

# **Generation and Propagation of Internal Solitary Waves on the Continental Shelf and Slope**

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## **LONG-TERM GOALS**

This project is a fundamental study of the basic dynamical mechanisms involved, and the consequent theoretical modelling approaches needed, in the generation and propagation of internal solitary waves across the continental shelf and slope.

## **OBJECTIVES**

There are two principal objectives. The first is to develop and refine amplitude evolution equations of the Korteweg-de Vries type to the point where they can be used as validated models for the propagation of internal solitary waves. The second is to undertake a major re-examination of the generation process, using a combination of theoretical and numerical analyses, and emphasising the distinction between two-dimensional and three-dimensional mechanisms.

## **APPROACH**

Our approach is to develop an understanding of the fundamental dynamical processes involved through a combination of theoretical analyses and numerical simulations. Our research group comprises post-doctoral fellows, research students and international collaborators who make long-term visits. We maintain contact with those making field and laboratory observations, with the aim of establishing an ongoing interactive collaboration on data interpretation, model development and validation.

## **WORK COMPLETED**

For the first objective, the development and refinement amplitude evolution equations of the Korteweg-de Vries type, the main focus to this point has been on understanding the role of cubic nonlinearity vis-à-vis that of quadratic nonlinearity in several contexts. Thus, we have obtained a correct asymptotic derivation of the coefficients for the quadratic and cubic nonlinear terms in the extended Korteweg-de Vries (eKdV) equation for background flows which allow for arbitrary density and current stratification, and importantly, allow for a free surface. Next we have examined the effect of various frictional processes on the family of solitary wave solutions of this eKdV equation, using primarily asymptotic techniques. Further, we have incorporated topographic forcing into this eKdV equation, and examined both the upstream and downstream wavetrains generated by transcritical flow

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interaction with an isolated topographic feature, and the interaction of an internal solitary wave with an isolated topographic feature.

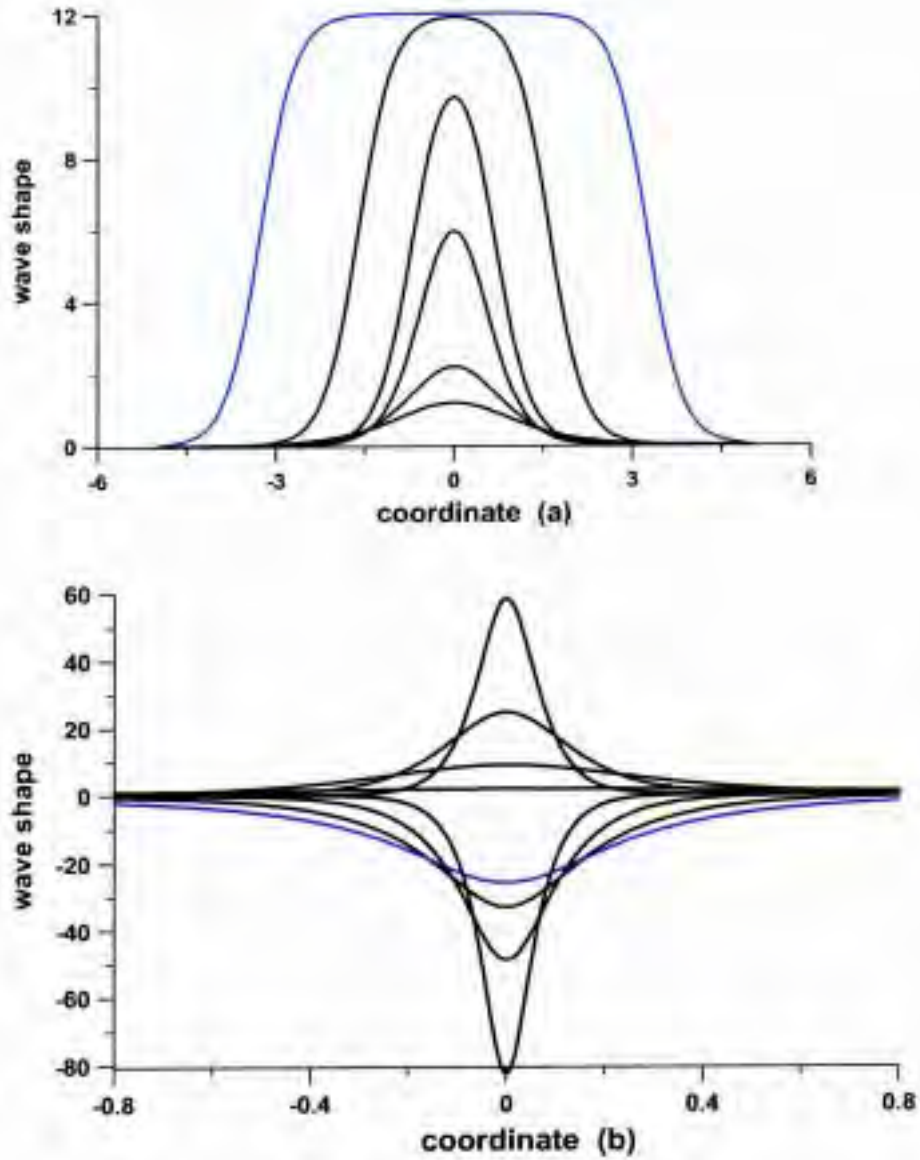
For the second objective, that is the utilisation of a combination of theoretical analyses and numerical simulations to re-examine the principal generation processes of internal solitary waves, we have focussed on the development of a suite of two- and –three-dimensional numerical codes for this purpose, and at the same time have developed a theoretical model of the Boussinesq-type, which can incorporate arbitrary stratification and topography. Our two-dimensional codes for the Euler equations have been extended to allow for the inertial effects of density stratification (i.e. we have not used the Boussinesq approximation), and we have commenced using a hydrostatic code (again without the Boussinesq approximation) to study the effects of alongshore variability in the generation of the internal tide by the interaction of the barotropic tide with the continental shelf and slope.

