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14. ABSTRACT
Research partially supported by this grant culminated in the submission of twenty eight new research papers twenty three of which have now been accepted for publication. Topics covered in this research include theory of large deviations, stochastic differential games, stochastic control, stochastic networks, random graphs, mean field models, and diffusion approximations.
Four Ph.D. students (Xin Liu, Dominik Reinhold, Jiang Chen and Xuan Wang) completed their dissertation under the direction of the PI. Three other Ph.D. students (Abhishek Pal Majumder, Ruoyu Wu and Eric Friedlander)

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Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received	Paper
07/31/2014 28.00	Arka Ghosh, Xin Liu, Amarjit Budhiraja. Scheduling control for Markov-modulated single-server multiclass queueing systems in heavy traffic, Queueing Systems, (03 2014): 0. doi: 10.1007/s11134-014-9396-8

TOTAL: 1

Number of Papers published in peer-reviewed journals:

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Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

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TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

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Received		Paper
07/05/2013	4.00	Chihoon Lee, Arka Ghosh, Amarjit Budhiraja. ERGODIC RATE CONTROL PROBLEM FOR SINGLE CLASS QUEUEING NETWORKS , SIAM Journal of Control and Optimization ( )
07/05/2013	19.00	Shankar Bhamidi, Amarjit Budhiraja, Xuan Wang. Bounded-size rules: The barely subcritical regime., math arXiv:1212.5480 (12 2012)
07/05/2013	16.00	Sergio Almada, Amarjit Budhiraja. Infinite Dimensional Forward-Backward Stochastic Differential Equations andthe KPZ Equation [, Math arXiv:1207.5568 (07 2012)
07/05/2013	20.00	Shankar Bhamidi, Amarjit Budhiraja, Xuan Wang. The augmented multiplicative coalescent and critical dynamic random graph models., math arXiv:1212.548093v1 (12 2012)
07/05/2013	18.00	Arka Ghosh, Xin Liu, Amarjit Budhiraja. Dynamic Scheduling for Markov Modulated Single-server Multiclass Queueing Systems in Heavy Traffic, math arXiv:1211.6831 (11 2012)
07/05/2013	17.00	Amarjit Budhiraja, Vladas Pipiras, Xiaoming Song. Admission Control for Multidimensional Workload with Heavy Tails and Fractional Ornstein-Uhlenbeck Process., math arXiv:1210.7781 (10 2012)
07/05/2013	13.00	Amarjit Budhiraja, Dominik Reinhold. Near Critical Catalyst Reactant Branching Processes with Controlled Immigration , Annals of Applied Probability (03 2012)
07/31/2014	22.00	Amarjit Budhiraja, Rami Atar. On the multi-dimensional skew Brownian motion , ArXiv e-prints, 1402.5703 (02 2014)
07/31/2014	21.00	Amarjit Budhiraja, Elisabeti Kira, Subhamay Saha. Central Limit Results for Jump-Diffusions with Mean Field Interaction and a Common Factor, ArXiv e-prints, 1405.7682 (05 2014)
07/31/2014	23.00	Amarjit Budhiraja, Paul Dupuis, Arnab Ganguly. Moderate Deviation Principles for Stochastic Differential Equations with Jumps , ArXiv e-prints, 1401.7316 (01 2014)
07/31/2014	24.00	Amarjit Budhiraja, Abhishek Pal Majumdar. Long Time Results for a Weakly Interacting Particle System in Discrete Time , ArXiv e-prints, 1401.3423 (01 2014)
07/31/2014	31.00	Pierre Nyquist, Amarjit Budhiraja. Large deviations for multidimensional state-dependent shot noise processes , ArXiv e-prints, 1407.6651 (07 2014)
07/31/2014	26.00	Amarjit Budhiraja, Zhen-Qing Chen. On uniform positivity of transition densities of small noise constrained diffusions, Electronic Communications in Probability (01 2014)

07/31/2014	30.00	Michael Lamm, Shu Lu, Amarjit Budhiraja. Individual confidence intervals for true solutions to stochastic variational inequalities, Math Programming (06 2014)
08/05/2011	1.00	Amarjit Budhiraja, , Paul Dupuis,, Markus Fischer. Large Deviation Properties of Weakly Interacting Processes via Weak Convergence Methods, Annals of Probability (10 2010)
08/10/2011	2.00	Rami Atar, Amarjit Budhiraja. On Near Optimal Trajectories of Games Associated with the infinity-Laplacian, Probability Theory and Related Fields (04 2011)
08/10/2011	3.00	Paul Dupuis, Vaselios Maroulas, Amarjit Budhiraja. Variational Representations for Continuous Time Processes, Annales de l'Institut Henri Poincare (08 2011)
08/10/2011	5.00	Amarjit Budhiraja, Xin Liu. Multiscale diffusion approximations for stochastic networks in heavy traffic, Stochastic Processes and their Applications (10 2010)
08/10/2011	6.00	Xin Liu, Adam Schwartz, Amarjit Budhiraja. Action Time Sharing Policies for Ergodic Control of Markov Chains., Submitted (05 2010)
08/10/2011	7.00	Amarjit Budhiraja, Anup Biswas. Exit time and invariant measure asymptotics for small noise constrained diffusions , Stochastic Processes and their Applications (01 2011)
08/10/2011	8.00	Amarjit Budhiraja, Arka Ghosh. Controlled Stochastic Networks in Heavy Tra?c: Convergence of Value Functions. , Annals of Applied Probability (04 2011)
08/10/2011	9.00	Amarjit Budhiraja, Xin Liu. Stability of Constrained Markov Modulated Di?usions, Submitted (04 2011)
09/02/2012	10.00	Amarjit Budhiraja, Pierre Del Moral, Sylvain Rubenthaler. Discrete Time Markovian Agents Interacting Through a Potential, [object Object] (06 2011)
09/02/2012	11.00	Shankar Bhamidi, Amarjit Budhiraja, Xuan Wang. Bohman-Frieze processes at criticality and emergence of the giant component , arXiv:1106.1022 (06 2011)
09/02/2012	12.00	Amarjit Budhiraja, Jiang Chen, Sylvain Rubenthaler. A Numerical Scheme for Invariant Distributions of Constrained Diffusions, arXiv:1205.5083 (05 2012)
09/02/2012	14.00	Amarjit Budhiraja, Jiang Chen, Paul Dupuis. Large Deviations for Stochastic Partial Differential Equations Driven by a Poisson Random Measure , arXiv:1203.4020 (03 2012)
09/02/2012	15.00	Shu Lu, Amarjit Budhiraja. Confidence regions for stochastic variational inequalities, Mathematics of Operations Research (06 2012)

TOTAL: **27**

Number of Manuscripts:

Books

Received   Book

TOTAL:

Received   Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Fellow of the Institute of Mathematical Statistics (elected April 2013).

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Xin Liu	0.50	
Dominik Reinhold	0.20	
Jiang Chen	0.50	
Xuan Wang	0.50	
FTE Equivalent:	**1.70**	
Total Number:	**4**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
Xiaoming Song	0.50
Chia Y. Lee	0.20
Subhamay Saha	0.10
FTE Equivalent:	**0.80**
Total Number:	**3**

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Amarjit Budhiraja	0.20	
FTE Equivalent:	**0.20**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME	
Xin Liu	
Dominik Reinhold	
Jiang Chen	
Xuan Wang	
Total Number:	**4**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Please see the attached document

Technology Transfer

Stochastic Analysis and Applied Probability(3.3.1): Topics in the Theory and Applications of Stochastic Analysis, August 15, 2010 – March 15, 2015.

Proposal Number: 57927-MA, Agreement Number: W911NF- 10-1-0158
Amarjit Budhiraja
Department of Statistics and Operations Research
University of North Carolina at Chapel Hill

August 13, 2015

Research partially supported by this grant culminated in the submission of twenty eight new research papers twenty three of which have now been accepted for publication. A detailed description of these works is provided below. Four Ph.D. students (Xin Liu, Dominik Reinhold, Jiang Chen and Xuan Wang) completed their dissertation under the direction of the PI. The topics of their study are outlined below. Three other Ph.D. students (Abhishek Pal Majumdar, Ruoyu Wu and Eric Friedlander) continued work on their dissertations. Postdoctoral fellows C.Y.Lee, X.Song and S.Saha were partially supported by this grant. Invited presentations that were given during this period are also listed.

(I) Large Deviation Properties of Weakly Interacting Processes via Weak Convergence Methods [14]. Collections of weakly interacting random processes have long been of interest in statistical physics, and more recently have appeared in problems of engineering and operations research. A simple but important example of such a collection is a group of particles, each of which evolves according to the solution of an Ito type stochastic differential equation (SDE). All particles have the same functional form for the drift and diffusion coefficients. The coefficients of particle i are, as usual, allowed to depend on the current state of particle i, but also depend on the current empirical distribution of all particle locations. When the number of particles is large the contribution of any given particle to the empirical distribution is small, and in this sense the interaction between any two particles is considered *weak*. For various reasons, including model simplification and approximation, one may consider a functional law of large numbers (LLN) limit as the number of particles tends to infinity. The limit behavior of a single particle (under assumptions which guarantee that all particles are in some sense exchangeable) can be described by a two component

Markov process. One component corresponds to the state of a typical particle, while the second corresponds to the limit of the empirical measures. Again using that all particles are exchangeable, under appropriate conditions one can show that the second component coincides with the distribution of the particle component. The limit process, which typically has an infinite dimensional state, is sometimes referred to as a nonlinear diffusion. Because the particle's own distribution appears in the state dynamics, the partial differential equations that characterize expected values and densities associated with this process are nonlinear, and hence the terminology. In this paper we consider the large deviation properties of the particle system as the number of particles tends to infinity. Thus the deviations we study are those of the empirical measure of the prelimit process from the distribution of the nonlinear diffusion. In the paper we develop an approach which is very different from the one taken in previous works. Unlike classical techniques, our proofs do not involve any time or space discretization of the system and no exponential probability estimates are invoked. The main ingredients in the proof are weak convergence methods for functional occupation measures and certain variational representation formulas. Our proofs cover models with degenerate noise and allow for interaction in both drift and diffusion terms. In fact the techniques are applicable to a wide range of model settings and an example of stochastic delay equations is considered to illustrate the possibilities. The starting point of our analysis is a variational representation for moments of nonnegative functionals of a Brownian motion. Using this representation, the proof of the large deviation principle reduces to the study of asymptotic properties of certain controlled versions of the original process. The key step in the proof is to characterize the weak limits of the control and controlled process as the large deviation parameter tends to its limit and under the same scaling that applies to the original process. More precisely, one needs to characterize the limit of the empirical measure of a large collection of controlled and weakly interacting processes. In the absence of control this characterization problem reduces to an LLN analysis of the original particle system, which has been studied extensively. Our main tools for the study of the controlled analogue are functional occupation measure methods. Indeed, these methods have been found to be quite useful for the study of averaging problems, but where the average is with respect to a time variable. In the problem studied here the measure valued processes of interest are obtained using averaging over particles rather than the time variable. Variational representations for Brownian motions and Poisson random measures have proved to be extremely useful for the study of small noise large deviation problems and many recent papers have applied these results to a variety of infinite dimensional small noise systems. We expect the current work to be similarly a starting point for the study, using variational representations, of a rather different collection of large deviation problems, namely asymptotics of a large number of interacting particles.

(II) On Near Optimal Trajectories of Games Associated with the infinity-Laplacian [3]. Consider the equation

$$\begin{cases} -2\Delta_\infty u = h & \text{in } G, \\ \\ u = g & \text{on } \partial G, \end{cases} \tag{1}$$

where, for an integer $m \geq 2$, $G \subset \mathbb{R}^m$ is a bounded $\mathcal{C}^2$ domain, and $g \in \mathcal{C}(\partial G, \mathbb{R})$ and the functions $h \in \mathcal{C}(\bar{G}, \mathbb{R} \setminus \{0\})$ are given. The infinity-Laplacian is defined as

$$\Delta_\infty f = \frac{1}{|Df|^2} \sum_{i,j=1}^{m} D_i f \, D_{ij} f \, D_j f = \frac{Df'}{|Df|} D^2 f \, \frac{Df}{|Df|},$$

provided $Df \neq 0$, where for a $\mathcal{C}^2$ function f we denote by Df the gradient and by $D^2 f$ the Hessian matrix. This paper is motivated by recent work of Peres et. al. [35], where a discrete time random turn game, referred to as *Tug-of-War*, is developed in relation to (1). This game, parameterized by $\varepsilon > 0$, has the property that the vanishing-ε limit of the value function uniquely solves (1) in the viscosity sense. The stochastic differential equation (SDE)

$$dX_t = 2\bar{p}(X_t)dW_t + 2q(X_t)dt, \tag{2}$$

where

$$\bar{p} = \frac{Du}{|Du|}, \qquad q = \frac{1}{|Du|^2}(D^2 u\, Du - \Delta_\infty u\, Du), \tag{3}$$

is suggested in [35] as the game's dynamics in the vanishing-ε limit. The relation is rigorously established in examples, but only heuristically justified in general. Recently, in [2], a two-player zero-sum stochastic differential game (SDG) is considered, for which the value function uniquely solves (1) in the viscosity sense. The dynamics of the state process are given as

$$X_t = x + \int_0^t (A_s - B_s)dW_s + \int_0^t (C_s + D_s)(A_s + B_s)ds, \qquad t \in [0, \infty), \tag{4}$$

and (A, C) and (B, D) are control processes, chosen by the two players, taking values in $\mathcal{S}^{m-1} \times [0, \infty)$ where $\mathcal{S}^{m-1}$ is the unit sphere in $\mathbb{R}^m$. Defined in the Elliott-Kalton sense, the SDG of [2] is formulated in such a way that one of the players selects a strategy, and then the other selects a control process. The payoff functional of interest is an exit time cost of the form $\boldsymbol{E}[\int_0^\tau h(X_s)ds + g(X_\tau)]$, where $\tau = \inf\{t : X_t \notin G\}$ (with an appropriate convention regarding $\tau = \infty$).

