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Technical Memorandum

A PRACTICAL APPROACH TO THE ESTIMATION OF
AMPLITUDE AND TIME DELAY PARAMETERS OF A
COMPOSITE SIGNAL

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ABSTRACT

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I. INTRODUCTION

A problem of practical concern that has received a great deal of attention is that of time delay estimation. Most of this past research [1], [2], [3], [4], has focused on problems of the form

$$r_1(t) = s(t) + n_1(t)$$

$$r_2(t) = s(t-D) + n_2(t)$$

Another more complicated problem of research interest is one in which the model is

$$r_1(t) = s(t)$$

$$r_2(t) = n(t) + \sum_{i=1}^M A_i s(t-D_i)$$

where $s(t)$ and M are known and we desire to estimate the scaling amplitudes A_1, A_2, \dots, A_M and delays D_1, D_2, \dots, D_M . For example, this could be a problem where a known probe signal $s(t)$ excites a linear time invariant filter whose output is observed in the presence of non-white Gaussian noise, and where the impulse response of the filter is a

finite sum of time-shifted delta functions with different amplitudes.

Closely related problems have been studied and reported in [5], [6],[7] and [8]. Here, we take a different approach in the derivation that results in a practical, easily interpretable, and computationally fast solution to the problem. We view this as being a useful contribution to an important engineering problem.

II. APPROACH/SOLUTION

First, we are given that $s(t)$ and $n(t)$ are time-limited ($0 \leq t \leq T$) sample functions of non-white, zero mean Gaussian processes uncorrelated with each other, $D_i \geq 0$, and $D_1 = 0$. Furthermore, $s(t)$ is completely known as is the power spectral density of its process, $G_{SS}(f)$. We also require that the maximum difference between any two delays is less than the total observation time. That is

$$\max |D_i - D_j| < T \text{ for } i \neq j$$

Then we derive the maximum likelihood (ML) estimator making approximations and dropping unnecessary terms.

Using the results of Carter [2], we can show that the ML estimator is to minimize

$$J_A = \int_{-\infty}^{\infty} \left[\frac{\hat{G}_{11}(f)}{G_{11}(f)} + \frac{\hat{G}_{22}(f)}{G_{22}(f)} \right] \frac{1}{[1 - C_{12}(f)]} df$$

$$- 2 \int_{-\infty}^{\infty} \frac{\hat{G}_{12}(f)}{G_{nn}(f)} \sum_{i=1}^M A_i e^{+j2\pi f D_i} df$$

through the selection of A_i 's and D_i 's. The letter G denotes power spectra, f is frequency and the hat denotes an estimate based on observed data. Using the fact that C_{12} is the magnitude squared coherence, letting L replace the entire summation term and switching the minimization to a maximization by changing signs, we find should maximize

$$J_B = 2 \int_{-\infty}^{\infty} \frac{\hat{G}_{12}}{G_{nn}} L^* df - \int_{-\infty}^{\infty} \frac{\hat{G}_{11} [G_{ss}/L^2 + G_{nn}]}{G_{nn} G_{ss}} df$$

$$- \int_{-\infty}^{\infty} \frac{\hat{G}_{22}}{G_{nn}} df$$

where we have dropped the explicit notational reminder of "f" dependence and used the term

$$L(f) = \sum_{i=1}^M A_i e^{-j2\pi f D_i}$$

By expanding and dropping terms not dependent upon the A_i 's and D_i 's we obtain the new expression to be maximized as:

$$J_c = 2 \int_{-\infty}^{\infty} \frac{\hat{G}_{12} L^*}{G_{nn}} df - \int_{-\infty}^{\infty} \frac{\hat{G}_{11} |L|^2}{G_{nn}} df$$

As will be seen, this frequency domain formulation is closely related to the time domain formulation of Ehrenberg, et. al. [6], when the appropriate assumptions are made.

Estimating the Amplitudes

^

We observe in J_c that $G_{11} = G_{ss}$ and test how J_c works for estimating amplitudes in the following simple example:

$$r_1(t) = s(t)$$

$$r_2(t) = A_1 s(t-D_1) + n(t)$$

Assume (for this example) that $D_1 \neq 0$ and $n(t)$ is white with unit power. Then we maximize:

$$J_c = 2 \int_{-\infty}^{\infty} \hat{G}_{12}(f) A_1 e^{j2\pi f D_1} df - \int_{-\infty}^{\infty} G_{ss}(f) A_1^2 df$$

$$= 2 A_1 \hat{R}_{12}(D_1) - A_1^2 R_{ss}(0)$$

^

where R_{12} is the estimated cross-correlation function between $r_1(t)$ and $r_2(t)$, and R_{ss} is the autocorrelation function of $s(t)$.

