

DETERMINING THE TEMPERATURE AND DENSITY DISTRIBUTIONS FROM A Z-PINCH RADIATION SOURCE

W. Matuska and H. Lee
Los Alamos National Laboratory
Los Alamos, NM 87545 USA

High temperature radiation sources exceeding one hundred eV can be produced via z-pinches using currently available pulsed power. The usual approach to compare the z-pinch simulation and experimental data is to convert the radiation output at the source, whose temperature and density distributions are computed from the 2-D MHD code, into simulated data such as a spectrometer reading. This conversion process involves a radiation transfer calculation through the axially symmetric source, assuming local thermodynamic equilibrium (LTE), and folding the radiation that reaches the detector with the frequency-dependent response function. In this paper we propose a different approach by which we can determine the temperature and density distributions of the radiation source directly from the spatially resolved spectral data. This unfolding process is reliable and unambiguous for the ideal case where LTE holds and the source is axially symmetric. In reality, imperfect LTE and axial symmetry will introduce inaccuracies into the unfolded distributions. We use a parameter optimization routine to find the temperature and density distributions that best fit the data. We know from our past experience that the radiation source resulting from the implosion of a thin foil does not exhibit good axial symmetry. However, recent experiments carried out at Sandia National Laboratory using multiple wire arrays were very promising to achieve reasonably good symmetry. For these experiments our method will provide a valuable diagnostic tool.

Introduction

Knowing the time dependent temperature and density distributions of a radiation source generated by a z-pinch machine is necessary for understanding the physics of this dynamic, hot plasma. Typically 2-D RMHD (radiation magneto-hydrodynamic) codes are used to generate these distributions, but in general, the resulting simulated data does not match the actual data in detail. It is highly desirable that we can develop a reconstruction method to solve for these distributions from the experimental data directly and complement the traditional 2-D RMHD approach. Previously we have used a reconstruction method where we modified, by iteration, the temperature and density distributions obtained from a 2-D code simulation until the transport radiation output best fit the measured spectrometer data (1,2). The resulting temperature and density distributions to a large extent seemed reasonable, but of course, we have no way to verify that it is correct. Here we apply a different reconstruction method to simple 1-D test cases for which the exact solution is known, in order to demonstrate that this method works. This method assumes local thermodynamic equilibrium (LTE) but does not use the results from a 2-D code simulation. Later, after experience with more complex test cases, we hope to apply this method with confidence to real data. If we have image data that are spatially resolved in both the radial and axial directions, then in principle it should be possible to extend this reconstruction method to provide a unique solution for temperature and density distributions in 2-D.

Traditional Forward Z-Pinch Calculations

Calculations for z-pinches are traditionally performed in a forward sense. For a set of initial conditions, material perturbations and other "knob" settings, a 2-D RMHD code is used to calculate the time-dependent temperature and density distributions of a z-pinch. Next the transport radiation spectra,

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE JUN 1997		2. REPORT TYPE N/A		3. DATES COVERED -	
4. TITLE AND SUBTITLE Determining The Temperature And Density Distributions From A Z-Pinch Radiation Source				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Los Alamos National Laboratory Los Alamos, NM 87545 USA				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release, distribution unlimited					
13. SUPPLEMENTARY NOTES See also ADM002371. 2013 IEEE Pulsed Power Conference, Digest of Technical Papers 1976-2013, and Abstracts of the 2013 IEEE International Conference on Plasma Science. Held in San Francisco, CA on 16-21 June 2013. U.S. Government or Federal Purpose Rights License.					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT SAR	18. NUMBER OF PAGES 6	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

both spatially dependent for images and integrated over space for other diagnostics, can be calculated using a separate code. Simulated detector data can then be generated by folding the calculated spectra with detector response functions. In practice however, simulated data is often determined by simpler methods. By comparing the simulated data with the actual data, a judgement is made as to how well one believes the calculated, time-dependent temperature and density distributions represent the actual z-pinch. If the agreement is judged not to be very good, the “knob” settings are adjusted and the above process is repeated in an attempt to better match the actual data. The term “knob” refers to any part of a computer calculation for which the user makes a judgement how something is done, even if by default. In the same spirit, this includes the decision of which computer code to use if the codes give different results.

This approach has the good property that the time evolution of a z-pinch can be calculated from first principles. If perturbations and other “knob” settings can be determined independently, or if they are determined by the above iterative procedure and hold true for a wide class of z-pinch conditions, then time-dependent temperature and density distributions can be predicted by calculation for new z-pinch conditions.

