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Improved chaff solution algorithm

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Abstract

During the Shipboard Integration of Sensor and Weapons Systems (SISWS) Technology Demonstration Project (TDP), an algorithm was developed to automatically generate the optimal chaff solution as a function of missile initial range and velocity, wind velocity, and engagement geometry. The chaff solution algorithm was developed using customized score functions and simple models. The SISWS chaff seduction algorithm was improved by taking a more physical approach in the selection of chaff solutions, including a monopulse antenna model. The document starts with the scenario being modeled, followed by models for chaff, intersect ratio, monopulse seeker, and probability of soft-kill.

Résumé

Dans le cadre du Programme de démonstration de technologies (PDT) sur l'intégration de capteurs et de systèmes d'armes embarqués (SISWS), un algorithme a été élaboré pour déterminer automatiquement la composition optimale des paillettes en fonction de la portée et de la vitesse initiales d'un missile, du vecteur vent et de la géométrie d'engagement. L'algorithme de détermination du largage des paillettes a été élaboré à l'aide de fonctions de caractérisation adaptées et de modèles simples. L'algorithme de séduction par paillettes du SISWS a été amélioré grâce à une approche plus concrète pour la sélection de la composition des paillettes, notamment grâce à un modèle d'antenne monopulsé. Le document débute par la modélisation du scénario, pour ensuit présenter les modèles des paillettes, des rapports d'intersection, de l'autodirecteur monopulsé et de la probabilité de neutralisation par déroutement.

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Executive summary

Improved chaff solution algorithm

Sylvain Gauthier; DRDC Ottawa TM 2008-324; Defence R&D Canada – Ottawa, March 2009.

During the Shipboard Integration of Sensor and Weapons Systems (SISWS) Technology Demonstration Project (TDP), an algorithm was developed to automatically generate the optimal chaff solution as a function of missile initial range and velocity, wind velocity, and engagement geometry. The chaff solution algorithm was developed using customized score functions and simple models.

The SISWS chaff solution algorithm was improved as described, in this document, by adding a monopulse antenna model to the seeker, and by using a more physical approach to the selection of solutions. The document starts with the scenario being modeled, followed by models for chaff, intersect ratio, monopulse seeker, and probability of soft-kill. The generated chaff solutions should provide more accurate responses for optimal survivability.

The next step would consist of analysing the efficiency of the chaff solutions in SADM and also comparing the results with the SISWS chaff solution algorithm. After that, the algorithm can be modified further to include the tactic of combining Continuous Wave (CW) jamming with chaff solutions.

Sommaire

Improved chaff solution algorithm

Sylvain Gauthier; DRDC Ottawa TM 2008-324; Defence R&D Canada – Ottawa, Mars 2009.

Dans le cadre du Programme de démonstration de technologies (PDT) sur l'intégration de capteurs et de systèmes d'armes embarqués (SISWS), un algorithme a été élaboré pour déterminer automatiquement la composition optimale des paillettes en fonction de la portée et de la vitesse initiales d'un missile, du vecteur vent et de la géométrie d'engagement. L'algorithme de détermination du largage des paillettes a été élaboré à l'aide de fonctions de caractérisation adaptées et de modèles simples.

L'algorithme de composition des paillettes du SISWS a été amélioré, tel qu'indiqué dans le présent document, grâce à l'ajout d'un modèle d'antenne monopulsée dans l'autodirecteur et à l'utilisation d'une approche plus concrète pour la sélection de la composition des paillettes. Le document débute par la modélisation du scénario, pour ensuit présenter les modèles des paillettes, des rapports d'intersection, de l'autodirecteur monopulsé et de la probabilité de neutralisation par déroutement. Les compositions de paillettes rendues devraient offrir une sensibilité plus précise en vue d'optimiser la capacité de survie.

L'étape suivante consisterait à analyser l'efficacité des compositions de paillettes des munitions de destruction atomique spéciale et de comparer les résultats à l'algorithme de composition des paillettes du SISWS. Par la suite, l'algorithme pourra être modifié de nouveau pour y inclure un brouillage tactique, où le brouillage à ondes entretenues (CW) précéderait le largage des paillettes.

