

T. W. Hussey, S. A. Slutz, and M. P. Desjarlais
Sandia National Laboratories, Albuquerque, NM 87185

Abstract

We have performed a series of idealized MHD calculations of the response of an anode plasma layer to the rising magnetic field in an applied-B ion diode. The purposes of this study were to consider Joule heating of the plasma layer, which could pose potential problems for lithium ion diodes, and the leakage of flux behind the ion-emitting surface, which is related to impedance collapse. We find that, as induced $\mathbf{J} \times \mathbf{B}$ forces move the plasma toward the wall, the entrained, initial field is compressed until it just equals the rising external field. Achieving such equilibrium decreases both Joule heating and flux leakage into the plasma. For relatively thin (~ 1 mm) plasma layers at moderate densities ($10^{16} - 10^{17} \text{ cm}^{-3}$) we calculate that this equilibrium is achieved quite rapidly, leading to acceptably low levels of both flux penetration and Joule heating.

Introduction

In a recent paper Slutz¹ concluded that Joule heating resulting from a rising diamagnetic current in an applied-B ion diode would be sufficient to quickly bring the anode plasma to a multi-keV temperature. This would, for densities above $\sim 10^{16}$ lead to a considerable fraction of collisional ionization of Li^+ to Li^{++} and Li^{+++} . Thus, it was concluded that ideal initial conditions for such an anode plasma would be a relatively thick (to allow a sufficient supply of ions) layer of low density ($< 10^{16} \text{ cm}^{-3}$) plasma. We have found from a series of idealized MHD calculations, however, that, if the anode surface has a conducting substrate, then much of the diamagnetic current can flow there, flattening the magnetic field distribution in the anode plasma. This shunting of diamagnetic current to the wall, shown to be more effective for thinner plasma layers, considerably reduces Joule heating. Furthermore, Desjarlais^{2,3} has shown that impedance collapse may be related to the rate of flux loss across the ion-emitting surface, which would also be altered by reduced field gradients in the plasma.

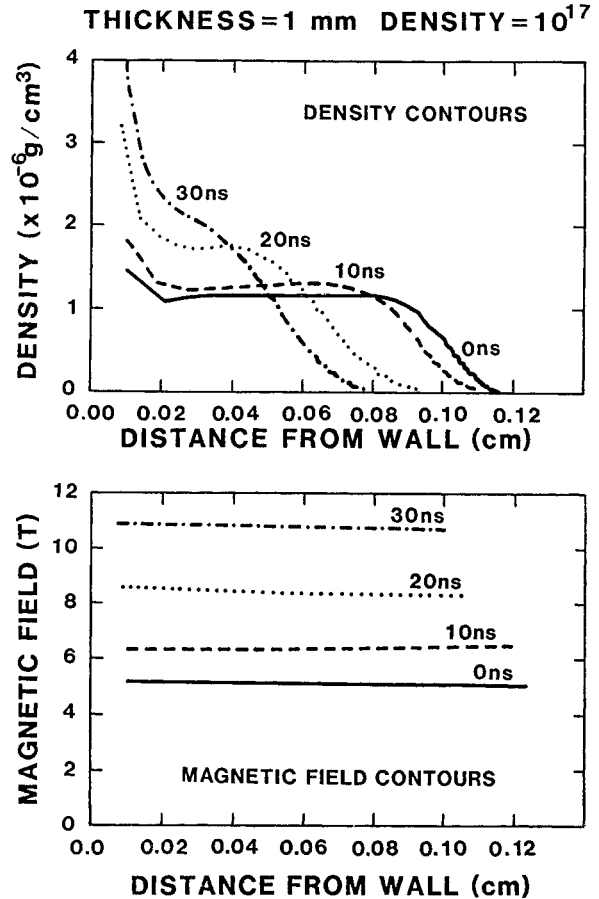
1-D MHD Calculations

A series of idealized 1-D MHD calculations was performed utilizing the following approximations:

- 1) A plasma of uniform density, n_0 , temperature, T_0 , extending a distance r_0 from the anode was assumed; ablation of new material from the substrate was ignored, as was the ionization of any neutrals that might be present in the gap.
- 2) An ideal gas equation of state for singly ionized Li was used.
- 3) An initially uniform, $B_0 = 5$ T external magnetic field was applied, rising exponentially (time constant = 40 ns) to ~ 10 T in 30 ns. Note that the rising component of field is here assumed to be in the same direction as the applied field, whereas experimentally, it may have a perpendicular component.
- 4) A highly conductive substrate was assumed, but heating and blow-off from this layer were ignored.
- 5) Erosion of ions from the plasma surface was ignored.

