\theta_j + \varphi_j) = 0 \quad (10.1)$$

$$\dot{x}_j^r \sin \theta_j - \dot{y}_j^r \cos \theta_j = 0 \quad (10.2)$$

where (x_j^r, y_j^r) are the cartesian coordinates of the rear wheel, $x_j^r = x_j - l_j \cos \theta_j$, and $y_j^r = y_j - l_j \sin \theta_j$ where l_j is the distance between the wheels. Substituting x_j^r and y_j^r into (10.2), the constraints (10.1)-(10.2) can be written as

$$J_j(q_{*j})\dot{q}_{*j} = 0 \quad (10.3)$$

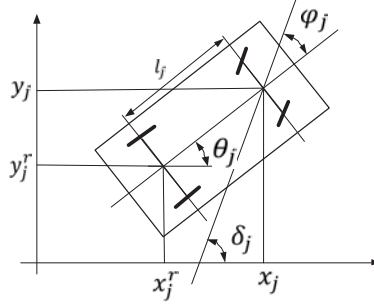


Figure 10.1: Configuration of a simplified car-like robot.

where

$$J_j(q_{*j}) = \begin{bmatrix} \sin(\theta_j + \varphi_j) & -\cos(\theta_j + \varphi_j) & 0 & 0 \\ \sin \theta_j & -\cos \theta_j & l_j & 0 \end{bmatrix}. \quad (10.4)$$

The dynamics of robot j can be obtained with the aid of the D'Alembert's principle [115] and is written as

$$M_j(q_{*j})\ddot{q}_{*j} + C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j} + G_j(q_{*j}) = B_j(q_{*j})\tau_j + J_j^\top(q_{*j})\lambda_j, \quad (10.5)$$

where $M_j(q_{*j})$ is an 4×4 bounded positive-definite symmetric matrix, $C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j}$ is centripetal and Coriolis force, $G_j(q_{*j})$ is gravitational force, $B_j(q_{*j})$ is an 4×2 input transformation matrix, τ_j is control input, λ_j is the constraint force on system j , and the superscript $^\top$ denotes the transpose operator. It is well-known that the following properties hold for the dynamics (10.5): 1. $(\dot{M}_j - 2C_j)$ is a skew-symmetric matrix; and 2. For any vector $\xi \in R^4$, $M_j(q_{*j})\dot{\xi} + C_j(q_{*j}, \dot{q}_{*j})\xi + G_j(q_{*j}) = Y_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})\beta_j$, where $Y_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})$ is a matrix function of q_{*j} , $\dot{q}_{*j}$, ξ , and $\dot{\xi}$ and is called the regressor matrix, and β_j is the inertia parameter vector. In this paper, it is assumed that the regressor matrix $Y_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})$ is a known matrix and the inertia parameter vector β_j is unknown and needs to be estimated.

The robots in the cluster are assumed to exchange data between them using on-board sensors and wireless communication. If each robot is considered as a node, the communication between robots can be described by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \dots, m\}$ is a node set, $\mathcal{E}$ is an edge set with element e_{ij} that describes the information flow from node i to node j . Robot i is referred to as a *neighbor* of robot j if the information of robot i is available to robot j . For robot j , the indices of its neighbors form a set denoted by $\mathcal{N}_j$. This study considers a bidirectional communication between a robot and its neighbors. A graph $\mathcal{G}$ is called to be connected if for any two nodes there is a set of edges which connect the two nodes.

For m robots, a desired formation $\mathcal{F}$ is defined by coordinates (p_{xj}, p_{yj}) ($1 \leq j \leq m$) that satisfy $\sum_{j=1}^m p_{xj} = 0$ and $\sum_{j=1}^m p_{yj} = 0$. Fig. 10.2 shows an example of a desired formation of a cluster of five robots ($m = 5$).

The cluster of m robots is said to be in the desired formation $\mathcal{F}$ if $x_j - x_i = p_{xj} - p_{xi}$ and $y_j - y_i = p_{yj} - p_{yi}$ for $1 \leq j \neq i \leq m$.

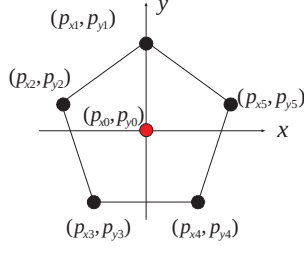


Figure 10.2: An example of desired rigid formation. The virtual robot occupies the centroid position.

In order to telecommand a cluster of mobile robots, a human operator controls a system whose position states are the desired trajectories of the centroid of the robot cluster. The system that is controlled directly by a human operator is called a virtual mobile robot. The virtual mobile robot is represented by:

$$J_0(q_{*0})\dot{q}_{*0} = 0 \quad (10.6)$$

$$\begin{aligned} M_0(q_{*0})\ddot{q}_{*0} + C_0(q_{*0}, \dot{q}_{*0})\dot{q}_{*0} + G_0(q_{*0}) \\ = B_0(q_{*0})\tau_h + J_0^\top(q_{*0})\lambda_0, \end{aligned} \quad (10.7)$$

where $q_{*0} = [x_0, y_0, \theta_0, \theta_0 + \varphi_0]^\top$ is the generalized coordinates of the virtual mobile robot, J_0 is defined in (10.4) with $j = 0$, M_0 , C_0 , G_0 , and B_0 are known matrices, τ_h is the external input of a human operator. System (10.6)-(10.7) is a virtual system. The state q_{*0} and its derivatives are available to one or more mobile robots via wireless communication. For convenience, the virtual robot in (10.6)-(10.7) is labeled as robot 0. The virtual robot is assumed to communicate to a subset of the mobile robots. If the virtual robot and the m mobile robots are considered as a group of $(m + 1)$ robots, the communications between robots can be described by a graph $\mathcal{G}^e$ with $(m + 1)$ nodes. A neighbor set of robot j is denoted by $\mathcal{N}_j^e$. The problem considered in this paper is defined as follows.

Control Problem: For a desired formation $\mathcal{F}$ and a desired trajectory (x_0, y_0) , the control problem is to design a distributed cooperative control law for robot j using its own state q_{*j} , its neighbors' state q_{*i} for $i \in \mathcal{N}_j^e$, and the desired formation information (p_{xj}, p_{yj}) and (p_{xi}, p_{yi}) for $i \in \mathcal{N}_j^e$ such that the m robots come into the desired formation and the centroid of the position of the mobile robots converges to the position of the virtual robot, i.e.,

$$\lim_{t \rightarrow \infty} (x_j - x_i) = p_{xj} - p_{xi}, \quad (10.8)$$

$$\lim_{t \rightarrow \infty} (y_j - y_i) = p_{yj} - p_{yi}, \text{ for } 1 \leq i \neq j \leq m. \quad (10.9)$$

$$\lim_{t \rightarrow \infty} \left(\frac{1}{m} \sum_{j=1}^m x_j(t) - x_0(t) \right) = 0 \quad (10.10)$$

$$\lim_{t \rightarrow \infty} \left(\frac{1}{m} \sum_{j=1}^m y_j(t) - y_0(t) \right) = 0. \quad (10.11)$$

The considered control problem introduces key challenges that make the proposed study different from existing research in bilateral teleoperation of a master system and multiple slave systems. These key challenges are: 1) the model of each robot is a *nonholonomic dynamic system*; (2) the model of each robot is *not exactly known*; (3) the state of the virtual robot is not available to each robot; and 4) there are communication delays between systems.

10.3 Controller Design

The system described by (10.3)-(10.5) is transformed into a cascade structure as discussed next. It is assumed that each robot is front wheel driving. Hence, the constraint (10.3) can be written as

$$\dot{q}_{*j} = \begin{bmatrix} \cos \delta_j & 0 \\ \sin \delta_j & 0 \\ \sin \varphi_j / l_j & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{1j} \\ v_{2j} \end{bmatrix} = g_j(q_{*j})v_{*j} \quad (10.12)$$

where $\delta_j = \theta_j + \varphi_j$, v_{1j} and v_{2j} are the driving input and the steering velocity input, respectively. Differentiating both sides of (10.12) and substituting it into (10.5) and multiplying both sides by $g_j(q_{*j})^\top$, one has

$$\bar{M}_j(q_{*j})\dot{v}_{*j} + \bar{C}_j(q_{*j}, \dot{q}_{*j})v_{*j} + \bar{G}_j(q_{*j}) = \bar{B}_j(q_{*j})\tau_j \quad (10.13)$$

where it is applied the fact that $g_j^\top(q_{*j})J_j^\top(q_{*j}) = 0$, and $\bar{M}_j = g_j^\top M_j g_j$, $\bar{C}_j = g_j^\top M_j \dot{g}_j + g_j^\top C_j g_j$, $\bar{G}_j = g_j^\top G_j$, and $\bar{B}_j = g_j^\top B_j$.

System (10.12)-(10.13) describes the motion of system (10.3) and (10.5). Therefore, the defined control problem can be solved based on system (10.12)-(10.13) instead of system (10.3) and (10.5). System (10.12) is called the kinematics of system j . System (10.13) is called the dynamics of system j .

Noting that the system in (10.12)-(10.13) has a cascade structure, a backstepping based approach with two steps is proposed. In the first step, the dynamics (10.13) are ignored and v_{*j} is assumed to be a virtual control input. Distributed control laws v_{*j} will be proposed for a group of m systems ($1 \leq j \leq m$) in (10.12) such that (10.8)-(10.9) are satisfied. In the second step, the dynamics (10.13) are taken into consideration and distributed control laws τ_j will be designed for the systems in (10.12)-(10.13) with the aid of the results in the first step and backstepping techniques such that eqn. (10.8)-(10.9) are satisfied. In order to highlight the basic ideas, communication delays are not considered.

10.3.1 Step 1: Controller Design for Kinematic Systems

In order to design distributed control laws v_{*j} ($1 \leq j \leq m$) for systems (10.12) such that (10.9) is satisfied, we introduce the following new variables

$$z_{1j} = x_j - p_{xj} + r \cos \delta_j \quad (10.14)$$

$$z_{2j} = y_j - p_{yj} + r \sin \delta_j \quad (10.15)$$

for $0 \leq j \leq m$, where $r (\neq 0)$ is a small positive number which can be chosen by the designer, $p_{x0} = 0$ and $p_{y0} = 0$. Then

$$\begin{bmatrix} \dot{z}_{1j} \\ \dot{z}_{2j} \end{bmatrix} = \Psi_j \begin{bmatrix} v_{1j} \\ v_{2j} \end{bmatrix} \quad (10.16)$$

where

$$\Psi_j = \begin{bmatrix} \cos \delta_j & -r \sin \delta_j \\ \sin \delta_j & r \cos \delta_j \end{bmatrix}.$$

It can be verified that Ψ_j is nonsingular if $r \neq 0$.

Lemma 10.1. *For the m systems in (10.16), if the communication graph $\mathcal{G}$ is connected, the control laws*

$$v_{1j} = \eta_{1j}, \quad v_{2j} = \eta_{2j} \quad (10.17)$$

for $1 \leq j \leq m$ guarantee that

$$\lim_{t \rightarrow \infty} (z_{1j} - z_{1i}) = 0, \quad \lim_{t \rightarrow \infty} (z_{2j} - z_{2i}) = 0 \quad (10.18)$$

for $0 \leq i \neq j \leq m$, where

$$\begin{bmatrix} \eta_{1j} \\ \eta_{2j} \end{bmatrix} = \Psi_j^{-1} \begin{bmatrix} -\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i}) \\ \rho_{1j} \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i}) \\ -\frac{\rho_{1j} \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i})}{\sqrt{\left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i})\right)^2 + h(t)}} \\ \dots \\ -\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i}) \\ \rho_{2j} \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i}) \\ -\frac{\rho_{2j} \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i})}{\sqrt{\left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i})\right)^2 + h(t)}} \end{bmatrix} \quad (10.19)$$

$a_{ji} = a_{ij} > 0$, ρ_{1j} and ρ_{2j} ($1 \leq j \leq m$) are sufficiently large constants, $h(t)$ is nonnegative and $\sqrt{h(t)}$ is an integrable time function.