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1 Introduction

During the Shipboard Integration of Sensor and Weapons Systems (SISWS) Technology Demonstration Project (TDP), an algorithm was developed to automatically generate the optimal chaff solution as a function of missile initial range and velocity, wind velocity, and engagement geometry [1]-[2]. An initial analysis on chaff effectiveness was conducted using the Ship Air Defence Missile (SADM) simulator against given set chaff deploy parameters (launcher number and fuse time). Characteristics of the most successful chaff solutions were identified from the simulated scenarios. The chaff solution algorithm was then developed using customized score functions and simple models.

Figure 1 shows a high level algorithm developed under SISWS for the generation of an optimal seduction chaff solution. The parameters of the scenarios needed to generate the chaff solutions are first extracted. After that, a set of candidate solutions for chaff launcher and fuse time is generated. These candidate solutions are then graded against customized score functions for visibility, miss distance, and so on. The best solution is selected by finding the one with the highest probability of capture and miss distance.

The SISWS chaff seduction algorithm can be improved by directly taking into account the physics of the problem of generating the chaff solutions. This is the subject of this document.

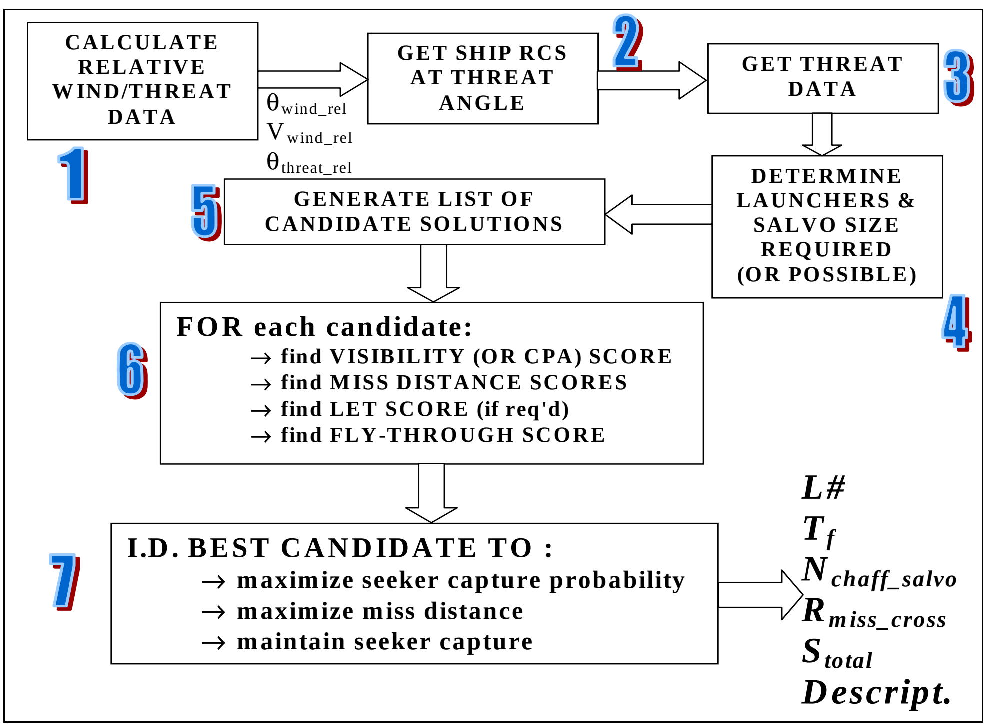


Figure 1: Overall Seduction Chaff Solution Algorithm

2 Modeling

This section describes the models used in the improved chaff solution algorithm. The main difference in modeling, compared to the SISWS algorithm, is the addition of a monopulse antenna model for the missile seeker. A more physical approach was also used in the selection of chaff solutions. This section starts with the scenario being modeled, followed by models for chaff, intersect ratio, seeker monopulse antennas, and probability of soft-kill.

2.1 Scenario

An antiship missile (ASM) is flying at constant speed and constant height directly toward a ship and its seeker is locked onto the ship. The seeker range-gate width (RGW) is centered on the ship. The ship is assumed to have uniform Radar Cross Section (RCS) of 37dB for all look angles.

