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NAVAL UNDERWATER SYSTEMS CENTER
NEW LONDON LABORATORY
NEW LONDON, CONNECTICUT 06320

Technical Memorandum

DETERMINING THE OPERATIONAL MARGIN OF A FIBER OPTIC DATA LINK

Date: 28 February 1985

Prepared by: John R. Ball
John R. Ball
Electro-Optic Systems

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Use of the Wald Sequential Probability Ratio Test to determine the operational margin of the data link is considered. Monte Carlo simulations of link operation are used to examine the suitability of the Wald test. It is found that the Sequential Probability Ratio Test will perform adequately for the task, given certain error levels and sufficient time to gather data.

ADMINISTRATIVE INFORMATION

This memorandum was prepared during the course of the SubACS Development Engineering - Fiber Optics project, under Job Order Number B47230, John R. Ball (Code 3422), Principal Investigator. The sponsoring organization was Naval Sea Systems Command, PMS-409, sub-project S1347; the Program Manager for SubACS BASIC is Nancy Cook.

INTRODUCTION

For some time, it was assumed that part of the Performance Monitor and Fault Location (PM/FL) operations for SubACS fiber optics would involve changing the sensitivity of the fiber optic receiver in such a way as to ensure that some operational margin remained in the system. At the same time, it has been unclear how it would be determined that the system is operating at the specified Bit Error Rate of 1×10^{-10}. Since the practical determination of Bit Error Rate involves reduction of either source signal level or receiver sensitivity, it was thought that this problem might somehow be resolved by use of the PM/FL software. The author has undertaken some investigation of the relationship between operational margin and BER. This memorandum outlines this investigation and its results.

SIMPLIFIED DESCRIPTION OF THE SYSTEM

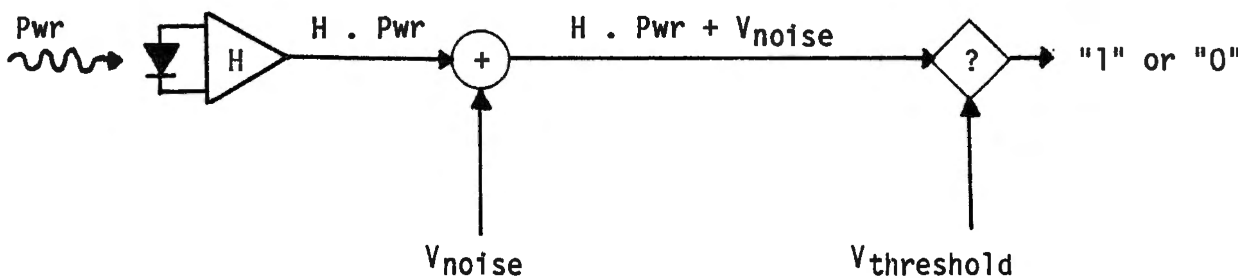


FIGURE 1

Figure 1 shows the idealized block diagram of the receiver circuit. Optical power, Pwr, is incident upon the photodiode. The electrical signal produced by the photodiode is sent to a noiseless amplifier. The transfer function of the photodiode-amplifier combination is linear and has a constant value of H (mV/mW). To the output of the amplifier is added a Gaussian noise voltage whose RMS value is Vn. The combination of the two signals, H Pwr + Vn, is one input to the comparator. The second comparator input is the threshold voltage, Vt. The comparator, errorless in itself, compares H Pwr + Vn to Vt, and if H Pwr + Vn > Vt, outputs a "1"; else, the output of the comparator is a "0".

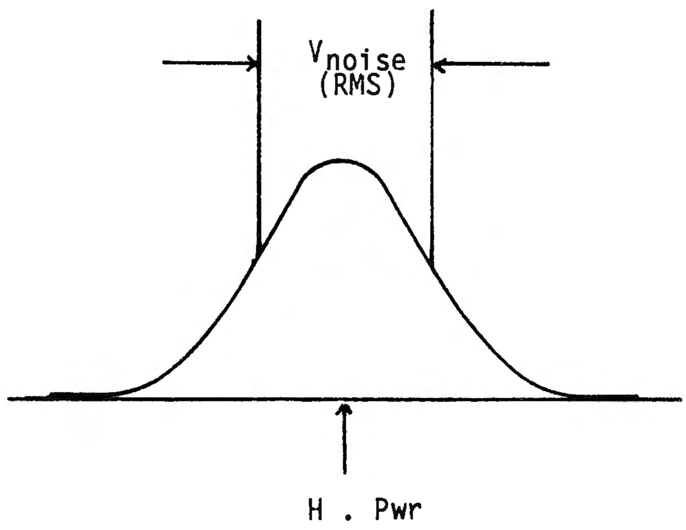


FIGURE 2

Figure 2 shows the probabilistic range of the signal applied to the comparator. Because the noise voltage fluctuates in time, at any arbitrary sampling time the signal can be any value. The noise is Gaussian, though, so the mean value of the signal is H•Pwr, and has a standard deviation of Vn. It is the relationship between Bit Error Rate (BER), noise, optical power, and threshold voltage that forms the heart of the problem.

DERIVATION OF THE MEAN BIT ERROR RATE

To be specific, there are two possible errors that can occur. If the comparator takes its sample at a time when no signal is being sent (Pwr = 0) but at the same instant the noise voltage spikes higher than the threshold voltage, the comparator will signal a "1" to the following circuitry although a "0" would be correct. Similarly, should a level of H Pwr be sent to signify a "1", but the noise spikes down sufficiently when the comparator samples, a "0" is output from the comparator. (We assume that Pwr = 0 means a "0" and that we send some higher level of Pwr to imply a "1".) If the number of errors of the first type are counted over a suitable interval, and divided by the time interval, the result is called the Bit Error Rate for false alarms; the notation for this is BER[1/0]. For the second type of error, we have the Bit Error Rate for failed detection, BER[0/1]. The Mean Bit Error Rate, MBER, is the probable mean of the two:

$$\text{MBER} = P(0)\,\text{BER}[1/0] + P(1)\,\text{BER}[0/1],$$

where P(0) is the probability of an intended "0" and P(1) is probability of an intended "1".