Proof: Define $\bar{z}_{1j} = z_{1j} - z_{10}$, then

$$\begin{aligned} \dot{\bar{z}}_{1j} &= -\sum_{i \in \mathcal{N}_j^e} a_{ji}(\bar{z}_{1j} - \bar{z}_{1i}) \\ &\quad -\frac{\rho_{1j} \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i})}{\sqrt{\left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i})\right)^2 + h}} - \dot{x}_{10} \mathbf{1} \end{aligned}$$

for $1 \leq j \leq m$. Let a Lyapunov function $V = \bar{z}_{1*}^\top (\mathcal{L} + \text{diag}[a_{10}, a_{20}, \dots, a_{m0}]) \bar{z}_{1*}$ where $\bar{z}_{1*} = [\bar{z}_{11}, \bar{z}_{12}, \dots, \bar{z}_{1m}]$, $\mathcal{L}$ is the Laplacian matrix of the communication graph $\mathcal{G}$, differentiate V along the solutions of the systems, with the aid of the results in [179] we have $\dot{V} \leq -\bar{z}_{1*}^\top (\mathcal{L} + \text{diag}[a_{10}, a_{20}, \dots, a_{m0}])^2 \bar{z}_{1*} + \sqrt{h} \sum_{j=1}^m \rho_{1j}$ if ρ_{1j} is sufficiently large. It can be proved that $\bar{z}_{1j}$ converges to zero. Similarly, we can show that $\bar{z}_{2j}$ converges to zero if ρ_{1j} is sufficiently large. $\blacksquare$

In Lemma 10.1, $h(t)$ can be $1/t^\alpha$ for $\alpha > 2$, $e^{-\alpha t}$ for $\alpha > 0$, or other functions. For simplicity, we can choose h to be a small positive constant in Lemma 10.1 and it can be shown that $(z_{1j} - z_{10})$ and $(z_{2j} - z_{20})$ converge to a small neighborhood of the origin.

In Lemma 10.1, ρ_{1j} and ρ_{2j} should be large enough. If the upper bounds of ρ_{1j} and ρ_{2j} are not known in advance, they can be estimated on-line as follows.

Lemma 10.2. *For the m systems in (10.16), if the communication graph $\mathcal{G}$ is connected, the control laws (10.17) for $1 \leq j \leq m$ guarantee that (10.18) hold for $0 \leq i \neq j \leq m$, where*

$$\begin{bmatrix} \eta_{1j} \\ \eta_{2j} \end{bmatrix} = \Psi_j^{-1} \begin{bmatrix} -\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i}) \\ \hat{\rho}_{1j} \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i}) \\ -\frac{\left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i})\right)^2}{\sqrt{\left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i})\right)^2 + h(t)}} \\ \dots \\ -\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i}) \\ \hat{\rho}_{2j} \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i}) \\ -\frac{\left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i})\right)^2}{\sqrt{\left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i})\right)^2 + h(t)}} \end{bmatrix} \quad (10.20)$$

$$\dot{\hat{\rho}}_{1j} = \frac{\gamma_{1j} \left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i})\right)^2}{\sqrt{\left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i})\right)^2 + h(t)}} \quad (10.21)$$

$$\dot{\hat{\rho}}_{2j} = \frac{\gamma_{2j} \left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i})\right)^2}{\sqrt{\left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i})\right)^2 + h(t)}} \quad (10.22)$$

$a_{ji} > 0$, γ_{1j} and γ_{2j} are positive constants, $h(t)$ is nonnegative and $\sqrt{h(t)}$ is an integrable time function.

The proof of Lemma 10.2 is omitted due to space limitation.

10.3.2 Step 2: Controller Design for Dynamics Systems

In this step, the dynamics (10.13) are taken into consideration. The control laws will be designed with the aid of backstepping techniques and robust control theory. In order to design τ_j , let $\tilde{v}_{1j} = v_{1j} - \eta_{1j}$ and $\tilde{v}_{2j} = v_{2j} - \eta_{2j}$, then the systems in (10.16) and (10.13) can be written as

$$\begin{aligned} \dot{z}_{1j} = & - \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i}) + \tilde{v}_{1j} \cos \delta_j - \tilde{v}_{2j} r \sin \delta_j \\ & - \frac{\rho_{1j} \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i})}{\sqrt{\left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i}) \right)^2 + h(t)}} \end{aligned} \quad (10.23)$$

$$\begin{aligned} \dot{z}_{2j} = & - \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i}) + \tilde{v}_{1j} \sin \delta_j + \tilde{v}_{2j} r \cos \delta_j \\ & - \frac{\rho_{2j} \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i})}{\sqrt{\left(\sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i}) \right)^2 + h(t)}} \end{aligned} \quad (10.24)$$

$$\begin{aligned} \tilde{M}_j \dot{\tilde{v}}_{*j} + \tilde{C}_j \tilde{v}_{*j} = & \tilde{B}_j \tau_j - (\tilde{M}_j \dot{\eta}_{*j} + \tilde{C}_j \eta_{*j} + \tilde{G}_j) \\ & \text{for } j \neq 0 \end{aligned} \quad (10.25)$$

$$\begin{aligned} \tilde{M}_0 \dot{\tilde{v}}_{*0} + \tilde{C}_0 \tilde{v}_{*0} = & \tilde{B}_0(\tau_0 + \tau_h) - (\tilde{M}_0 \dot{\eta}_{*0} \\ & + \tilde{C}_0 \eta_{*0} + \tilde{G}_0) \end{aligned} \quad (10.26)$$

where $\tilde{v}_{*j} = [\tilde{v}_{1j}, \tilde{v}_{2j}]^\top$, $\tilde{M}_j = (g_j \Psi_j)^\top M_j g_j \Psi_j$, $\tilde{C}_j = (g_j \Psi_j)^\top M_j \frac{d}{dt}(g_j \Psi_j) + (g_j \Psi_j)^\top C_j g_j \Psi_j$, $\tilde{G}_j = (g_j \Psi_j)^\top G_j$, and $\tilde{B}_j = (g_j \Psi_j)^\top B_j$. For the systems in (10.26), the following two properties can be proven: 1. $(\tilde{M}_j - 2\tilde{C}_j)$ is a skew-symmetric matrix; and 2. For any vector $\xi \in R^2$, $\tilde{M}_j(q_{*j})\dot{\xi} + \tilde{C}_j(q_{*j}, \dot{q}_{*j})\xi + \tilde{G}_j(q_{*j}) = \tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})\beta_j$, where $\tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})$ is a matrix function of q_{*j} , $\dot{q}_{*j}$, ξ , and $\dot{\xi}$.

Lemma 10.3. *For $(m+1)$ systems, if the communication graph $\mathcal{G}^e$ is connected, the control laws*

$$\begin{aligned} \tau_j = & \tilde{B}_j^{-1} \left[-K_j \tilde{v}_{*j} + \tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \dot{\eta}_{*j}, \eta_{*j}) \hat{\beta}_j \right. \\ & \left. - \Lambda_{*j} \right] \end{aligned} \quad (10.27)$$

$$\dot{\hat{\beta}}_j = -\Gamma_j \tilde{Y}_j(q_j, \dot{q}_j, \dot{\eta}_{*j}, \eta_{*j})^\top \tilde{v}_{*j} \quad (10.28)$$

guarantee that z_{*j} converges to z_{*0} and that $\hat{\beta}_j$ is bounded, where Γ_j is a positive constant matrix, $\Lambda_{*j} = [\Lambda_{1j}, \Lambda_{2j}]^\top$, $\Lambda_{1j} = \cos \delta_j \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i}) + \sin \delta_j \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i})$, $\Lambda_{2j} = -r \sin \delta_j \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{1j} - z_{1i}) + r \cos \delta_j \sum_{i \in \mathcal{N}_j^e} a_{ji}(z_{2j} - z_{2i})$, η_{1j} and η_{2j} are defined in (10.19) or (10.20).

Proof: If η_{1j} and η_{2j} are defined in (10.19), let

$$\begin{aligned} V &= \bar{z}_{1*}^\top (\mathcal{L} + \text{diag}[a_{10}, \dots, a_{m0}]) \bar{z}_{1*} + \sum_{j=1}^m \tilde{v}_{*j}^\top \tilde{M}_j \tilde{v}_{*j} \\ &\quad + \bar{z}_{2*}^\top (\mathcal{L} + \text{diag}[a_{10}, \dots, a_{m0}]) \bar{z}_{2*} \\ &\quad + \sum_{j=1}^m (\beta_j - \hat{\beta}_j)^\top \Gamma_j^{-1} (\beta_j - \hat{\beta}_j) \end{aligned} \quad (10.29)$$

Differentiating it along the solution of the system (10.23)-(10.26), we have

$$\begin{aligned} \dot{V} &\leq -\bar{z}_{1*}^\top (\mathcal{L} + \text{diag}[a_{10}, \dots, a_{m0}])^2 \bar{z}_{1*} \\ &\quad -\bar{z}_{2*}^\top (\mathcal{L} + \text{diag}[a_{10}, \dots, a_{m0}])^2 \bar{z}_{2*} \\ &\quad -2 \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} + \sum_{j=1}^m (\rho_{1j} + \rho_{2j}) \sqrt{h} \end{aligned} \quad (10.30)$$

it can be shown that V is bounded, which means that $\hat{\beta}_j$ is bounded. Furthermore, it can be shown that $(z_{1j} - z_{10})$, $(z_{2j} - z_{20})$, and $\tilde{v}_{*j}$ converge to zero.

If η_{1j} and η_{2j} are defined in (10.20), the results can be proved similarly. ■

With the aid of Lemmas 10.3, the following results are obtained.

Theorem 10.1. *For m robots and a virtual system operated by a human operator, if the communication graph $\mathcal{G}^e$ is connected, the control laws in Lemma 10.3 guarantee that*

$$\lim_{t \rightarrow \infty} |x_j - x_i - p_{xj} + p_{xi}| \leq 2r, \quad (10.31)$$

$$\lim_{t \rightarrow \infty} |y_j - y_i - p_{yj} + p_{yi}| \leq 2r, \quad \text{for } 1 \leq i \neq j \leq m. \quad (10.32)$$

$$\lim_{t \rightarrow \infty} \left| \frac{1}{m} \sum_{j=1}^m x_j(t) - x_0(t) \right| \leq 2r \quad (10.33)$$

$$\lim_{t \rightarrow \infty} \left| \frac{1}{m} \sum_{j=1}^m y_j(t) - y_0(t) \right| \leq 2r \quad (10.34)$$

and that $\hat{\beta}_j$, $\hat{\rho}_{1j}$, and $\hat{\rho}_{2j}$ are bounded, where the control parameters are defined in Lemma 10.3.

Theorem 10.1 can be proved with the aid of Lemma 10.3 and the proof is omitted here.

Remark 10.1. *In Theorem 10.1, r can be chosen as small as possible. However, small r value leads to large control inputs. In practice, r should be chosen according to the tradeoff of magnitudes of control inputs and tracking errors between systems.*

Remark 10.2. *In this paper, the regressor matrix of each system is assumed to be known. If the regressor matrix is unknown, distributed robust adaptive controller can be proposed with the aid of robust and adaptive control theory. If approximation theory is applied to estimate unknown terms, distributed approximation-based adaptive controllers can be proposed. Due to space limitation, we omit them here.*

10.4 Simulation

To show the effectiveness of the proposed control design, two simulation cases of the teleoperation of five mobile robots are considered. The dynamics of each mobile robot can be found in [180] and are not presented here due to space limitation. The virtual robot is described by (10.6)-(10.7). For simplicity, we assume that $M_0 = I$, $C_0 = \mathbf{0}$, $G_0 = 0$, and $B_0 = \text{diag}[1, 1]$. The communication graph between the five robots is illustrated in Fig. 10.3, where the information about the location of the virtual robot (robot 0) is assumed to be available only to robot 5. The desired robot formation is specified in Fig. 10.6. The control laws of Theorem 10.1 are implemented to solve the control problems. In the first simulation example, the robot cluster is tele-commanded so that it forms the desired rigid formation and moves along a circular path while maintaining that desired formation. Figs. 10.4-10.5 show the response of transformed position vector $[z_{1j}, z_{2j}]^T$. Fig. 10.6 shows that starting from an initial formation, the robot cluster moves to form the desired formation, and then moves along the desired circular path while maintaining the desired formation.

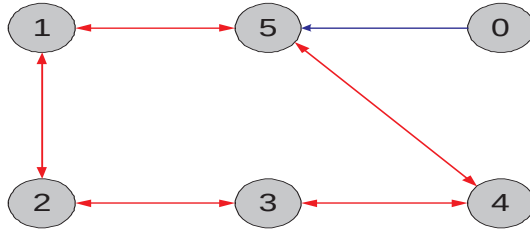


Figure 10.3: Communication graph $\mathcal{G}$.

