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Successive Over-Relaxation Technique for High-Performance Blind Image Deconvolution

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14. ABSTRACT

This award has allowed to develop a new, mathematically sound algorithm of the blind image deconvolution, which demonstrates a superior performance when compared with existing techniques. The concept change in our approach to blind image deconvolution is in the basic strategy of addressing sensible approximate solutions to the ill-posed nonlinear inverse problem. These solutions are addresses as fixed points of the iteration which consists in alternating approximations (AA) for the object and for the PSF performed with a prescribed number of inner iterative descents from trivial (zero) initial guess. This approach shall be contrasted with traditional inexact alternating minimization (AM) approach, where a stationary point of the objective function is being targeted by the global descent trajectory, and monotonic descent to this point is terminated to achieve regularization. Artificial inversions with noise-free images show that the new approach allows the successful deconvolution of data of higher complexity, where the AM approach suffers from stagnation. In our vision, the stagnation is caused by approaching a local critical point of the cost function; these points are not necessarily the fixed points of our iteration. Inversions with real data produce solutions which are more accurate, free from the artifacts, and do not depend on the initial guess.

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S. V. Vorontsov and S. M. Jefferies

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Below is a brief overview of the algorithm. Its complete description can be found in S. V. Vorontsov and S. M. Jefferies 2015 "A new approach to blind deconvolution of astronomical images". This paper has just been submitted for publication to the *Inverse Problems*, and is attached to this Report.

THE ALGORITHM {Alternating Approximation method for blind deconvolution $\mathbf{x}*\mathbf{y}=\mathbf{f}$}
  Choose initial pair of regularization parameters (k_x,k_y)
  Choose non-zero initial guess for point-spread function **y**
  `REPEAT` {Graduated optimization}
    `REPEAT` {Fixed point iteration}
      $\mathbf{x}=\mathbf{x}(\mathbf{f};\mathbf{y})$ using k_x descents with +SOR from zero guess
      $\mathbf{y}=\mathbf{y}(\mathbf{f};\mathbf{x})$ using k_y descents with +SOR from zero guess
    `UNTIL` convergence
    Increase (k_x,k_y)
  `UNTIL` stopping condition is met.

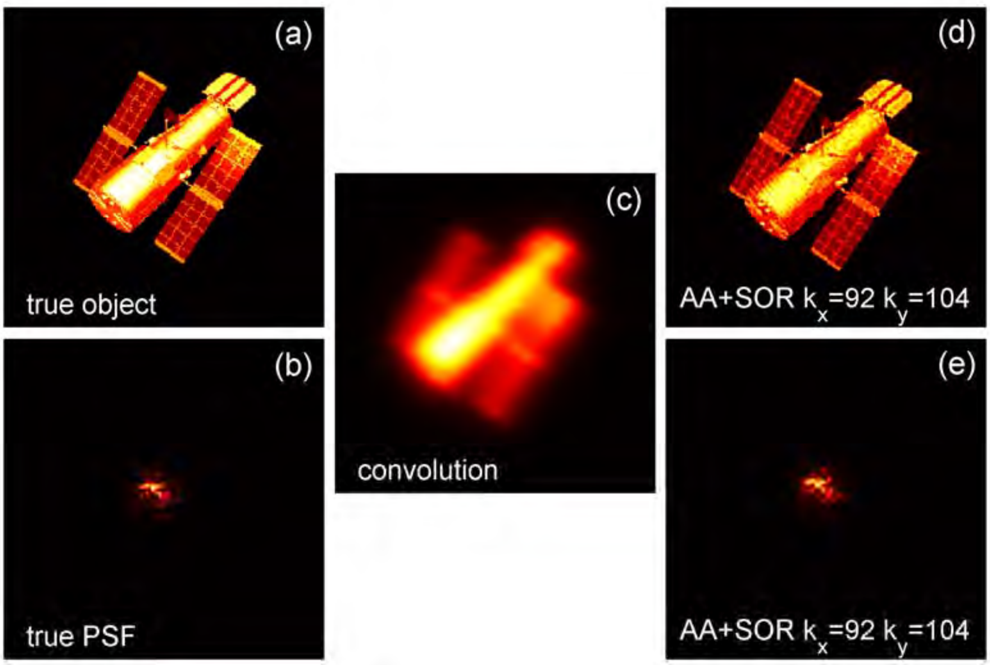
Inner minimization routine performs standard (non-blind) deconvolution, using non-negativity-constrained successive over-relaxation technique (+SOR) with adaptive scheduling of the relaxation parameter. This routine was inherited from our pre-award research, and was added with some few improvements and with deeper convergence analysis. In our numerical experience, this routine is exceptionally efficient and robust when working with data of arbitrary complexity. A particularly attractive feature of the routine is that it is free from any tunable parameters---apart, of coarse, from the number of iterations where the descent is terminated to prevent excessive noise magnification. A natural stopping condition is the flattening of the Fourier amplitude of the residuals to near the noise level.