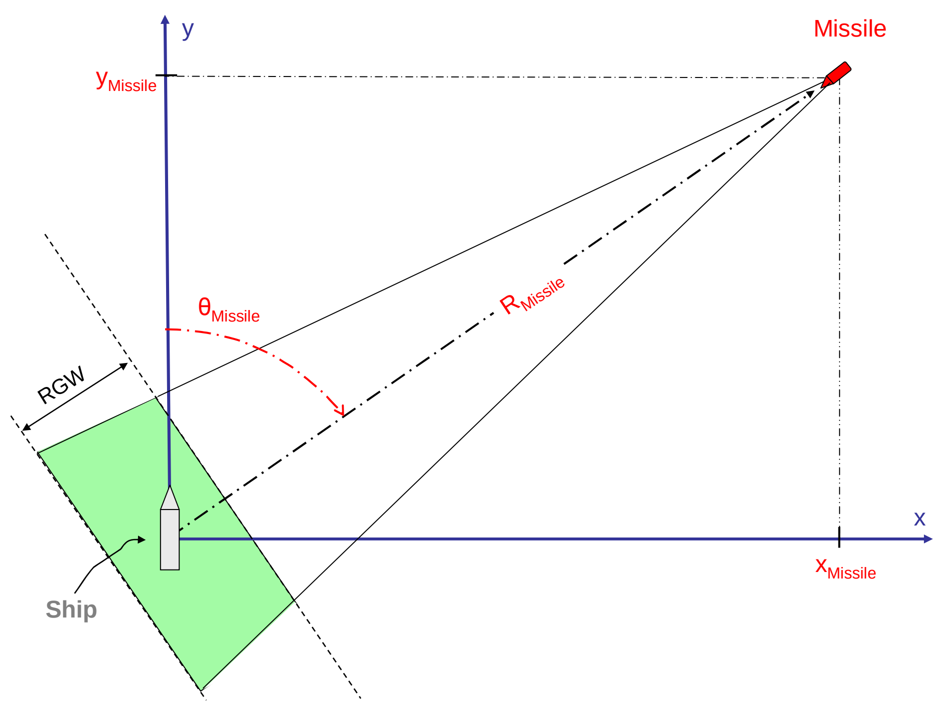


Figure 2: Scenario overview

2.2 Coordinate system transformations

The position of the missile in the ship coordinate system can be described in Cartesian or polar coordinates as shown in Figure 3. The y axis is along the ship heading and the x axis is perpendicular to the y axis toward the right. In polar coordinates, the position of the missile is defined by its range to the ship and the direction angle relative to the y axis. The coordinates of a point p can be transformed from one system to the other as follows.

$$x = R_p \cos(\theta_p); \quad y = R_p \sin(\theta_p) \tag{1}$$

$$R = \sqrt{x_p^2 + y_p^2}; \quad \theta = \tan(x_p / y_p) \tag{2}$$

The position of the missile can also be described into the RGW coordinate system which is centered on the ship but with its axis as shown in Figure 4 and Figure 5. The down range axis denoted by “d” is along the line joining the missile and the ship, and is oriented away from the missile. The cross range axis denoted by “c” is perpendicular to the d axis and positive to the right. The coordinates of a point (x_p, y_p) can be transformed into the RGW coordinates as shown in equation (3).

$$c = - x_p \cos(\theta_{Missile}) + y_p \sin(\theta_{Missile}); \quad d = - y_p \cos(\theta_{Missile}) - x_p \sin(\theta_{Missile}) \tag{3}$$