The goal of this paper is to show that, with appropriate conditions, (2) can be rigorously interpreted as the optimal dynamics of the SDG. We will assume in this paper that the equation possesses a classical solution u i.e., $\mathcal{C}^2$ with non-vanishing gradient. Under this assumption we specify, for each $\delta > 0$, a δ-optimal strategy β^δ, and a control process Y^δ that is δ-optimal for play against β^δ, in terms of first and second derivatives of u. We then identify the limit law, as $\delta \to 0$, of the state process under (β^δ, Y^δ), as the solution X to the SDE (2), stopped when X hits the boundary ∂G.

The construction of near optimal strategy-control pairs, that is of interest in its own right, is based on an interpretation of (1) as the following Bellman-Isaacs type equation

$$\sup_{|b|=1, d\geq 0} \inf_{|a|=1, c\geq 0} \Big\{ -\frac{1}{2}(a-b)'(D^2 u)(a-b) - (c+d)(a+b)\cdot Du \Big\} = h.$$

In this form there is a natural way to construct strategy and control, by associating the supremum and infimum with the two players. The variables a, b, c and d selected by the players dictate the coefficients of the game's state process, and, we prove, the coefficients converge to those of equation (2) in the limit as the supremum and infimum are achieved. This convergence is then lifted to the convergence of the underlying state process in (4) to the diffusion (2).

(III) Variational Representations for Continuous Time Processes [18]. In this paper we prove a variational representation for positive measurable functionals of a Poisson random measure and an infinite dimensional Brownian motion. These processes provide the driving noises for a wide range of important process models in continuous time, and thus we also obtain variational representations for these processes when a strong solution exists. The representations have a number of uses, the most important being to prove large deviation estimates.

The theory of large deviations is by now well understood in many settings, but there remain some situations where the topic is not as well developed. These are often settings where technical issues challenge standard approaches, and the problem of finding nearly optimal or even reasonably weak sufficient conditions is hindered as much by technique of proof as any other issue.

Variational representations of the sort developed in this paper have been shown to be particularly useful, when combined with weak convergence methods, for analyzing such systems. For example, Brownian motion representations have been used by many authors (see references in [18]) in the large deviation analysis of solutions to SPDEs in the small noise limit. The usefulness of the representations is in part due to the fact that they avoid certain discretization and/or approximation arguments, which can be cumbersome for complex systems. Another reason is that exponential tightness, a property that is often required by other approaches and which often leads to artificial conditions, is replaced by ordinary tightness for controlled processes with uniformly bounded control costs. (Although exponential tightness can a posteriori be obtained as consequence of the large deviation principle (LDP) and properties of the rate function). What is required for the weak convergence approach, beyond the variational representations, is that basic qualitative properties (existence, uniqueness and law of large number limits) can be demonstrated for certain controlled versions of the original process.

Previous work on variational representations has focused on either discrete time processes, functionals of finite dimensional Brownian motion, or various formulations of infinite dimensional Brownian motion. An important class of processes that were not covered are continuous time Markov processes with jumps, e.g., Levy processes. In this paper we eliminate this gap, and in fact give variational representations for functionals of a fairly general Poisson random measure (PRM) plus an independent infinite dimensional Brownian motion (BM), thereby covering many continuous time models.

In [38] Zhang has also proved a variational representation for functionals of a PRM. The representation in [38] is given in terms of certain predictable transformations on the canonical Poisson space. Existence of such transformations relies on solvability of certain nonlinear partial differential equations from the theory of mass transportation. This imposes restrictive conditions on the intensity measure (e.g., absolute continuity with respect to Lebesgue measure) of the PRM, and even the very elementary setting of a standard Poisson process is not covered. Additionally, use of such a representation for proving large deviation results for general continuous time models with jumps appears to be unclear.

In contrast, we impose very mild assumptions on the intensity measure (namely, it is a σ-finite measure on a locally compact space), and establish a representation, that is given in terms of a fixed PRM defined on an augmented space. A key question in formulating the representation for PRM is "*what form of controlled PRM is natural for purposes of representation?*" In the Brownian case there is little room for discussion, since control by shifting the mean is obviously very appealing. In [38] the control moves the atoms of the Poisson random measure through a rather complex nonlinear transformation. The fact that atoms are neither created nor destroyed is partly responsible for the fact that the representation does not cover the standard Poisson process. In the representation obtained in our work the control process enters as a censoring/thinning function in a very concrete fashion, which in turn allows for elementary weak convergence arguments in proofs of large deviation results.

As an application of the representation, we establish a general large deviation principle (LDP) for functionals of a PRM and an infinite dimensional BM. A similar LDP for functionals of an infinite dimensional BM [16] has been used in recent years by numerous authors to study small noise asymptotics for a variety of infinite dimensional stochastic dynamical models (see references in [18]). The LDP obtained in the current paper is expected to be similarly useful in the study of infinite dimensional stochastic models with jumps (e.g., SPDEs with jumps). In our work we illustrate the use of the LDP in via a simple finite dimensional jump-diffusion model. The goal is to simply show how the approach can be used and no attempt is made to obtain the best possible conditions.

(IV) Ergodic Rate Control Problem for Single Class Queueing Networks [19]. We study Jackson networks with state dependent and dynamically controlled arrival and service rates. The network consists of K service stations, each of which has an associated infinite capacity buffer. Arrivals of jobs can be from outside the system and/or from internal routing. Upon completion of service at a station, the customer is routed to one of the other service stations (or exits the system) according to a probabilistic routing matrix. Network is assumed to be in heavy traffic in an appropriate sense. Roughly speaking, one considers a sequence of networks with identical topology, parametrized by $n \in \mathbb{N}$. Instantaneous arrival and service rates in the n-th network are queue length dependent and are of order $\mathcal{O}(n)$. Additionally, a system manager can exercise service rate controls of order $\mathcal{O}(n^{1/2})$. The heavy traffic assumption says that traffic intensity at each station is of the order $1 - \mathcal{O}(1/n^{1/2})$. In an uncontrolled setting, diffusion approximations of suitably scaled queue length processes for such networks have been studied in [37]. The limit stochastic process is a reflected diffusion in the nonnegative orthant. Dependence of arrival and service rates on queue length processes leads to drift and diffusion coefficients for the limit model that are state dependent. Existence and uniqueness of solutions to such reflected stochastic differential equations (SDE) follows from the classical theory and well understood regularity properties of the Skorohod map.

For the controlled setting considered here, a formal analysis along similar lines leads to controlled reflected diffusions with control entering linearly in the drift coefficient. Goal of this work is to use such a formal diffusion approximation in order to obtain provably asymptotically optimal rate control policies for the underlying physical networks. We are concerned with an average long term cost per unit time criterion. Cost function is a sum of two terms; the first term measures the cost for holding customers in the buffer, while the second term is the cost for exercising control. The holding cost is allowed to have a polynomial growth (as a function of queue lengths) while the control cost is assumed to be linear. Since the cost criterion involves an infinite time horizon, additional stability conditions are needed to ensure the finiteness of the cost. In our work these conditions are manifested in the definition of the class of admissible controls. The definition in particular ensures that the controls are "uniformly stabilizing", viz. all polynomial moments of the queue length process are bounded uniformly in time, the scaling parameter and the choice of admissible control sequence. This stability property is a key ingredient in our proofs.

Our main result relates the value function (i.e. the optimum value of the cost) of the queueing rate control problem with the value function associated with an ergodic cost problem for the formal controlled diffusion limit. Cost function for the diffusion model is analogous to that for the physical networks. It is shown that, under certain conditions, the value function for the n-th queueing network converges to that for the diffusion model, as $n \to \infty$. The theorem also shows that using an appropriately chosen ε-optimal feedback control for the diffusion model, one can construct an asymptotically ε-optimal rate control policy for the physical queueing network. Thus the theorem describes the precise sense in which the diffusion model, which is originally justified only formally, is a rigorous asymptotic approximation for the queueing model. Rate control problems with an ergodic cost criterion for single class open queueing networks in heavy traffic have been studied in several papers. Most works concern a one dimensional problem (i.e. a single server queue) and provide explicit expressions/recipes for asymptotically optimal policies. The paper [32] studies a general K dimensional Jackson type network with finite buffers. In this setting, due to compactness of the state space, all stability issues become trivial. Furthermore, convergence of value functions is established under the additional assumption that ε-optimal continuous feedback controls exist for the limit diffusion model– but no details on when such an assumption holds are provided.

One of the main steps in the proof of our main result is establishing existence of an ε-optimal continuous feedback control for the limit diffusion model. Existence of optimal feedback controls for the class of ergodic diffusion control problems studied here has been established in [10]. Our proof starts from such an optimal control b^* and constructs a sequence $\{b_n\}$ of continuous feedback controls in a manner such that an appropriately chosen measure of the set $\{b_n \neq b^*\}$ converges to 0 as $n \to \infty$. We show that, as long as initial distributions converge, the solution X^n of the reflected SDE that uses b_n as the feedback control converges weakly to X which solves the equation with b_n replaced with b^*. Proof of this result relies on certain representations and estimates for transition probability densities of reflected Brownian motions that are of independent interest. Once weak convergence of X^n to X is established, existence of an ε-optimal continuous feedback control is an immediate consequence of the linearity of the control cost. Next, an ε-optimal continuous feedback control b^ε, obtained from above result is used to define a rate control policy for the n-th network, for each $n \in \mathbb{N}$. The associated costs are shown to converge to the cost associated with b^ε in the diffusion control problem. As a consequence of this result one obtains that the value function for the limit diffusion model is an upper bound for any limit point of the sequence of value functions for the controlled queueing networks. We then establish the reverse inequality. Combining these bounds we have the main result of the paper. Proofs of these bounds use functional occupation measure methods developed in [32]. The main idea is to represent the cost associated with the n-th network in terms of integrals with respect to occupation measures associated with various stochastic processes that constitute the dynamical description of the network. Using stability estimates of this paper one can establish tightness of these occupation measures and then classical martingale characterization methods can be applied to identify the limit points in an appropriate manner.

Stability estimates obtained in this work are also useful for the study of the uncontrolled setting. We show that, under appropriate conditions, stationary distributions of scaled queue length processes (which are strong Markov processes when there are no controls) converge to the unique stationary distribution of the limit reflecting diffusion model. For the setting where the arrival and service rates are constant, such a result has been proved in [21]. To our knowledge the current paper is the first one to treat a setting with state dependent rates.

(V) Multiscale Diffusion Approximations for Stochastic Networks in Heavy Traffic[23].

Jackson networks have been very well studied and extensively used for modeling and analysis in a variety of disciplines. The elegant distributional theory and asymptotic properties of Jackson networks break down when one attempts to incorporate some more realistic features of specific application settings into such a model. For example, when the distributions of the primitives are relaxed to be more general than i.i.d. exponential, or the rates of interarrivals and services are allowed to depend on the state of the system, the tractability of such a generalized Jackson network fails and thus one seeks suitable approximate models. One class of such approximations are diffusion models that can be rigorously justified when networks are operating in the heavy traffic regime, i.e., when the network capacity is roughly balanced with network load. Attractiveness of such approximations primarily lies in the fact that, analogous to the central limit theory, the limit model is described only using a few important parameters of the underlying networks and the complex distributional properties of the primitives are averaged out. The first general result in the study of such diffusion approximations is due to Reiman[36] who considered the case where the arrivals and services are mutually independent renewal processes with square integrable summands. Queueing networks in which rates of arrival and service processes depend on the current state of network arise in many application areas. In particular, Yamada[37] has shown that, under appropriate heavy traffic conditions, suitably scaled queue length processes for such networks converge weakly to a reflected diffusion process with drift and diffusion coefficients that depend on the state of the process.