Thus, to compute the MBER in any case, we must consider both BER[1/0] and BER[0/1]. We will be very specific about the conditions and use the following assumptions about the SubACS fiber optics:

1) At an incident level of Pwr equivalent to -31.5 dBm, the MBER is less than or equal to 1 X 10^{-10};

2) The RMS noise voltage, Vn, is less than one-seventh of the threshold voltage, Vt;

3) The threshold voltage, in normal operation, is 100 mV;

4) During testing, Vt is raised to 200 mV;

5) P(0) = P(1) = 0.5

At this point, we must make a small mathematical digression. The general form for calculation of Gaussian probabilities is

$$P[x \leq X] = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{x} e^{-(x-\mu)^2/2\sigma^2}\, dx$$

where σ is the standard deviation and μ is the mean value. The probability that $X > x$ is $1 - P[x \leq X]$. However, this rather clumsy form is not commonly used in calculation. Rather, a change of variable is made of the form

$$t = (x-\mu)/\sigma$$

The result of this change of variables is "standard" normal distribution, which has a mean value of 0, and a standard deviation of 1. Virtually all calculations are commonly done using a standard normal distribution. In our case, we make the following identifications:

$$\sigma = V_{noise} \quad , \quad \mu = H \cdot Pwr \quad .$$

In general, the probability that the signal applied to the comparator is less than Vt is given by

$$P[v \leq V_t] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{V_t} e^{-t^2/2} dt$$

Consider the case where Pwr = 0; that is when a "0" is being signaled. The probability that, at sampling time, the voltage applied to the comparator is greater than Vt is given by

$$P[v > V_t] = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{V_t} e^{-t^2/2} dt$$

From assumption 2), Vt/Vn > 7, so when this integral is carried out, the corresponding probability is less than (roughly) 1×10^{-12}. If we set Vt/Vn = 7.03448, then BER[1/0] = 1×10^{-12}. From this, and assumption 3), we have Vn = 14.2157 mV.

If MBER = 1×10^{-10} and BER[1/0] = 1×10^{-12}, then BER[0/1] = 1.99×10^{-10}. If

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{V_t - H \cdot P_{wr}}{V_n}} e^{-t^2/2} dt = 1.99 \times 10^{-10} ,$$

then (Vt - H Pwr)/Vn = -6.25481. Using the level of power given by assumption 1), and the value of Vt given in assumption 3), we have H = 266,852 mV/mW.

If we now raise the level of Vt to 200 mV, Vt/Vn = 14.06896, and BER[1/0] drops to approximately 2.95 X 10^{-45}; for all practical purposes, MBER = BER[0/1]/2, so that BER[0/1] = 2 X 10^{-10}. In turn, this implies that (Vt - H Pwr)/Vn = -6.25403, which means that the value of Pwr corresponds to -29.66 dBm.

Note that while Vt has been doubled, the value of Pwr required to maintain the MBER at 1 X 10^{-10} has risen only by 1.84 dBm. Thus, this value of Vt during PM would only assure us that we have a margin of 1.84 dB. Further calculations show that the level of Vt should be 288 mV if we are to assure ourselves that 3 dB of margin remains. (At this value of Vt, BER[1/0] falls to about 1.471 X 10^{-91}.)

Figures 3 through 5 show the various BERs and MBER as a function of Pwr for the values of Vt discussed above. One of the most notable features is the way in which the MBER curve changes shape with the various values of Vt. When the comparator is operated with Vt = 100 mV, the rate of change of MBER about the MBER = 1 X 10^{-10} point is much more gradual than when Vt = 288 mV. A decrease in the level of Pwr by -0.5 dBm might not be readily detected when Vt = 100 mV; it should be quite apparent when Vt = 288 mV. Thus, changing the value of Vt would appear to be an acceptable method of determining that at least 3 dB of margin in the value of Pwr is available, if that is the only source of error. However, aging of the LED, cable, star, and connectors -- those factors which reduce the level of optical power reaching the receiver -- are not the only elements that can change the MBER.

EFFECT OF INTERNAL RECEIVER NOISE

Consider the effect of an increase in the RMS value of receiver noise, Vn, if applied to the equations given above. If the value of Vn doubles, the effect on bus operation is disastrous. The value of BER[1/0] rises to 2.2 X 10^{-4} and the asymptotic value of MBER is approximately 1 X 10^{-4}; these results are shown in Figure 6. If the threshold value, Vt, is now raised, and if the level of Pwr is at least -28.5 dBm, normal operation results, as is shown in Figure 7. If, however, the value of Vt during PM is modified to 288 mV, the level of Pwr required to restore normal operation is approximately -27.6 dBm (see Figure 8).

It is clear that a direct determination of MBER cannot be made by changing the threshold level. In the usual measurement of MBER, the level of optical power arriving at the receiver in externally varied until a measurable error rate is achieved. Then, from a knowledge of the MBER curves, the MBER can be extrapolated. But, as the foregoing analysis shows, varying the threshold level is not equivalent to varying the optical input level. Therefore, the problem of determining the system MBER remains unresolved.

DETERMINATION OF OPERATIONAL MARGIN

On the other hand, it may be practical and desirable to show that a certain amount of margin exits. For this purpose, IBM has suggested use of the Wald Sequential Probability Ratio Test, which we will outline shortly.