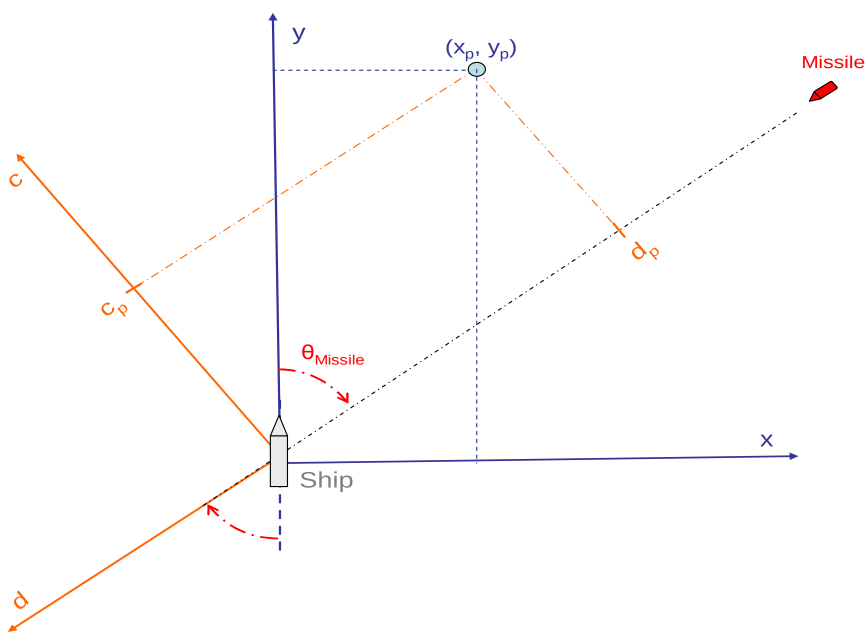


Figure 3: Cross range and down range coordinates in the RGW system

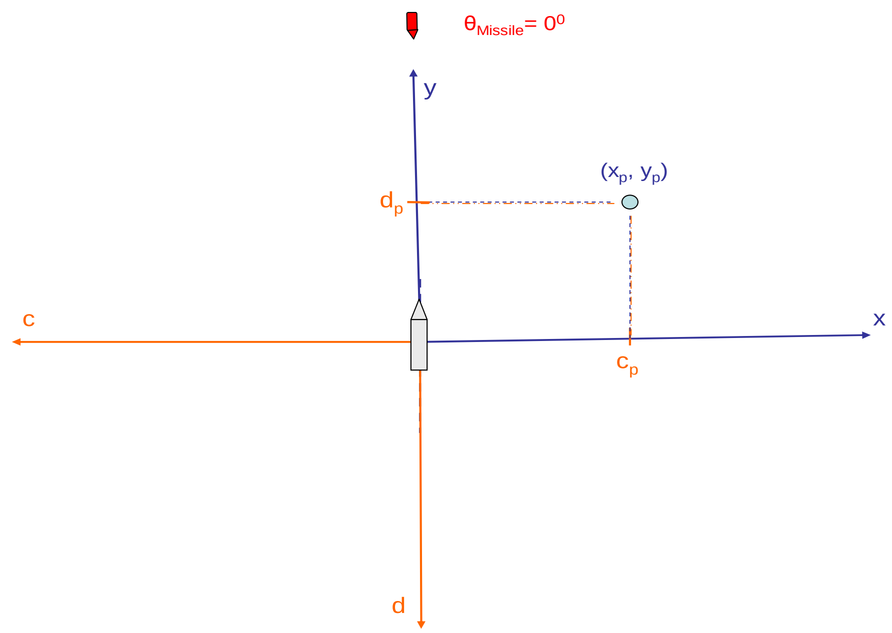


Figure 4: Cross range and down range coordinates in the RGW system ($\theta_{missile}$ = 0^0)

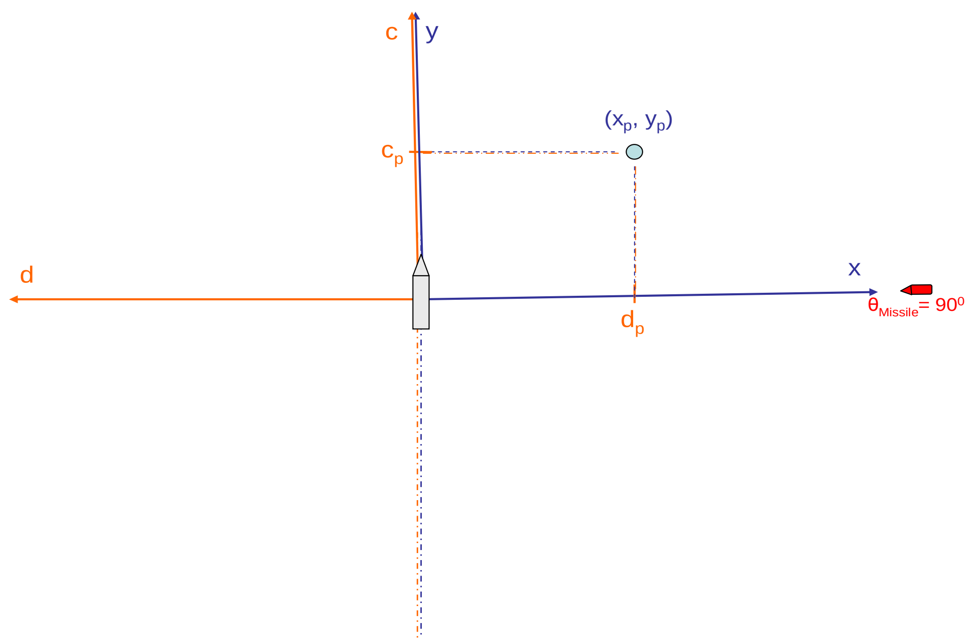


Figure 5: Cross range and down range coordinates in the RGW system ($\theta_{missile}$ = 90^0)