In models considered in works of Reiman and Yamada, the underlying topology of the network is the same as that of a Jackson network. In particular, there is a fixed probability routing matrix P which governs the routing of jobs in the networks. In this work, we are interested in settings where the routing structure and arrival/service rates may change over time, according to an extraneous finite state Markov process. This Markov process can be interpreted as the random environment in which the system is operating. We begin by considering a situation where the system performance is analyzed on time scales which are much larger than typical interarrival/service times of jobs, which in turn are significantly larger than typical time intervals at which the state of the random environment changes. Such multi-scale models are motivated by the study of large computer networks where one is interested in the behavior of modeling traffic of files with moderate size over a long period of time, for a small subset of nodes in the system. Denote by $\mathcal{E}$ the collection of all nodes in the network and $\mathcal{E}_0 \subset \mathcal{E}$ the subset of nodes of interest. Here, the size of $\mathcal{E}_0$ is much smaller than that of $\mathcal{E}$. One is interested in building a model for traffic between nodes in $\mathcal{E}_0$ without taking a very precise account of the interactions of such nodes with those in $\mathcal{E} \setminus \mathcal{E}_0$. A node $e \in \mathcal{E}_0$ may receive files from a large number of nodes, most of which one does not want to explicitly account for in the reduced model. The point of view taken here is to model the effect of nodes in $\mathcal{E} \setminus \mathcal{E}_0$ at a node $e \in \mathcal{E}_0$ by a rapidly varying channel capacity (at e), which is modulated by a Markov process. If a large number of nodes in $\mathcal{E} \setminus \mathcal{E}_0$ are connected to e, one expects that the rate at which the channel capacity changes is much greater than the transmission rate of a typical file through e. Rapid changes in channel capacity lead to variations in processing rates and available routing options for files accepted at nodes in $\mathcal{E}_0$. Thus we propose a traffic model for nodes in $\mathcal{E}_0$ in term of a Jackson type network where the arrival/service rates and routing probability matrices vary randomly over time according to a finite state Markov process.

Goal of this work is to study diffusion approximations for such multiscale models. We show that, under appropriate conditions, the state of the system can be well approximated by that of a reflected diffusion process with drift and diffusion coefficients depend on the equilibrium measure of the Markov process governing the random environment. A crucial requirement in typical heavy traffic approximation results is the traffic balance condition of the form $\lambda = [I - P]\alpha$. In the multi-scale setting considered here all these quantities in the above equation, i.e., P, λ, α, vary randomly over time according to the modulating Markov process. We find that if transition times of the Markov process are suitably fast, then the traffic balance condition can be relaxed to an average balance condition where the average is taken with respect to the equilibrium measure of the Markov process. Making this statement mathematically precise is one of the key contributions of this work.

In contrast to the scaling regime described above is the situation where the random environment changes very slowly relative to the rates of interarrival and service times. In such a regime, changes in the environment are revealed only when the system is viewed over long time intervals. Our second result in this work is to give a precise mathematical formulation and establish a suitable diffusion approximation for networks in such a scaling regime. For simplicity we restrict ourselves to a setting where the random environment affects only the arrival and service rates in the system while the routing probabilities stay the same (governed by a fixed non-random matrix P). We show that, with a suitable scaling, the state process for the network approaches a reflected diffusion with coefficients that are modulated by a Markov process that is independent of the driving Brownian motion.

The two scaling regimes considered above correspond to very different behaviors of the background environment process. In the first setting the environment changes at a much faster rate than the typical arrival/serive rates in the system, while in the second one the reverse is true. More generally, one can consider a setting where background variation of both types exists. Namely, there are two independent Markov processes governing the randomly varying environment, the first corresponds to fast changes while the second captures slow changes. We also establish an appropriate diffusion approximation result for such multi-scale queueing networks. This work is part of the dissertation work of Ms. Xin Liu.

(VI) Action Time Sharing Policies for Ergodic Control of Markov Chains [27]. Markov Decision processes are used extensively as the simplest models that involve both stochastic behavior and control. A common measure of performance is the long-time average (or ergodic) criterion. Given all relevant parameters, a typical goal is to find a simple (e.g. feedback, or deterministic stationary) policy that achieves the optimal value.

The goal of adaptive control is to obtain an optimal policy, when some relevant information concerning the behavior of the system is missing. The relevant information needs to be obtained while controls are chosen at each step. The classical approach is to design an algorithm which collects information, while at the same time choosing controls, in such a way that sufficient information is collected for making good control decisions, in the sense that the chosen controls "approach optimality over time." Existing results include general solutions for the case of countable state space, and specify an estimation and a control scheme. A different approach to this issue, including PAC criteria, can be found in the large literature on Reinforcement learning.

We are concerned with a more elementary question, namely: What are the basic controlled objects that determine the cost? Since the objective function is defined as a Cesaro limit, we can expect that a similar Cesaro definition of the choice of controls would suffice to determine the cost. For the case of countable state and action spaces one fundamental result says that the control decisions can deviate from those dictated by the Markov policy q, and still produce the same long term average cost, as long as the conditional frequencies converge to the correct values. This flexibility is useful in many estimation and sampling applications. In the current work we are concerned with a setting where the state and action spaces are not (necessarily) countable. Our main objective is to formulate an appropriate definition for an Action-Time Sharing (ATS) policy for a given Markov control which, similar to the countable case, on the one hand leads to long term costs that are identical to those for the corresponding Markov control, while on the other hand allows for flexible implementation well suited for various estimation and adaptive control goals. We show that, under suitable stability, irreducibility and Feller continuity conditions, occupation measures for state and action sequences, under an ATS policy (suitably defined), converge a.s. to the same (deterministic) measure as under the corresponding Markov control. Such a result in particular shows that long term costs for a broad family of one stage cost functions, under the two control policies, coincide. ATS policies can be used to develop a variety of variance reduction schemes for ergodic control problems. Additionally, ATS policies provide much flexibility for sampling (namely using controls without regards to the ensuing cost), for example for the purpose of collecting information. This could be information which is related to the main optimization objective, but could also be other information which is of interest. We show that how ATS policies introduced in this work can be used for consistent estimation of unknown model parameters and also for adaptive control problems. This paper is part of the dissertation work of Ms. Xin Liu.

(VII) Exit Time and Invariant Measure Asymptotics for Small Noise Constrained Diffusions [6]. Diffusions in polyhedral domains arise commonly as approximate models for stochastic processing networks in heavy traffic. In this work we consider a family of such constrained diffusions with a small parameter (denoted as ϵ) multiplying the diffusion coefficient. Goal of the work is the study of asymptotic properties of invariant measures and exit times from suitable domains, as $\varepsilon \to 0$. The classical reference for small noise asymptotics of diffusions in $\mathbb{R}^k$ is the book of Freidlin and Wentzell (1998). The basic object of study in this fundamental body of work is a collection of diffusion processes $\{X^\varepsilon\}_{\varepsilon>0}$, given as

$$dX^\varepsilon(t) = b(X^\varepsilon(t))dt + \varepsilon\sigma(X^\varepsilon(t))dW(t),\ X^\varepsilon(0) = x, \tag{5}$$

where W is a k dimensional standard Brownian motion and b, σ are suitable coefficients. The stochastic process $X^\varepsilon \equiv \{X^\varepsilon(t)\}_{0\le t\le T}$, for each $T \ge 0$ can be regarded as a $\mathcal{C}_T = C([0,T] : \mathbb{R}^k)$ (space of continuous functions from $[0,T]$ to $\mathbb{R}^k$ with the uniform topology) valued random variable and under suitable conditions on b, σ one can show that, as $\varepsilon \to 0$, X^ε converges in probability to ξ which is the unique solution of the ordinary differential equation (ODE)

$$\dot{\xi} = b(\xi),\ \xi(0) = x. \tag{6}$$

One of the basic results in the field says that for each $T > 0$, as $\varepsilon \to 0$, X^ε satisfies a large deviation principle (LDP) in $\mathcal{C}_T$, uniformly in the initial condition x in any compact set K, with an appropriate rate function $I_T : \mathcal{C}_T \to [0, \infty]$. This result is a starting point for the study of numerous asymptotic questions for such small noise diffusions. In particular, when the underlying diffusions have suitable stability properties, the above LDP plays a central role in the study of asymptotic properties of invariant measures and exit times from domains. The asymptotics are governed by the "quasi-potential" function V which is determined from the collection of rate functions $\{I_T : T > 0\}$.

Freidlin-Wentzell theory has been extended and refined in many different directions. One notable work is Day(1982) which studies asymptotics of solutions of Dirichlet problems associated with diffusions given by (5). To signify the dependence on the initial condition, denote the solution of (5) by X^ε_x. Let B be a bounded domain in $\mathbb{R}^k$ and K be an arbitrary compact subset of B. Under the assumption that all solutions of the ODE (6), with $x = \xi(0) \in B$, converge without leaving B, to a single linearly asymptotically stable critical point, Day shows that with suitable conditions on the coefficients, for all bounded measurable f

$$\sup_{x,y\in K} \left| I\!\!E \left(f(X^\varepsilon_x(\tau^\varepsilon_x))\right) - I\!\!E \left(f(X^\varepsilon_y(\tau^\varepsilon_y))\right) \right|$$

converges to 0 at an exponential rate. Here, $\tau^\varepsilon_x = \inf\{t : X^\varepsilon_x(t) \in B^c\}$. This property, usually referred to as "exponential leveling", says that although the exit time of the process from the domain approaches ∞, as $\varepsilon \to 0$, the moments of functionals of exit location, corresponding to distinct initial conditions, coalesce asymptotically, at an exponential rate. The key ingredient in the proof is the gradient estimate

$$\sup_{x\in K} |\nabla u^\varepsilon(x)| \le c\varepsilon^{-1/2}, \tag{7}$$

where u^ε is the solution of the Dirichlet problem on B associated with the diffusion (5) with boundary data f.

The goal of the current work is to develop the Freidlin-Wentzell small noise theory for a family of constrained diffusions in polyhedral cones. Let $G \subset \mathbb{R}^k$ be convex polyhedral cone in $\mathbb{R}^k$ with the vertex at origin given as the intersection of half spaces $G_i, i = 1, 2, ..., N$. Let n_i be the unit vector associated with G_i via the relation

$$G_i = \{x \in \mathbb{R}^k : \langle x, n_i\rangle \ge 0\}.$$

We will denote the set $\{x \in \partial G : \langle x, n_i\rangle = 0\}$ by F_i. With each face F_i we associate a unit vector d_i such that $\langle d_i, n_i\rangle > 0$. This vector defines the *direction of constraint* associated with the face F_i. Roughly speaking such a process evolves infinitesimally as a diffusion in $\mathbb{R}^k$ and is instantaneously pushed back using the oblique reflection direction d_i upon reaching the face F_i. Formally, such a process, denoted once more as X^ε_x, can be represented as a solution of a stochastic integral equation of the form

$$X^\varepsilon_x(t) = \Gamma\left(x + \int_0^\cdot b(X^\varepsilon(s))ds + \varepsilon \int_0^\cdot \sigma(X^\varepsilon(s))dW(s)\right)(t), \tag{8}$$

where Γ is the Skorohod map taking trajectories with values in $\mathbb{R}^k$ to those with values in G, consistent with the constraint vectors $\{d_i, i = 1, \cdots N\}$. Under certain regularity assumptions on the Skorohod map and the usual Lipschitz conditions on the coefficients b and σ, our first result establishes a (locally uniform) LDP for X^ε_x, in $C([0,T]:G)$ for each $T>0$. This result is the starting point for all exit time and invariant measure estimates obtained in this work.

Stability properties of constrained diffusions in polyhedral domains have been studied in several works. Let

$$\mathcal{C} = \{-\sum_{i=1}^{k} \alpha_i d_i : \alpha_i \geq 0; i \in \{1,...,k\}\}.$$

The paper [5] shows that under regularity of the Skorohod map, uniform non-degeneracy of σ and Lipschitz coefficients, if for some $\delta > 0$

$$b(x) \in \mathcal{C}(\delta) = \{v \in \mathcal{C} : \text{dist}(v, \partial\mathcal{C}) \geq \delta\} \text{ for all } x \in G, \tag{9}$$

then the constrained diffusion X^ε is positive recurrent and consequently admits a unique invariant probability measure μ^ε. In the current work we study asymptotic properties of μ^ε, as $\varepsilon \to 0$.

We also consider asymptotic properties of exit times from a bounded domain $B \subset G$ that contains the origin. One important feature in this analysis is that the stability condition (9), in general, does not ensure that the trajectories of the associated deterministic dynamical system

$$\xi = \Gamma(x + \int_0^\cdot b(\xi(s))ds) \tag{10}$$

with initial condition in B will stay within the domain at all times. However, a weaker stability property holds, namely, one can find domains $B_0 \subset B$ such that all trajectories of (10) starting in B_0 stay within B at all times.