The worst of these approximations is probably the first, since a non-uniform density and temperature distribution, a flux of material ablated off the anode surface, or the ionization of a significant number of

pre-existing neutrals would be required to give the observed late time expansion of plasma away from the anode³. Nevertheless, results of calculations employing these approximations are instructive in that they point to the existence of an effect that must be considered in more detailed models.

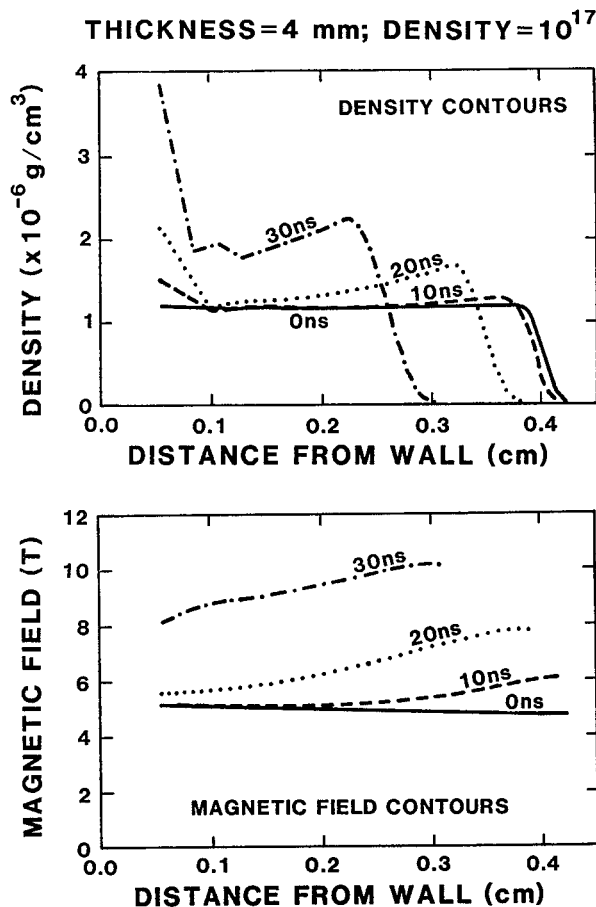


- 1) Calculated density and magnetic field contours at four different times from a 1-D MHD calculation of a 1 mm thick 10^{17} cm^{-3} Li^+ plasma compressed by a magnetic field that rises from 5 T to 10 T in 30 ns.

Figures 1-2 illustrate qualitatively the effect of anode plasma layer thickness on field penetration. Figure 1 shows density profiles at four different times, as well as magnetic field profiles at those same times, for a calculation of a 1-mm-thick 10^{17} cm^{-3} plasma. Figures 2 shows the same quantities for the same density plasma with a thickness of 4 mm. A key observation is that magnetic field profiles are nearly perfectly flat for the 1-mm-thick case, while for the 4-mm-thick case the field rises at the outer edge first and doesn't penetrate to the wall until later. We note that these plasmas are very low beta ($\beta \approx 0.01$) so that variations in field to account for pressure equilibrium would be nearly indistinguishable in the field plots. Since current density is proportional to the derivative of the magnetic field, we find that the thicker layer is heated to ~ 100 eV, while the thinner one is heated to only ~ 20 eV.