2.3 Chaff trajectory

2.3.1 Range height profile of chaff round

The range and height of the chaff round is a function of fuse time (Tf) as shown in Figure 6 (for three elevation barrels) [1]. Ignoring the rolls and pitches of the ship, the chaff barrel elevation is always 30^0. Table 1 shows the extracted values of chaff range and height for the 30^0 barrel elevation.

The interesting point here is the fact that chaff rounds, if not fused fall back to launcher height after about 19.5 seconds. This limits the range of possible fuse time solutions from zero to 19.5 seconds. Indeed, chaff launcher height is about 5 meters above sea level. The fall rate of chaff is typically between 0.6 to 0.9 m/s [1]. Hence a chaff cloud would fall to the sea surface about 6 seconds before it blooms to 90% of its full value.

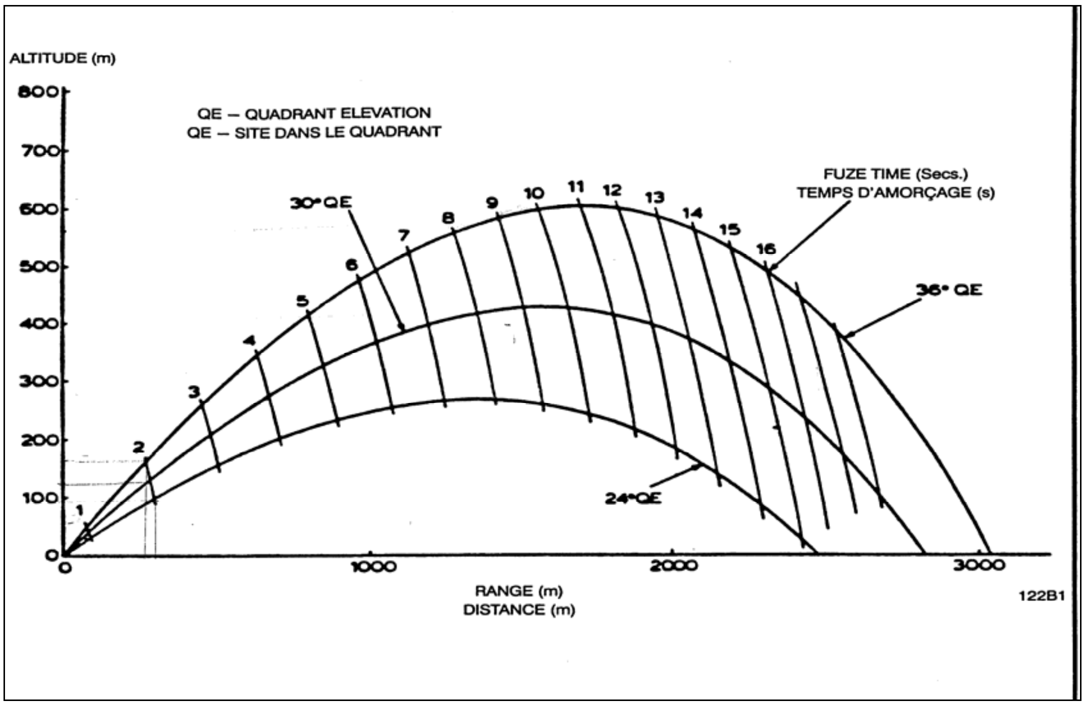


Figure 6: Chaff range and height as a function of fuse time T_f and barrel elevation

Table 1: The range and height of chaff round as a function of fuse time (30^0 in elevation)

TOF (sec)	Range (m)	Height (m)		TOF (sec)	Range (m)	Height (m)
0	0.0	0		10	1,659.1	420
1	90.9	50		11	1,795.5	410
2	295.5	120		12	1,931.8	390
3	500.0	210		13	2,045.5	360
4	681.8	270		14	2,181.8	320
5	863.6	320		15	2,318.2	280
6	1,045.5	360		16	2,431.8	220
7	1,204.5	390		17	2,522.7	160
8	1,363.6	410		18	2,659.1	110
9	1,522.7	420		19	2,750.0	60
				19.5	2,795.5	0

