

# **Predictability and Ensemble-Forecast Skill Enhancement Based on the Probability Density Function Estimation**

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## **LONG-TERM GOAL**

My long term goal is to make substantial contributions to enhancement of forecast skills in numerical prediction for the oceans and atmosphere, by deepening our knowledge concerning nature of predictability. I place my emphasis on coherent structures of the geophysical flows and transition mechanism associated with them.

## **OBJECTIVES**

I wish to address development of theoretical frameworks for enhancement of predictability by improving currently existing methodologies and constructing new theories, so that they can be combined systematically. I also wish to throw a bridge to dynamical systems theories that have not been used for predictability study in the past. To help enhance predictability of severe events, I wish to investigate the impact of the noises whose probability distribution have heavy-tails. Knowledge obtained by these predictability studies will naturally lead to a design of comprehensive ensemble-forecast systems and data-adaptive observing systems.

## **APPROACH**

I start from assessing and extending individual elements that are involved in the theoretical framework and developing new theories to fill the gaps between them as necessary. My approach involves: identification and detection of the predictability elements, Eulerian and Lagrangian descriptions of the probability density function evolution, nonlinear determinism as well as additive and multiplicative stochasticity, and understanding of their impacts on the forecast skills. There are several new paths we can take to enhance predictability with help of dynamical systems theories. One is based on the idea of Lagrangian transport theory to choose ensembles judiciously near the dynamical transition points. Geometry of such ensembles is governed by the invariant manifolds of distinguished hyperbolic trajectories (DHTs) in the phase space. Difficulty arises from the fact that DHTs are not identifiable in the instantaneous flow. Therefore I pursue analytical development and numerical implementation of DHT tracking methodology. While Lagrangian transport theory can offer an analytical tool to estimate probability density function for transition dynamics based on ensembles, it is computationally exhaustive to obtain the precise geometry. I therefore propose to take an alternate view and develop the Eulerian transport theory, which effectively provides the leading order estimation to the geometry of such ensembles. To improve forecast skills for severe events, I investigate role played by stochastic

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noises. Severe events can be viewed as rare extreme bursts and therefore may be related to stochastic noises whose probability distribution has heavy tail. I construct a new innovative methodology for data assimilation systems subject to dynamical and observational noises with heavy-tail distributions.

## **WORK COMPLETED**

Theory for DHT tracking was refined for a class of two-dimensional systems. The numerical codes have been implemented into Lagrangian transport software being developed at California Institute of Technology.

Eulerian transport theory was developed for the first time so as to efficiently estimate the transport across kinematically defined boundary in two-dimensional flows. Its relation to Lagrangian lobe dynamics was proved analytically. An application to a geophysical flow was made successfully.

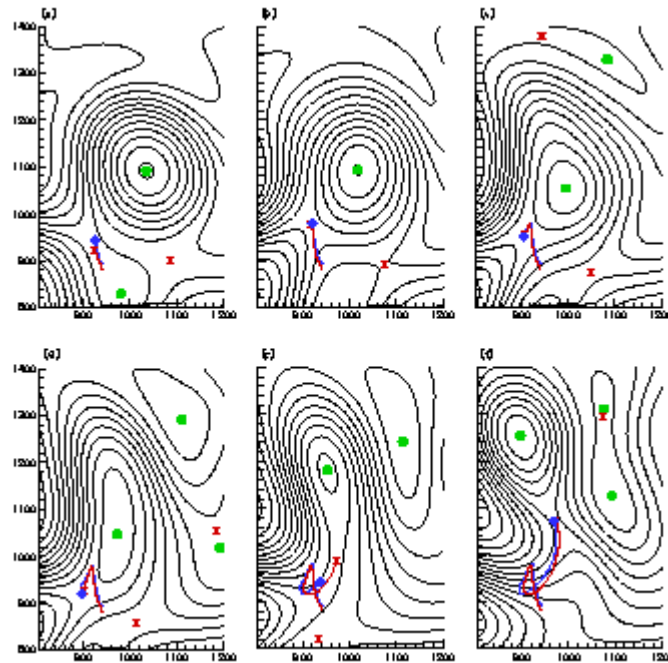
Mathematical framework of data assimilation system subject to additive noises with heavy-tail probability distribution was developed in the spirit of the Kalman filter when the underlying dynamics is linear.

## **RESULTS**

Instantaneous hyperbolic stagnation points and DHTs are in one-to-one correspondence if certain conditions are satisfied. Using instantaneous hyperbolic stagnation points that are observable, we developed a methodology, called Fourier DHT, to track DHTs that are not observable (Figure 1).

Eulerian transport theory efficiently estimate transport across kinematically defined boundary. Its framework is more flexible than the well-studied Lagrangian transport theory. In the case of heteroclinic connection, geometry and transport mechanism are shown to be closely related to those of Lagrangian transport theory with a concrete comparison to the results based on Lagrangian transport theory (Coulliette and Wiggins, 2000).