Taking the partial derivative with respect to A_1 and setting it equal to zero, we get:

$$\frac{\partial J_c}{\partial A_1} = 2 \hat{R}_{12}(D_1) - 2A_1 R_{ss}(0) = 0$$

So that

$$A_1 = \frac{\hat{R}_{12}(D_1)}{R_{ss}(0)}$$

^

In this example, if the noise were not white, $R_{12}(D_1)$ would be the noise prewhitened generalized cross-

correlation function evaluated at the delay, D_1 , and which we designate as $R_{12}'(D_1)$, and $R_{SS}(0)$ would be replaced by the noise prewhitened auto-correlation, $R_{SS}'(0)$, where

$$\hat{R}_{12}'(D_1) = \int_{-\infty}^{\infty} \frac{\hat{G}_{12}(f)}{G_{nn}(f)} e^{+j2\pi f D_1} df$$

and

$$R_{SS}'(0) = \int_{-\infty}^{\infty} \frac{G_{SS}(f)}{G_{nn}(f)} df$$

These results can be generalized by taking the partial derivatives of J_C with respect to A_j and setting each equal to zero,

$$\begin{aligned} \frac{\partial J_C}{\partial A_j} &= \frac{\partial}{\partial A_j} [2A_j \hat{R}_{12}'(D_j)] \\ &= \frac{\partial}{\partial A_j} \int_{-\infty}^{\infty} |L(f)|^2 \frac{G_{SS}(f)}{G_{nn}(f)} df = 0 \end{aligned} \quad j=1,2,\dots,M$$

Before taking the partial derivative of $|L(f)|^2$ we identify the critical portions of $|L(f)|^2$ depending on A_j that have a non-zero partial derivative. This yields

$$|L(f)|_{A_j}^2 = \left[A_j e^{+j2\pi f D_j} \sum_{i=1}^M A_i e^{+j2\pi f D_i} + A_j e^{-j2\pi f D_j} \sum_{i=1}^M A_i e^{+j2\pi f D_i} \right]$$

and hence

$$\frac{\partial}{\partial A_j} \int_{-\infty}^{\infty} |L(f)|^2 \frac{G_{ss}(f)}{G_{nn}(f)} df = 2 \sum_{i=1}^M A_i R_{ss}'(D_j - D_i)$$

In order to make useful progress from here, we further restrict our problem to the case when we can resolve the delays relative to the correlation time of $s(t)$. That is, the difference between any two delay values must be greater than the correlation time of $s(t)$. To help clarify this, we determine the cross-correlation between $r_1(t)$ and $r_2(t)$:

$$R_{12}(d) = E [r_1(t) r_2(t+d)]$$

$$= E [s(t) \left[\sum_{i=1}^M A_i s(t-D_i+d) + n(t+d) \right]]$$

$$= \sum_{i=1}^M A_i R_{ss}(d-D_i)$$

So we see that $R_{12}(d)$ is a sum of shifted replicas of $R_{SS}(d)$ with amplitude weightings A_i . Now if $R_{SS}(d) \approx 0$ for $|d| > d_c/2$ we say that values of $s(t)$ are uncorrelated for time differences greater than d_c , and loosely refer to d_c as the correlation time of $s(t)$. Thus we can state our restriction on the differences in the delays as

$$|D_i - D_j| > d_c \quad i \neq j$$

With this restriction we can make the observation that:

$$R_{SS}'(D_j - D_i) \approx 0, \quad i \neq j$$

Therefore, we conclude that

$$\frac{\partial J_c}{\partial A_j} \approx 2 \hat{R}_{12}'(D_j) - 2 A_j R_{SS}'(0) = 0$$

so that

$$A_j = \frac{\hat{R}_{12}'(D_j)}{R_{SS}'(0)}$$

Thus the only thing we need to estimate the attenuations is the noise prewhitened generalized cross correlation function

at the correct time delays and the noise prewhitened signal power.

For nearly white noise

$$R_{SS}'(0) \approx R_{SS}(0)/G_{nn}$$

where G_{nn} is the noise power spectrum level. In this case

$$A_j \approx \frac{\Lambda R_{12}'(D_j) G_{nn}}{R_{SS}(0)}$$

Estimating the Delays

We now obtain equations for D_j 's. Expand J_C to get

$$J_D = \int_{-\infty}^{\infty} \frac{\hat{G}_{12} L^*}{G_{nn}} df + \int_{-\infty}^{\infty} \frac{\hat{G}_{12} L^*}{G_{nn}} df - \int_{-\infty}^{\infty} \frac{\hat{G}_{11}}{G_{nn}} |L|^2 df$$

We note that maximizing J_D is equivalent to maximizing J_C .
At this point we derive the cross power spectrum:

$$\begin{aligned} G_{12}(f) &= F[R_{12}(\tau)] \\ &\quad M \\ &= F[E\{s(t) \sum_{i=1}^M A_i s(t-D_i + \tau)\}] \\ &\quad i=1 \\ &\quad M \\ &= G_{SS}(f) \sum_{i=1}^M A_i \exp(-j2\pi f D_i) \\ &\quad i=1 \\ &= G_{SS}(f) L(f) \end{aligned}$$

Where F denotes taking the Fourier Transform, and E is the expectation.