In actuality, “knob” settings, including perturbations, can make calculations agree with integral data (3,4); but these calculations are not unique since, for the cases where more detailed data exists, “knob” settings cannot be found which reproduce the detailed data (5,6,7). Here integral data refers to the current waveform, the bolometer measurement, and some XRD currents (at least shape if not amplitude). Detailed data refers to shape and amplitude of all XRD currents as well as time-resolved x-ray images. Only a relative comparison can be made between actual and calculated x-ray images since the camera amplitudes are not calibrated. With a visual comparison alone, actual and calculated x-ray images sometimes look very much alike; however, a detailed comparison of the spatially dependent intensities show large differences. Specifically, the on-axis hot spots are much more intense in the calculated images, and consequently the radial dependence of the intensity is generally much different between calculated images and actual data. Since the “knob” setting which matches the integral data does not also reproduce the detailed data, there must be some different setting which will if the z-pinch is ever to be calculated correctly. Hence at least to the level of integral data, the “knob” settings for the traditional forward calculations are not unique.

Another difficulty with forward modeling is that correctly modeling a z-pinch implosion requires many aspects of physics, some of which are extremely difficult to incorporate accurately. An example is the fact that the z-pinch implosion is slightly 3-D. The 2-D code forces the implosion to converge with exact symmetry on axis, which is unrealistic. This exactly symmetric convergence in calculations is strongly suspected to produce the artificially intense hot spots seen on axis in the simulated x-ray images (6). Even though the implosion has 3-D aspects, the resulting pinch is to a large degree 2-D. Hence a reconstruction method, such as the one discussed below, circumvents such difficulties. The reconstruction method does not require any of the complex physics needed to accurately implode the pinch, only an accurate calculation of the spectra emitted by any defined temperature and density distributions that represent the pinch.

Reconstruction Method for Hot Z-Pinches

Reconstruction methods for radiation images are nothing new. Familiar examples are holography and 3-D tomography, which use a back lighter source and the object to be measured has no self-emission. Density reconstruction from x-ray radiographic data of non-radiating z-pinch liners (8), which employs Abel inversion, is another example. This last example depends upon radiation attenuation only to solve for density as a function of radius, assuming axial symmetry. Examples also exist for emission only reconstruction methods. Plasma emission distributions can be reconstructed in a tokamak from spectral data (9), and emission line ratios can be used to obtain information about temperatures and densities in a plasma (10,11).

In this paper we reconstruct the temperature and density distributions of a source which is both attenuating and self-emitting. This reconstruction method is based on a parameter optimization technique (12) described as follows: First, the temperature or density distribution at a given time, as a function of the radius and axial position of the source, is expressed in some functional form of our choice by a set of parameters $\{p_j\}$ to be optimized. The radiation from this source so characterized will then be transported, with proper self-emission and attenuation, to the detector location. Simulated detector data (SDD) are obtained by folding the transported radiation spectrum with the detector response function. For each type of detector, we define a function $b(\{p_j\})$ as the figure-of-merit measure of the relative difference between the SDD and measured detector data (MDD). Finally, we take the sum of the squares of these b functions, denoted by $E(\{p_j\})$, and minimize it by the optimization code to obtain the best-fitted solution for $\{p_j\}$.

The starting information required for this reconstruction method is the MDD and the suitable functional form the temperature and density distributions (the solution) will have. The reconstruction method does not require any particular functional form nor that a particular set of data be used; however, image data should give the most detailed information. The test calculations below use time-gated images for different response functions, XRD currents, a bolometer current, and the total mass in the z-pinch. Additional data, such as spectrometer signals and emission line ratios, could also have been included. Straight-line interpolation, linear-linear for temperature and semi-logarithmic for the density, is used in the test calculations. The independent parameters $\{p_j\}$ are the temperatures and densities, at fixed radii, which define the line segments.

Spectra, both spatially dependent and integrated over space, are calculated, using opacities and the transfer equation, for the temperatures and densities defined by the functional form. The simulated data is generated by folding the calculated spectra with detector responses and simply summing volumes time densities to get the mass. For XRD currents, bolometer currents, and mass, b is defined as

$$b_c = w_c (I_{dat} - I_{cal}) / \max(I_{dat}, I_{cal}) \quad (1)$$

where I_{cal} is either a calculated current or a mass and I_{dat} is the corresponding data. For images, b is defined as

$$b_i = w_i \sum_y \sum_z w(y,z) (I_{dat}(y,z) - I_{cal}(y,z)) / \max(I_{dat}(y,z), I_{cal}(y,z)) \quad (2)$$

where I_{cal} and I_{dat} are the calculated and measured image intensities, respectively. The variables y and z are the positions normal to and along the axis of the pinch, respectively, in the image plane. All w 's are user supplied weights. The function to be minimized, E , is then the sum of the squares

$$E = \sum b_c^2 + \sum b_i^2. \quad (3)$$

Starting with a guessed parameter set $\{p_j\}$, the optimization code will evaluate all the partial derivatives $\partial b / \partial p_j$ to determine the direction to proceed in the parameter space. Discrete steps are taken along this direction until the function E assumes a minimum value, at which point we have an updated parameter set. The whole process is repeated until we reach the convergent solution.