Testing for a system's operational margin presents difficulties. Examination of the BER curve for normal operation will show that the relationship between MBER and input optical power, in the region where MBER $\approx 1 \times 10^{-10}$, is not particularly monotonic. That is, we cannot distinguish easily between 0.5 dB of margin and 3 dB of margin. In both cases, the MBER is very nearly 5×10^{-13}.

In the case of "modified PM" operation, when the threshold voltage is raised to 288 mV, the problem is simplified. In this case, a system with the required 3 dB margin will exhibit an MBER of 1×10^{-10} while a system with only 2.8 dB of margin would have an MBER of approximately 1.032×10^{-7}, a change of almost 3 orders of magnitude.

However, it must be noted that the DSDB does not record bit errors, but, rather, packet error. Therefore, we must consider the Packet Error Rate (PER). We will assume that, during testing (FL), each packet will consist of 2048 bytes, each byte having 8 bits. (The fact that each byte has a ninth bit, and inverted version of the eighth bit of the byte, will change only the details of the following calculations.) Thus, each packet will consist of 16,384 bits. Then

$$\begin{aligned} PER[MBER = 1 \times 10^{-10}] &= 1 - P[\text{at least one error in 16,384 bits}] \\ &= 1.6384 \times 10^{-6} \quad , \end{aligned}$$

and

$$\begin{aligned} PER[MBER = 1.032 \times 10^{-7}] &= 1 - P[\text{at least one error in 16,384 bits}] \\ &= 1.6894 \times 10^{-3} \quad . \end{aligned}$$

* This description of the Wald Sequential Probability Ratio Test follows that found in Ostle, Statistics in Research, Iowa State University Press, Ames, Iowa, 1963, PP 140-142.

APPLICATION OF THE WALD SEQUENTIAL PROBABILITY RATIO TEST

The Wald Sequential Probability Ratio Test (PRT) is an efficient statistical method used to test simple hypotheses*. We may frame the hypothesis to be tested as follows. If the system has at least 3 dB of margin, when the threshold voltage is raised to 288 mV, the PER = 1.6384×10^{-6} or less. On the other hand, if the margin is less than 3 dB, PER $> 1.6894 \times 10^{-3}$. (Specifically, this occurs at 2.8 dB of margin.) This hypothesis is virtually the classical example of the application of the Wald Sequential Probability Ratio Test to a test of binomial probability.

To be specific, we make the following hypothesis:

Hypothesis: Probability of packet error = 1.6384×10^{-6},

Alternate: Probability of packet error = 1.6894×10^{-3}.

In testing these hypotheses, we would like the probability of rejecting a true hypothesis (Type I error) and the probability of accepting a false hypothesis (Type II error) to be small, say, 0.001. One advantage of the Wald Sequential Probability Ratio Test is that these probabilities may be assigned, a priori.

The test, as applied to the matter at hand, is quite simple. A packet is passed over the system, and, after the packet is received, one of the following decisions is made:

Accept the hypothesis (which implies 3 dB margin),

or, Accept the alternate (which implies 3 dB margin),

or, Send another packet.

The decision is based on the calculation of the "Likelihood Ratio", R_n, where

$$R_n = \prod_{i=1}^{n} \frac{f_1(x_i)}{f_0(x_i)}$$

$f_0(x_i)$ is the probability function under the assumption that the hypothesis is true, and $f_1(x_i)$ is the probability function under the assumption that the alternate is true. Let A be the probability of a Type I error, and B be the probability of a Type II error. Then, if

$R_n \leq B / (1 - A)$, we accept the hypothesis;

$R_n \geq (1 - B) / A$, we accept the alternate;

$B / (1 - A) < R_n < (1 - B) / A$, we send another packet.

It should be noted that the number of packets that must be sent is random number, not a fixed number.

This procedure may be put on a more practical basis. We can use the equation for R_n given above, substitute the binomial probability functions, and solve for acceptance and rejection values; thus,

$$\text{Accept} = \frac{\ln[B/(1-A)] + n \ln[1-p0)/(1-p1)]}{\ln[p1/p0] + \ln[1-p0)/(1-p1)]},$$

$$\text{Reject} = \frac{\ln[(1-B)] + n \ln[1-p0)/(1-p1)]}{\ln[p1/p0] + \ln[1-p0)/(1-p1)]},$$

where p0 is the probability associated with the hypothesis, p1 is the probability associated with the alternate, and n is the number of samples sent. The test statistic is the number of failed packets. If this number is less than the "Accept" value, then the hypothesis (that the link has 3 dB of margin) is accepted. If the number of failures is greater than the "Reject" value, the alternate hypothesis (that the link has 2.8 dB or less margin) is accepted. It might be thought that the number of packets may be very large, and this is possible. However, the number is finite and bounded**.

It must be emphasized that the comparison of the test statistic to the acceptance and rejection values is made after each packet is sent. When and if the statistic equals or exceeds one of the criteria, the test halts. This is important. If a fixed number of packets were to be sent, without intermediate examination of the test statistic, it is possible for the test statistic to "wander" in and out of the bounds set by the accept and reject criteria. Sending a fixed number of packets and looking to see where the test statistic is in relation to the accept/reject bounds after sending this number defeats the raison d'etre of the Wald Sequential Probability Ratio Test and is invalid**.

MONTE CARLO SIMULATION OF THE PRT

This procedure was modeled on a digital computer for various Packet Error Rates. The procedure followed was quite simple. To test the performance of the PRT, a value of Packet Error Rate was chosen, e.g., 1×10^{-2}. To simulate packets being passed with that error rate, a computer algorithm generated a random number whose value lay between 0 and 1. (The random numbers generated by this algorithm have a uniform distribution.) The random number represented a trial packet. If the random number had a value less than the PER, the packet was presumed to have failed; if the number was larger, the packet was considered to have been received successfully. The acceptance and rejection values were calculated after each packet was tried (random number was generated). If the total number of failed packets did not pass either bound, another packet (random number) was tried. This procedure continued until one bound or the other was exceeded.