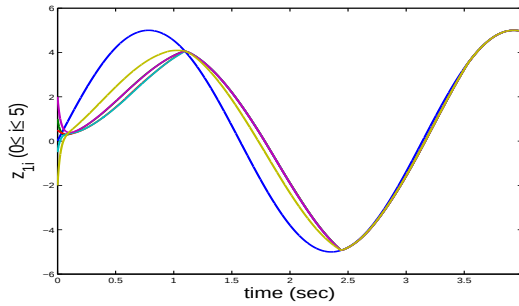


Figure 10.4: Response of z_{1j} ($0 \leq j \leq 5$).

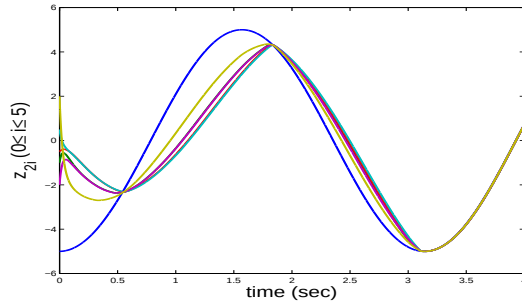


Figure 10.5: Response of z_{2j} ($0 \leq j \leq 5$).

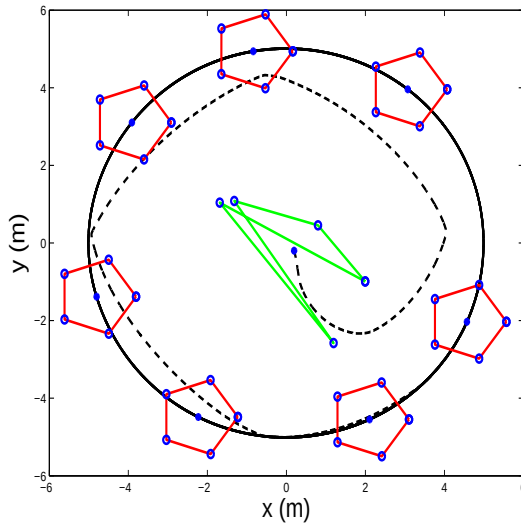


Figure 10.6: Path of the centroid of the robot cluster (dashed line). The green pentagon represents the initial robot formation and the red pentagons represent the desired rigid formation maintained by the cluster while moving along the desired circular path.

10.5 Conclusion

This chapter considered teleoperation of one master system and multiple slave systems. Distributed adaptive controllers were proposed with the aid of backstepping techniques and results of graph theory. Simulation results have demonstrated the effectiveness of the proposed control approach. In this paper, the effects of communication delays were not discussed. Future work will take the communication delays into consideration.

Chapter 11

Consensus of Multiple Nonholonomic Mechanical Systems with Non-ideal Nonholonomic Constraints

In this chapter, we consider Problem 3 defined in the last chapter, i.e., the consensus problem of multiple nonholonomic mechanical systems in (1.1) and (1.10). Since (1.11) and (1.4) are equivalent to (1.1) and (1.10), the consensus problem is considered based on (1.11) and (1.4). In order to design distributed control laws, we solve the problem in two steps. In the first step, we assume that u_{*j} are control input and design distributed algorithms using neighbors' information such (1.8) is satisfied. In the second step, we design control law τ_j using neighbors' information such that (1.8) is satisfied.

11.1 Distributed Controller Design for Kinematic Systems

Consider m systems in (1.12), i.e.,

$$\dot{x}_{1j} = v_{1j} + \phi_{1j}(x_{1j}) \quad (11.1)$$

$$\dot{x}_{2j} = v_{2j} + \phi_{2j}(\bar{x}_{2j}) \quad (11.2)$$

$$\dot{x}_{ij} = v_{1j}x_{i-1,j} + \phi_{ij}(\bar{x}_{ij}), \quad 3 \leq i \leq n \quad (11.3)$$

The communication between systems is defined by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. For simple presentation, it is assumed that the communication between systems is bi-directional in this chapter.

The considered problem is defined as follows.

Consensus of Multiple Chained Systems: For a group of m systems in (11.1)-(11.3), the problem is how to design a distributed control law (v_{1j}, v_{2j}) for system j based on its own information and its neighbors' information such that

$$\lim_{t \rightarrow \infty} (x_{*j} - c) = 0, \quad 1 \leq j \leq m, \quad (11.4)$$

where $x_{*j} = [x_{1j}, \dots, x_{nj}]^\top$ and c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

The system (11.1)-(11.3) has a cascade structure. (11.1) is a linear system with a perturbation term. For controller design, it is assumed that the uncertain terms in (11.1)-(11.3) satisfy the following conditions:

$$|\phi_{1j}(x_{1j})| \leq \gamma_{1j}(x_{1j}), \quad |\phi_{2j}(\bar{x}_{2j})| \leq \gamma_{2j}(\bar{x}_{2j}), \quad |\phi_{ij}(\bar{x}_{ij})| \leq \gamma_{ij}(\bar{x}_{ij}), \quad 3 \leq i \leq n \quad (11.5)$$

where β_{ij} are known positive functions.

With the aid of the results in [7], we have the following results.

Lemma 11.1. *For the system in (11.1)-(11.3), there exists a function $f_j(\beta_{*j}, \epsilon_j) \in R^n$ such that the matrix $G_j(\beta_{*j}) = [g_{1j}(f_j), g_{2j}(f_j), \frac{\partial f_j}{\partial \beta_{1j}}, \dots, \frac{\partial f_j}{\partial \beta_{n-2,j}}]$ is nonsingular for any β_{*j} and $\epsilon_j > 0$, where $g_{1j} = [1, 0, x_{2j}, \dots, x_{n-1,j}]^\top$, $g_{2j} = [0, 1, 0, \dots, 0]^\top$, $\beta_{*j} = [\beta_{1j}, \dots, \beta_{n-2,j}]^\top$, $\beta \in R^{n-2}$ and the function f_j has the following properties:*

1. f_j is bounded for any β_{*j} ;
2. $\lim_{\epsilon_j \rightarrow 0} f_j(\beta_{*j}, \epsilon_j) = 0$.

The proof of Lemma 11.1 can be found in [7, 8]. The construction of f_j can be found also in [7, 8]. The function f_j is called the transverse function.

With the aid of Lemma 11.1 and the notations in [7], it can be found the function $f_j(\beta_j, \epsilon_j)$ such that G_j is nonsingular. Let

$$z_{*j} = x_{*j} f_j(\beta_{*j})^{-1}$$

then, we have the augmented system

$$\dot{z}_{*j} = dr_{f_j(\beta_{*j})^{-1}}(x_{*j}) dl_{z_{*j}}(f_j(\beta_{*j})) G_j(\beta_{*j}) [v_{1j}, v_{2j}, -\dot{\beta}_{1j}, \dots, -\dot{\beta}_{n-2,j}]^\top \quad (11.6)$$

We define the neighbors's difference as

$$e_{*j} = [e_{1j}, \dots, e_{nj}] = \sum_{i \in \mathcal{N}_j} a_{ji} (z_{*j} - z_{*i}). \quad (11.7)$$

If $\dot{\beta}_{*j}$ is considered as an additional input, we propose the following distributed control law.

Theorem 11.1. *For the m systems in (11.1)-(11.3), if the communication graph has a spanning tree, the control law*

$$v_{1j} = \eta_{1j} \quad (11.8)$$

$$v_{2j} = \eta_{2j} \quad (11.9)$$

$$\begin{bmatrix} \eta_{1j} \\ \eta_{2j} \\ -\dot{\beta}_{*j} \end{bmatrix} = -G_j^{-1} dl_{z_{*j}^{-1}}(x_{*j}) dr_{f_j(\beta_{*j})}(z_{*j}) \left[\sum_{i \in \mathcal{N}_j} a_{ji} (z_{*j} - z_{*i}) - \Delta_{*j} \right] \quad (11.10)$$

ensures that

$$\lim_{t \rightarrow \infty} \|x_{*j} - x_{*i}\| \leq \delta_{ji}(\epsilon_j, \epsilon_i) \quad (11.11)$$

where δ_{ji} is a nonnegative continuous function of ϵ_j and ϵ_i and δ_{ji} converges to zero when ϵ_j and ϵ_i converge to zero, and

$$\Delta_{*j} = \begin{bmatrix} \frac{\gamma_{1j}e_{1j}}{\sqrt{e_{1j}^2 + e^{-t}}} \\ \vdots \\ \frac{\gamma_{nj}e_{nj}}{\sqrt{e_{nj}^2 + e^{-t}}} \end{bmatrix} \quad (11.12)$$

Proof: By Lemma 11.1, G_j is nonsingular. So, the control law exists. Substitute the control law into the system, we have

$$\dot{z}_{1j} = -e_{1j} - \frac{\gamma_{1j}e_{1j}}{\sqrt{e_{1j}^2 + e^{-t}}} + \phi_{1j} \quad (11.13)$$

$$\vdots \quad (11.14)$$

$$\dot{z}_{nj} = -e_{nj} - \frac{\gamma_{nj}e_{nj}}{\sqrt{e_{nj}^2 + e^{-t}}} + \phi_{nj} \quad (11.15)$$

Choose a function

$$V_i = z_{i*}^\top \mathcal{L} z_{i*}$$

where $z_{i*} = [z_{i1}, \dots, z_{in}]^\top$ and $\mathcal{L}$ is the Laplacian matrix, we have

$$\begin{aligned} \dot{V}_i &= -z_{i*}^\top \mathcal{L}^2 z_{i*} - \sum_{j=1}^m \frac{\gamma_{ij}e_{ij}^2}{\sqrt{e_{ij}^2 + e^{-t}}} + \sum_{j=1}^m e_{ij}\phi_{ij} \\ &\leq -z_{i*}^\top \mathcal{L}^2 z_{i*} - \sum_{j=1}^m \gamma_{ij}\sqrt{e_{ij}^2 + e^{-t}} + \sum_{j=1}^m \frac{e^{-t}}{\sqrt{e_{ij}^2 + e^{-t}}} + \sum_{j=1}^m \gamma_{ij}|e_{ij}| \\ &\leq -z_{i*}^\top \mathcal{L}^2 z_{i*} + me^{-t/2} \end{aligned}$$

By integrating both sides of the above inequality, it can be shown that z_{i*} is bounded and $\mathcal{L}z_{i*}$ converges to zero, which means that $(z_{ij} - z_{il})$ converges to zero for $1 \leq j \neq l \leq m$. Therefore, (11.11) holds. $\blacksquare$

Remark 11.1. If ϵ_j ($1 \leq j \leq m$) are chosen to be small constants, $\delta_{ji}(\epsilon_j, \epsilon_i)$ is a small constant, which means that $\|x_{*j} - x_{*i}\|$ converges to a small neighborhood of the origin. We say (11.4) is achieved practically.

Remark 11.2. In Theorem 11.1, nothing is said about β_{*j} . So, β_{*j} may be bounded or unbounded. Thanks to the properties of the function f_j , the boundedness of β_{*j} plays no role in the consensus problem.

11.2 Distributed Controller Design for Dynamical Systems

We considered m dynamical systems. The j -th system is defined as

$$\dot{x}_{1j} = v_{1j} + \phi_{1j}(x_{1j}) \quad (11.16)$$

$$\dot{x}_{2j} = v_{2j} + \phi_{2j}(\bar{x}_{2j}) \quad (11.17)$$

$$\dot{x}_{ij} = v_{1j}x_{i-1,j} + \phi_{ij}(\bar{x}_{ij}), \quad 3 \leq i \leq n \quad (11.18)$$

$$\tilde{M}_j \dot{v}_{*j} + \tilde{C}_j v_{*j} + \tilde{G}_j + \tilde{D}_j = \tilde{B}_j \tau_j \quad (11.19)$$

where $v_{*j} = [v_{1j}, v_{2j}]^\top$ and $x_{*j} = [x_{1j}, \dots, x_{nj}]^\top$. The following properties are satisfied.

Property 11.1. $\tilde{M}_j$ is bounded and $\dot{\tilde{M}}_j - 2\tilde{C}_j$ is skew-symmetric.

Property 11.2. For any differentiable vector $\xi \in R^2$,

$$\tilde{M}_j \dot{\xi} + \tilde{C}_j \xi + \tilde{G}_j = \tilde{Y}(x_{*j}, \dot{x}_{*j}, \xi, \dot{\xi}) a_j$$

where $\tilde{Y}_j$ is a known function of x_{*j} , $\dot{x}_{*j}$, ξ , and $\dot{\xi}$, and a_j is the inertia parameter vector.

The problem considered in this section is defined as follows.