## RESULTS

First, we have examined various aspects of the eKdV model for the propagation of internal solitary waves. For the present discussion this is given by,

$$A_t + \Delta A_x + \mu A A_x + \nu A^2 A_x + \lambda A_{xxx} + \Gamma(A) + f_x(x) = 0, \quad (1)$$

where  $A(x, t)$  is the amplitude of a representative isopycnal displacement,  $t$  is a time-like variable describing the evolution of a solitary wave, and  $x$  is a phase variable describing the shape of the solitary wave. The coefficients  $\Delta, \mu, \nu, \lambda$  are determined by the linear long-wave modal function, which in turn depends on the background density and current stratification. The expression  $\Gamma(A)$  is a dissipative term, which can take several forms, and  $f(x)$  is a term representing the local effects of bottom topography. Explicit expressions for the key coefficients  $\mu, \nu, \lambda$ , and also those of the higher-order linear dispersive term, and the nonlinear dispersive terms not shown in (1), have now been obtained for arbitrary density and current stratification, and for a free surface. One of the most important features to emerge from our work so far is the role of the cubic nonlinear term with coefficient  $\nu$  vis-à-vis that of the quadratic nonlinear term with coefficient  $\mu$ . This is immediately evident in the richer structure of the solitary wave solutions supported by (1) when the coefficients are all constants, and there are no dissipative or forcing terms, when the compared with the corresponding family of solitary wave solutions of the KdV equation (i.e. (1) with only quadratic nonlinearity). These are shown in Figure 1, where, without any loss of generality, it has been assumed that  $\mu, \lambda$  are both positive. Thus, when the coefficient  $\nu$  of the cubic nonlinear term is negative (Figure 1a) we see that the solitary waves resemble those of the KdV equation for small amplitudes, but for large amplitudes, they are much thicker and reach a limiting amplitude of  $-\mu/\nu$ , known as the “thick” wave. On the other hand, when the coefficient  $\nu$  of the cubic nonlinear term is positive (Figure 1b), there are two families of solitary waves. That with positive polarity resembles the KdV family, but that with negative polarity is quite different and in particular has no small-amplitude limit; instead, there is a lower bound for the amplitude of  $-2\mu/\nu$ , and solutions of (1) with lower energy are represented by breathers, that is, solutions which resemble pulsating solitary waves. Given that observed internal solitary waves are often quite large, these two key differences from the familiar KdV theory, are likely to be very significant. Further, it is pertinent to point out that while in the literature there has been a tendency to assume that  $\nu$  has the opposite sign to  $\mu$  (i.e Figure 1a holds), the newly-available complete expression for  $\nu$  has revealed that it can easily have either sign for realistic oceanic conditions.



**Figure 1a: (top) Solitary wave shape for negative cubic nonlinearity**  
**Figure 1b: (bottom) Solitary wave shape for positive cubic nonlinearity**

For instance, consider the effect of dissipation on a solitary wave, for various forms of the dissipative term  $\Gamma(A)$  representing Newtonian damping, laminar or turbulent boundary layer damping, or damping due to interior turbulence. This work is in progress, but some new features are already evident. Thus, for a negative coefficient  $\nu$  (with  $\mu, \lambda$  both positive) we find that the decay of a “thick” wave can lead to the formation of secondary wave packets, while for a positive coefficient  $\nu$ , the decay of a wave of negative polarity leads to the formation of breather states, which resemble a wave packet. Next, analogous new features have emerged from our on-going study of the interaction between internal solitary waves and topography, modelled in this case by (1) with the topography

represented by  $f(x)$ , and with no dissipative term. Thus, in some circumstances for a negative value of  $v$ , the interaction can cause a small-amplitude KdV-like solitary wave to transform into a “thick” wave, while for a positive value of  $v$  a solitary wave of negative polarity can be transformed into a breather.