A significant part of this work is devoted to the proof of the exponential leveling property for constrained diffusions. We recall that the key ingredient in the proof of such a result for diffusions in $\mathbb{R}^k$ is the gradient estimate (7) for solutions of the associated Dirichlet problem. For diffusions in domains with corners and with oblique reflection fields that change discontinuously, there are no regularity (eg. C^1 solutions) results known for the associated partial differential equations(PDE). Our proof of the exponential leveling property is purely probabilistic and bypasses all PDE estimates. The main step in the proof is the construction of certain (uniform in ε) Lyapunov functions which are then used to construct a coupling of the processes X^ε_x, X^ε_y with explicit uniform estimates on exponential moments of time to coupling. The key ingredient in this coupling construction is a minorization condition on transition densities of reflected diffusions. We give some examples where such a minorization property holds. Obtaining general conditions under which this estimate on transition densities of reflected diffusions holds is a challenging open problem. The minorization property allows for the construction of a pseudo-atom, for each fixed $\varepsilon > 0$, using split chain ideas of Athreya-Ney-Nummelin. Coupling based on pseudo-atom constructions have been used by many authors for the study of a broad range of stability and control problems, however the current paper appears to be the first to bring these powerful techniques to bear for the study of exit time asymptotics of small noise Markov processes.

As a second consequence of our coupling constructions we show that difference of moments of an exit time functional with a sub-logarithmic growth corresponding to distinct initial conditions in

B_0 is asymptotically bounded. Note that for typical unbounded functionals the associated moments will approach ∞, as $\varepsilon \to 0$ thus the result provides a rather non-trivial "leveling estimate". The third important consequence of our approach says that as initial conditions approach 0 at rate ε^2 the corresponding moments of exit time functionals with sub-logarithmic growth asymptotically coalesce at an exponential rate. To the best of our knowledge, analogous estimates are not known even for the setting of small noise diffusions (with suitable stability properties) in $\mathbb{R}^k$.

These estimates are interesting for one dimensional models as well. Let X be a one dimensional reflected Brownian motion, starting from $x \in [0,1)$, with drift $b \in (-\infty, 0)$ and variance $\sigma^2\varepsilon^2 \in (0,\infty)$, given on some probability space $(\Omega, \mathcal{F}, \boldsymbol{P}_x^\varepsilon)$. I.e., $\boldsymbol{P}_x^\varepsilon$ a.e.,

$$X(t) = x + bt + \varepsilon\sigma B(t) - \inf_{0\le s\le t}\{(x + bs + \varepsilon\sigma B(s)) \wedge 0\},\ t \ge 0$$

where B is a standard Brownian motion. Let $\tau = \inf\{t : X(t) = 1\}$. Let $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ be a measurable map and let $\varphi_\varepsilon(x) = \mathbb{E}_x^\varepsilon \psi(\tau)$. Our results for this one dimensional setting says that if ψ has a sub-logarithmic growth then for some $c, \delta \in (0,\infty)$, as $\varepsilon \to 0$,

$$e^{\delta/\varepsilon}|\varphi_\varepsilon(c\varepsilon^2) - \varphi_\varepsilon(0)| \to 0. \tag{11}$$

If $\psi(t) = t$, $t \ge 0$, then φ_ε is the unique solution to the boundary value problem

$$\varepsilon^2\sigma^2\varphi_\varepsilon'' + b\varphi_\varepsilon' = -1,\ \varphi_\varepsilon'(0) = 0,\quad \varphi_\varepsilon(1) = 0,$$

whose solution is given as $\varphi_\varepsilon(x) = \frac{\varepsilon^2\sigma^2}{b^2}[e^{\frac{-b}{\varepsilon^2\sigma^2}} - e^{\frac{-bx}{\varepsilon^2\sigma^2}}] + \frac{1}{b}(1-x)$. It is easily verified that if $x_\varepsilon = c\varepsilon^2$, for some $c > 0$, then $|\varphi_\varepsilon(x_\varepsilon) - \varphi_\varepsilon(0)| \to 0$ at rate ε^2, as $\varepsilon \to 0$. In particular, the decay is not exponential. Furthermore, for $x \neq y$ in $[0,1)$, $|\varphi_\varepsilon(x) - \varphi_\varepsilon(y)| \to \infty$ as $\varepsilon \to 0$. For general ψ a similar ODE analysis is less tractable and thus the coupling techniques developed here give a powerful approach (especially in higher dimensions) to the study of such asymptotic estimates.

(VIII) Controlled Stochastic Networks in Heavy Traffic: Convergence of Value Functions [29]. As an approximation to control problems for critically-loaded stochastic networks, Harrison (1988) has formulated a stochastic control problem in which the state process is driven by a multi-dimensional Brownian motion along with an additive control that satisfies certain feasibility and non-negativity constraints. This control problem that is usually referred to as the Brownian Control Problem (BCP) has been one of the key developments in the heavy traffic theory of controlled stochastic processing networks (SPN). BCPs can be regarded as formal scaling limits for a broad range of scheduling and sequencing control problems for multiclass queuing networks. Finding optimal (or even near-optimal) control policies for such networks – which may have quite general non-Markovian primitives, multiple server capabilities and rather complex routing geometry – is prohibitive. In that regard BCPs that provide significantly more tractable approximate models are very useful. In this diffusion approximation approach to policy synthesis, one first finds an optimal (or near-optimal) control for the BCP which is then suitably interpreted to construct a scheduling policy for the underlying physical network. In recent years there have been many works that consider specific network models for which the associated BCP is explicitly solvable (i.e. an optimal control process can be written as a known function of the driving Brownian motions) and, by suitably adapting the solution to the underlying network, construct control policies that are asymptotically (in the heavy traffic limit) optimal. Although now there are several papers which establish a rigorous connection between a network control problem and its associated BCP by exploiting the explicit form of the solution of the latter, a systematic theory which justifies the use of BCPs as approximate models has been missing. In a recent work [28] it was shown that for a large family of Unitary Networks, with general interarrival and service times, probabilistic routing, and an infinite horizon discounted linear holding cost, the cost associated with any admissible control policy for the network is asymptotically, in the heavy traffic limit, bounded below by the value function of the BCP. This inequality, which provides a useful bound on the best achievable asymptotic performance for an admissible control policy (for this large family of models), was a key step in developing a rigorous general theory relating BCPs with SPN in heavy traffic. The current paper is devoted to the proof of the reverse inequality. The network model is required to satisfy assumptions made in [28]. In addition we impose a non-degeneracy condition, a condition on the underlying renewal processes regarding probabilities of deviations from the mean and regularity of a certain Skorohod map. Under these assumptions we prove that the value function of the BCP is bounded below by the heavy traffic limit (limsup) of the value functions of the network control problem. Combining this with the result obtained in [28] we obtain the main result of the paper. This theorem says that, under broad conditions, the value function of the network control problem converges to that of the BCP. This result provides, under general conditions, a rigorous basis for regarding BCPs as approximate models for critically loaded stochastic networks. Conditions imposed in this paper allow for a wide range of open multi-class queuing networks, parallel server networks and job shop type models. We note that our approach does not require the BCP to be explicitly solvable and the result covers many settings where explicit solutions are unavailable. Previous works noted earlier, that treat the setting of explicitly solvable BCP, do much more than establish convergence of value functions. In particular, these works give an explicit implementable control policy for the underlying network that is asymptotically optimal in the heavy traffic limit. In the generality treated in the current work, giving explicit recipes (eg. threshold type policies) is unfeasible, however the policy sequence that we construct, suggests a general approach for building near asymptotically optimal policies for the network given a near optimal control for the BCP. Obtaining near optimal controls for the BCP in general requires numerical approaches, study of which will be a topic for future work.

(IX) Stability of Constrained Markov Modulated Diffusions [30].

A family of constrained diffusions in a random environment is considered. Constraint set is a polyhedral cone and coefficients of the diffusion are governed by, in addition to the system state, a finite state Markov process that is independent of the driving noise. Such models arise as limit objects in the heavy traffic analysis of stochastic processing networks (SPN) with Markov modulated arrival and processing rates. We give sufficient conditions (which in particular includes a requirement on the regularity of the underlying Skorohod map) for positive recurrence and geometric ergodicity. When the coefficients only depend on the modulating Markov process (i.e. they are independent of the system state), a complete characterization for stability and transience is provided. The case, where the pathwise Skorohod problem is not well-posed but the underlying reflection matrix is completely-$\mathcal{S}$, is treated as well. As consequences of geometric ergodicity various results, such as exponential integrability of invariant measures and CLT for fluctuations of long time averages of process functionals about their stationary values, are obtained. Conditions for stability are formulated in terms of the averaged drift, where the average is taken with respect to the stationary distribution of the modulating Markov process. Finally, steady state distributions of the underlying SPN are considered and it is shown that, under suitable conditions, such distributions converge to the unique stationary distribution of the constrained random environment diffusion.

(X) Discrete Time Markovian Agents Interacting Through a Potential [15]. A discrete time stochastic model for a multiagent system given in terms of a large collection of interacting Markov chains is studied. The evolution of the interacting particles is described through a time inhomogeneous transition probability kernel that depends on the 'gradient' of the potential field. The particles, in turn, dynamically modify the potential field through their cumulative input. Interacting Markov processes of the above form have been suggested as models for active biological transport in response to external stimulus such as a chemical gradient. One of the basic mathematical challenges is to develop a general theory of stability for such interacting Markovian systems and for the corresponding nonlinear Markov processes that arise in the large agent limit. Such a theory would be key to a mathematical understanding of the interactive structure formation that results from the complex feedback between the agents and the potential field. It will also be a crucial ingredient in developing simulation schemes that are faithful to the underlying model over long periods of time. The goal of this work is to study qualitative properties of the above stochastic system as the number of particles (N) and the time parameter (n) approach infinity. In this regard asymptotic properties of a deterministic nonlinear dynamical system, that arises in the propagation of chaos limit of the stochastic model, play a key role. We show that under suitable conditions this dynamical system has a unique fixed point. This result allows us to study stability properties of the underlying stochastic model. We show that as $N \to \infty$, the stochastic system is well approximated by the dynamical system, uniformly over time. As a consequence, for an arbitrarily initialized system, as $N \to \infty$ and $n \to \infty$, the potential field and the empirical measure of the interacting particles are shown to converge to the unique fixed point of the dynamical system. In general, simulation of such interacting Markovian systems is a computationally daunting task. We propose a particle based approximation for the dynamic potential field which allows for a numerically tractable simulation scheme. It is shown that this simulation scheme well approximates the true physical system, uniformly over an infinite time horizon.

(XI) Bohman-Frieze processes at criticality and emergence of the giant component [7].

The percolation phase transition and the mechanism of the emergence of the giant component through the critical scaling window for random graph models, has been a topic of great interest in many different communities ranging from statistical physics, combinatorics, computer science, social networks and probability theory. The last few years have witnessed an explosion of models which couple random aggregation rules, that specify how one adds edges to existing configurations, and choice, wherein one selects from a limited set of edges at random to use in the configuration. These models exhibit fascinating new phenomenon, ranging from delaying or speeding up the emergence of the giant component, to explosive percolation, where the diameter of the scaling window is several orders of magnitude smaller than that for standard random graph models. While an intense study of such models has ensued, understanding the actual emergence of the giant component and merging dynamics in the critical scaling window has remained impenetrable to a rigorous analysis. In this work we take an important step in the analysis of such models by studying one of the standard examples of such processes, namely the Bohman-Frieze model, and provide first results on the asymptotic dynamics, through the critical scaling window, that lead to the emergence of the giant component for such models. We identify the scaling window and show that through this window, the component sizes properly rescaled converge to the standard multiplicative coalescent. Proofs hinge on a careful analysis of certain infinite-type branching processes with types taking values in the space of RCLL paths, and stochastic analytic techniques to estimate susceptibility functions of the components all the way through the scaling window where these functions explode. Previous approaches for analyzing random graphs at criticality have relied largely on classical breadth-first search techniques that exploit asymptotic connections with Brownian excursions. For dynamic random graph models evolving via general Markovian rules, such approaches fail and we develop a quite different set of tools that can potentially be used for the study of critical dynamics for all bounded size rules.

(XII) A Numerical Scheme for Invariant Distributions of Constrained Diffusions [12].