Various values of PER were tried (the values of A and B were set at 0.001), and at each rate, five hundred trial tests were performed. The results are shown in the following table:

Packet Error Rate	Accepted	Rejected	Average Number of Packets	
1.000×10^{-2}	0	500	182	
1.689×10^{-3}	0	500	1148	(2.8 dB Margin)
1.000×10^{-3}	9	491	1950	
1.000×10^{-4}	446	54	5364	
6.466×10^{-5}	485	15	4881	(2.9 dB Margin)
1.000×10^{-5}	500	0	4294	
1.638×10^{-6}	500	0	4121	(3.0 dB Margin)
1.000×10^{-6}	500	0	4105	
1.000×10^{-7}	500	0	4089	

**Lindgren, Statistical Theory, MacMillan, New York, 1960, PP 255-267.

These results indicate that the test is not as clean-cut as might be desired. Clearly, if the system possesses 2.8 dB or less margin, the test would reject the system. However, if the system has more than 2.8 dB of margin, yet less than 3 dB, there is a very real possibility that the test will pass the system. An error of 0.2 dB may be acceptable, but one may wonder if it is possible to improve upon this.

As a trial, the testing hypothese were re-framed:

Hypothesis: Probability of packet error = 1.6384×10^{-6},

Alternate: Probability of packet error = 6.4663×10^{-5}.

The null hypothesis corresponds to a margin of 3.0 dB; the alternate hypothesis corresponds to 2.9 dB of margin. The following table shows the results.

Packet Error Rate	Accepted	Rejected	Average Number of Packets	
1.000×10^{-2}	0	500	200	
1.689×10^{-3}	0	500	1193	(2.8 dB Margin)
1.000×10^{-3}	0	500	2058	
1.000×10^{-4}	0	500	28262	
6.466×10^{-5}	0	500	47876	(2.9 dB Margin)
1.000×10^{-5}	449	51	209247	
1.638×10^{-6}	500	0	121598	(3.0 dB Margin)
1.000×10^{-6}	500	0	115767	
1.000×10^{-7}	500	0	109818	

This would seem more promising. For values of margin of 2.9 dB, or less, the test cleanly rejects the hypothesis of 3.0 dB of margin. For values of margin greater than, or equal to, 3.0 dB, the test accepts the hypothesis; the region of uncertainty has been reduced to about 0.1 dB. The price paid is the longer time required to run the test.***

*** The author is sensitive to the length of time required. It required some thirty-six minutes of CPU time on a VAX computer to perform the Monte Carlo simulation reflected in the first table. The second table required some 318 million random numbers. The second table was prepared by using two modern Hewlett-Packard desk-top computers, which ran non-stop for eleven days!

The reader may ask why a similar PRT could not be tried when using a comparator threshold voltage of 200 mV. The answer may be seen by comparing the MBER curves for threshold voltages of 200 mV and 288 mV (Figures 4 and 5). It will be observed that the MBER for a threshold of 200 mV and an incident optical power of -28.5 dBm is extremely small, somewhere below 1×10^{-30}. This would make the Packet Error Rate correspondingly small, and would require a very long time to complete a Sequential Probability Ratio Test.

All of the foregoing, of course, will be academic if the PM/FL system is incapable of distinguishing between true packet failures and "queue busy" failures.

SUMMARY

It has been shown that raising the comparator threshold is not equivalent to reducing the incident optical power. Thus, attempting to determine the MBER of a fiber optic bus link by raising the threshold voltage is not appropriate.

However, it would appear that, by raising the comparator threshold to 288 mV, and applying the Wald Sequential Probability Ratio Test, it should be possible to determine that a link has 3 dB of margin.