Consensus of Multiple Systems: For a group of m systems in (11.16)-(11.19), the problem is how to design a distributed control law τ_j for system j based on its own information and its neighbors' information such that

$$\lim_{t \rightarrow \infty} (x_{*j} - c) = 0, \quad 1 \leq j \leq m, \quad (11.20)$$

where $x_{*j} = [x_{1j}, \dots, x_{nj}]^\top$ and c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

In the dynamics (11.19), we first assume that the inertia parameter vector a_j is a constant and is unknown.

In order to design controllers, we let

$$\tilde{v}_{1j} = v_{1j} - \eta_{1j} \quad (11.21)$$

$$\tilde{v}_{2j} = v_{2j} - \eta_{2j}, \quad 1 \leq j \leq m \quad (11.22)$$

where η_{1j} and η_{2j} are defined in (11.8) and (11.9), respectively. Let

$$\chi_j = [\chi_{i,l,j}] = r_{f_j(\beta_{*j})^{-1}}(x_{*j}) dl_{z_{*j}}(f_j(\beta_{*j})) G_j(\beta_{*j})$$

then we have

$$\dot{z}_{1j} = -e_{1j} - \frac{\gamma_{1j} e_{1j}}{\sqrt{e_{1j}^2 + e^{-t}}} + \phi_{1j} + \chi_{1,1,j} \tilde{v}_{1j} + \chi_{1,2,j} \tilde{v}_{2j} \quad (11.23)$$

$$\dot{z}_{2j} = -e_{2j} - \frac{\gamma_{2j} e_{2j}}{\sqrt{e_{2j}^2 + e^{-t}}} + \phi_{2j} + \chi_{2,1,j} \tilde{v}_{1j} + \chi_{2,2,j} \tilde{v}_{2j} \quad (11.24)$$

$$\dot{z}_{ij} = -e_{ij} - \frac{\gamma_{ij} e_{ij}}{\sqrt{e_{ij}^2 + e^{-t}}} + \phi_{ij} + \chi_{i,1,j} \tilde{v}_{1j} + \chi_{i,2,j} \tilde{v}_{2j}, \quad 3 \leq i \leq n \quad (11.25)$$

$$\tilde{M}_j \dot{\tilde{v}}_{*j} + \tilde{C}_j \tilde{v}_{*j} = \tilde{B}_j \tau_j - (\tilde{M}_j \dot{\eta}_{*j} + \tilde{C}_j \eta_{*j} + \tilde{G}_j + \tilde{D}_j) \quad (11.26)$$

where $\tilde{v}_{*j} = [\tilde{v}_{1j}, \tilde{v}_{2j}]^\top$.

For the dynamics of each system, we have

$$\tilde{M}_j \dot{\eta}_{*j} + \tilde{C}_j \eta_{*j} + \tilde{G}_j = \tilde{Y}_j(x_{*j}, \dot{x}_{*j}, \eta_{*j}, \dot{\eta}_{*j}) a_j \quad (11.27)$$

where a_j is the inertia parameter. For the disturbance $\tilde{D}_j$, it is assumed that

$$\|\tilde{D}_j\| \leq \rho_j(x_{*j}) \quad (11.28)$$

where ρ_j is a known function of x_{*j} . If the inertia parameter a_j is a constant and is unknown, we have the following results.

Theorem 11.2. *For the m systems in (11.16)-(11.19), if the communication graph has a spanning tree, the control law*

$$\tau_j = \tilde{B}_j^{-1} \left[-K_j \tilde{v}_{*j} + \tilde{Y}_j \hat{a}_j - \rho_j \text{sign}(\tilde{v}_{*j}) - \Lambda_j \right] \quad (11.29)$$

$$\dot{\hat{a}}_j = -\Gamma_j \tilde{Y}_j^\top \tilde{v}_{*j} \quad (11.30)$$

ensures that $\hat{a}_j$ is bounded and (11.11) holds, where K_j and Γ_j are positive constant matrices, and

$$\Lambda_j = \begin{bmatrix} \sum_{i=1}^n e_{ij} \chi_{i,1,j} \\ \sum_{i=1}^n e_{ij} \chi_{i,2,j} \end{bmatrix} \quad (11.31)$$

Proof: Let

$$V = \sum_{i=1}^n z_{i*}^\top \mathcal{L} z_{i*} + \sum_{j=1}^m \frac{1}{2} \tilde{v}_{*j}^\top \tilde{M}_j \tilde{v}_{*j} + \sum_{j=1}^m \frac{1}{2} (\hat{a}_j - a_j)^\top \Gamma_j^{-1} (\hat{a}_j - a_j)$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned} \dot{V} &\leq -\sum_{i=1}^n z_{i*}^\top \mathcal{L}^2 z_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} - \sum_{j=1}^m \rho_j \tilde{v}_{*j}^\top \text{sign}(\tilde{v}_{*j}) - \sum_{j=1}^m \tilde{v}_{*j}^\top \tilde{D}_j + m e^{-t/2} \\ &\leq -\sum_{i=1}^n z_{i*}^\top \mathcal{L}^2 z_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} + m e^{-t/2} \end{aligned}$$

By integrating both sides of the above inequality, it can be shown that V is bounded, which means that z_{i*} , $\tilde{v}_{*j}$ and $\hat{a}_j$ are bounded. Furthermore, it can be shown that by Barbalat's lemma that e_{i*} and $\tilde{v}_{*j}$ converge to zero. Therefore, (11.11) holds. $\blacksquare$

In Theorem 11.2, the unknown inertia parameter is estimated by an adaptive control law. If an estimate of a_j is $\bar{a}_j$ and

$$\|a_j - \bar{a}_j\| \leq \gamma_j$$

for $1 \leq j \leq m$ and γ_j is a known constant, we propose the following robust control laws.

Theorem 11.3. *For the m systems in (11.16)-(11.19), if the communication graph has a spanning tree, the control law*

$$\tau_j = \tilde{B}_j^{-1} \left[-K_j \tilde{v}_{*j} + \tilde{Y}_j \bar{a}_j - \gamma_j \tilde{Y}_j \text{sign}(\tilde{Y}_j^\top \tilde{v}_{*j}) - \rho_j \text{sign}(\tilde{v}_{*j}) - \Lambda_j \right] \quad (11.32)$$

ensures that (11.11) holds, where K_j is a positive constant matrix and Λ_j is defined in (11.31).

Proof: Let

$$V = \sum_{i=1}^n z_{i*}^\top \mathcal{L} z_{i*} + \sum_{j=1}^m \frac{1}{2} \tilde{v}_{*j}^\top \tilde{M}_j \tilde{v}_{*j}$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned} \dot{V}_j &\leq -\sum_{i=1}^n z_{i*}^\top \mathcal{L}^2 z_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} - \sum_{j=1}^m \rho_j \tilde{v}_{*j}^\top \text{sign}(\tilde{v}_{*j}) - \sum_{j=1}^m \tilde{v}_{*j}^\top \tilde{D}_j \\ &\quad + \sum_{j=1}^m \tilde{v}_{*j}^\top \tilde{Y}_j (\bar{a}_j - a_j) - \sum_{j=1}^m \gamma_j \tilde{v}_{*j}^\top \tilde{Y}_j \text{sign}(\tilde{Y}_j^\top \tilde{v}_{*j}) + m e^{-t/2} \\ &\leq -\sum_{i=1}^n z_{i*}^\top \mathcal{L}^2 z_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} + m e^{-t/2} \end{aligned}$$

Therefore, V is bounded, which means that z_{i*} and $\tilde{v}_{*j}$ are bounded. By Barbalat's lemma, it can be shown that e_{i*} and $\tilde{v}_{*j}$ converge to zero. $\blacksquare$

In Theorem 11.3, the unknown inertia parameter a_j is not required to be a constant. In the control laws, γ_j is required to be known. It is possible to estimate it.

11.3 Simulation

We considered three nonholonomic wheeled mobile robots considered in Section 3.5.

The constraint on the front wheels can be written as

$$\dot{x}_j \sin \theta_j - \dot{y}_j \cos \theta_j = P_j(x_j, y_j) \quad (11.33)$$

where (x_j, y_j) is the position of robot j , θ_j is the orientation of robot j , and P_j denotes slight slipping along the axis of the wheels. The dynamics of robot j are described by the following differential equations

$$\begin{cases} m_j \ddot{x}_j &= \lambda_j \cos \theta_j + \frac{1}{R_j} (\tau_{1j} + \tau_{2j}) \cos \theta_j \\ m_j \ddot{y}_j &= -\lambda_j \sin \theta_j + \frac{1}{R_j} (\tau_{1j} + \tau_{2j}) \sin \theta_j \\ I_j \ddot{\theta}_j &= \frac{L_j}{R_j} (\tau_{1j} - \tau_{2j}) \end{cases} \quad (11.34)$$

where m_j is the mass of robot j , and I_j is its inertia moment around the vertical axis at point Q. R_j is the radius of the wheels and $2L_j$ the length of the axis of the front wheels, and τ_{1j} and τ_{2j} are the torques provided by the motors.

Let $q_{*j} = [x_j, y_j, \theta_j]^\top$,

$$M_j(q_{*j}) = \begin{bmatrix} m_j & 0 & 0 \\ 0 & m_j & 0 \\ 0 & 0 & I_j \end{bmatrix}, C_j(q_{*j}, \dot{q}_{*j}) = 0, G_j(q_{*j}) = 0$$

$$B_j(q_{*j}) = \frac{1}{R_j} \begin{bmatrix} \cos \theta_j & \cos \theta_j \\ \sin \theta_j & \sin \theta_j \\ L_j & -L_j \end{bmatrix}, J_j = [\sin \theta_j, -\cos \theta_j, 0]$$

The system (11.33)-(11.34) is in the form of (11.1)-(11.2).

Let

$$g_{*j} = \begin{bmatrix} \cos \theta_j & 0 \\ \sin \theta_j & 0 \\ 0 & 1 \end{bmatrix}$$

then Equation (11.33) and (11.34) are converted into

$$\begin{cases} \dot{x}_j &= u_{1j} \cos \theta_j + P_j \sin \theta_j \\ \dot{y}_j &= u_{1j} \sin \theta_j - P_j \cos \theta_j \\ \dot{\theta}_j &= u_{2j} \\ m_j \dot{u}_{1j} &= \frac{1}{R_j} (\tau_{1j} + \tau_{2j}) \\ I_j \dot{u}_{2j} &= \frac{L_j}{R_j} (\tau_{1j} - \tau_{2j}) \end{cases} \quad (11.35)$$

With the transformation

$$\begin{cases} x_{1j} &= -\theta_j \\ x_{2j} &= x_j \cos \theta_j + y_j \sin \theta_j \\ x_{3j} &= -x_j \sin \theta_j + y_j \cos \theta_j \\ v_{1j} &= -u_{2j} \\ v_{2j} &= u_{1j} - x_{3j} v_{1j} \end{cases}$$

Equation (11.35) can be converted into the following standard form

$$\begin{cases} \dot{x}_{1j} = v_{1j} \\ \dot{x}_{2j} = v_{2j} \\ \dot{x}_{3j} = x_{2j} v_{1j} - P_j \\ \tilde{M}_j \dot{v}_{*j} + \tilde{C}_j v_{*j} = \tilde{B}_j \tau_{*j} \end{cases} \quad (11.36)$$

where

$$\tilde{M}_j = \begin{bmatrix} I_j + m_j x_{3j}^2 & m_j x_{3j} \\ m_j x_{3j} & m_j \end{bmatrix}, \tilde{C}_j = \begin{bmatrix} m_j x_{3j} \dot{x}_{3j} & 0 \\ m_j \dot{x}_{3j} & 0 \end{bmatrix}, \tilde{B}_j = \frac{1}{R_j} \begin{bmatrix} x_{3j} + L_j & x_{3j} - L_j \\ 1 & 1 \end{bmatrix}$$

and

$$\tilde{M}_j(q_{*j}) \dot{\xi} + \tilde{C}_j(q_{*j}, \dot{q}_{*j}) \xi = \tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi}) a_j$$

where the inertia parameter vector $a_j = [m_j, I_j]^T$,

$$\tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi}) = \begin{bmatrix} x_{3j}^2 \dot{\xi}_1 + x_{3j} \dot{\xi}_2 + x_{3j} \dot{x}_{3j} \xi_1 & \dot{\xi}_1 \\ x_{3j} \dot{\xi}_1 + \dot{\xi}_2 + \dot{x}_{3j} \xi_1 & 0 \end{bmatrix}$$

The consensus problem of the kinematics in (11.36) can be solved with the aid of the results proposed in Theorem 11.1. By the results in [7], we choose

$$f_{*j} = \begin{bmatrix} \epsilon_j \sin \beta_j \\ \epsilon_j \cos \beta_j \\ \frac{\epsilon_j^2}{4} \sin 2\beta_j \end{bmatrix}, \quad \epsilon_j > 0 \quad (11.37)$$

Then

$$\frac{df_{*j}}{d\beta_{1j}} = \begin{bmatrix} \epsilon_j \cos \beta_j \\ -\epsilon_j \sin \beta_j \\ \frac{\epsilon_j^2}{2} \cos 2\beta_j \end{bmatrix} \dot{\beta}_{1j}. \quad (11.38)$$

It can be verified that f_j satisfies the properties in Lemma 11.1.

