

| Report Documentation Page                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                   |                                    |                                                           | Form Approved<br>OMB No. 0704-0188                  |                                 |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------|------------------------------------|-----------------------------------------------------------|-----------------------------------------------------|---------------------------------|
| Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. |                                   |                                    |                                                           |                                                     |                                 |
| 1. REPORT DATE<br><b>30 SEP 2001</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                   | 2. REPORT TYPE                     |                                                           | 3. DATES COVERED<br><b>00-00-2001 to 00-00-2001</b> |                                 |
| 4. TITLE AND SUBTITLE<br><b>Wave Chaos and Chaotic Transmission of Waves</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                   |                                    |                                                           | 5a. CONTRACT NUMBER                                 |                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                   |                                    |                                                           | 5b. GRANT NUMBER                                    |                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                   |                                    |                                                           | 5c. PROGRAM ELEMENT NUMBER                          |                                 |
| 6. AUTHOR(S)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                   |                                    |                                                           | 5d. PROJECT NUMBER                                  |                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                   |                                    |                                                           | 5e. TASK NUMBER                                     |                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                   |                                    |                                                           | 5f. WORK UNIT NUMBER                                |                                 |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br><b>Courant Institute of Mathematical Sciences,,New York University,251 Mercer St,,New York,,NY, 10012</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                   |                                    |                                                           | 8. PERFORMING ORGANIZATION REPORT NUMBER            |                                 |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                   |                                    |                                                           | 10. SPONSOR/MONITOR'S ACRONYM(S)                    |                                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                   |                                    |                                                           | 11. SPONSOR/MONITOR'S REPORT NUMBER(S)              |                                 |
| 12. DISTRIBUTION/AVAILABILITY STATEMENT<br><b>Approved for public release; distribution unlimited</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                   |                                    |                                                           |                                                     |                                 |
| 13. SUPPLEMENTARY NOTES                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                   |                                    |                                                           |                                                     |                                 |
| 14. ABSTRACT<br><b>Our goal is to develop an analytical and numerical approach to diagnose arrival sound waves from long-distanced sources on the basis of the most advanced methods of dynamics that include ray dynamics, wave dynamics reconstruction on the basis of ray dynamics, specific asymptotic solutions in the short-wave approximation, and specific diagnostic codes adjusted to the wave-chaos analysis.</b>                                                                                                                                                                                                                                                                                                                                                                                                                           |                                   |                                    |                                                           |                                                     |                                 |
| 15. SUBJECT TERMS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                   |                                    |                                                           |                                                     |                                 |
| 16. SECURITY CLASSIFICATION OF:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                   |                                    | 17. LIMITATION OF ABSTRACT<br><b>Same as Report (SAR)</b> | 18. NUMBER OF PAGES<br><b>3</b>                     | 19a. NAME OF RESPONSIBLE PERSON |
| a REPORT<br><b>unclassified</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | b ABSTRACT<br><b>unclassified</b> | c THIS PAGE<br><b>unclassified</b> |                                                           |                                                     |                                 |

# **Wave Chaos and Chaotic Transmission of Waves**

George M. Zaslavsky  
Courant Institute of Mathematical Sciences  
New York University  
251 Mercer St.  
New York, New York 10012  
phone: 212-998-3259 fax: (212) 995-4121 email: [zaslav@cims.nyu.edu](mailto:zaslav@cims.nyu.edu)

Award Number: N00014-97-1-0426

## **LONG-TERM GOALS**

Our goal is to develop an analytical and numerical approach to diagnose arrival sound waves from long-distanced sources on the basis of the most advanced methods of dynamics that include ray dynamics, wave dynamics reconstruction on the basis of ray dynamics, specific asymptotic solutions in the short-wave approximation, and specific diagnostic codes adjusted to the wave-chaos analysis.

## **OBJECTIVES**

Long-range sound propagation in ocean can be studied using contemporary methods of nonlinear dynamics, resonance theory, theory of chaos and, particularly, quantum chaos.

Our research is directed towards developing and understanding of new phenomenon: wave-chaos. The Results can be applied to wave propagation in plasma layers and toroidal plasma devices, to mesoscopic systems (electron transport and conductivity of quantum wires and dots).

## **APPROACH**

In our research we use:

1. WKB-method
2. Topological analysis of chaotic orbits in the phase space
3. Kinetic theory and fractional kinetics
4. High-performance simulations

## **WORK COMPLETED**

It has been demonstrated that the mechanism of emergence of ray chaos due to overlapping of nonlinear ray-medium resonances should play an important role in long range sound propagation.

We considered quantum breaking time scaling in superdiffusive dynamics.

We considered weak mixing and anomalous kinetics along filamented surfaces.

## RESULTS

The properties of ray travel times, including fractal properties of the timefront fine structure, under condition of ray chaos have been investigated. It has been shown that coexistence of chaotic and regular rays, typical for dynamical chaos, leads to the appearance of gaps in ray travel time distributions, which are absent in unperturbed waveguides.

We demonstrate by simulation that quantum breaking time scales as  $\tau_h \sim (1/\hbar)^{1/\mu}$  with  $\mu > 1$  that corresponds to superdiffusive dynamics.

We showed that in a square billiard with a horizontal bar in the middle the transport is anomalous and that its properties can be linked to the ergodic properties of continued fractions.

## IMPACT/APPLICATIONS

General theory of wave propagation in waveguides.

## REFERENCES

1. “Theory and application of ray chaos to the underwater acoustics” by I.P. Smirnov, A.L. Virovlyansky, and G.M. Zaslavsky (Published in Phys. Rev. E, Vol. 64, 036221 (2001))
2. “Quantum breaking time scaling in superdiffusive dynamics” by A.Iomin and G.M. Zaslavsky (Published in Phys. Rev. E, Vol. 63, 047203 (2001))
3. “Weak mixing and anomalous kinetics along filamented surfaces” by G.M. Zaslavsky and M. Edelman (Published in Chaos, Vol. 11, pp. 295-305 (June 2001))