A mathematical model of the chaff round is range/height profile in Table 1 was produced by using a Matlab function called polyfit, with the degree set to six. The resulting polynomial coefficients that fit these data define the chaff range / height profile, as shown below. The range/height profiles of the chaff round obtained from equations (4) and (5) compare very well with the original data as shown in Figure 7.

$$r_{chaff} = 0.003t^6 - 0.0225t^5 + 0.5891t^4 - 7.4036t^3 + 41.8246t^2 + 91.1741t \quad (4)$$

$$h_{chaff} = 0.0001t^6 - 0.0077t^5 + 0.2041t^4 - 2.6225t^3 + 11.3826t^2 + 52.4026t \quad (5)$$

Once the chaff round explodes, its dimension reaches 100m along the launch axis [1]. The exploding rounds will be spread, centered on its fuse position. Hence, the chaff round should not be fused before traveling at least 50 meters. Using equations (4) and (5), this minimum distance corresponds to a fuse time of 0.41 seconds. Typical tactics have a minimum fuse time of 0.75s, which corresponds to a round distance of 99.5 meters.

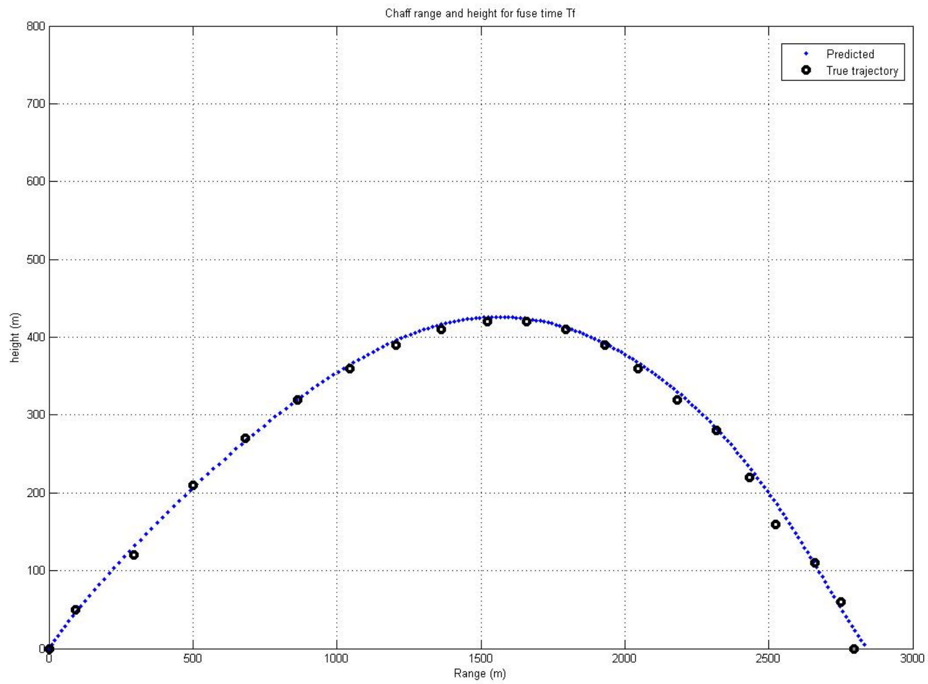


Figure 7: Chaff range and height vs fuse time T_f and barrel elevation

2.3.2 Round position at fuse time relative to ship x, y coordinates

The positions of each chaff launcher are given in Table 2 and shown in Figure 8 [1]. The initial position of the chaff round at fuse time in x, y coordinates are then given in equations (6), (7) and (8).

$$x_{chaff\ init}(L\#, T_f) = x_{L\#} + r_{chaff}(T_f) * \cos(90 - \theta_{az}) \tag{6}$$

$$y_{chaff\ init}(L\#, T_f) = y_{L\#} + r_{chaff}(T_f) * \sin(90 - \theta_{az}) \tag{7}$$

$$z_{chaff\ init}(L\#, T_f) = z_{L\#} + h_{chaff}(T_f) \tag{8}$$

Table 2: Position of each launch, in ship coordinates

Launcher #	x (m)	y (m)	z(m)	θ_{az} (degree)
1	8	15	5	30
2	-8	15	5	-30
3	8	-15	5	110
4	-8	-15	5	-110

Height of launcher is assumed to be 5 meters above sea level.