Now we focus attention on the last two terms of J_D . If we assume $r_1(t)$ is a noise free copy of $s(t)$ then we do not need to estimate $G_{11}(f)$ since it is simply $G_{SS}(f)$. When we get in the neighborhood of the correct estimate of G_{12} such that $G_{12}L^* = G_{SS}L L^*$ we see that J_D is approximately equal to

$$J_E = \int_{-\infty}^{\infty} \frac{\hat{G}_{12} L^*}{G_{nn}} df + \int_{-\infty}^{\infty} \frac{G_{SS} |L|^2}{G_{nn}} df - \int_{-\infty}^{\infty} \frac{G_{SS} |L|^2}{G_{nn}} df$$

We note that this step is an approximation since we have only time limited signal and noise sample functions. With the approximation, the last two terms cancel. Hence, for problems of practical interest, the function to be maximized can be written as

$$J_F = \int_{-\infty}^{\infty} \frac{\hat{G}_{12}(f)}{G_{nn}(f)} \sum_{i=1}^M A_i e^{+j\omega f D_i} df$$

Even though J_F looks complicated we will now show how this

form of the answer leads to a significant simplification in the practical engineering solution of this problem.

The maximization of J_F can be rewritten as

$$J_F = \sum_{i=1}^M A_i \int_{-\infty}^{\infty} \frac{\hat{G}_{12}(f)}{G_{nn}(f)} e^{+j2\pi f D_i} df$$

We see that the term to be maximized can be interpreted as a weighted finite sum of generalized cross-correlation functions with noise prewhitening. Note also that the integration term is a Fourier transform of a weighted cross power spectrum; once it has been computed (e.g., via an FFT) it does not have to be recomputed for other D_i 's. In terms of the noise prewhitened cross-correlation function the maximization is of

$$J_F = \sum_{i=1}^M A_i R_{12}'(D_i)$$

by selection of D_i 's.

Now we substitute the expression for A_i derived earlier and we have

$$J_F = \frac{1}{R_{ss}'(0)} \sum_{i=1}^M [R_{12}'(D_i)]^2$$

Finally, noting that $R_{ss}'(0)$ is just a constant, we see that maximizing J_F is equivalent to choosing the D_i 's which maximize

$$J_G = \sum_{i=1}^M [\hat{R}_{12}'(D_i)]^2$$

The selected D_i 's are also subject to the constraint that for $i \neq j$, then $D_j \neq D_i$ and $|D_j - D_i|$ must be greater than the correlation time of $s(t)$. To put it simply, we find the M highest peaks of $[R_{12}'(d)]^2$ that are separated by at least the correlation time of $s(t)$. If we make use of our earlier assumption that $D_1=0$, and introduce an additional constraint that $D_i > D_j$ for $i > j$ we can have a meaningful ordering of our A_i and D_i estimates without affecting the generality of our solutions. Note that the squaring and summing of the generalized cross-correlation function takes care of negative peaks possibly occurring at some of the D_i 's.

IV. COMPARISON WITH OTHER RESULTS

Here, we make comparisons between our results and previously published works that deal with problems closely

related, but not identical to the one we have just addressed.

Comparison to Ehrenberg, Ewart, and Morris [6]

It can readily be shown that our results and those of [6] become identical when we impose our constraint of

$$R_{SS}(D_i - D_j) = 0 \quad \text{for } i \neq j$$

and make their assumption of white noise with two-sided power spectral density $N_0/2$. So for more restrictive assumptions on delay resolvability but less restrictive assumptions on noise complexity, we obtain an easy to implement processor.

Comparison to Knapp and Carter [3]

For a single time delay modelled by

$$r_1(t) = s(t) + n_1(t)$$

$$r_2(t) = s(t-D) + n_2(t)$$

and under typical simplifying assumptions, Knapp and Carter[3] have shown that the ML delay estimator is the delay at which the generalized cross-correlator with proper prefilters is a maximum. In particular, for the single delay model the solution was shown to be

$$R_{12}(\tau) = F^{-1}[W(f) G_{12}(f)]$$

where F^{-1} denotes the inverse Fourier transform and the conjugate product of the prefilters of $r_1(t)$ and $r_2(t)$ is

$$W(f) = H_1(f) H_2^*(f) = \frac{1}{|G_{12}(f)|} \frac{C(f)}{[1 - C(f)]}$$

and $C(f)$ is the magnitude-squared coherence (i.e., the magnitude squared cross-power spectrum divided by the product of the two autospectra). Now for the model here with $n_1(t) = 0$ and $n_2(t) = n(t)$, we find