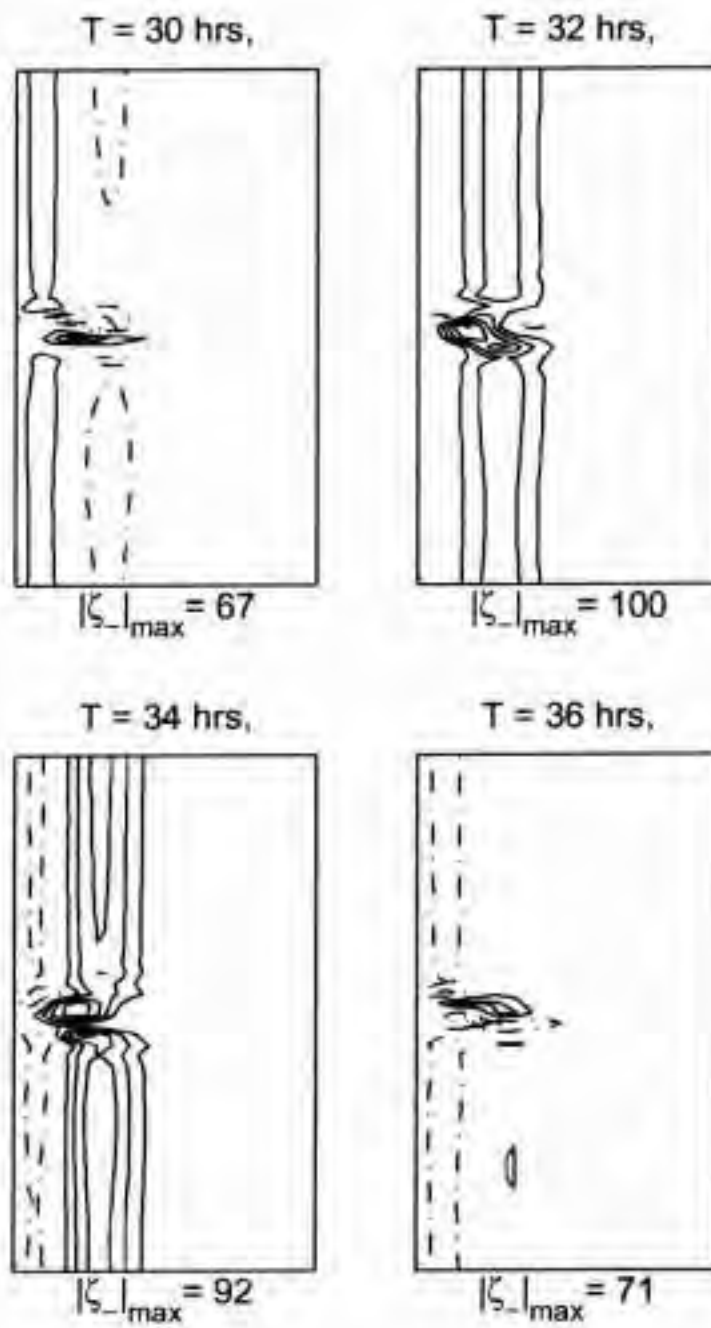
Next, we are developing a suite of numerical codes to solve the Euler equations, with the aim of examining the process by which the barotropic tide generates an internal tide, which in turn may then form a train of solitary waves. The most interesting results obtained so far are from a three-dimensional hydrostatic model which uses a two-layer formulation for the density stratification. Unlike most previous simulations, this code has a free surface and hence the barotropic and baroclinic tides are allowed to interact. A typical simulation forces a barotropic tide at the deep ocean boundary, and then allows an internal, or baroclinic, tide to be generated over the continental slope, modelled by a “hyperbolic- tangent” profile. We conducted simulations both with and without a submarine canyon inset into the continental shelf. A typical result containing a submarine canyon is shown in Figure 2, where one can see a substantial enhancement of the internal tide over the canyon. The canyon dimensions, both along-slope and across-slope, are chosen to scale with the internal Rossby radius (about 50 km in Figure 2), and the enhancement is reduced if either dimension is scaled differently. Of course, being a hydrostatic model, these simulations cannot allow for the formation of any internal solitary waves, but are nonetheless encouraging in demonstrating that submarine canyons of the appropriate dimensions may be significant for the generation of internal solitary waves.

## **IMPACT/APPLICATIONS**

We anticipate that the results obtained will inform the scientific community about the structure of internal solitary waves, their behaviour under such environmental impacts as friction and topography, and the processes which favour their generation.

## **PUBLICATIONS (2000-)**

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*Figure 2. Contours of the internal tide (isopycnal displacement) over a submarine canyon*