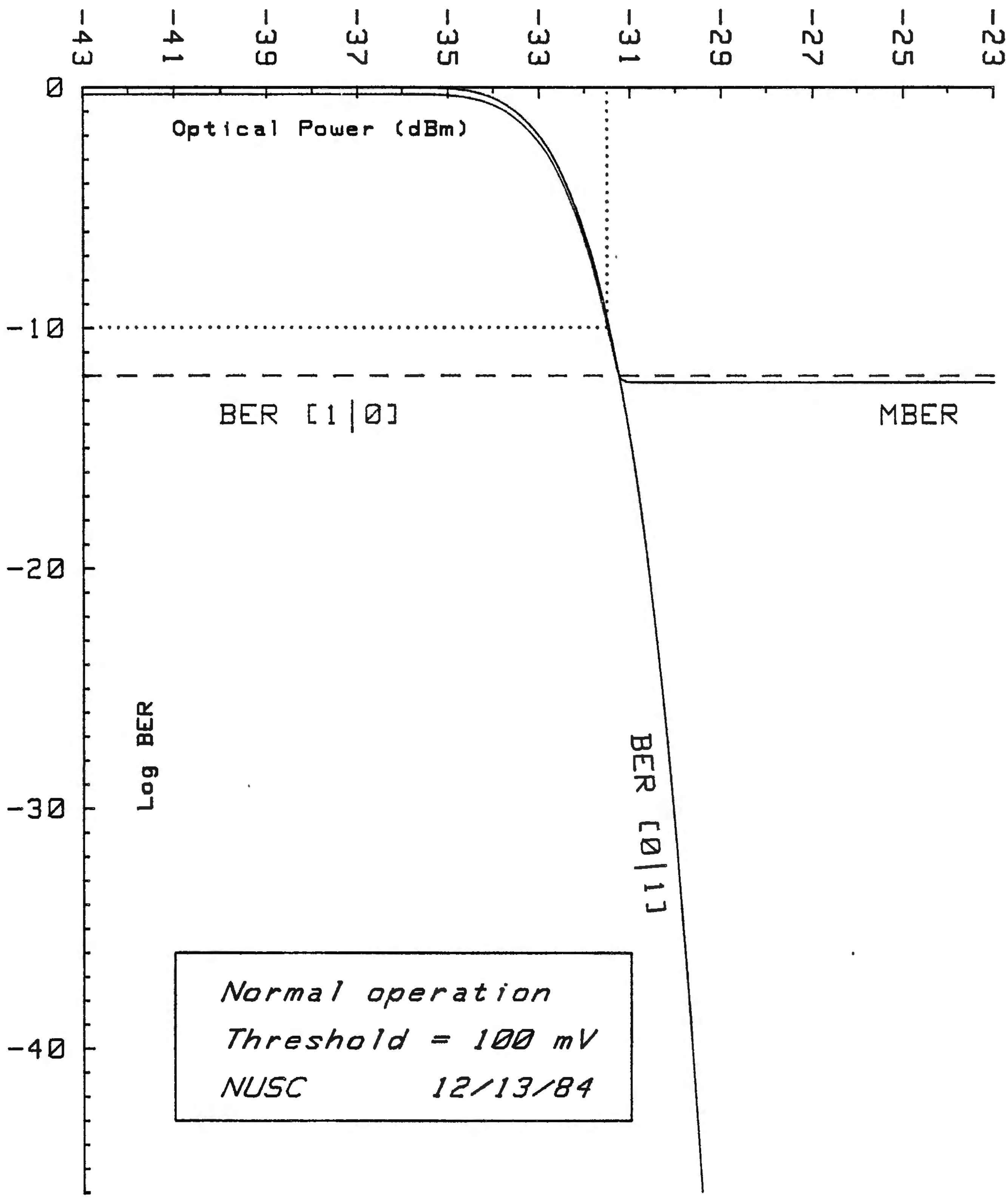
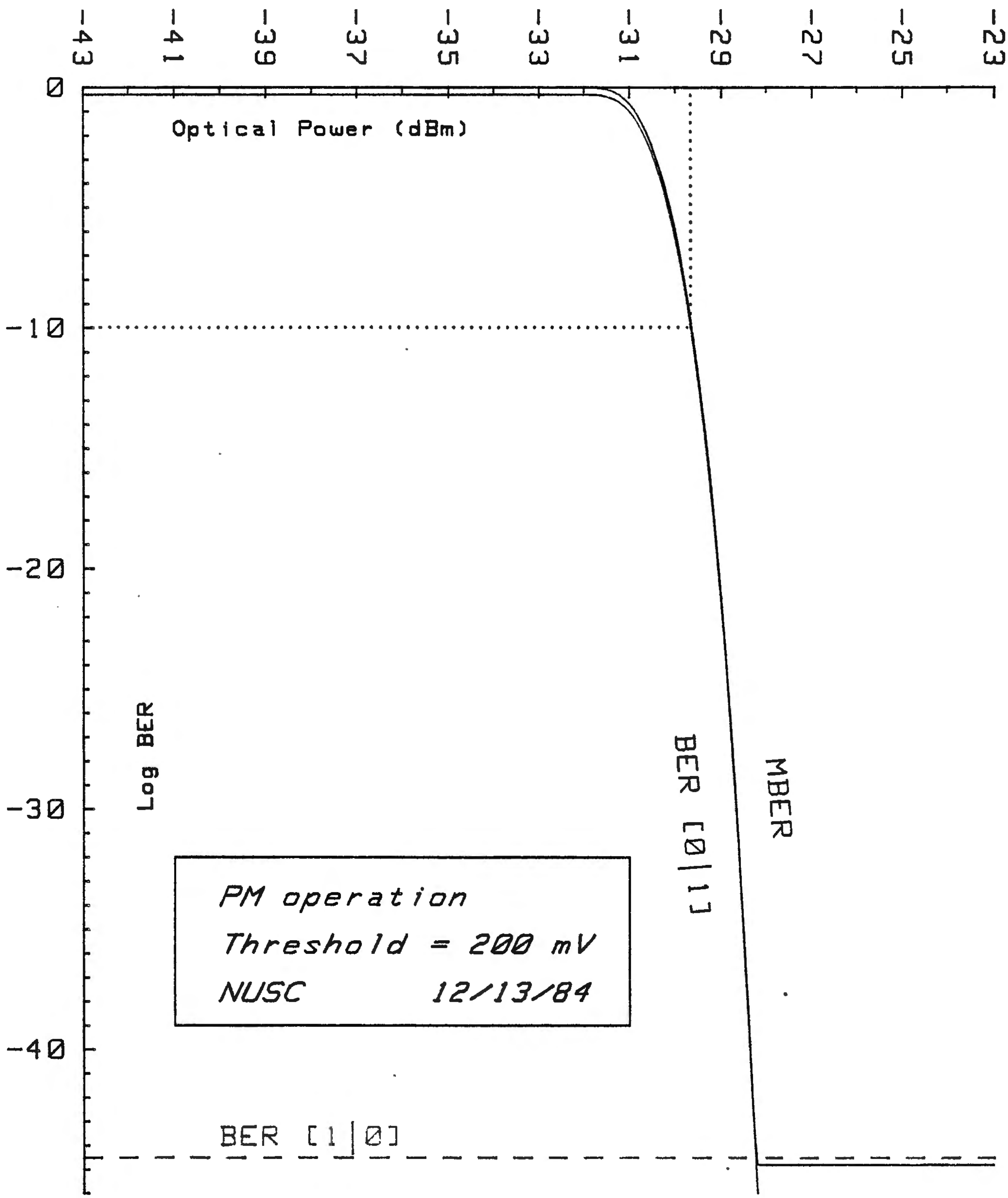