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14. ABSTRACT We have performed a series of idealized MHD calculations of the response of an anode plasma layer to the rising magnetic field in an applied-B ion diode. The purposes of this study were to consider Joule heating of the plasma layer, which could pose potential problems for lithium ion diodes, and the leakage of flux behind the ion-emitting surface, which is related to impedance collapse. We find that, as induced $J \times B$ forces move the plasma toward the wall, the entrained, initial field is compressed until it just equals the rising external field. Achieving such equilibrium decreases both Joule heating and flux leakage into the plasma. For relatively thin (- 1 mm) plasma layers at moderate densities (10^{16} - 10^{17} cm⁻³) we calculate that this equilibrium is achieved quite rapidly, leading to acceptably low levels of both flux penetration and Joule heating.					
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It is tempting, based on simply comparing the field profiles of Fig. 1, to conclude that rapid diffusion of field through the thinner layer (Fig. 1) causes it to be uniform in that case. If that were



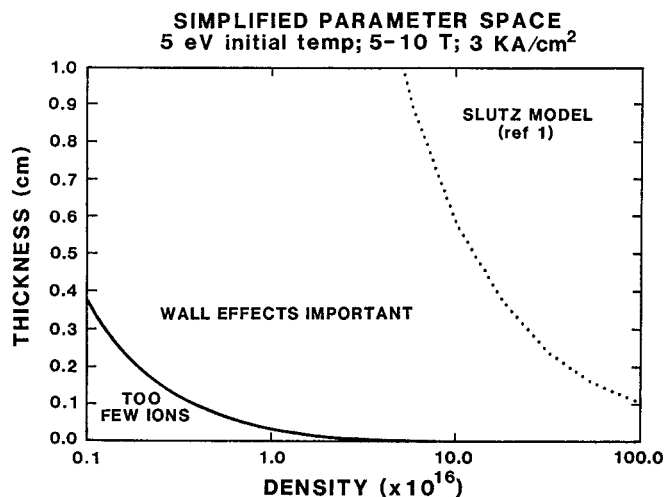
2) Calculated density and magnetic field contours at four different times for a 4 mm thick, 10^{17}cm^{-3} plasma layer.

so, however, there would be little in the way of $\mathbf{J} \times \mathbf{B}$ force to push the plasma toward the wall, and the density distribution would appear to change little in time. In fact, we observe a shelf in density that increases approximately linearly with field, suggesting that a frozen field approximation is more appropriate. The 4-mm-thick case (Fig. 2) best illustrates this effect. As the field rises initially, a gradient (i.e. current) is set up which begins to accelerate the plasma toward the wall. As the plasma compresses, so does the applied B-field entrained within it (initially 5 T in these calculations). When the compressed field equals the field outside the plasma, all the current flows in the wall and the plasma density rises linearly with field, implying little further acceleration. A comparison of Figs 1 and 2 indicates that this transition from acceleration phase to the bulk compression phase occurs nearly instantaneously in the thin case, while it doesn't occur until about 30 ns in the thick case. This is because for a thicker plasma this bulk compression requires a much higher velocity. Also, we note that for a given thickness this transition occurs more rapidly for lower density than for higher density. Since significant Joule heating will occur only when there are significant field gradients in the plasma, a rapid transition to the compression phase leads to little heating.

For low densities, in which the transition to the compression phase comes early for a wide range of thicknesses, we would expect Joule heating to be small and to vary little with thickness. Calculations show, however, that in this regime shock heating associated with the rapid acceleration of plasma to compression velocity, as well as the subsequent adiabatic compression can dominate. This component of heating is found to increase with thickness until the time required to transition from the shock phase to the compression phase just equals the pulse length. For thicknesses greater than this, final temperature depends only on density, reproducing the result of Ref. 1.

Analytic Model

We consider here, within the context of the above approximations, the two-dimensional parameter space in which initial plasma thickness, r_0 , and initial plasma density, n_0 , are varied. In principal, initial plasma temperature, T_0 , is also a variable, but we find that this affects the final temperature only modestly, so



3) A map of the regions in the initial density - plasma layer thickness parameter space where various models apply.

we have taken $T_0 = 5$ eV in all cases. Figure 3 shows the parameter range considered by the calculations, with the Li ion density extending from 10^{15} to 10^{18}cm^{-3} and plasma thickness up to 1 cm. At low density and small radius we see the region of parameter space corresponding to there being insufficient mass to supply a constant 3 kA/cm 2 beam of ions for 20 ns. For higher densities and greater thicknesses, neither the shock wave nor the diffusion wave reaches the wall, and thickness plays no role (i.e. the analysis of Ref. 1 applies). It is the intermediate region for which the present analysis can be expected to apply.