Let

$$z_{*j} = x_{*j} f_{*j}^{-1} = \begin{bmatrix} x_{1j} - f_{1j} \\ x_{2j} - f_{2j} \\ x_{3j} - f_{3j} - f_{1j}(x_{2j} - f_{2j}) \end{bmatrix}$$

the controller is proposed as

$$v_{1j} = \eta_{1j} \quad (11.39)$$

$$v_{2j} = \eta_{2j} \quad (11.40)$$

$$\begin{bmatrix} \eta_{1j} \\ \eta_{2j} \\ -\dot{\beta}_j \end{bmatrix} = \begin{bmatrix} 1 & 0 & \epsilon_j \cos \beta_j \\ 0 & 1 & -\epsilon_j \sin \beta_j \\ \epsilon_j \cos \beta_j & 0 & \frac{\epsilon_j^2}{2} \cos 2\beta_j \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ z_{2j} & -\epsilon_j \sin \beta_j & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} \sum_{i \in \mathcal{N}_j} a_{ji}(z_{*j} - z_{*i}) + \Delta_{*j} \end{bmatrix} \quad (11.41)$$

where $a_{ji} > 0$ and Δ_j is defined in (11.12) with $n = 3$.

In the simulation, we choose $P_j(x_j, y_j) = 0.2 \sin t$. The communication graph is shown as in Fig. 11.1. The communication graph is shown as in Fig. 12.1. Figs. 11.2-11.4 show the time response of x_{1*} , x_{2*} , and x_{3*} , respectively. It is shown that the state of three systems reach consensus.

The consensus problem of the dynamics in (11.36) can be solved with the aid of the results in Theorem 11.2. The controller is proposed as in (11.29)-(11.30) if the inertia parameter vector a_j is a constant and is unknown. Figs. 11.5-11.7 show the time response of x_{1*} , x_{2*} , and x_{3*} , respectively. It is shown that the state of three systems reach consensus.

If the inertia parameter vector a_j is a constant and is unknown, we can also solve the consensus problem by the robust control algorithms in Theorem 11.3. Figs. 11.8-11.10 show the time response of x_{1*} , x_{2*} , and x_{3*} , respectively. It is shown that the state of three systems reach consensus.

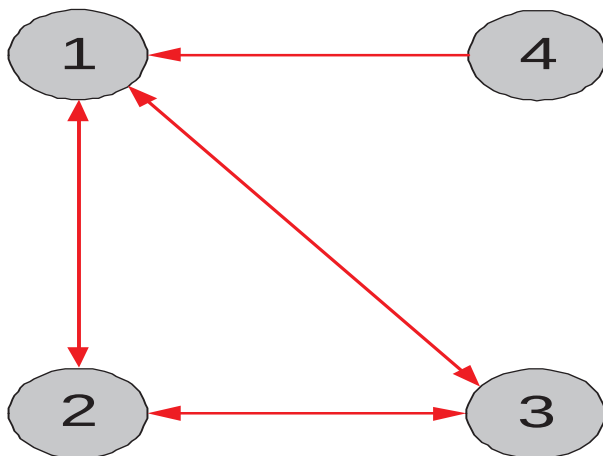


Figure 11.1: Communication graph.

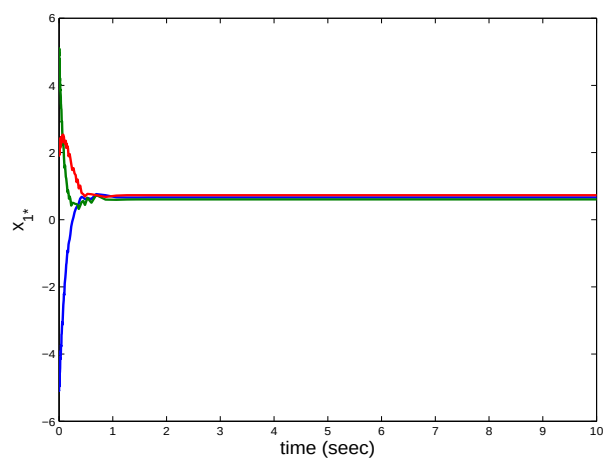


Figure 11.2: Response of x_{1*} .

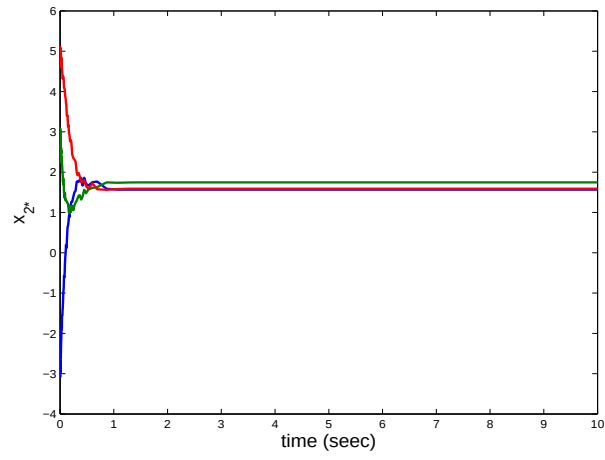


Figure 11.3: Response of x_{2*} .

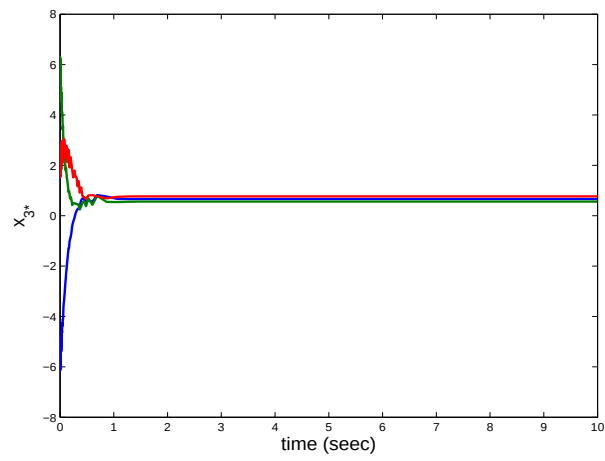


Figure 11.4: Response of x_{3*} .

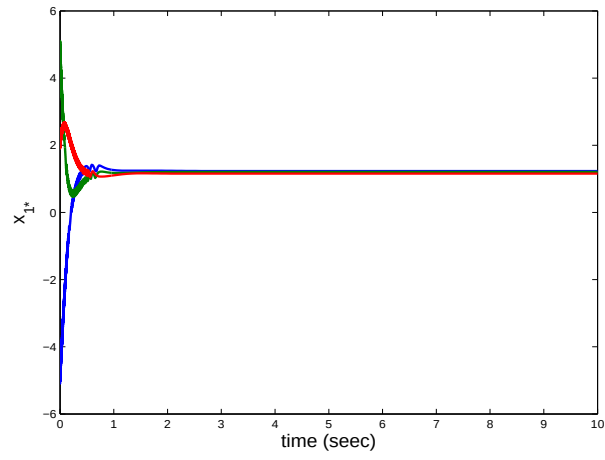


Figure 11.5: Response of x_{1*} .

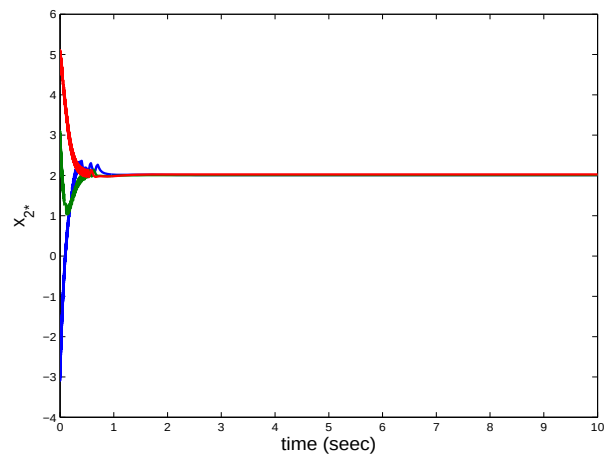


Figure 11.6: Response of x_{2*} .

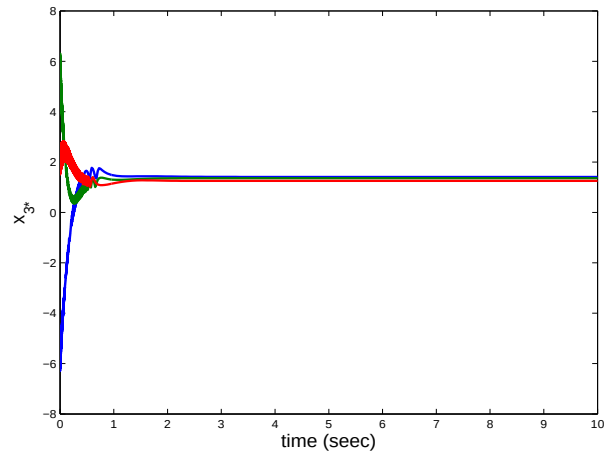


Figure 11.7: Response of x_{2*} .

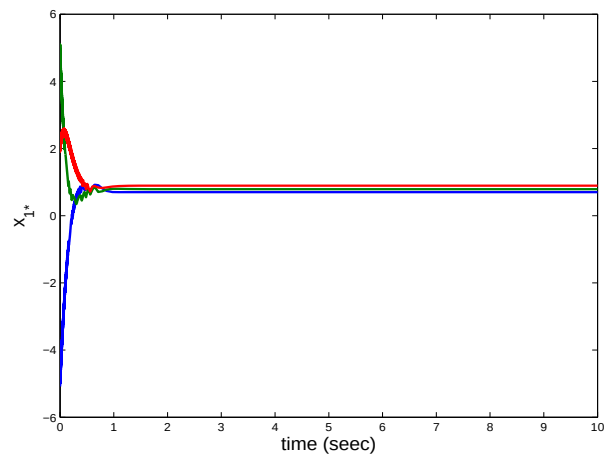


Figure 11.8: Response of x_{1*} .

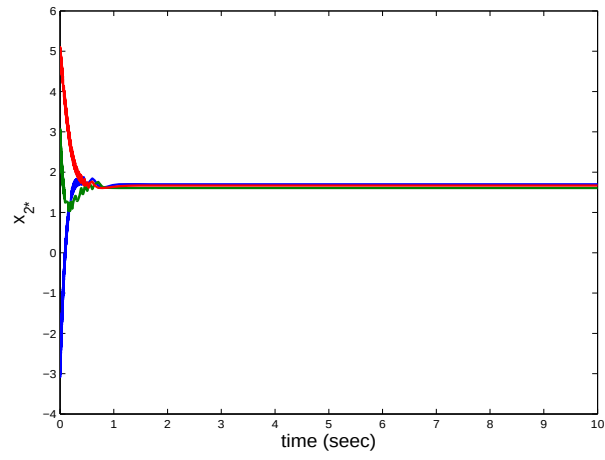


Figure 11.9: Response of x_{2*} .

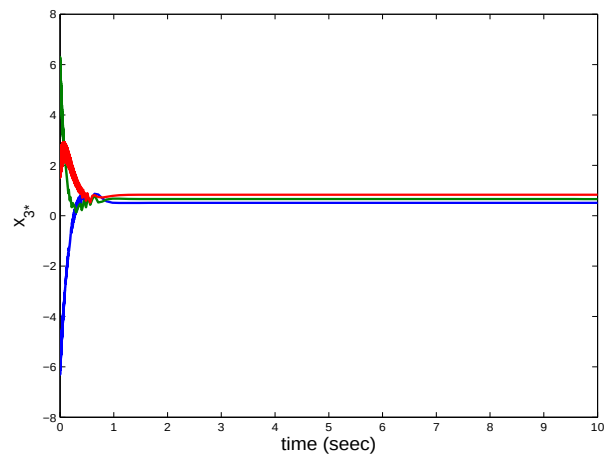


Figure 11.10: Response of x_{2*} .