-43
-41
-39
-37
-35
-33
-31
-29
-27
-25
-23
Optical Power (dBm)
0
-10
-20
-30
-40
Log BER
BER [1|0]
MBER
BER [0|1]
Normal operation
Threshold = 100 mV
NUSC 12/13/84

FIGURE 3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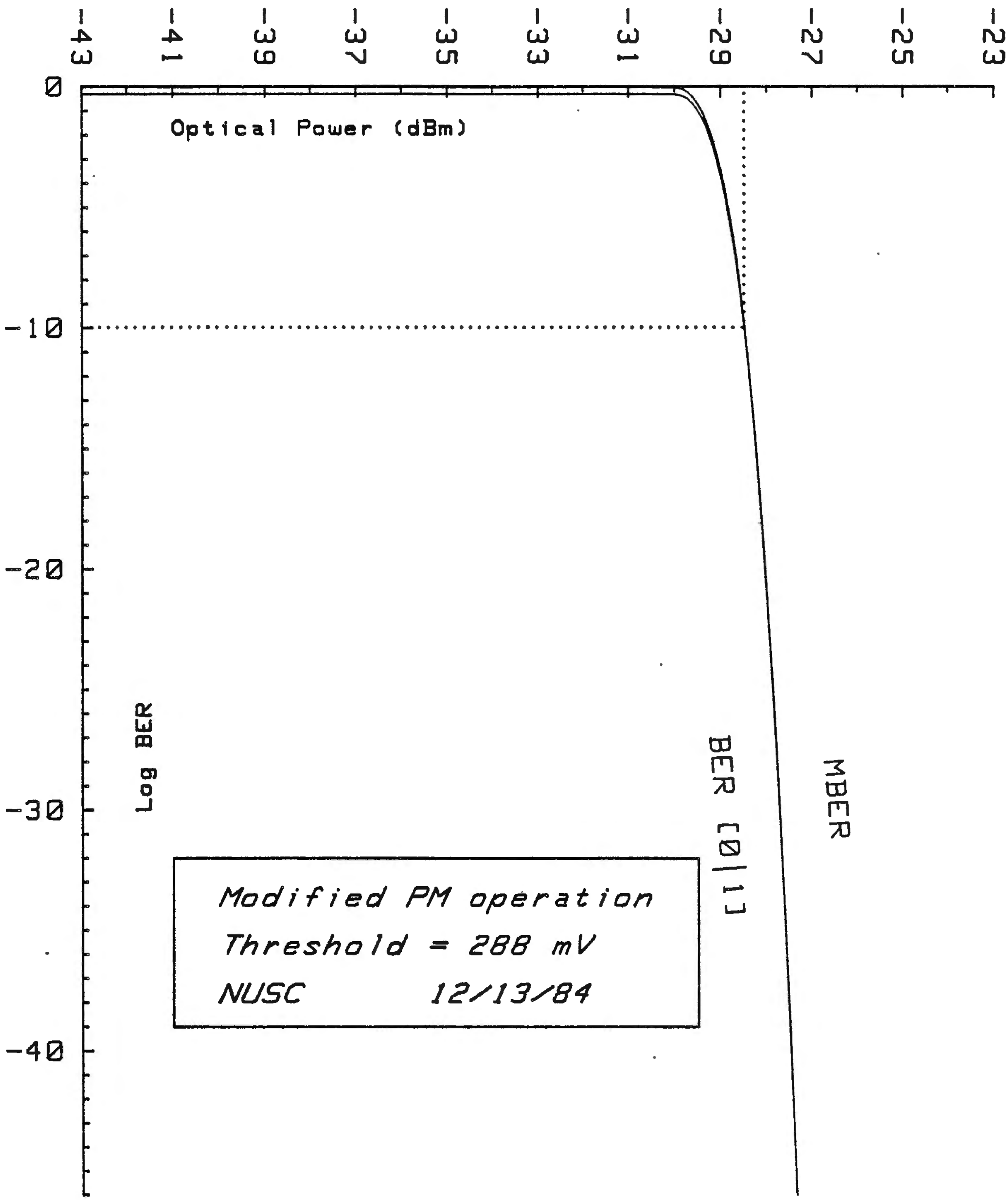
0
-10
-20
-30
-40
Optical Power (dBm)
Log BER
BER [0|1]
MBER
PM operation
Threshold = 200 mV
NUSC 12/13/84
BER [1|0]