The reconstruction method has its own set of possible difficulties. It requires assumptions about the functional form of the temperature and density distributions, which can bias the solution; however, a bad functional form will probably never give a good fit to the data. Also, this method cannot determine the temperature and density distributions for parts of the z-pinch from which the emission effectively does not reach the detectors. Perhaps not all initial guesses converge to the correct temperature and density distributions. (This is analogous to forward calculations where a "knob" setting matches integral data, but a different setting is required to match all data.) So far we have only tried examples with a few independent parameters, and this method may have difficulties with a large number of free parameters. Some actual z-pinchs are so 3-dimensional that no consistent 2-D solution can approximate them, and extending this method to 3-D would be very complex.

Another reconstruction method, which we have yet to try, would be similar to Abel inversion, where one starts by solving for the density at the outer radius of a liner and then solves for the density at successively smaller radii. This outside-in approach could be modified to solve for a temperature and density pair, which best fit x-ray image data (for two or more images), by starting at the outside radius and progressing in steps to successively smaller radii as in Abel inversion.

Test Calculations

The test problems considered here are 1-D, the temperatures and densities are a function of radius but are constant in z. To demonstrate that the reconstruction method can find a known solution, we start with a distribution defined by temperatures and densities at three different radii and straight-line-interpolation in between. The material is assumed to be tungsten. Spectra are calculated for a line-of-sight normal to the z-axis. Spatially dependent spectra are folded with the two responses in Fig. 1 to generate two simulated images, and the total spectrum is folded with the four responses in Fig. 2 to generate four simulated XRD currents. The bolometer power is the total spectrum integrated over photon energy, and the total mass is the radial integral of volume times density. The first test problem is shown in Fig. 3. Here we start with guesses at temperature and density, which are different from the known solution, and let the parameter optimization technique iterate until it converges to the solution. In this example the temperatures converge to the solution in 20 iterations, but the densities require 37 iterations to reach convergence. In Fig. 4 we have the same solution as before but start with a different guess. For this guess convergence is much slower, demonstrating that the convergence rate can be dependent upon the guess. After 780 iterations the temperature has, for all practical purposes, converged to the solution. The density at this point appears to be converging, but it will require many more iterations to reach the solution.

Extending this method to 2-D can be done one of two ways, depending upon the situation. If only data such as images, which is a function of y and z, are used and the line-of-sight axis is normal to the axis of the pinch, then the 2-D problem reduces to a series of mutually independent 1-D problems along the axis of the pinch. If however the line-of-sight axis is not normal to the axis of the pinch and/or some

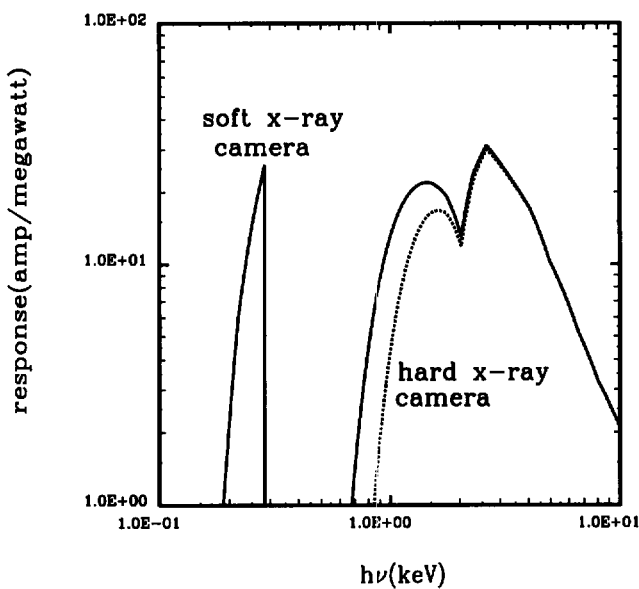


Fig. 1. Response functions used to generate the 2 x-ray images in the test problems.