The following mechanisms are found to dominate the heating of the plasma in the calculations:

- 1) Joule heating prior to the transition to the compression phase,
 - 2) Shock heating associated with acceleration of plasma to the compression velocity, and
 - 3) Adiabatic compression after the compression phase.
- Each of these contributions to the final plasma electron temperature can be analytically estimated separately and their sum is found to agree well with calculations.

We first note that each of these three heating mechanisms requires that we know the time of transition from the shock phase to the compression

phase. Consider the simplified equation of motion for a layer of density, ρ , accelerated by $J \times B$ forces, in CGS units,

$$\frac{d}{dt} (\rho v) = \frac{1}{c} J \times B, \quad (1)$$

where $B_0 = 5$ T is the initially applied field, $B = B_0 \exp(\alpha t)$ is the rising anode field, and $\alpha^{-1} = 40$ ns (in this case) is the time constant. One may approximate the expression for current by

$$J = \frac{c}{4\pi} (B - B_0)/\delta, \quad (2)$$

where $\delta = (\eta c^2 t / \pi)^{1/2}$ cm is the approximate diffusion length and η is resistivity. We assume the plasma to be anomalously resistive¹ and set the anomalous collision frequency to ω_{pi} , the ion plasma frequency, giving

$$\delta \approx 1.68 \times 10^7 t^{1/2} n_i^{-1/4} = 31. t^{1/2} \rho^{-1/4} \text{ cm}, \quad (3)$$

where we assume singly ionized lithium ($A = 7$). By combining the above equations, integrating, and setting $d\rho/dt = 0$, we obtain, for small αt (i.e. at early time before the plasma has moved much)

$$v \approx 2.73 \times 10^{14} B_0^2 n_i^{-3/4} \alpha t^{3/2} \text{ cm/sec}. \quad (4)$$

Then we note that during the uniform field, bulk compression phase, assuming an exponentially rising field, the thickness must decrease as $r = r_0 e^{-\alpha t}$, corresponding to a velocity of

$$v = \alpha r_0 e^{-\alpha t}. \quad (5)$$

The time at which Eq. 4 exceeds Eq. 5 corresponds to the time of transition from the shock phase to the compression phase.

We may now estimate the shock heating that must occur as the velocity increases to that given by Eq. 5. The kinetic energy per unit area is just $\rho \alpha^2 r_0^3 e^{-2\alpha t}/6$. If we estimate that approximately this amount of energy goes into internal energy and use $\Delta E = \rho c_v \Delta T$, $c_v \approx 2.0 \times 10^{11}$, and $\alpha = (4. \times 10^{-8})$, we have (temperature in eV)

$$\Delta T_{\text{shock}} \approx 500. r_0^2 \exp(-2\alpha t_t), \quad (6)$$

where t_t is the transition time obtained from Eqs 4 and 5.

Next we consider the Joule heating that occurs prior to the transition time, t_t , when field gradients still persist in the plasma. The temperature change rate associated with this process is

$$dT_J/dt \approx 1.24 \times 10^{10} B_0^2 (e^{\alpha t} - 1)^2 / n_0 t \text{ eV/sec}.$$

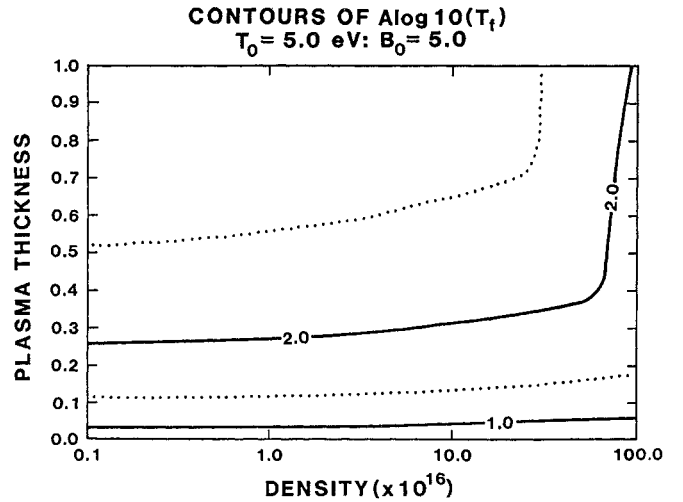
To integrate this from 0 to t we note that only a layer of thickness, $\delta(t)$ is heated. Since we would like to calculate the mean heating of the entire anode layer, we reduce this instantaneous heating rate by the factor, $\delta(t)/r(t)$, if $\delta(t) < r(t)$, giving