FIGURE 4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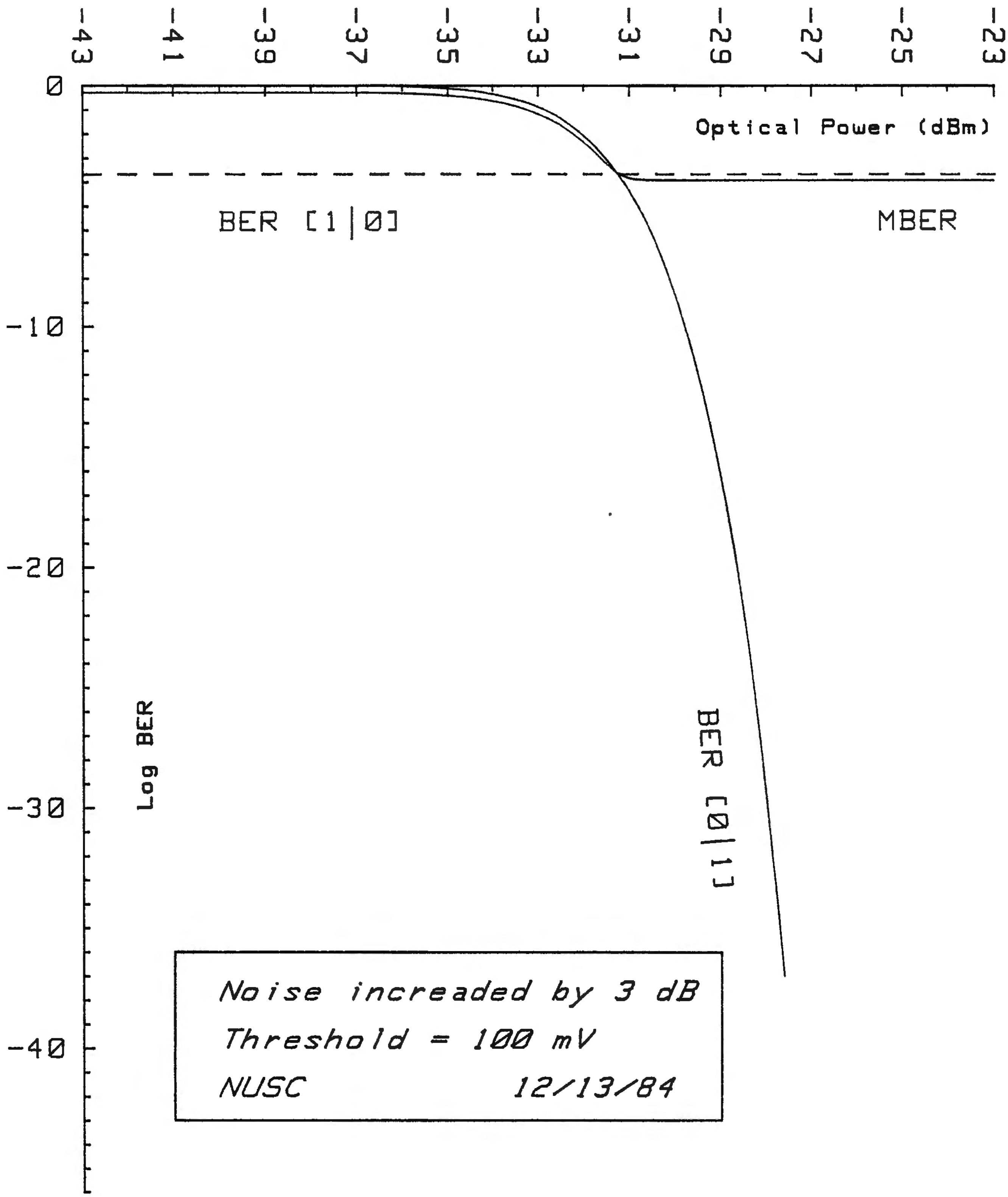
0
-10
-20
-30
-40
Optical Power (dBm)
Log BER
BER [0|1]
MBER
Modified PM operation
Threshold = 288 mV
NUSC 12/13/84

FIGURE 5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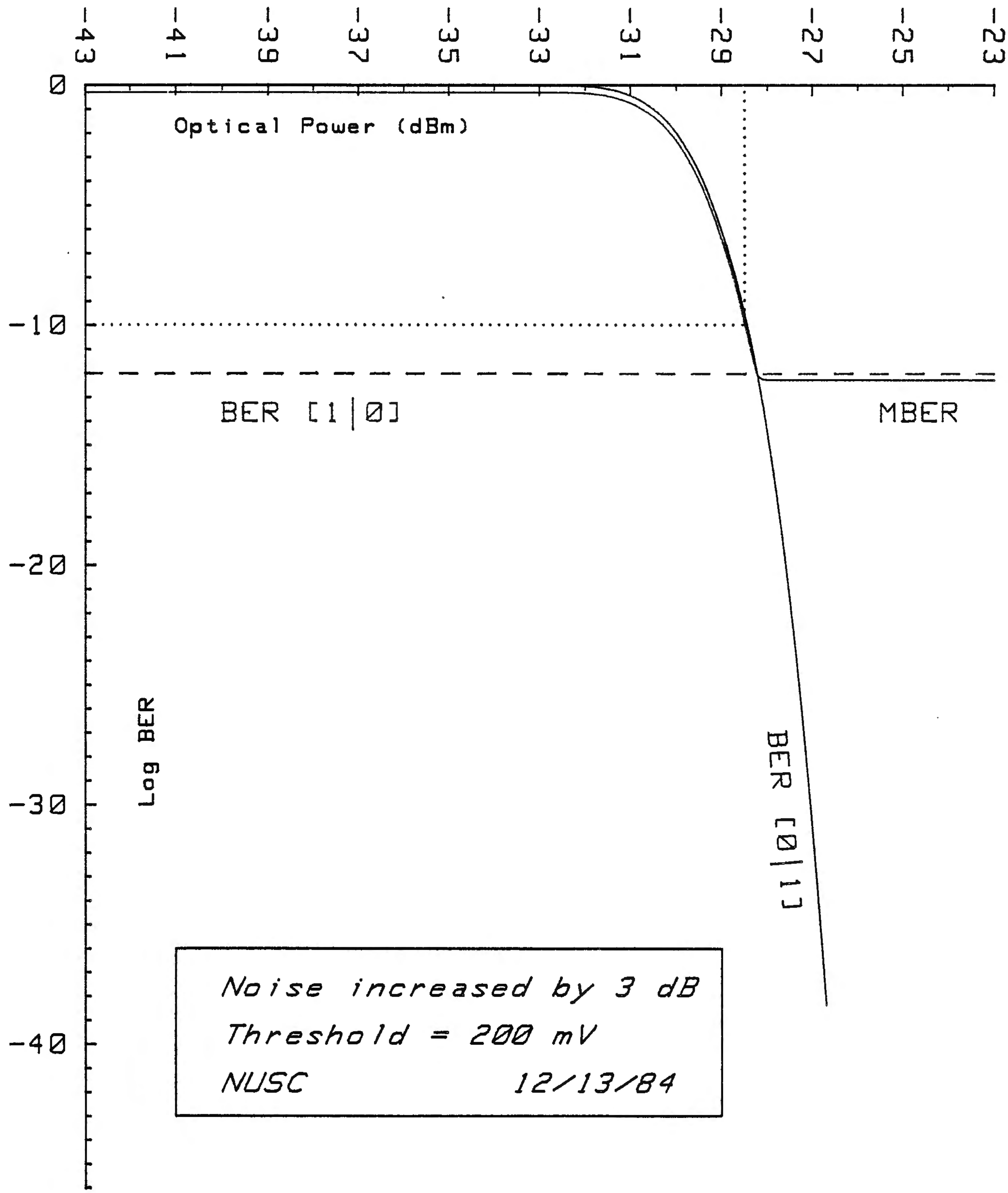
-23
Optical Power (dBm)
0
-10
-20
-30
-40
Log BER
BER [1|0]
MBER
BER [0|1]
Noise increaded by 3 dB
Threshold = 100 mV
NUSC 12/13/84