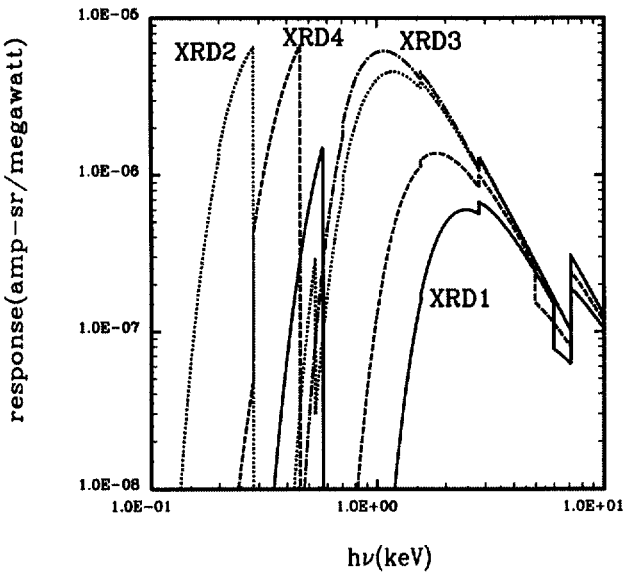


Fig. 2. Response functions used to generate the 4 XRD currents in the test problems.

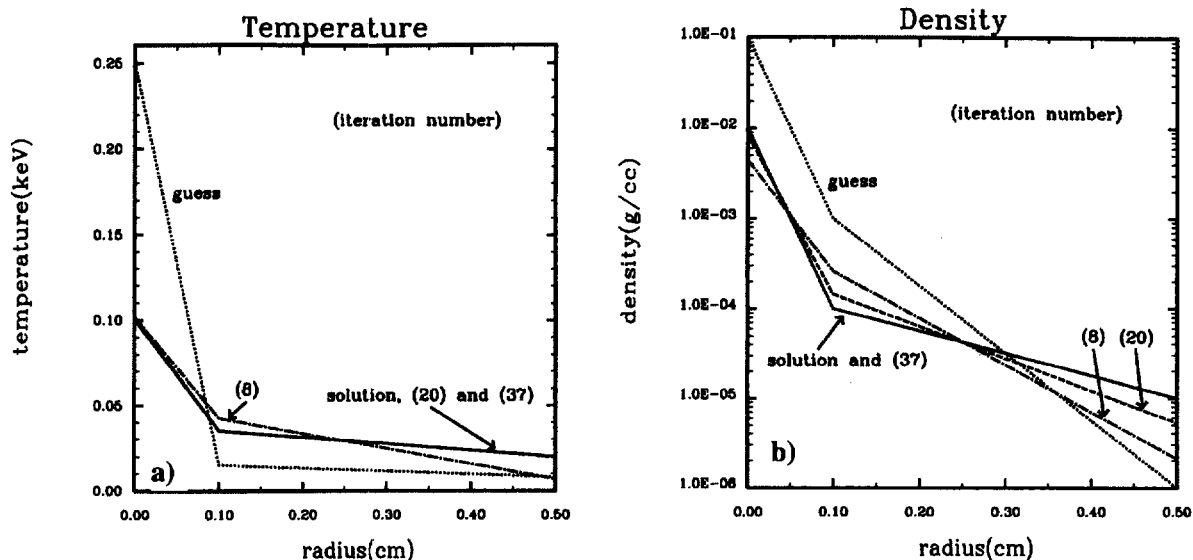


Fig. 3. A test problem showing the reconstruction method starting with a guess and iterating until it converges to the known solution. The functional form for the temperature distribution is in a), the functional form for the density distribution is in b). Distributions at representative iterations are also shown.