Fixed-point iteration. The result of the non-blind deconvolution for object **x** with given PSF **y** using k_x iterations of +SOR, $\mathbf{x}=\mathbf{x}(\mathbf{f};\mathbf{y})$ is used in non-blind deconvolution for **y** with k_y descents, $\mathbf{y}=\mathbf{y}(\mathbf{f};\mathbf{x})$, and the process is repeated until convergence. In our algorithm, **x** belongs to a compact convex set, and the existence of a fixed point of function $\mathbf{x}(\mathbf{f};\mathbf{y}(\mathbf{f};\mathbf{x}))$ is warranted by the Brouwer fixed point theorem. We consider attractive fixed points corresponding to different values of k_x and k_y as candidates for a sensible solution to the blind-deconvolution problem. The solution becomes independent on the initial guess when the first attractor is reached. To make a distinction with AM methods, our approach to blind deconvolution in its general form (+SOR can be replaced by another non-blind deconvolution routine) shall be probably designated as an "alternating approximation" (AA) method. Interestingly, we have not met any reports on implementing this approach in literature on nonnegative matrix factorization - a wide area of research with numerous applications, our blind-deconvolution problem being just a particular example.

Graduated optimization. For a given pair of regularization parameters (k_x,k_y), the iteration $\mathbf{x}\leftarrow\mathbf{x}(\mathbf{f};\mathbf{y}(\mathbf{f};\mathbf{x}))$ may have multiple attractive fixed points. Our numerical experiments show that multiplicity of the attractors grows when regularization is relaxed (bigger k_x,k_y), and when addressing data of higher complexity (e.g. with a PSF of more complicated morphology). To find those attractors which correspond to sensible approximate solutions, we start with an easier problem of finding an appropriate attractor at small values of (k_x,k_y), and then relax the regularization gradually by going to bigger values of (k_x,k_y), using the PSF of the previous fixed point as the initial guess. This approach, which is known in the literature as graduated optimization, is closely related with approach known as deterministic annealing.

When working with data of moderately small complexity, this approach demonstrates excellent results. Like with any other methods, it has to reveal its own problems and limitations when the data complexity becomes high. We have addressed these limitations by performing numerous artificial inversions. When image complexity is characterized by the parameter D/r_0, where D is the aperture size and r_0 is a coherence scale of atmospheric turbulence, the technique allows decent deconvolution of single images with D/r_0 up to about 20. This limitation is apparently related with either some imperfections of the inner minimization routine, or with too wide steps in the regularization parameter space (integer k_x,k_y). Further enhancement of the performance of the technique needs more efforts. We note, however, that these efforts are only relevant to practical inversions when data has an exceptionally high signal-to-noise ratio.

Numerical results. We were focused largely on the most difficult problem of single-frame blind deconvolution, which is important for data sets that have objects that change pose quickly and for which multi-frame approaches are not valid.



This figure illustrates one of the results of our artificial experiments with simulated data (here, D/r_0 =10), targeted at examining the potential capability of the deconvolution technique to accurately split the information, contained in the blurred images, between the object and the PSF. More results can be found in our paper. There, we also report the results obtained with two sets of satellite images acquired using ground-based telescopes with and without adaptive optics compensation at the Air Force's AMOS site on Maui, and extend the technique to multi-frame deconvolution. The results are compared with those obtained by alternating minimization (AM) using the positivity-constrained conjugate-gradient version of the PCID algorithm, which is in routine use at the AMOS facility on Maui, demonstrating much better performance of the new technique.

Further efforts. There are at least three directions where the technique calls for extensions. First, the algorithm in its current form implements least-squares (LS) approximations, which maximize the solution likelihood only when the noise statistics is stationary Gaussian. In reality, noise in the images is close to Poissonian (photon shot noise). Despite the observations that accounting for the correct noise statistics does not change the results of image deconvolution significantly when compared with the LS approach, adaptation of the technique to proper statistics has to be done, as otherwise the results can not be of minimum variance because they are not based on a correct data weighting. The main difficulty is in modifying the theoretical recipe for scheduling the relaxation parameter of the +SOR routine, which is used in minimization.

When applied to multi-frame deconvolution, our technique (together with others) attributes equal weights to different data frames, and treats them on an equal basis. Adaptation has to be made to account for the fact that separate frames are often distorted by the atmospheric turbulence to a very different extent. This problem is related with a similar one in the single-frame deconvolution, where an optimal relation between the number of descents in the object- and in the PSF space has to be addressed from the theoretical standpoint.

The band-limited nature of astronomical imagery (which comes from the finite aperture size) has to exploited by the algorithm. This is particularly important for attempts of improving the resolution of distant objects to beyond the diffraction limit by using over-sampling (collecting data on a grid finer than the diffraction limit). An obvious way is re-parameterization of the PSF in terms of the wave-front distortion at the aperture plane. The difficulty to overcome is in the additional source of nonlinearity introduced to the inverse problem.

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7 June 2015