FIGURE 6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Optical Power (dBm)
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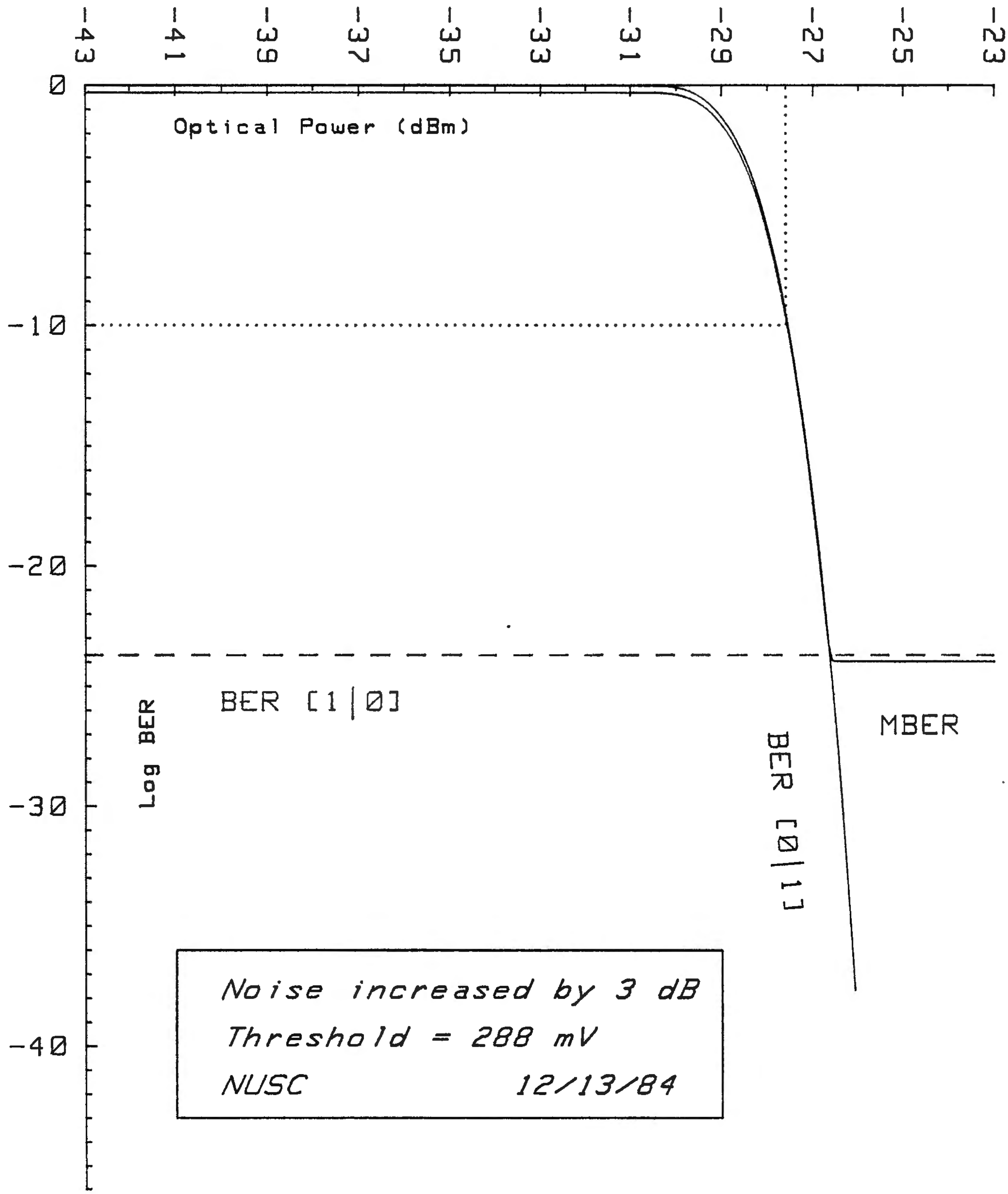
-10
-20
-30
-40
Log BER
BER [1|0]
MBER
BER [0|1]
Noise increased by 3 dB
Threshold = 200 mV
NUSC 12/13/84

FIGURE 7



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Optical Power (dBm)
0
-10
-20
-30
-40
Log BER
BER [1|0]
BER [0|1]
MBER
Noise increased by 3 dB
Threshold = 288 mV
NUSC 12/13/84

FIGURE 8

TM No. 851028

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J. R. Ball
Electro-Optic Systems
28 February 1985
UNCLASSIFIED

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