Chapter 12

Distributed Tracking Control of Multiple Nonholonomic Mechanical Systems with Non-ideal Nonholonomic Constraints

In this chapter, we consider Problem 4 defined in the last chapter, i.e., the tracking control problem of multiple nonholonomic mechanical systems in (1.1) and (1.10).

12.1 Distributed Controller Design for Kinematic Systems

Consider m systems where the j -th system is defined by

$$\dot{x}_{1j} = v_{1j} + \phi_{1j}(x_{1j}) \quad (12.1)$$

$$\dot{x}_{2j} = v_{2j} + \phi_{2j}(\bar{x}_{2j}) \quad (12.2)$$

$$\dot{x}_{ij} = v_{1j}x_{i-1,j} + \phi_{ij}(\bar{x}_{ij}), \quad 3 \leq i \leq n \quad (12.3)$$

The communication between systems is defined by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. For simple presentation, it is assumed that the communication between systems is bi-directional in this chapter.

It is given a desired trajectory $x^d = [x_{1d}, \dots, x_{nd}]^\top$ which is generated by

$$\dot{x}_{1d} = v_{1d} \quad (12.4)$$

$$\dot{x}_{2d} = v_{2d} \quad (12.5)$$

$$\dot{x}_{id} = v_{1d}x_{i-1,d}, \quad 3 \leq i \leq n \quad (12.6)$$

where v_{1d} and v_{2d} are known functions. It is assumed that

$$\max_{t \in [0, \infty)} |x_{id}| \leq \delta_i, \quad 1 \leq i \leq n.$$

where δ_i is a positive constant.

The problem considered in this chapter is defined as follows.

Tracking Control of Multiple Chained Systems: For a group of m systems in (12.1)-(12.3), it is given a desired trajectory x^d , the problem is how to design a distributed control law (v_{1j}, v_{2j}) for system j based on its own information and its neighbors' information such that

$$\lim_{t \rightarrow \infty} (x_{*j} - x^d) = 0, \quad 1 \leq j \leq m, \quad (12.7)$$

where $x_{*j} = [x_{1j}, \dots, x_{nj}]^\top$.

It is assumed that the uncertain terms in (12.1)-(12.3) satisfy the following conditions:

$$|\phi_{1j}(x_{1j})| \leq \gamma_{1j}(x_{1j}), \quad |\phi_{2j}(\bar{x}_{2j})| \leq \gamma_{2j}(\bar{x}_{2j}), \quad |\phi_{ij}(\bar{x}_{ij})| \leq \gamma_{ij}(\bar{x}_{ij}), \quad 3 \leq i \leq n \quad (12.8)$$

where β_{ij} are known positive functions.

With the aid of the results in [7], we have the following results.

Lemma 12.1. *For the system in (12.1)-(12.3), there exists a function $f_j(\beta_{*j}, \epsilon_j) \in R^n$ such that the matrix $G_j(\beta_{*j}) = [g_{1j}(f_j), g_{2j}(f_j), \frac{\partial f_j}{\partial \beta_{1j}}, \dots, \frac{\partial f_j}{\partial \beta_{n-2,j}}]$ is nonsingular for any β_{*j} and $\epsilon_j > 0$, where $g_{1j} = [1, 0, x_{2j}, \dots, x_{n-1,j}]^\top$, $g_{2j} = [0, 1, 0, \dots, 0]^\top$, $\beta_{*j} = [\beta_{1j}, \dots, \beta_{n-2,j}]^\top$, $\beta \in R^{n-2}$ and the function f_j has the following properties:*

1. f_j is bounded for any β_{*j} ;
2. $\lim_{\epsilon_j \rightarrow 0} f_j(\beta_{*j}, \epsilon_j) = 0$.

The proof of Lemma 12.1 can be found in [7, 8]. The construction of f_j can be found also in [7, 8]. The function f_j is called the transverse function.

With the aid of Lemma 12.1 and the notations in [7], it can be found the function $f_j(\beta_j, \epsilon_j)$ such that G_j is nonsingular. Let

$$z_{*j} = x_{*j} f_j(\beta_{*j})^{-1}$$

then, we have the augmented system

$$\dot{z}_{*j} = dr_{f_j(\beta_{*j})^{-1}}(x_{*j}) dl_{z_{*j}}(f_j(\beta_{*j})) G_j(\beta_{*j}) [v_{1j}, v_{2j}, -\dot{\beta}_{1j}, \dots, -\dot{\beta}_{n-2,j}]^\top \quad (12.9)$$

We define the neighbors' difference as

$$e_{*j} = [e_{1j}, \dots, e_{nj}] = \sum_{i \in \mathcal{N}_j} a_{ji}(z_{*j} - z_{*i}) + b_j(z_{*j} - x^d). \quad (12.10)$$

where $b_j = 1$ if the desired trajectory is available to system j and $b_j = 0$ if x^d is not available to system j . If $\dot{\beta}_{*j}$ is considered as an additional input, we propose the following distributed control law.

Theorem 12.1. *For the m systems in (12.1)-(12.2), if the communication graph has a spanning tree, the control law*

$$v_{1j} = \eta_{1j} \quad (12.11)$$

$$v_{2j} = \eta_{2j} \quad (12.12)$$

$$\begin{bmatrix} \eta_{1j} \\ \eta_{2j} \\ -\dot{\beta}_{*j} \end{bmatrix} = -G_j^{-1} dl_{z_{*j}^{-1}}(x_{*j}) dr_{f_j(\beta_{*j})}(z_{*j}) \left[\sum_{i \in \mathcal{N}_j} a_{ji}(z_{*j} - z_{*i}) + b_j(z_{*j} - x^d) - \Delta_{*j} \right] \quad (12.13)$$

ensures that

$$\lim_{t \rightarrow \infty} \|x_{*j} - x^d\| \leq \delta_j(\epsilon_j) \quad (12.14)$$

where δ_j is a nonnegative continuous function of ϵ_j and δ_j converges to zero when ϵ_j converges to zero, and

$$\Delta_{*j} = \begin{bmatrix} \frac{(\gamma_{1j} + \delta_1)e_{1j}}{\sqrt{e_{1j}^2 + e^{-t}}} \\ \vdots \\ \frac{(\gamma_{nj} + \delta_n)e_{nj}}{\sqrt{e_{nj}^2 + e^{-t}}} \end{bmatrix} \quad (12.15)$$

Proof: By Lemma 12.1, G_j is nonsingular. So, the control law exists. Substitute the control law into the system and define $\tilde{z}_{ij} = z_{ij} - x_{id}$, we have

$$\dot{\tilde{z}}_{1j} = -e_{1j} - \frac{\gamma_{1j}e_{1j}}{\sqrt{e_{1j}^2 + e^{-t}}} + \phi_{1j} - \dot{x}_{1d} \quad (12.16)$$

$$\vdots \quad (12.17)$$

$$\dot{\tilde{z}}_{nj} = -e_{nj} - \frac{\gamma_{nj}e_{nj}}{\sqrt{e_{nj}^2 + e^{-t}}} + \phi_{nj} - \dot{x}_{nd} \quad (12.18)$$

Choose a function

$$V_i = \tilde{z}_{i*}^\top \mathcal{L}^e \tilde{z}_{i*}$$

where $\tilde{z}_{i*} = [\tilde{z}_{i1}, \dots, \tilde{z}_{in}]^\top$ and $\mathcal{L}$ is the Laplacian matrix, we have

$$\begin{aligned} \dot{V}_i &= -\tilde{z}_{i*}^\top \mathcal{L}^e \mathcal{L}^e \tilde{z}_{i*} - \sum_{j=1}^m \frac{\gamma_{ij}e_{ij}^2}{\sqrt{e_{ij}^2 + e^{-t}}} + \sum_{j=1}^m e_{ij}(\phi_{ij} - \dot{x}_{id}) \\ &\leq -\tilde{z}_{i*}^\top \mathcal{L}^e \mathcal{L}^e \tilde{z}_{i*} - \sum_{j=1}^m \gamma_{ij} \sqrt{e_{ij}^2 + e^{-t}} + \sum_{j=1}^m \frac{e^{-t}}{\sqrt{e_{ij}^2 + e^{-t}}} + \sum_{j=1}^m \gamma_{ij} |e_{ij}| \\ &\leq -\tilde{z}_{i*}^\top \mathcal{L}^e \mathcal{L}^e \tilde{z}_{i*} + m e^{-t/2} \end{aligned}$$

By integrating both sides of the above inequality, it can be shown that $\tilde{z}_{i*}$ is bounded and $\mathcal{L}^e \tilde{z}_{i*}$ converges to zero, which means that $\tilde{z}_{*j}$ converges to zero. Therefore, (12.14) holds. ■

Remark 12.1. If ϵ_j ($1 \leq j \leq m$) are chosen to be small constants, $\delta_{ji}(\epsilon_j, \epsilon_i)$ is a small constant, which means that $\|x_{ij} - x_{id}\|$ converges to a small neighborhood of the origin. We say (12.7) is achieved practically.

Remark 12.2. In Theorem 12.1, nothing is said about β_{*j} . So, β_{*j} may be bounded or unbounded. Thanks to the properties of the function f_j , the boundedness of β_{*j} plays no role in the consensus problem.

12.2 Distributed Controller Design for Dynamical Systems

We considered m dynamical systems. The j -th system is defined as

$$\dot{x}_{1j} = v_{1j} + \phi_{1j}(x_{1j}) \quad (12.19)$$

$$\dot{x}_{2j} = v_{2j} + \phi_{2j}(\bar{x}_{2j}) \quad (12.20)$$

$$\dot{x}_{ij} = v_{1j}x_{i-1,j} + \phi_{ij}(\bar{x}_{ij}), \quad 3 \leq i \leq n \quad (12.21)$$

$$\tilde{M}_j \dot{v}_{*j} + \tilde{C}_j v_{*j} + \tilde{G}_j + \tilde{D}_j = \tilde{B}_j \tau_j \quad (12.22)$$

where $v_{*j} = [v_{1j}, v_{2j}]^\top$ and $x_{*j} = [x_{1j}, \dots, x_{nj}]^\top$. The following properties are satisfied.

Property 12.1. $\tilde{M}_j$ is bounded and $\dot{\tilde{M}}_j - 2\tilde{C}_j$ is skew-symmetric.

Property 12.2. For any differentiable vector $\xi \in R^2$,

$$\tilde{M}_j \dot{\xi} + \tilde{C}_j \xi + \tilde{G}_j = \tilde{Y}(x_{*j}, \dot{x}_{*j}, \xi, \dot{\xi}) a_j$$

where $\tilde{Y}_j$ is a known function of x_{*j} , $\dot{x}_{*j}$, ξ , and $\dot{\xi}$, and a_j is the inertia parameter vector.

The problem considered in this section is defined as follows.

Tracking Control of Multiple Systems: For a group of m systems in (12.19)-(12.22), the problem is how to design a distributed control law τ_j for system j based on its own information and its neighbors' information such that

$$\lim_{t \rightarrow \infty} (x_{*j} - c) = 0, \quad 1 \leq j \leq m, \quad (12.23)$$

where $x_{*j} = [x_{1j}, \dots, x_{nj}]^\top$ and c is an unprescribed constant vector which depends on the initial condition of each system and the topology of digraph $\mathcal{G}$.

In the dynamics (12.22), we first assume that the inertia parameter vector a_j is a constant and is unknown.

In order to design controllers, we let

$$\tilde{v}_{1j} = v_{1j} - \eta_{1j} \quad (12.24)$$

$$\tilde{v}_{2j} = v_{2j} - \eta_{2j}, \quad 1 \leq j \leq m \quad (12.25)$$

where η_{1j} and η_{2j} are defined in (12.11) and (12.12), respectively. Let

$$\chi_j = [\chi_{i,l,j}] = r_{f_j(\beta_{*j})^{-1}}(x_{*j}) dl_{z_{*j}}(f_j(\beta_{*j})) G_j(\beta_{*j})$$

then we have

$$\dot{z}_{1j} = -e_{1j} - \frac{\gamma_{1j}e_{1j}}{\sqrt{e_{1j}^2 + e^{-t}}} + \phi_{1j} + \chi_{1,1,j}\tilde{v}_{1j} + \chi_{1,2,j}\tilde{v}_{2j} \quad (12.26)$$

$$\dot{z}_{2j} = -e_{2j} - \frac{\gamma_{2j}e_{2j}}{\sqrt{e_{2j}^2 + e^{-t}}} + \phi_{2j} + \chi_{2,1,j}\tilde{v}_{1j} + \chi_{2,2,j}\tilde{v}_{2j} \quad (12.27)$$

$$\dot{z}_{ij} = -e_{ij} - \frac{\gamma_{ij}e_{ij}}{\sqrt{e_{ij}^2 + e^{-t}}} + \phi_{ij} + \chi_{i,1,j}\tilde{v}_{1j} + \chi_{i,2,j}\tilde{v}_{2j}, \quad 3 \leq i \leq n \quad (12.28)$$

$$\tilde{M}_j\dot{\tilde{v}}_{*j} + \tilde{C}_j\tilde{v}_{*j} = \tilde{B}_j\tau_j - (\tilde{M}_j\dot{\eta}_{*j} + \tilde{C}_j\eta_{*j} + \tilde{G}_j + \tilde{D}_j) \quad (12.29)$$

where $\tilde{v}_{*j} = [\tilde{v}_{1j}, \tilde{v}_{2j}]^\top$.