$$\frac{C(f)}{[1 - C(f)]_{n1} = 0} = \frac{G_{ss}(f)}{G_{nn}(f)}$$

Also for the model here

$$|G_{12}(f)| = G_{ss}(f)$$

So the estimator is to find the delay that causes the maximum of

$$R_{12}(\tau) = F^{-1} \left[\frac{1}{G_{nn}(f)} G_{12}(f) \right]$$

This is exactly the J_F term in our problem if there were only one delay and if $A_1 = 1$. There are two other points, of interest. First, Knapp and Carter [3] introduced a squaring of the correlation function in their figure showing a practical realization of picking a maxima; this, in part, takes care of negative attenuation in one receiver channel. Second, if we wanted, we could filter $s(t)$ by $H_1(f) = 1/[G_{nn}(f)]^{1/2}$ and $r_2(t)$ by the same function.

Whalen [10] has derived the ML solution for a simple case of a single attenuation. In particular, given

$$r(t) = A s(t) + n(t)$$

where $n(t)$ is white noise the ML estimate of A is

$$\hat{A} = \frac{\int_0^T r(t) s(t) dt}{\int_0^T s^2(t) dt} \cong \frac{\int_0^T r(t) s(t) dt}{R_{ss}(0)}$$

so we correlate $r(t)$ with $s(t)$; no noise prefilters are

required, since $n(t)$ is white; then we read off the correlation function at zero delay and normalize by the signal power. If we expand the solution above we see

$$\hat{A} = \frac{A \int_0^T s^2(t) dt + \int_0^T n(t)s(t) dt}{\int_0^T s^2(t) dt}$$

and that by taking the expectation the answer is unbiased.

Now if $n(t)$ is not white then from $r(t) = As(t) + n(t)$ we form

$$r'(t) = As'(t) + n'(t)$$

where r' , s' and n' are convolution outputs with a prewhitening filter

$$|H(f)|^2 = \frac{1}{G_{nn}(f)}$$

Then

$$\hat{A} = \frac{\int_0^T r'(t) s'(t) dt}{\int_0^T [s'(t)]^2 dt} \cong \frac{G_{nn} \int_0^T r'(t) s'(t) dt}{R_{ss}(0)}$$

so both $s(t)$ and $r(t)$ are subjected to a prefilter

$$|H(f)|^2 = \frac{1}{G_{nn}(f)}$$

and thus the normalization takes into account both the noise and the signal power.

V. RE-CAP OF STEPS

1. Estimate the cross power spectrum between stored replica (reference) $r_1(t) = s(t)$ and received signal $r_2(t)$.
2. Estimate the noise power spectrum $G_{nn}(f)$ by noting

$$G_{22}(f) = G_{SS}(f) |L(f)|^2 + G_{nn}(f)$$

and

$$\frac{|G_{12}(f)|^2}{G_{SS}(f)} = G_{SS}(f) |L(f)|^2$$

and computing

$$G_{nn}(f) = G_{22}(f) - [|G_{12}(f)|^2 / G_{SS}(f)]$$

3. Compute the total signal power $R_{SS}(0)$ and the total noise prewhitened signal power $R_{SS}'(0)$.
4. Compute (e.g., via an FFT) the inverse transform of the cross power spectrum divided by the noise power spectrum to get the generalized cross-correlation function. Also compute the square of this function.

5. Find the M largest peaks of the squared generalized cross-correlation function, which are separated by at least the correlation time of $s(t)$. The corresponding delay values at these peaks are the M delay estimates.

6. Determine the value of the generalized cross-correlation function at each of the M delay estimates.

7. Adjust the generalized cross-correlation values found in step 6 by dividing by the noise prewhitened signal power. These adjusted M correlation peak heights then become ML estimates of the corresponding amplitude scalings. If the noise spectrum is relatively flat (i.e., nearly white) then the estimates can be approximated by multiplying the values found in step 6 by the noise power spectrum level, then dividing them by the total signal power. There should be good agreement in the two estimates for this case.

VI. SUMMARY

We have derived a practical method for estimating the amplitudes and delays for a composite signal in additive, non-white Gaussian noise, the signal being composed of a sum of amplitude-scaled and time-delayed replicas of a completely known sample function of a Gaussian process that is uncorrelated with the corrupting noise. We have

constrained the problem to the case where the composite time-delays are resolvable with respect to the correlation time of the basis signal.

The form of our solution is easily interpretable and convenient to implement by means of generalized correlation functions. We have found that our results are consistent with related results of previous work when appropriate adjustments of constraints and assumptions are made. While our approach has been based on the principles of maximum likelihood estimation, the simplifications made in the derivation and practical limitations in observing a stochastic signal make our resultant formulation approximate maximum likelihood estimates.

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