$$\Delta T_J = 1.24 \times 10^{10} B_0^2 / n_0 \int_0^{t_t} dt \frac{(e^{\alpha t} - 1)^2}{t} \chi, \quad (7)$$

where χ is the minimum of $\delta(t)/r(t)$ and one.

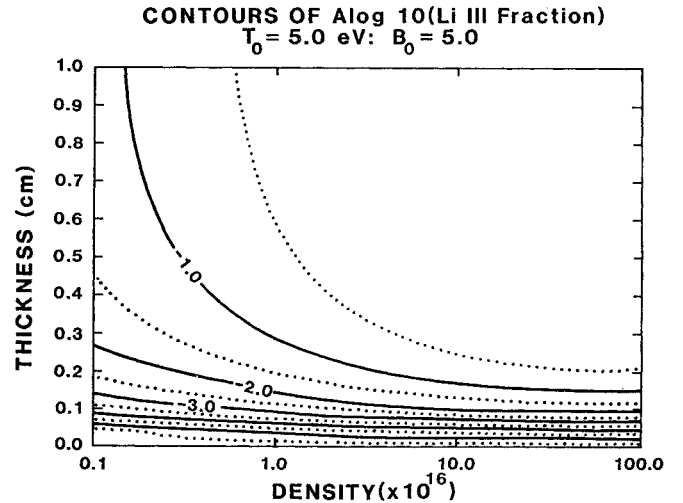
Heating due to adiabatic compression after the transition time is trivially obtained by noting that $PV^{5/3}$ is constant, giving

$$\Delta T_c = \exp(0.67 \alpha (t_{\text{final}} - t_t)). \quad (8)$$



4) Contours of the base-10 logarithm (contour spacing = $10^{1/2}$) of the fraction of the final plasma electron temperature in the initial density - plasma layer thickness parameter space of Fig. 3.

Figure 4 plots the sum of Eqs. 6, 7, and 8 over the same parameter space as Fig. 3. Note that the knees in the isotherm curves are associated with the transition from being shock dominated to being Joule heating dominated.



5) Contours of the base-10 logarithm (contour spacing = $10^{1/2}$) of the fraction of the Li^+ plasma that is collisionally ionized to Li^{++} in 30 ns.

Figure 5 gives contour plots over the $n_0 - r_0$ parameter space of the fraction of Li^+ that is collisionally ionized to Li^{++} , obtained by integrating the cross section from Lotz⁴ over a Maxwellian velocity distribution and over time. From this figure we note that this ionization fraction depends primarily on density for large r_0 , while for small r_0 it depends primarily on thickness. In addition, we note that for thicknesses less than about 1 mm this fraction is less than $\sim 1\%$, a level that should be acceptable for diode operation.

Conclusions

The results shown in Figs. 4 and 5 differ from those of Ref. 1 in that here we find that the key parameter is layer thickness, not density. The analysis of Ref. 1 concluded that the anode plasma density should be less than 10^{16}cm^{-3} , and the role of plasma layer thickness was not considered. Here, Fig 5 suggests that densities of 10^{17}cm^{-3} or greater are acceptable so long as the layer is ≤ 1 mm thick.

Finally, it is important to note that this model would predict little in the way of flux loss across the ion-emitting surface for a thin plasma layer due to the very small field gradients. Desjarlais³ has shown, however, that the impedance collapse observed in ion diode experiments may be related to such a flux loss. Since we have shown here that this is inconsistent with a thin, relatively uniform plasma layer of constant total mass, we find, as in Ref. 3, that we must invoke the infusion of new material. This would result either from the ionization of neutrals already present in the gap or from a significant flux of thermal-conduction-driven ablated material from the anode surface. Since diode performance without this infusion of new material seems promising, an attempt to understand and decrease it should be advantageous.

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