For the dynamics of each system, we have

$$\tilde{M}_j\dot{\eta}_{*j} + \tilde{C}_j\eta_{*j} + \tilde{G}_j = \tilde{Y}_j(x_{*j}, \dot{x}_{*j}, \eta_{*j}, \dot{\eta}_{*j})a_j \quad (12.30)$$

where a_j is the inertia parameter. For the disturbance $\tilde{D}_j$, it is assumed that

$$\|\tilde{D}_j\| \leq \rho_j(x_{*j}) \quad (12.31)$$

where ρ_j is a known function of x_{*j} . If the inertia parameter a_j is a constant and is unknown, we have the following results.

Theorem 12.2. *For the m systems in (12.19)-(12.22), if the communication graph has a spanning tree, the control law*

$$\tau_j = \tilde{B}_j^{-1} \left[-K_j\tilde{v}_{*j} + \tilde{Y}_j\hat{a}_j - \rho_j\text{sign}(\tilde{v}_{*j}) - \Lambda_j \right] \quad (12.32)$$

$$\dot{\hat{a}}_j = -\Gamma_j\tilde{Y}_j^\top\tilde{v}_{*j} \quad (12.33)$$

ensures that $\hat{a}_j$ is bounded and (12.14) holds, where K_j and Γ_j are positive constant matrices, and

$$\Lambda_j = \begin{bmatrix} \sum_{i=1}^n e_{ij}\chi_{i,1,j} \\ \sum_{i=1}^n e_{ij}\chi_{i,2,j} \end{bmatrix} \quad (12.34)$$

Proof: Let

$$V = \sum_{i=1}^n \tilde{z}_{i*}^\top \mathcal{L}^e \tilde{z}_{i*} + \sum_{j=1}^m \frac{1}{2} \tilde{v}_{*j}^\top \tilde{M}_j \tilde{v}_{*j} + \sum_{j=1}^m \frac{1}{2} (\hat{a}_j - a_j)^\top \Gamma_j^{-1} (\hat{a}_j - a_j)$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned}\dot{V} &\leq -\sum_{i=1}^n \tilde{z}_{i*}^\top \mathcal{L}^e \mathcal{L}^e \tilde{z}_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} - \sum_{j=1}^m \rho_j \tilde{v}_{*j}^\top \text{sign}(\tilde{v}_{*j}) - \sum_{j=1}^m \tilde{v}_{*j}^\top \tilde{D}_j + m e^{-t/2} \\ &\leq -\sum_{i=1}^n \tilde{z}_{i*}^\top \mathcal{L}^e \mathcal{L}^e \tilde{z}_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} + m e^{-t/2}\end{aligned}$$

By integrating both sides of the above inequality, it can be shown that V is bounded, which means that $\tilde{z}_{i*}$, $\tilde{v}_{*j}$ and $\hat{a}_j$ are bounded. Furthermore, it can be shown that by Barbalat's lemma that e_{i*} and $\tilde{v}_{*j}$ converge to zero. Therefore, (12.14) holds. $\blacksquare$

In Theorem 12.2, the unknown inertia parameter is estimated by an adaptive control law. If an estimate of a_j is $\bar{a}_j$ and

$$\|a_j - \bar{a}_j\| \leq \gamma_j$$

for $1 \leq j \leq m$ and γ_j is a known constant, we propose the following robust control laws.

Theorem 12.3. *For the m systems in (12.19)-(12.22), if the communication graph has a spanning tree, the control law*

$$\tau_j = \tilde{B}_j^{-1} \left[-K_j \tilde{v}_{*j} + \tilde{Y}_j \bar{a}_j - \gamma_j \tilde{Y}_j \text{sign}(\tilde{Y}_j^\top \tilde{v}_{*j}) - \rho_j \text{sign}(\tilde{v}_{*j}) - \Lambda_j \right] \quad (12.35)$$

ensures that (12.14) holds, where K_j is a positive constant matrix and Λ_j is defined in (12.34).

Proof: Let

$$V = \sum_{i=1}^n \tilde{z}_{i*}^\top \mathcal{L}^e \tilde{z}_{i*} + \sum_{j=1}^m \frac{1}{2} \tilde{v}_{*j}^\top \tilde{M}_j \tilde{v}_{*j}$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned}\dot{V}_j &\leq -\sum_{i=1}^n \tilde{z}_{i*}^\top \mathcal{L}^e \mathcal{L}^e \tilde{z}_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} - \sum_{j=1}^m \rho_j \tilde{v}_{*j}^\top \text{sign}(\tilde{v}_{*j}) - \sum_{j=1}^m \tilde{v}_{*j}^\top \tilde{D}_j \\ &\quad + \sum_{j=1}^m \tilde{v}_{*j}^\top \tilde{Y}_j (\bar{a}_j - a_j) - \sum_{j=1}^m \gamma_j \tilde{v}_{*j}^\top \tilde{Y}_j \text{sign}(\tilde{Y}_j^\top \tilde{v}_{*j}) + m e^{-t/2} \\ &\leq -\sum_{i=1}^n \tilde{z}_{i*}^\top \mathcal{L}^e \mathcal{L}^e \tilde{z}_{i*} - \sum_{j=1}^m \tilde{v}_{*j}^\top K_j \tilde{v}_{*j} + m e^{-t/2}\end{aligned}$$

Therefore, V is bounded, which means that $\tilde{z}_{i*}$ and $\tilde{v}_{*j}$ are bounded. By Barbalat's lemma, it can be shown that e_{i*} and $\tilde{v}_{*j}$ converge to zero. $\blacksquare$

In Theorem 12.3, the unknown inertia parameter a_j is not required to be a constant. In the control laws, γ_j is required to be known. It is possible to estimate it.

12.3 Simulation

We considered three nonholonomic wheeled mobile robots considered in Section 3.5.

The constraint on the front wheels can be written as

$$\dot{x}_j \sin \theta_j - \dot{y}_j \cos \theta_j = P_j(x_j, y_j) \quad (12.36)$$

where (x_j, y_j) is the position of robot j , θ_j is the orientation of robot j , and P_j denotes slight slipping along the axis of the wheels. The dynamics of robot j are described by the following differential equations

$$\begin{cases} m_j \ddot{x}_j &= \lambda_j \cos \theta_j + \frac{1}{R_j}(\tau_{1j} + \tau_{2j}) \cos \theta_j \\ m_j \ddot{y}_j &= -\lambda_j \sin \theta_j + \frac{1}{R_j}(\tau_{1j} + \tau_{2j}) \sin \theta_j \\ I_j \ddot{\theta}_j &= \frac{L_j}{R_j}(\tau_{1j} - \tau_{2j}) \end{cases} \quad (12.37)$$

where m_j is the mass of robot j , and I_j is its inertia moment around the vertical axis at point Q. R_j is the radius of the wheels and $2L_j$ the length of the axis of the front wheels, and τ_{1j} and τ_{2j} are the torques provided by the motors.

Let $q_{*j} = [x_j, y_j, \theta_j]^\top$,

$$M_j(q_{*j}) = \begin{bmatrix} m_j & 0 & 0 \\ 0 & m_j & 0 \\ 0 & 0 & I_j \end{bmatrix}, C_j(q_{*j}, \dot{q}_{*j}) = 0, G_j(q_{*j}) = 0$$

$$B_j(q_{*j}) = \frac{1}{R_j} \begin{bmatrix} \cos \theta_j & \cos \theta_j \\ \sin \theta_j & \sin \theta_j \\ L_j & -L_j \end{bmatrix}, J_j = [\sin \theta_j, -\cos \theta_j, 0]$$

The system (12.36)-(12.37) is in the form of (12.1)-(12.2).

Let

$$g_{*j} = \begin{bmatrix} \cos \theta_j & 0 \\ \sin \theta_j & 0 \\ 0 & 1 \end{bmatrix}$$

then Equation (12.36) and (12.37) are converted into

$$\begin{cases} \dot{x}_j &= u_{1j} \cos \theta_j + P_j \sin \theta_j \\ \dot{y}_j &= u_{1j} \sin \theta_j - P_j \cos \theta_j \\ \dot{\theta}_j &= u_{2j} \\ m_j \dot{u}_{1j} &= \frac{1}{R_j}(\tau_{1j} + \tau_{2j}) \\ I_j \dot{u}_{2j} &= \frac{L_j}{R_j}(\tau_{1j} - \tau_{2j}) \end{cases} \quad (12.38)$$

It is given a desired trajectory $q^d = [q_{1d}, q_{2d}, q_{3d}]^\top$ which is generated by

$$\dot{q}_{1d} = v_d \cos q_{3d}, \quad \dot{q}_{2d} = v_d \sin q_{3d}, \quad \dot{q}_{3d} = \omega_d$$

where v_d and ω_d are time-varying functions.

With the transformation

$$\begin{cases} x_{1j} &= -\theta_j \\ x_{2j} &= x_j \cos \theta_j + y_j \sin \theta_j \\ x_{3j} &= -x_j \sin \theta_j + y_j \cos \theta_j \\ v_{1j} &= -\dot{\theta}_j \\ v_{2j} &= \dot{x}_j - x_{3j} \dot{\theta}_j \end{cases}$$

Equation (12.38) can be converted into the following standard form

$$\begin{cases} \dot{x}_{1j} = v_{1j} \\ \dot{x}_{2j} = v_{2j} \\ \dot{x}_{3j} = x_{2j} v_{1j} - P_j \\ \tilde{M}_j \dot{v}_{*j} + \tilde{C}_j v_{*j} = \tilde{B}_j \tau_{*j} \end{cases} \quad (12.39)$$

where

$$\tilde{M}_j = \begin{bmatrix} I_j + m_j x_{3j}^2 & m_j x_{3j} \\ m_j x_{3j} & m_j \end{bmatrix}, \tilde{C}_j = \begin{bmatrix} m_j x_{3j} \dot{x}_{3j} & 0 \\ m_j \dot{x}_{3j} & 0 \end{bmatrix}, \tilde{B}_j = \frac{1}{R_j} \begin{bmatrix} x_{3j} + L_j & x_{3j} - L_j \\ 1 & 1 \end{bmatrix}$$

and

$$\tilde{M}_j(q_{*j})\dot{\xi} + \tilde{C}_j(q_{*j}, \dot{q}_{*j})\xi = \tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})a_j$$

where the inertia parameter vector $a_j = [m_j, I_j]^T$,

$$\tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi}) = \begin{bmatrix} x_{3j}^2 \dot{\xi}_1 + x_{3j} \dot{\xi}_2 + x_{3j} \dot{x}_{3j} \xi_1 & \dot{\xi}_1 \\ x_{3j} \dot{\xi}_1 + \xi_2 + \dot{x}_{3j} \xi_1 & 0 \end{bmatrix}$$

By the transformation

$$\begin{cases} x_{1d} &= -q_{3d} \\ x_{2d} &= q_{1d} \cos q_{3d} + q_{2d} \sin q_{3d} \\ x_{3d} &= -q_{1d} \sin q_{3d} + q_{2d} \cos q_{3d} \\ v_{1d} &= -\dot{q}_{3d} \\ v_{2d} &= \dot{q}_{1d} - x_{3d} \dot{q}_{3d} \end{cases}$$

we have

$$\dot{x}_{1d} = v_{1d}, \quad \dot{x}_{2d} = v_{2d}, \quad \dot{x}_{3d} = v_{1d} x_{2d}.$$

The tracking control problem of the kinematics in (12.39) can be solved with the aid of the results proposed in Theorem 12.1. By the results in [7], we choose

$$f_{*j} = \begin{bmatrix} \epsilon_j \sin \beta_j \\ \epsilon_j \cos \beta_j \\ \frac{\epsilon_j^2}{4} \sin 2\beta_j \end{bmatrix}, \quad \epsilon_j > 0 \quad (12.40)$$

Then

$$\frac{df_{*j}}{d\beta_{1j}} = \begin{bmatrix} \epsilon_j \cos \beta_j \\ -\epsilon_j \sin \beta_j \\ \frac{\epsilon_j^2}{2} \cos 2\beta_j \end{bmatrix} \dot{\beta}_{1j}. \quad (12.41)$$

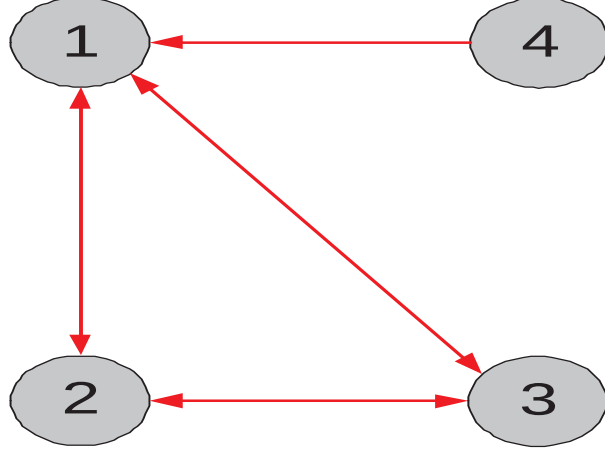


Figure 12.1: Communication graph.