Reflected diffusions in polyhedral domains are commonly used as approximate models for stochastic processing networks in heavy traffic. Stationary distributions of such models give useful information on the steady state performance of the corresponding stochastic networks and thus it is important to develop reliable and efficient algorithms for numerical computation of such distributions. In this work we propose and analyze a Monte- Carlo scheme based on an Euler type discretization of the reflected stochastic dierential equation using a single sequence of time discretization steps which decrease to zero as time approaches infinity. Appropriately weighted empirical measures constructed from the simulated discretized reflected diffusion are proposed as approximations for the invariant probability measure of the true diffusion model. Almost sure consistency results are established that in particular show that weighted averages of polynomially growing continuous functionals evaluated on the discretized simulated system converge a.s. to the corresponding integrals with respect to the invariant measure. Proofs rely on constructing suitable Lyapunov functions for tightness and uniform integrability and characterizing almost sure limit points through an extension of Echeverrias criteria for reflected diffusions. Regularity properties of the underlying Skorohod problems play a key role in the proofs. Rates of convergence for suitable families of test functions are also obtained. A key advantage of Monte-Carlo methods is the ease of implementation, particularly for high dimensional problems. A numerical example of a eight dimensional Skorohod problem is presented to illustrate the applicability of the approach.

(XIII) Near Critical Catalyst Reactant Branching Processes with Controlled Immigration [31]. Near critical catalyst-reactant branching processes with controlled immigration are studied. The reactant population evolves according to a branching process whose branching rate is proportional to the total mass of the catalyst. The bulk catalyst evolution is that of a classical continuous time branching process; in addition there is a specific form of immigration. Immigration takes place exactly when the catalyst population falls below a certain threshold, in which case the population is instantaneously replenished to the threshold. Such models are motivated by problems in chemical kinetics where one wants to keep the level of a catalyst above a certain threshold in order to maintain a desired level of reaction activity. A diffusion limit theorem for the scaled processes is presented, in which the catalyst limit is described through a reflected diffusion, while the reactant limit is a diffusion with coefficients that are functions of both the reactant and the catalyst. Stochastic averaging principles under fast catalyst dynamics are established. In the case where the catalyst evolves "much faster" than the reactant, a scaling limit, in which the reactant is described through a one dimensional SDE with coefficients depending on the invariant distribution of the reflected diffusion, is obtained. Proofs rely on constrained martingale problem characterizations, Lyapunov function constructions, moment estimates that are uniform in time and the scaling parameter, and occupation measure techniques.

(XIV) Large Deviations for Stochastic Partial Differential Equations Driven by a Poisson Random Measure [11]. Stochastic partial differential equations driven by Poisson random measures (PRM) have been proposed as models for many different physical systems, where they are viewed as a refinement of a corresponding noiseless partial differential equations (PDE). A systematic framework for the study of probabilities of deviations of the stochastic PDE from the deterministic PDE is through the theory of large deviations. The goal of this work is to develop the large deviation theory for small Poisson noise perturbations of a general class of deterministic infinite dimensional models. Although the analogous questions for finite dimensional systems have been well studied, there are currently no general results in the infinite dimensional setting. This is in part due to the fact that in this setting solutions may have little spatial regularity, and thus classical approximation methods for large deviation analysis become intractable. The approach taken here, which is based on a variational representation for nonnegative functionals of general PRM, reduces the proof of the large deviation principle to establishing basic qualitative properties for controlled analogues of the underlying stochastic system. As an illustration of the general theory, we consider a particular system that models the spread of a pollutant in a waterway.

(XV) Confidence regions for stochastic variational inequalities [34]. The sample average approximation (SAA) method is a basic approach for solving stochastic variational inequalities. It is well known that the SAA solutions provide good point estimators for the true solution to a stochastic variational inequality. It is of fundamental interest to use such point estimators along with the central limit theory to develop confidence regions of prescribed level of significance for the true solution. However, the non-smooth nature of variational inequalities makes it difficult to construct the confidence regions by following standard procedures. This paper overcomes such difficulty and proposes a method to build asymptotically exact confidence regions for the true solution that are computable from the SAA solutions. We justify this method theoretically by establishing a precise limit theorem, apply it to complementarity problems, and test it with a linear complementarity problem.

(XVI) Infinite Dimensional Forward-Backward Stochastic Differential Equations and the KPZ Equation [1]. Kardar-Parisi-Zhang (KPZ) equation is a quasilinear stochastic partial differential equation(SPDE) driven by a space-time white noise. In recent years there have been several works directed towards giving a rigorous meaning to a solution of this equation. Bertini, Cancrini and Giacomin have proposed a notion of a solution through a limiting procedure and a certain renormalization of the nonlinearity. In this work we study connections between the KPZ equation and certain infinite dimensional forward-backward stochastic differential equations. Forward-backward equations with a finite dimensional noise have been studied extensively, mainly motivated by problems in mathematical finance. Equations considered here differ from the classical works in that, in addition to having an infinite dimensional driving noise, the associated SPDE involves a non-Lipschitz (namely a quadratic) function of the gradient. Existence and uniqueness of solutions of such infinite dimensional forward-backward equations is established and the terminal values of the solutions are then used to give a new probabilistic representation for the solution of the KPZ equation.

(XVII) Admission Control for Multidimensional Workload with Heavy Tails and Fractional Ornstein-Uhlenbeck Process. [26]. The infinite source Poisson arrival model with heavy-tailed workload distributions has attracted much attention, especially in the modeling of data packet traffic in communication networks. In particular, it is well known that under suitable assumptions on the source arrival rate, the centered and scaled cumulative workload process for the underlying processing system can be approximated by fractional Brownian motion. In many applications one is interested in the stabilization of the work inflow to the system by modifying the net input rate, using an appropriate admission control policy. In this work we study a natural family of admission control policies which keep the associated scaled cumulative workload asymptotically close to a pre-specified linear trajectory, uniformly over time. Under such admission control policies and with natural assumptions on arrival distributions, suitably scaled and centered cumulative workload processes are shown to converge weakly in the path space to the solution of a d-dimensional stochastic differential equation (SDE) driven by a Gaussian process. It is shown that the admission control policy achieves moment stabilization in that the second moment of the solution to the SDE (averaged over the d-stations) is bounded uniformly for all times. In one special case of control policies, as time approaches infinity, we obtain a fractional version of a stationary Ornstein-Uhlenbeck process that is driven by fractional Brownian motion with Hurst parameter $H > 1/2$.

(XVIII) Dynamic Scheduling for Markov Modulated Single-server Multiclass Queueing Systems in Heavy Traffic. [20]. This paper studies a scheduling control problem for a single-server multiclass queueing network in heavy traffic, operating in a changing environment. The changing environment is modeled as a finite state Markov process that modulates the arrival and service rates in the system. Various cases are considered: fast changing environment, fixed environment and slow changing environment. In each of the cases, using weak convergence analysis, in particular functional limit theorems for renewal processes and ergodic Markov processes, it is shown that an appropriate "averaged" version of the classical $c\mu$ -policy (the priority policy that favors classes with higher values of the product of holding cost c and service rate μ) is asymptotically optimal for an infinite horizon discounted cost criterion.

(XIX) Bounded-size rules: The barely subcritical regime. [8] Bounded-size rules are dynamic random graph processes which incorporate limited choice along with randomness in the evolution of the system. One starts with the empty graph and at each stage two edges are chosen uniformly at random. One of the two edges is then placed into the system according to a decision rule based on the sizes of the components containing the four vertices. For bounded-size rules, all components of size greater than some fixed $K \geq 1$ are accorded the same treatment. Writing $\mathrm{BS}(t)$ for the state of the system with $nt/2$ edges, Spencer and Wormald proved that for such rules, there exists a critical time t_c such that when $t < t_c$ the size of the largest component is of order $\log n$ while for $t > t_c$, the size of the largest component is of order n. In this work we obtain upper bounds (that hold with high probability) of order $n^{2\gamma} \log^4 n$, on the size of the largest component, at time instants $t_n = t_c - n^{-\gamma}$, where $\gamma \in (0, 1/4)$. This result for the barely subcritical regime forms a key ingredient in the study undertaken in [9], of the asymptotic dynamic behavior of the process describing the vector of component sizes and associated complexity of the components for such random graph models in the critical scaling window. The proof uses a coupling of BSR processes with a certain family of inhomogeneous random graphs with vertices in the type space $\mathbb{R}_+ \times D([0, \infty) : \mathbb{N}_0)$ where $D([0, \infty) : \mathbb{N}_0)$ is the Skorohod D-space of functions that are right continuous and have left limits equipped with the usual Skorohod topology. The coupling construction also gives an alternative characterization (than the usual explosion time of the susceptibility function) of the critical time t_c for the emergence of the giant component in terms of the operator norm of integral operators on certain L^2 spaces.

(XX) The augmented multiplicative coalescent and critical dynamic random graph models. [9] Random graph models with limited choice have been studied extensively with the goal of understanding the mechanism of the emergence of the giant component. One of the standard models are the Achlioptas random graph processes on a fixed set of n vertices. Here at each step, one chooses two edges uniformly at random and then decides which one to add to the existing configuration according to some criterion. An important class of such rules are the bounded-size rules where for a fixed $K \geq 1$, all components of size greater than K are treated equally. While a great deal of work has gone into analyzing the subcritical and supercritical regimes, the nature of the critical scaling window, the size and complexity (deviation from trees) of the components in the critical regime and nature of the merging dynamics has not been well understood. In this work we study such questions for general bounded-size rules. Our first main contribution is the construction of an extension of Aldous's standard multiplicative coalescent process which describes the asymptotic evolution of the vector of sizes and surplus of all components. We show that this process, referred to as the standard augmented multiplicative coalescent (AMC) is 'nearly' Feller with a suitable topology on the state space. Our second main result proves the convergence of suitably scaled component size and surplus vector, for any bounded-size rule, to the standard AMC. The key ingredients here are a precise analysis of the asymptotic behavior of various susceptibility functions near criticality and certain bounds from [8], on the size of the largest component in the barely subcritical regime.

(XXI) Central Limit Results for Jump-Diffusions with Mean Field Interaction and a Common Factor [22] A system of N weakly interacting particles whose dynamics is given in terms of jump-diffusions with a common factor is considered. The common factor is described through another jump-diffusion and the coefficients of the evolution equation for each particle depend, in addition to its own state value, on the empirical measure of the states of the N particles and the common factor. Systems with a common factor arise in many different areas. In Mathematical Finance, they have been used to model correlations between default probabilities of multiple firms. In neuroscience modeling these arise as systematic noise in the external current input to a neuronal ensemble. For particle approximation schemes for stochastic partial differential equations (SPDE), the common factor corresponds to the underlying driving noise in the SPDE. The goal of this work is to study a general family of weakly interacting jump-diffusions with a common factor. Our main objective is to establish a suitable Central Limit Theorem(CLT). A key point here is that due to the presence of the common factor, the limit of particle empirical measures will in general be a random measure. This in particular means that the centering in the fluctuation theorem will typically be random as well and one expects the limit law for such fluctuations to be not Gaussian but rather a 'Gaussian mixture'. Our main result provides a CLT under quite general conditions . The summands in this CLT can be quite general functionals of the trajectories of the particles with suitable integrability properties. The key idea is to first consider a closely related collection of N stochastic processes that, conditionally on a common factor, are independent and identically distributed. By introducing a suitable Radon-Nikodym derivative one can evaluate the expectations associated with a perturbed form of the original scaled and centered sum in terms of the conditionally i.i.d. collection. The asymptotics of the latter quantity are easier to analyze using, in particular, the classical limit theorems for symmetric statistics. The perturbation arises due to the fact that in the original system the evolution of the common factor jump-diffusion depends on the empirical measure of the states of the N-particles whereas in the conditionally i.i.d. construction the common factor evolution is determined by the large particle limit of the empirical measures. Estimating the error introduced by this perturbation is one of the key technical challenges in the proof. An application to models in Mathematical Finance of self-excited correlated defaults is described.

(XXII) On the multi-dimensional skew Brownian motion [**4**]. We provide a new, concise proof of weak existence and uniqueness of solutions to the stochastic differential equation for the multidimensional skew Brownian motion. We also present an application to Brownian particles with skew-elastic collisions.

Let Σ denote the hyperplane $\{x \in \mathbb{R}^d : x_1 = 0\}$ in $\mathbb{R}^d$, $d \geq 1$, and let a vector field $b : \Sigma \to \mathbb{R}^d$ be given on it. Consider the stochastic differential equation (SDE), for a process X taking values in $\mathbb{R}^d$, of the form

$$X(t) = x + W(t) + \int_0^t b(X(s))dL(s), \qquad t \geq 0, \tag{12}$$

where $x \in \mathbb{R}^d$, W is a standard d-dimensional Brownian motion and L is the local time of X at Σ, and b is bounded and Lipschitz, and satisfies $b_1(x) \in [-1, 1]$ for all $x \in \Sigma$. This paper provides a new proof of weak existence and uniqueness of solutions to (12). A more general equation that allows for a bounded, measurable drift coefficient is also treated. These results are known by the work of Takanobu(1986,1987). Our purpose here is to provide a much shorter proof. Takanobu studies in a sequence of two papers an equation of the form (12) with general drift and diffusion coefficients. The first paper treats existence of weak solutions and the second proves uniqueness. Existence is shown by constructing a suitable approximating diffusion with a non-singular drift while uniqueness relies on rather involved arguments from Brownian excursion theory. While our results are a special case of those established by Takanobu, the proof (of both existence and uniqueness) provided here is much shorter and more elementary. Existence of solutions is established by constructing a 'skew random walk' and showing that in diffusion scaling it converges in distribution to a weak solution of the equation. The proof of weak uniqueness provided here is inspired by the technique used in Weinryb(1984) in a one-dimensional setting with time-varying skewness. As an application we consider a model introduced by Fernholz et al. (2013) for the dynamics of a pair of 1-dimensional Brownian particles X_1 and X_2 that exhibit various possible types of interaction when they collide. The equations involve the local time at zero of the relative position, and the types of interaction are determined by the coefficients in front of the local time terms. In contrast to Fernholz et al. our work allows for state-dependent local time coefficients. This allows one to model variability in the type of collision, where the type is determined by the collision position.