By introducing the concept of a “tail covariance” that generalizes the standard notion of the covariance, we constructed the Kalman-Levy filter. We demonstrated that, when the standard Kalman filter is applied to a system with heavier-tail noise distribution than the Gaussian, the predication can be significantly deteriorated (Figure 2).



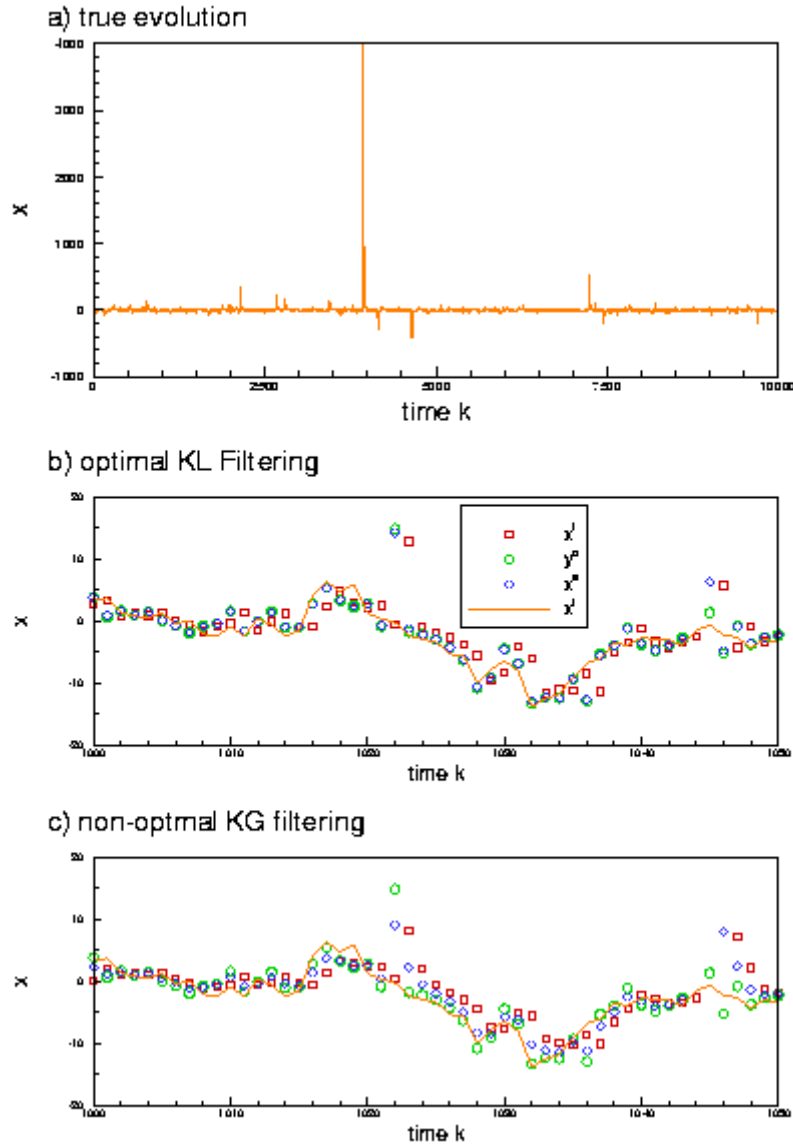
**Figure 1.** Time sequence of the Fourier DHT (blue diamond with solid line for trajectory) and corresponding instantaneous SP (red cross with trajectory) in application of the DHT theory to a numerical simulation of wind-driven mid-latitude ocean circulation. The DHT controls the boundaries of the eddy, which is pinching off from the main jet. The time interval between consequent panels is two weeks.

## IMPACT/APPLICATION

While Lagrangian transport theory offers exact and precise information concerning geometry of ensembles involved in transport, its computation is significantly intensive and fine-scale may be fragile subject to the stochastic noises. Eulerian transport theory provides with a robust alternative mathematical framework to compute transport. In data assimilation for severe events, the Kalman-Levy filter can be used to design observing systems.

## TRANSITIONS

DHT tracking methodology for one-dimensional boundary implemented in the Caltech's Lagrangian transport toolkit is extended to compute a class of DHTs in two-dimensional flows.



**Figure 2. Numerical experiments for KL filtering: a) true evolution of the heavy tail system with convergent dynamics; b) optimal KL filtering; c) non-optimal filtering using conventional Kalman filter. Solid line is true evolution, circle is observation ( $y_o$ ) based on true evolution, square is forecast ( $x_f$ ) and diamond is analysis ( $x_a$ ).**

## RELATED PROJECTS

1– NASA data assimilation on ocean-atmosphere coupled predictability in collaboration with Michael Ghil (UCLA).

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