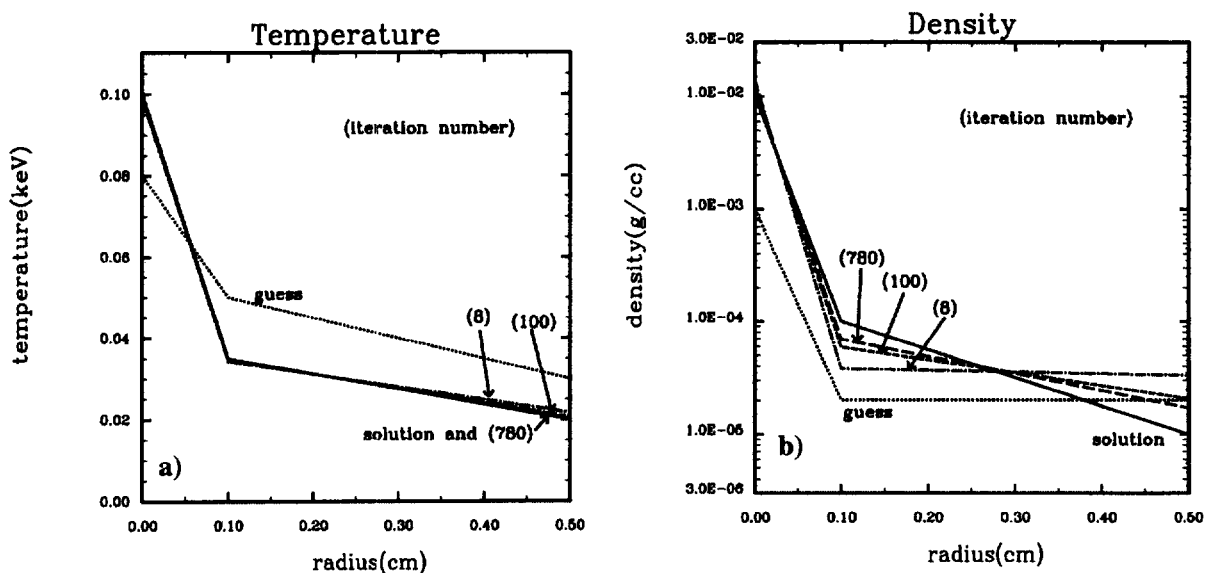


Fig. 4. A test problem showing the reconstruction method starting with a different guess than in Fig. 3 and approaching the same solution as in Fig. 3. Temperature and density distributions are in a) and b), respectively. Distributions at representative iterations are also shown.

integral data is used, then 1-D problems would no longer be mutually independent. In this case the functional form needs to be defined in two dimensions, with some form of interpolation between the points in this 2-D grid. The position of these points need not be held fixed as in the test problems above, these positions can also be treated as free parameters in the iteration.

Future Directions and Conclusions

The reconstruction method in this paper has been successfully tested on several 1-D radiation sources that have both emission and attenuation. Through experience we should learn which diagnostics

to weight more heavily and perhaps which to ignore. After extensive testing has been done where the solution is known, including cases where random noise has been introduced into the data, this method should be applied to actual data. This should include extending this method to 2-D, first normal to the z-axis, and then perhaps to cases not normal to the z-axis. A 3-D version of this method, which would require data from more than one azimuthal direction, would also be interesting to try. In addition, the outside-in approach should also be tried. Since forward calculations can rarely be made to match detailed data, this reconstruction method should provide a valuable diagnostic tool for real data.

Acknowledgements

The computer routine, which implements the parameter optimization technique, was written by the Department of Nuclear Engineering Sciences, University of Florida, Gainesville. We also thank R. Bowers and D. L. Peterson for discussions relating to areas where this reconstruction method could experience difficulties.

References

1. Matuska, *et al.*, "Source to detector spectrum transformation and its inverse for the Pegasus z-pinch," *3rd International Conference on Dense Z-Pinches*, 19-23 April 1993, Imperial College, London.
2. Matuska, *et al.*, *Phys. Plasmas* **3**, 1415-1429 (1996).
3. Peterson, *et al.*, *Phys. Plasmas* **3**, 368-381 (1996).
4. Peterson, *et al.*, *Proceedings of the Fourth International Conf. On Dense Z-Pinches*, 29-30 May 1997, Vancouver, Canada.
5. Matuska, *et al.*, "Comparing calculated and measured x-ray images," *10th IEEE International Pulsed Power Conference*, 3-6 July 1995, Hyatt Regency Hotel, Albuquerque.
6. Matuska, *et al.*, "Modeling x-ray data for the Saturn z-pinch machine," *24th IEEE International Conference on Plasma Science*, 19-22 May 1997, Catamaran Hotel, San Diego (no proceedings published, request copies of poster from author).
7. Derzon, *et al.*, (to be published).
8. Lee, "Method to unfold the density distribution of any axial symmetric liner from radiographic data," *24th IEEE International Conference on Plasma Science*, 19-22 May 1997, Catamaran Hotel, San Diego (to be published).
9. Decoste and Noel, *Proc. SPIE*, Vol. 661, p. 50-57, 3-6 June 1986, Quebec City, Canada.
10. Coulter, *et al.*, *J. Quant. Spectrosc. Radiat. Transfer* **44**, 443 (1990).
11. Deeney, *et al.*, *Phys. Rev. A* **44**, 6762 (1991).
12. Wilde and Beightler, *Foundations of Optimization* (Prentice-Hall, Englewood Cliffs, NJ, 1967), p. 298.