(XXIII) Moderate Deviation Principles for Stochastic Differential Equations with Jumps [17]. Large deviation principles for small noise diffusion equations have been extensively studied in the literature. Since the original work of Freidlin and Wentzell(1970), model assumptions have been significantly relaxed and many extensions have been studied in both finite-dimensional and infinite-dimensional settings.

The goal of this work is to study moderate deviation problems for stochastic dynamical systems. In such a study one is concerned with probabilities of deviations of a smaller order than in large deviation theory. Consider for example an independent and identically distributed (iid) sequence $\{Y_i\}_{i\geq 1}$ of $\mathbb{R}^d$-valued zero mean random variables with common probability law ρ. A large deviation principle (LDP) for $S_n = \sum_{i=1}^n Y_i$ will formally say that for $c > 0$

$$\mathbb{P}(|S_n| > nc) \approx \exp\{-n \inf\{I(y) : |y| \geq c\}\},$$

where for $y \in \mathbb{R}^d$, $I(y) = \sup_{\alpha\in\mathbb{R}^d}\{\langle \alpha, y\rangle - \log \int_{\mathbb{R}^d} \exp\langle \alpha, y\rangle \rho(dy)\}$. Now let $\{a_n\}$ be a positive sequence such that $a_n \uparrow \infty$ and $n^{-1/2}a_n \to 0$ as $n \to \infty$ (e.g. $a_n = n^{1/4}$). Then a moderate deviation principle (MDP) for S_n will say that

$$\mathbb{P}(|S_n| > n^{1/2}a_n c) \approx \exp\{-a_n^2 \inf\{I^0(y) : |y| \geq c\}\},$$

where $I^0(y) = \frac{1}{2}\left\langle y, \Sigma^{-1}y\right\rangle$ and $\Sigma = \mathrm{Cov}(Y)$. Thus the moderate deviation principle gives estimates on probabilities of deviations of order $n^{1/2}a_n$ which is of lower order than n and with a rate function that is a quadratic form. Since $a_n \to \infty$ as slowly as desired, moderate deviations bridge the gap between a central limit approximation and a large deviations approximation. Moderate deviation principles for discrete time systems have been extensively studied in Mathematical Statistics (see references in [17]), however for continuous time stochastic dynamical systems such principles are less well studied and none of the prior results consider stochastic dynamical systems with jumps or infinite dimensional models.

In this paper we study moderate deviation principles for finite and infinite dimensional SDE with jumps. For simplicity we consider only settings where the noise is given in terms of a PRM and there is no Brownian component. However, the more general case where both Poisson and Brownian noises are present can be treated similarly. In finite dimensions, the basic stochastic dynamical system we study takes the form

$$X^\varepsilon(t) = x_0 + \int_0^t b(X^\varepsilon(s))ds + \int_{\mathbb{X}\times[0,t]} \varepsilon G(X^\varepsilon(s-), y)N^{\varepsilon^{-1}}(dy, ds).$$

Here $b : \mathbb{R}^d \to \mathbb{R}^d$ and $G : \mathbb{R}^d \times \mathbb{X} \to \mathbb{R}^d$ are suitable coefficients and $N^{\varepsilon^{-1}}$ is a Poisson random measure on $\mathbb{X}_T = \mathbb{X}\times[0, T]$ with intensity measure $\varepsilon^{-1}\nu_T = \varepsilon^{-1}\nu\otimes\lambda_T$, where $\mathbb{X}$ is a locally compact

Polish space, ν is a locally finite measure on $\mathbb{X}$, λ_T is the Lebesgue measure on $[0,T]$ and $\varepsilon > 0$ is the scaling parameter. Under conditions X^ε will converge in probability (in a suitable path space) to X^0 given as the solution of the ODE

$$\dot{X}^0(t) = b(X^0(t)) + \int_{\mathbb{X}} G(X^0(t), y)\nu(dy), \; X^0(0) = x_0.$$

The moderate deviations problem for $\{X^\varepsilon\}_{\varepsilon>0}$ corresponds to studying asymptotics of

$$(\varepsilon/a^2(\varepsilon)) \log \mathbb{P}(Y^\varepsilon \in \cdot),$$

where $Y^\varepsilon = (X^\varepsilon - X^0)/a(\varepsilon)$ and $a(\varepsilon) \to 0$, $\varepsilon/a^2(\varepsilon) \to 0$ as $\varepsilon \to 0$. In this paper we establish a moderate deviations principle under suitable conditions on b and G. We in fact give a rather general sufficient condition for a moderate deviation principle to hold for systems driven by Poisson random measures. This sufficient condition covers many finite and infinite dimensional models of interest. A typical infinite dimensional model corresponds to the SPDE

$$\begin{aligned} dX^\varepsilon(u,t) &= (LX^\varepsilon(u,t) + \beta(X^\varepsilon(u,t)))dt + \varepsilon \int_{\mathbb{X}} G(X^\varepsilon(u,t-),u,y) N^{\varepsilon^{-1}}(ds,dy) \qquad (13) \\ X^\varepsilon(u,0) &= x(u), \quad u \in O \subset \mathbb{R}^d. \end{aligned}$$

where L is a suitable differential operator, O is a bounded domain in $\mathbb{R}^d$ and the equation is considered with a suitable boundary condition on ∂O. Here $N^{\varepsilon^{-1}}$ is a PRM as above. The solution of such a SPDE has to be interpreted carefully, since typically solutions for which $LX^\varepsilon(u,t)$ can be defined classically do not exist. We follow the framework of Kallianpur and Xiong(1995), where the solution space is described as the space of RCLL trajectories with values in the dual of a suitable nuclear space. We establish a MDP for such infinite dimensional systems by verifying the sufficient condition given in the work.

(XXIV) Long Time Results for a Weakly Interacting Particle System in Discrete Time [25]. Stochastic dynamical systems that model the evolution of a large collection of weakly interacting particles have long been studied in statistical mechanics, however in recent years such models have been considered in many other application areas as well, some examples include, chemical and biological systems(e.g. biological aggregation, chemotactic response dynamics), mathematical finance (e.g. mean field games, default clustering in large portfolios), social sciences (e.g. opinion dynamics models), communication systems etc. Starting from the work of Sznitman(1991) there has been an extensive body of work that studies law of large number behavior (Propagation of Chaos), central limit theory (normal fluctuations from the mean) and large deviation principles for such models. All of these results concern the behavior of the system over a finite time horizon. In many applications the time asymptotic behavior of the system is of central concern. For example, stability of a communication system, steady state aggregation and self organization in biological and chemical systems, long term consensus formation mechanisms in opinion dynamics modeling, particle based approximation methods for invariant measures all rely on a careful analysis of the time asymptotic behavior of such systems. Such behavior for special families of weakly interacting particle systems has been considered by several authors (see [25] for references). In the current work we consider a discrete time weakly interacting particle system and the corresponding nonlinear Markov process in $\mathbb{R}^d$, described in terms of a general stochastic evolution equation. Denoting by $X_n^i \equiv X_n^{i,N}$ the state of the i-th particle ($i = 1, ..., N$) at time instant n, the evolution is given as

$$X_{n+1}^i = AX_n^i + \delta f(X_n^i, \mu_n^N, \epsilon_{n+1}^i), \qquad i = 1, .., N, \quad n \in \mathbb{N}_0 \tag{14}$$

Here $\mu_n^N := \frac{1}{N}\sum_{i=1}^N \delta_{X_n^i}$ is the empirical measure of the particle values at time instant n, A is a $d \times d$ matrix, δ is a small parameter, $\{\epsilon_n^i, i = 1, ..., N, \quad n \geq 1\}$ is an i.i.d array of $\mathbb{R}^m$ valued random variables with common probability law θ and $f : \mathbb{R}^d \times \mathcal{P}(\mathbb{R}^d) \times \mathbb{R}^m \to \mathbb{R}^d$ is a measurable function. Also, $\{X_0^i, i = 1, ..., N\}$ are taken to be exchangeable with common distribution μ_0. The following nonlinear Markov chain corresponds to the $N \to \infty$ limit of (14).

$$X_{n+1} = AX_n + \delta f(X_n, \mu_n, \epsilon_{n+1}), \quad \mathcal{L}(X_n) = \mu_n, \ n \in \mathbb{N}_0. \tag{15}$$

where $\mathcal{L}(X)$ denotes the probability distribution of a random variable X with values in some Polish space S. Under conditions on f, θ, δ and A we study several long time properties of the N-particle system and the associated nonlinear Markov chain. Our starting point is the evolution equation for the law of the nonlinear Markov chain given by the equation

$$\mu_{n+1} = \Psi(\mu_n). \tag{16}$$

where $\Psi : \mathcal{P}(\mathbb{R}^d) \to \mathcal{P}(\mathbb{R}^d)$ is defined as $\Psi(\mu) = \mu P^\mu$.

We show that under conditions, that include a Lipschitz property of f with the Wasserstein-1($\mathcal{W}_1$) distance on the space of probability measures, contractivity of A and δ being sufficiently small, (16) has a unique fixed point and starting from an arbitrary initial condition convergence to the fixed point occurs at an exponential rate. Using this result we next argue that under an additional integrability condition, as $N \to \infty$, the empirical measure μ_n^N of the N-particles at time n converges to the law μ_n of the nonlinear Markov process at time n, in the $\mathcal{W}_1$ distance, in L^1, *uniformly* in n. This result in particular shows that the $\mathcal{W}_1$ distance between μ_n^N and the unique fixed point μ_∞ of (16) converges to zero as $n \to \infty$ and $N \to \infty$ in any order. This result is key in developing particle based numerical schemes for approximating the fixed point of the evolution equation (16). We next show that under an irreducibility condition on the underlying Markovian dynamics the unique invariant measure Π_∞^N of the N-particle dynamics is μ_∞-chaotic, namely as $N \to \infty$, the projection of Π_∞^N on the first k-coordinates converges to $\mu_\infty^{\otimes k}$ for every $k \geq 1$. This propagation of chaos property all the way to $n = \infty$ crucially relies on the uniform in time convergence of μ_n^N to μ_∞. The next three results study the rate of this uniform convergence by developing suitable probability concentration estimates. The first result, under an assumption of polynomial moments on the initial data and noise sequence establishes a corresponding uniform in time polynomial concentration bound. The proof relies on an idea of restricting measures to a compact set and estimates on metric entropy introduced in Bolley et. al.(2007). The basic idea is to first obtain a concentration bound for the $\mathcal{W}_1$ distance between the truncated law and its corresponding empirical law in a compact ball of radius R along with an estimate on the contribution from the region outside the ball and finally optimize suitably over R. The last two results are concerned with exponential concentration. These impose much stronger integrability conditions on the problem data. The first considers the setting where the initial random variables form a general exchangeable sequence and gives a concentration bound with an exponential decay rate of $N^{\frac{1}{d+2}}$. The second result uses exponential concentration estimates for empirical measures of i.i.d. sequences based on recent results on transportation inequalities In this case the concentration bound gives an exponential decay rate of order N.

(XXV) On Uniform Positivity of Transition Densities of Small Noise Constrained Diffusions [13].