It can be verified that f_j satisfies the properties in Lemma 12.1.

Let

$$z_{*j} = x_{*j} f_{*j}^{-1} = \begin{bmatrix} x_{1j} - f_{1j} \\ x_{2j} - f_{2j} \\ x_{3j} - f_{3j} - f_{1j}(x_{2j} - f_{2j}) \end{bmatrix}$$

the controller is proposed as

$$v_{1j} = \eta_{1j} \tag{12.42}$$

$$v_{2j} = \eta_{2j} \tag{12.43}$$

$$\begin{bmatrix} \eta_{1j} \\ \eta_{2j} \\ -\beta_j \end{bmatrix} = \begin{bmatrix} 1 & 0 & \epsilon_j \cos \beta_j \\ 0 & 1 & -\epsilon_j \sin \beta_j \\ \epsilon_j \cos \beta_j & 0 & \frac{\epsilon_j^2}{2} \cos 2\beta_j \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ z_{2j} & -\epsilon_j \sin \beta_j & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} \sum_{i \in \mathcal{N}_j} a_{ji}(z_{*j} - z_{*i}) + \Delta_{*j} \end{bmatrix} \tag{12.44}$$

where $a_{ji} > 0$ and Δ_j is defined in (12.15) with $n = 3$.

In the simulation, we choose $P_j(x_j, y_j) = 0.2 \sin t$. The desired trajectory x^d is assumed to be $x^d = [x_{1d}, x_{2d}, x_{3d}]^\top =$. Figs. 12.2-12.4 show the time response of x_{1*} , x_{2*} , and x_{3*} , respectively. It is shown that the state of three systems reach consensus.

The tracking control problem of the dynamics in (12.39) can be solved with the aid of the results in Theorem 12.2. The controller is proposed as in (12.32)-(12.33) if the inertia parameter vector a_j is a constant and is unknown. Figs. 12.5-12.7 show the time response of x_{1*} , x_{2*} , and x_{3*} , respectively. It is shown that the state of three systems reach consensus.

If the inertia parameter vector a_j is a constant and is unknown, we can also solve the consensus problem by the robust control algorithms in Theorem 12.3. Figs. 12.8-12.10 show the time response of x_{1*} , x_{2*} , and x_{3*} , respectively. It is shown that the state of three systems reach consensus.

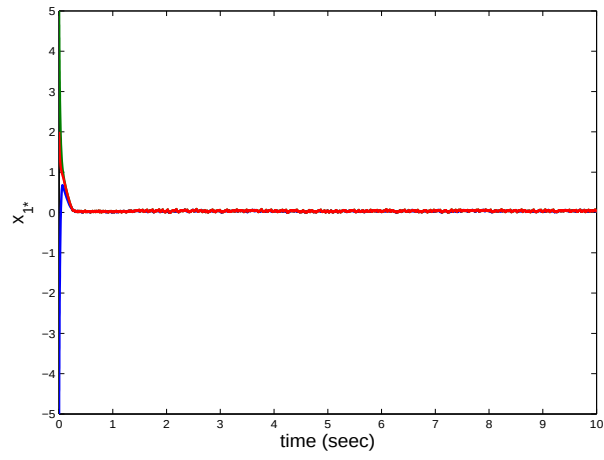


Figure 12.2: Response of x_{1*} .

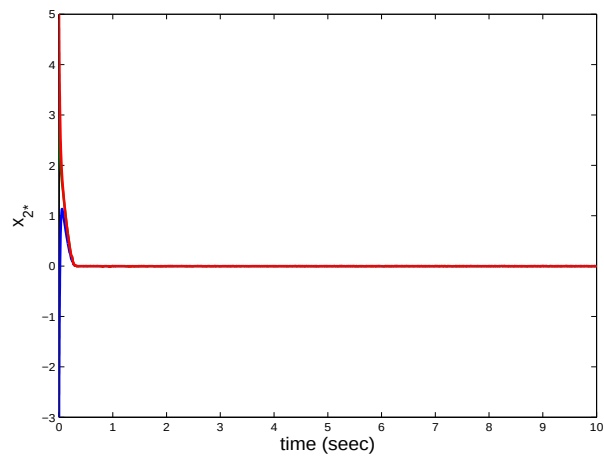


Figure 12.3: Response of x_{2*} .

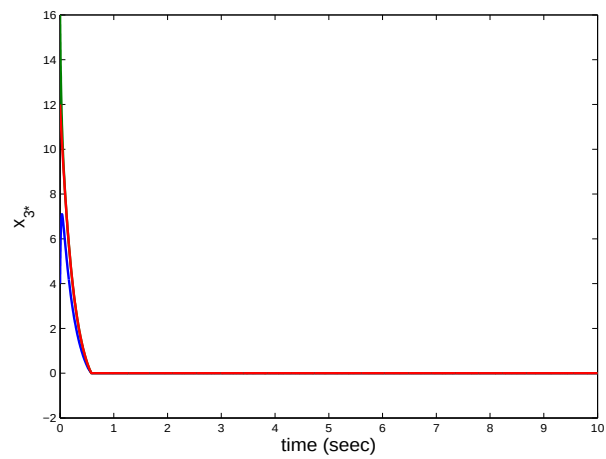


Figure 12.4: Response of x_{2*} .

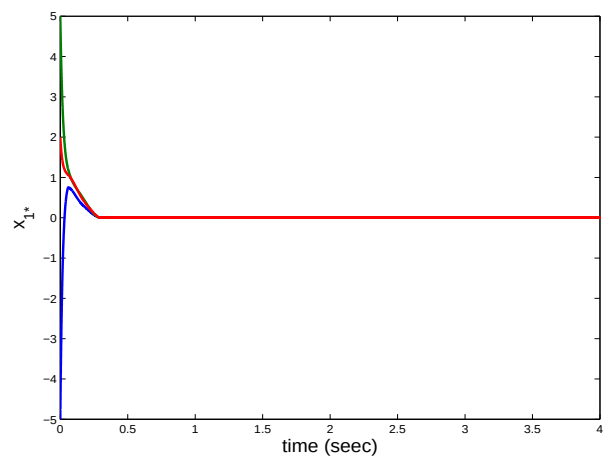


Figure 12.5: Response of x_{1*} .

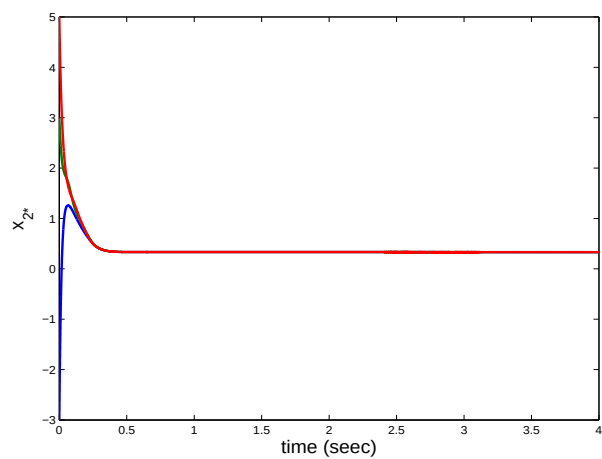


Figure 12.6: Response of x_{2*} .

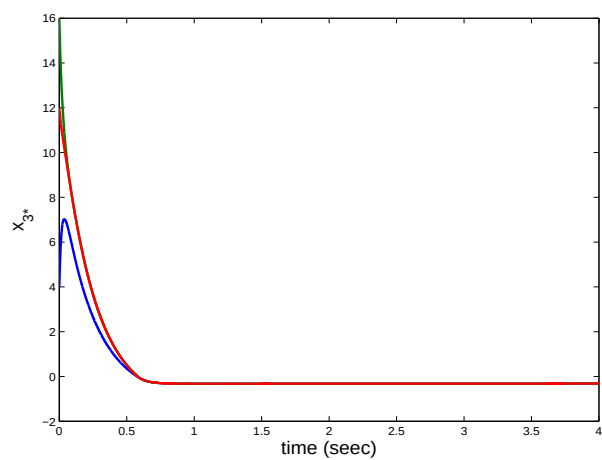


Figure 12.7: Response of x_{2*} .

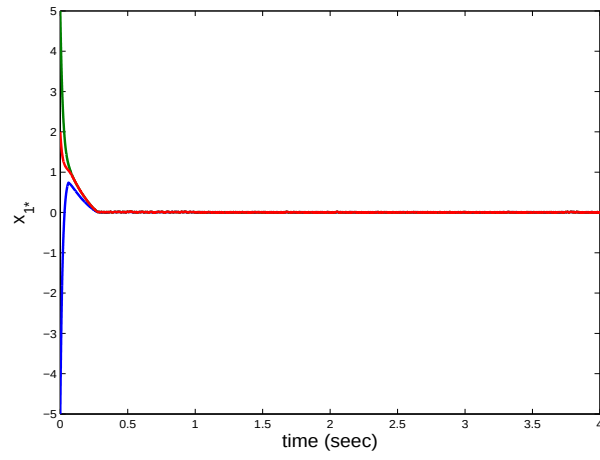


Figure 12.8: Response of x_{1*} .

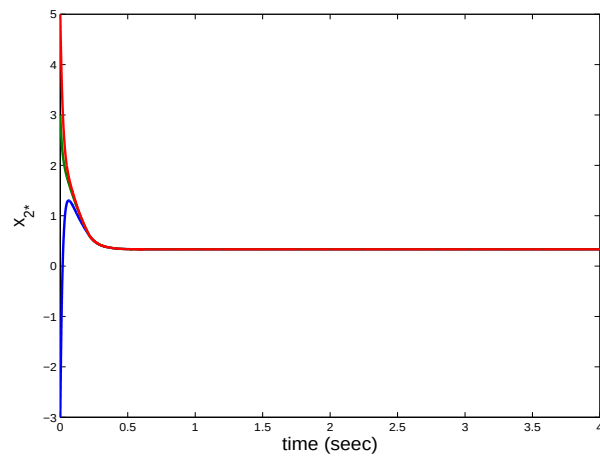


Figure 12.9: Response of x_{2*} .

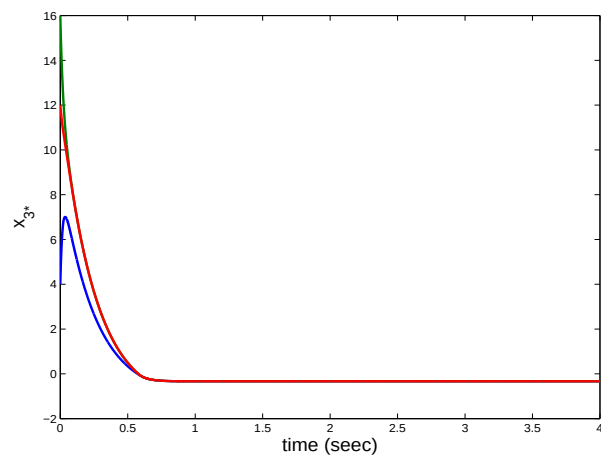


Figure 12.10: Response of x_{2*} .

Chapter 13

Conclusions

In this report, we summarized our achievements in research and education. The PI, the graduate students, and our institute have been benefitted from the support of this project. We thank the financial support of the ARO very much and hope in the future we can receive financial report again.

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