Diffusions in polyhedral domains have been extensively studied in the heavy traffic limit theory for stochastic processing networks (see references in [13]). In a recent work [6] small noise asymptotics for a general family of such constrained diffusions have been studied. The precise setting there is as follows. Let $G \subset \mathbb{R}^k$ be convex polyhedral cone with a nonempty interior with the vertex at origin given as the intersection of half spaces $G_i, i = 1, 2, ..., N$. Let n_i be the unit inward normal vector associated with G_i via the relation

$$G_i = \{x \in \mathbb{R}^k \, : \, \langle x, n_i \rangle \geq 0\}.$$

We will denote the set $\{x \in \partial G \, : \, \langle x, n_i \rangle = 0\}$ by F_i. With each face F_i we associate a unit vector d_i such that $\langle d_i, n_i \rangle > 0$. This vector defines the *direction of constraint* associated with the face F_i. Roughly speaking, a constrained diffusion considered in this work is a process that evolves infinitesimally as a diffusion in $\mathbb{R}^k$ and is instantaneously pushed back using the oblique reflection direction d_i upon reaching the face F_i. Formally, such a process can be represented as a solution of a stochastic integral equation of the form

$$X_x^\varepsilon(t) = \Gamma \left(x + \int_0^\cdot b(X^\varepsilon(s))ds + \varepsilon \int_0^\cdot \sigma(X^\varepsilon(s))dW(s) \right)(t), \tag{17}$$

where Γ is the Skorohod map taking trajectories with values in $\mathbb{R}^k$ to those with values in G, consistent with the constraint vectors $\{d_i, i = 1, \cdots N\}$. Under certain regularity assumptions on the Skorohod map and the usual Lipschitz conditions on the coefficients b and σ, the above integral equation has a unique pathwise solution. One of the main results of [6] is an 'exponential leveling' property of exit times from bounded domains for such small noise diffusions. Such results for diffusions in $\mathbb{R}^k$ have been obtained in Day(1982) which is concerned with asymptotics of Dirichlet problems in bounded domains associated with diffusions with infinitesimal generator of the form

$$\mathcal{L}_\varepsilon f(x) = \frac{\varepsilon^2}{2} \text{Tr}(\sigma(x) D^2 f(x) \sigma(x)^{tr}) + b(x) \cdot \nabla f(x), \; f \in C_b^2(\mathbb{R}^k). \tag{18}$$

Here $\sigma(x)^{tr}$ is the transpose of the matrix $\sigma(x)$. The precise result of Day is as follows. Denote by Z_x^ε the diffusion process governed by the generator $\mathcal{L}_\varepsilon$ and initial distribution δ_x. Let B be a bounded domain in $\mathbb{R}^k$ and K be an arbitrary compact subset of B. Suppose that all solutions of the ODE $\dot{\xi} = b(\xi)$ with $x = \xi(0) \in B$ converge, without leaving B, to a single linearly asymptotically stable critical point in B. Then, with suitable conditions on the coefficients of the diffusion, for all bounded measurable f

$$\sup_{x,y \in K} |\mathbb{E}\left(f(Z_x^\varepsilon(\tau_x^\varepsilon))\right) - \mathbb{E}\left(f(Z_y^\varepsilon(\tau_y^\varepsilon))\right)|$$

converges to 0 at an exponential rate. Here, $\tau_x^\varepsilon = \inf\{t : Z_x^\varepsilon(t) \in B^c\}$. This property is a statement on the long time behavior of the diffusion and says that although, as $\varepsilon \to 0$, the exit time of the process from the domain approaches ∞, the expected values of functionals of exit location, corresponding to distinct initial conditions, coalesce asymptotically, at an exponential rate. The key ingredient in the proof is the gradient estimate

$$\sup_{x\in K} |\nabla u^{\varepsilon}(x)| \leq c\varepsilon^{-1/2}, \tag{19}$$

where u^{ε} is the solution of the Dirichlet problem

$$\begin{cases} \mathcal{L}_{\varepsilon} u^{\varepsilon}(x) = 0, & x \in B \\ u(x) = f(x), & x \in \partial B. \end{cases} \tag{20}$$

Diffusions of interest in the current work are constrained to take values in domains with corners and where the constraining mechanism is governed by an oblique reflection field that changes discontinuously from one face of the boundary to another. To the best of our knowledge there are no regularity (e.g. C^1 solutions) results known for the associated partial differential equations(PDE) with oblique reflecting boundary condition. In view of this, a probabilistic approach for the study of exponential leveling property for such constrained settings, that 'almost' bypasses all PDE estimates was developed in [6]. The main step in the proof is the construction of certain (uniform in ε) Lyapunov functions under a suitable stability condition which are then used to construct a coupling of the processes X^{ε}_x, X^{ε}_y with explicit uniform estimates on exponential moments of time to coupling. The key ingredient in this coupling construction is a, uniform in ε, minorization condition on transition densities of the reflected diffusions. The paper [6] gave one simple example with constant drift and diffusion coefficients where such a condition is satisfied. However the question of when such a minorization property is available was left as an open problem.

The objective of this work is to answer this question and give general conditions under which the minorization property holds. The main result of the paper shows that under mild conditions the desired minorization statement holds. This result, together with results of [6] then gives general sufficient conditions for an exponential leveling property to hold for a broad family of constrained diffusion processes. The proof at one step crucially relies on an interior lower bound estimate for the Dirichlet heat kernel of $\mathcal{L}_{\varepsilon}$ over bounded domains (Riahi(2005)). In this sense the proofs are not fully probabilistic.

(XXVI) Dynamic Scheduling for Markov Modulated Single-server Multiclass Queueing Systems in Heavy Traffic [20].

Heavy traffic formulation provides tractable approximations for complex queueing systems that capture broad qualitative features of the networks. While most works in the literature deal with fixed (or internal network-state-dependent) rates, with the advent of modern wireless networks there is an explosion of research on models for networks where external factors such as meteorological variables (temperature, humidity, etc) can affect the transmission rates to and from the servers. The current paper deals with the simplest such setting of a multiclass network, namely a single server multiclass queueing system in heavy traffic, where the arrival and services fluctuate according to an environment process. In the classical constant rate setting this model has been studied in van Mieghem(1995), in the conventional heavy traffic regime. The queueing system consists of a single server which can process K different classes of jobs ($K \geq 1$). The K classes represent different types of jobs (voice traffic, data traffic etc.), and they have different arrival and service rate functions.

We assume that these functions are modulated by a finite state Markov process that represents the background environment. The arrival and service rates satisfy a suitable heavy traffic assumption which, loosely speaking, says that the network capacity and service requirements are balanced in the long run. We consider a scheduling control problem, where the controller decides (dynamically, at each time point $t \geq 0$) which class of jobs should the server process so as to minimize an infinite horizon discounted cost function, which involves a linear holding cost per job per unit time for each of the K different classes. The jump rates of the modulating Markov process are modeled by a scaling parameter ν. The value of ν can vary from negative to positive – the larger the value of ν, the faster the environment changes – and we consider three distinct regimes for ν. We show that under different values of ν, with suitably scaling, queue length processes stabilize leading to different diffusion approximations. Main result of the paper shows that, in each of these regimes, an asymptotically optimal scheduling policy is a variation of the classical and intuitively appealing $c\mu$-rule. Classical $c\mu$-rule is a simple priority policy, where the priority is always given to the class (of nonempty queue) with the highest $c\mu$-value where c and μ represent the holding cost and the service rate of that class, respectively. It is well known that $c\mu$-rule is optimal in many cases, see van Mieghem(1995) and references therein. In our model, the arrival rates change randomly according to the background Markov process, and it is far from clear if the classical $c\mu$-rule would be optimal. In addition, in one of the regimes we consider, the service rates are allowed to be modulated by the random environment as well, which means that the order of the $c\mu$-values keeps changing according to the environment. In particular, in situations where the environment process is not directly observable and one only knows (or can estimate) its statistical properties, the classical $c\mu$-rule cannot be implemented. We propose a modified $c\mu^*$-rule, where μ^* represents the average service rate of each class and the average is taken with respect to the stationary distribution of the environment process. Under the $c\mu^*$-rule, the server always processes jobs from the nonempty queue with the highest $c\mu^*$-value. We show that, under an appropriate heavy traffic condition and a suitable scaling, the $c\mu^*$-rule is asymptotically optimal for the chosen cost functional for this environment-dependent queueing model, in each of the three regimes for ν.

(XXVII) Large deviations for Multidimensional State-Dependent Shot Noise Processes [24].

Shot noise processes are used in applied probability to model a variety of physical systems in, for example, teletraffic theory, insurance and risk theory and in the engineering sciences. In this work we prove a large deviation principle for the sample-paths of a general class of multidimensional state-dependent Poisson shot noise processes. The result covers previously known large deviation results for one dimensional state-independent shot noise processes with light tails. We use the weak convergence approach to large deviations, which reduces the proof to establishing the appropriate convergence of certain controlled versions of the original processes together with relevant results on existence and uniqueness.

(XXVIII) Individual confidence intervals for true solutions to stochastic variational inequalities [33]. Stochastic variational inequalities (SVI) provide a means for modeling various optimization and equilibrium problems where data are subject to uncertainty. Often it is necessary to estimate the true SVI solution by the solution of a sample average approximation (SAA) problem. This paper proposes three methods for building confidence intervals for components of the true solution, and those intervals are computable from a single SAA solution. The first two methods use an "indirect approach" that requires initially computing asymptotically exact confidence intervals for the solution to the normal map formulation of the SVI. The third method directly constructs confidence intervals for the true SVI solution; intervals produced with this method meet a minimum specified level of confidence in the same situations for which the first two methods are applicable. We justify the three methods theoretically with weak convergence results, discuss how to implement these methods, and test their performance using two numerical examples.

Ph.D. Dissertation Abstract of Xin Liu. Applications arising from computer, telecommunications, and manufacturing systems lead to many challenging problems in the simulation, stability, control, and design of stochastic models of networks. The networks are usually too complex to be analyzed directly and thus one seeks suitable approximate models. One class of such approximations are diffusion models that can be rigorously justified when networks are operating in heavy traffic, i.e., when the network capacity is roughly balanced with network load. We study stochastic networks with time varying arrival and service rates and rout- ing structure. Time variations are governed, in addition to the state of the system, by two independent finite state Markov processes X and Y. Transition times of X are significantly smaller than the typical interarrival and processing times whereas the reverse is true for the Markov process Y. We first establish a diffusion approximation for such multiscale queueing networks in heavy traffic. The result shows that, under appropriate heavy traffic conditions, properly normalized queue length processes converge weakly to a Markov modulated reflected diffusion process. More precisely, the limit process is a reflected diffusion with drift and diffusion coefficients that are functions of the state process, the invariant distribution of X and a finite state Markov process which is independent of the driving Brownian motion. We then study the stability properties of such Markov modulated reflected diffusion processes and establish positive recurrence and geometric ergodicity properties under suitable stability conditions. As consequences, we obtain results on the moment generating function of the invariant probability measure, uniform in time moment estimates and functional central limit results for such processes. We also study relationship between invariant measures of the Markov modulated constrained diffusion processes and that of the underlying queueing network. It is shown that, under suitable heavy traffic stability conditions, the invariant probability measure of the queueing network converges to that of the corresponding Markov modulated reflected diffusion model. The last part of this dissertation focuses on ergodic control problems for discrete time controlled Markov chains with a locally compact state space and a compact action space under suitable stability, irreducibility and Feller continuity conditions. We introduce a flexible family of controls, called action time sharing (ATS) policies, associated with a given continuous stationary Markov control. It is shown that the long term average cost for such a control policy, for a broad range of one stage cost functions, is the same as that for the associated stationary Markov policy. Through examples we illustrate the use of such ATS policies for parameter estimations and adaptive control problems.

Ph.D. Dissertation Abstract of Dominik Reinhold. This dissertation is composed of two parts, a theoretical part, in which certain asymptotic properties of near critical branching processes are studied, and an applied part, consisting of statistical analysis of cell growth data. First, near critical single type Bienaym e-Galton-Watson (BGW) processes are con- sidered. It is shown that, under appropriate conditions, Yaglom distributions of suitably scaled BGW processes converge to that of the corresponding diffusion approximation. Convergences of stationary distributions for Q-processes and models with immigration to the corresponding distributions of the associated diffusion approximations are estab- lished as well. Moreover, convergence of Yaglom distributions of suitably scaled multitype subcritical BGW processes to that of the associated diffusion model is established. Next, near critical catalyst-reactant branching processes with controlled immigration are considered. The catalyst population evolves according to a classical continuous time branching process, while the reactant population evolves according to a branching process whose branching rate is proportional to the total mass of the catalyst. Immigration takes place exactly when the catalyst population falls below a certain threshold, in which case the population is instantaneously replenished to the threshold to ensure a certain activity level. A diffusion limit theorem for the scaled processes is established, in which the catalyst limit is a reflected diffusion with reflection at 1, and the reactant limit is a diffusion with coefficients depending on the reactant. The driving Brownian motions in the two diffusions are independent.

Stochastic averaging under fast catalyst dynamics are considered next. In the setting where both catalyst and reactant evolve according to the above described (reflected) diffusions, but where the evolution of the catalyst is accelerated by a factor of n, we establish a scaling limit theorem, as $n \to \infty$, in which the reactant process is asymp- totically described through a one dimensional SDE with coefficients depending on the invariant distribution of the catalyst reflected diffusion. Convergence of the stationary distribution of the scaled catalyst branching process (with immigration) to that of the limit reflected diffusion is established as well. Finally, results from a collaborative proof-of-principle study, relating cell growth to the stiffness of the surrounding environment, are presented.

Ph.D. Dissertation Abstract of Jiang Chen. This dissertation studies large deviations problems for stochastic dynamical systems. First, a family of Stochastic Partial Differential Equations (SPDE) driven by a Poisson Random Measure (PRM) that are motivated by problems of chemical/pollutant dispersal are considered. A Large Deviation Principle (LDP) for the long time profile of the chemical concentration using techniques based on variational representations for nonnegative functionals of general PRM is established. Second, a LDP for small Poisson noise perturbations of a general class of deterministic infinite dimensional models is studied. SPDEs driven by PRM have been proposed as models for many different physical systems. The approach taken here, which is based on variational representations, reduces the proof of the LDP to establishing basic qualitative properties for controlled analogues of the underlying stochastic system. Third, stochastic systems with two time scales are studied. Such multiscale systems arise in many applications in engineering, operations research and biological and physical sciences. The models considered in this dissertation are usually referred to as systems with full dependence, which refers to the feature that the coefficients of both the slow and the fast processes depend on both variables. A LDP for such systems with degenerate diffusion coefficients is established. The last part of this dissertation focuses on numerical schemes for computing invariant measures of reflected diffusions. Reflected diffusions in polyhedral domains are commonly used as approximate models for stochastic processing networks in heavy traffic. Stationary distributions of such models give useful information on the steady state performance of the corresponding stochastic networks and thus it is important to develop reliable and efficient algorithms for numerical computation of such distributions. A Monte-Carlo scheme based on an Euler type discretization is proposed and analyzed. An almost sure consistency result that proves the convergence of the appropriately weighted empirical measures constructed from the simulated discretized reflected diffusion to the true diffusion model is established. Rates of convergence are also obtained for certain class of test functions. Some numerical examples are presented to illustrate the applicability of this approach.

Ph.D. Dissertation Abstract of Xuan Wang. This dissertation consists of two parts. The first part studies phase transition of a class of dynamical random graph processes, that evolve via the addition of new edges in a manner that incorporates both randomness as well as limited choice. As the density of edges increases, the graphs display a phase transition from the subcritical regime, where all components are small, to the supercritical regime, where a giant component emerges. The goal is to understand the behavior at criticality. First, the simplest model of this kind, namely the Bohman-Frieze process is considered. It is shown that the stochastic process of component sizes, in the critical window for the Bohman-Frieze process after proper scaling, converges to the standard multiplicative coalescent. Next, a more general family of dynamical random graph models is studied, namely, the bounded-size-rule processes. A useful upper bound on the size of the largest component in the barely subcritical regime is established. Next, this upper bound is used to study both sizes and surplus of the components of the bounded-size-rule processes in the critical window. In order to describe the joint evolution of sizes and surplus, the augmented multiplicative coalescent is introduced. The main result shows that the vector of suitably scaled component sizes and surplus converges in distribution to the augmented multiplicative coalescent. In the second part of this dissertation, a large deviation problem related to the configuration model with a given degree distribution is studied. A random walk associated with the depth-first-exploration of the random graph constructed from the configuration model is introduced. The large deviation principle of this random walk is studied using weak convergence techniques. Some large deviation bounds on the probabilities related to the sizes of the largest component are proved.

Invited Presentations.

- *On Some Free Boundary Problems for Stochastic Systems.* AMS Spring Eastern Sectional Meeting, University of Maryland, Baltimore County, Baltimore, MD, March 29-30, 2014.
- *Large Deviations for Stochastic Dynamical Systems Driven by a Poisson Noise.* 2014 SIAM Conference on Uncertainty Quantification, Hyatt Regency Savannah, Savannah, GA, March 31-April 3, 2014.
- *Moderate Deviations Principles for Stochastic Dynamical Systems.* AMS Southeastern Spring Sectional Meeting University of Tennessee, Knoxville, Knoxville, TN March 21-23, 2014.
- *Infinity Laplacian and Stochastic Differential Games.* Quasilinear PDEs and Game Theory, December 2-4, 2013, Institut Mittag-Leffler and Uppsala University, Sweden.
- *Asymptotics of Infinite Dimensional Small Noise Systems.* NSF/CBMS Conference: Analysis of Stochastic Partial Differential Equations, August 19-23, Michigan State University, 2013.
- *Infinity Laplacian and Stochastic Differential Games*, Probability and Mathematical Physics Seminar, Courant Institute, NYU, May 3, 2013.
- *Long Time Asymptotics of Constrained Diffusions in Polyhedral Cones*, Probability Seminar, Univ. Wash., Seattle, May 20, 2013.

- *Large Deviations and Variational Representations for Infinite Dimensional Systems*, IMA Annual Program Year Workshop, Theory and Applications of Stochastic PDEs January 14-18, 2013, IMA Minnesota.

- *Large deviation problems for infinite dimensional stochastic systems*, Math. Colloquium at Wayne State University, May 2012.

- *A Stochastic Differential Game for the Infinity Laplacian.*, Applied Probability Seminar at U. Texas at Austin, Jan 2012.

- *Some Variational Formulas with Applications to Large Deviations.*, Mathematical Finance and Probability Seminar at Rutgers University, Nov 2011.

- *A Stochastic Differential Game for the Infinity Laplacian.* Special Session on Stochastic Analysis, 2012 Spring Central Section Meeting University of Kansas, Lawrence, March 30 - April 1, 2012.

- *A Stochastic Differential Game for the Infinity Laplacian.* Special Session on Game Theory, U.S. Army Conference on Applied Statistics, 19-21 October, 2011 Annapolis, MD.

- *Some Variational Formulas for Space-Time Brownian Motions and Poisson Random Measures*, Thirty Third Midwest Probability Colloquium, October 13-15, 2011, Northwestern University, Evanston, IL.

- *Variational representations for Poisson random measures and infinite dimensional Brownian motions*, 35th Conference on Stochastic Processes and their Applications, 19–24 June, 2011, Oaxaca, Mexico.

- *A Stochastic Differential Game for the Inhomogeneous Infinity-Laplace Equation*, Institute of Mathematical Statistics, 73rd Annual Meeting, August 2010, Gothenburg, Sweden.

- *A stochastic differential game for the inhomogeneous infinity-Laplace equation*, Seminar on Probability and its Applications, Penn State University, October 2010.

- *A stochastic differential game for the inhomogeneous infinity-Laplace equatio*, Stochastics Seminar at Georgia Tech, September 2010.

References

[1] S. A. Almada and A. Budhiraja. Infinite Dimensional Forward-Backward Stochastic Differential Equations and the KPZ Equation *Electron. J. Probab.*, 19 (2014), no. 40, 121.

[2] R. Atar and A. Budhiraja. A stochastic differential game for the inhomogeneous ∞-Laplace equation. *Ann. Prob.*, 38 (2010), no. 2, 498–531.

[3] R. Atar and A. Budhiraja. On near optimal trajectories for a game associated with the ∞-Laplacian. *Prob. Th. Rel. Fields*, 151 (2011), no. 3–4, 509–528.

[4] R. Atar and A. Budhiraja . On the multi-dimensional skew Brownian motion. *Submitted*

[5] R. Atar, A. Budhiraja and P.Dupuis On Positive Recurrence of Constrained Diffusion Processes. *Annals of Probability*, 29 (2001), no. 2, 979-1000.

[6] A. Biswas and A.Budhiraja. Exit Time and Invariant Measure Asymptotics for Small Noise Constrained Diffusions, *Stoch. Proc. App.*, 121 (2011), no. 5, 899–924.

[7] S. Bhamidi, A. Budhiraja and X. Wang Bohman-Frieze processes at criticality and emergence of the giant component *To Appear in Rand. Str. Alg.*.

[8] S. Bhamidi, A. Budhiraja and X. Wang Bounded-size rules: The barely subcritical regime. *Submitted (arXiv:1212.5480).*

[9] S. Bhamidi, A. Budhiraja and X. Wang The augmented multiplicative coalescent and critical dynamic random graph models *Submitted (arXiv:1212.5493).*

[10] A. Budhiraja. An ergodic control problem for constrained diffusion processes: Existence of optimal Markov control. *SIAM J. Control Optim*, 42(2):532–558, 2003.

[11] A. Budhiraja, J. Chen and P. Dupuis Large Deviations for Stochastic Partial Differential Equations Driven by a Poisson Random Measure. *To Appear in Stoch. Proc. App.*.

[12] A. Budhiraja, J. Chen and S. Rubenthaler A Numerical Scheme for Invariant Distributions of Constrained Diffusions. *To Appear in Math. Oper. Res.*.

[13] A. Budhiraja and Z.Q. Chen On Uniform Positivity of Transition Densities of Small Noise Constrained Diffusions. *Electronic Communications in Probability*, 19 (2014), no. 1, 1-9.

[14] A. Budhiraja, P..Dupuis and M. Fischer Large deviation properties of weakly interacting processes via weak convergence methods. *Ann. Prob.*, 40 (2012), no. 1, 74–102.

[15] A. Budhiraja, P. Del Moral and S. Rubenthaler Discrete Time Markovian Agents Interacting Through a Potential. *To Appear in ESAIM: Probability & Statistics.*

[16] A. Budhiraja and P. Dupuis. A variational representation for positive functional of infinite dimensional Brownian motions. *Probability and Mathematical Statistics*, 20:39–61, 2000.

[17] A. Budhiraja, P..Dupuis and A. Ganguly Moderate Deviation Principles for Stochastic Differential Equations with Jumps. *Submitted.*

[18] A. Budhiraja, P. Dupuis and V. Maroulas. Variational Representations for Continuous Time Processes, *Annales de l'Institut Henri Poincaré, Probabilités et Statistiques*, 47 (2011), no. 3, 725–747.

[19] A. Budhiraja, A. Ghosh and C. Lee. Ergodic Rate Control Problem for Single Class Queueing Networks, *SIAM J. Cont. App.*, 47 (2011), no. 3, 725–747.

[20] A. Budhiraja, A. Ghosh and X.Liu. Dynamic Scheduling for Markov Modulated Single-server Multiclass Queueing Systems in Heavy Traffic, *Submitted.*

[21] A. Budhiraja and C. Lee. Stationary distribution convergence for generalized Jackson networks in heavy traffic. *Math. Oper. Res.* , Vol. 34, No. 1, February 2009, pp. 45-56.

[22] A. Budhiraja, E.Kira and S.Saha. Central limit results for jump-diffusions with mean field interaction and a common factor. *Submitted.*

[23] A. Budhiraja and X. Liu. Multiscale Diffusion Approximations for Stochastic Networks in Heavy Traffic, *Stoch. Proc. App.*, 121 (2011), no. 3, 630–656.

[24] A. Budhiraja and P. Nyquist. Large deviations for Multidimensional State-Dependent Shot Noise Processes, *Submitted.*

[25] A. Budhiraja and A. Pal Majumdar. Long Time Results for a Weakly Interacting Particle System in Discrete Time. *Submitted.*

[26] A. Budhiraja , V. Pipiras and X. Song. Admission Control for Multidimensional Workload with Heavy Tails and Fractional Ornstein-Uhlenbeck Process (with V. Pipiras and X. Song)., *Submitted*.

[27] A.Budhiraja, A. Shwartz and X.Liu. Action Time Sharing Policies for Ergodic Control of Markov Chains. *SIAM J. Cont. Opt.*, 50 (2012), no. 1, 171–195.

[28] A.Budhiraja and A.Ghosh. Diffusion approximations for controlled stochastic networks: An asymptotic bound for the value function. *Ann. App. Prob.*, 16, no. 4 (2006), 1962–2006.

[29] A.Budhiraja and A.Ghosh. Controlled Stochastic Networks in Heavy Traffic: Convergence of Value Functions, *Ann. App. Prob.*, Volume 22, Number 2 (2012), 734–791.

[30] A. Budhiraja and X. Liu. Stability of Constrained Markov Modulated Diffusions, *To appear in Math. Oper. Res.*.

[31] A. Budhiraja and D. Reinhold. Near Critical Catalyst Reactant Branching Processes with Controlled Immigration, *To Appear in Ann. App. Prob.*.

[32] H. J. Kushner and L. F. Martins. Limit theorems for pathwise average cost per unit time problems for controlled queues in heavy traffic. *Stochastics Stochastics Rep.*, 42(1):25–51, 1993.

[33] Individual confidence intervals for true solutions to stochastic variational inequalities . *Submitted*

[34] S. Lu and A. Budhiraja Confidence regions for stochastic variational inequalities *To appear in Math. Oper. Res.*.

[35] Y. Peres, O. Schramm, S. Sheffield and D. B. Wilson. Tug-of-war and the infinity Laplacian. *Jour. AMS*, to appear

[36] M. I. Reiman. Open queueing networks in heavy traffic. *Mathematics of Operations Research*, 9(3): 441–458, 1984.

[37] K. Yamada. Diffusion approximation for open state-dependent queueing networks in the heavy traffic situation. *Ann. Appl. Probab.*, 5(4):958–982, 1995.

[38] X. Zhang. Clark-Ocone formula and variational representation for Poisson functionals. To appear in *Annals of Probab.*