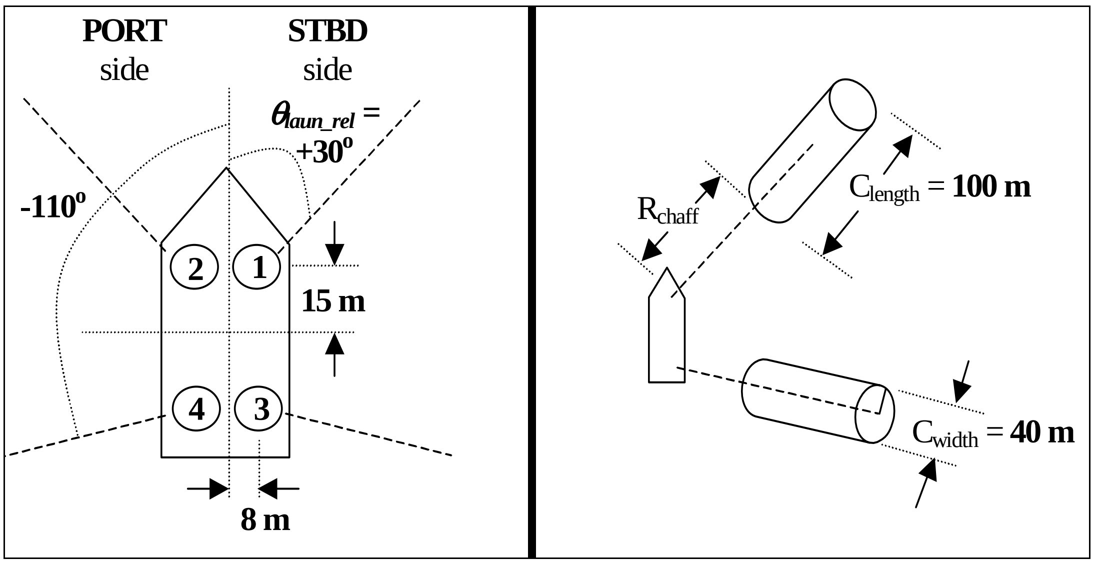


Figure 8: Chaff launcher numbers, orientation, and chaff cloud shape

2.3.3 Cloud Drift

We assume that the chaff cloud at fuse time is centered on the round position at T_f (mass center). Each part of the cloud then drifts along the wind direction at the same speed. Annex A shows distance traveled by a chaff cloud in function of wind speed and elapsed time. Chaff cloud typically has a fall rate between 0.6 to 0.9 m/s. Let us define dt as the elapsed time after fuse time. Then the position of the chaff center is given by equations (9), (10) and (11).

$$x_{chaff\ dt}(L\#, T_f) = x_{chaff\ init}(L\#, T_f) + V_{wind\ x} * dt \tag{9}$$

$$y_{chaff\ dt}(L\#, T_f) = y_{chaff\ init}(L\#, T_f) + V_{wind\ y} * dt \tag{10}$$

$$z_{chaff\ dt}(L\#,T_f) = z_{chaff\ init}(L\#,T_f) - V_{chaff\ \ fall} * dt \tag{11}$$

Using these equations, the trajectory of the chaff cloud for all acceptable fuse time solutions (0 to 19.5 seconds) and launcher number can be plotted.

The cross range and down range position of the chaff cloud in the RGW coordinate system can be calculated using previous equations of transformation coordinates resulting in equations (12) and (13).

$$c_{chaff\ dt}(L\#,T_f) = -\ x_{chaff\ dt}(L\#,T_f)\cos(\theta_{Missile}) + y_{chaff\ dt}(L\#,T_f)\sin(\theta_{Missile}) \tag{12}$$

$$d_{chaff\ dt}(L\#,T_f) = -\ y_{chaff\ dt}(L\#,T_f)\cos(\theta_{Missile}) - x_{chaff\ dt}(L\#,T_f)\sin(\theta_{Missile}) \tag{13}$$

2.4 Intersect Ratio

This subsection calculates the amount of chaff cloud that intersects with the seeker RGW which is centered on the targeted ship. Figure 9 shows an example of chaff cloud center trajectories from launchers 1 and 4, which intersect with the missile seeker RGW. This subsection starts by modeling the physical extent of the chaff cloud and associated trajectory in the RGW coordinate system. The amount of intersection with the RGW is then calculated using equation (13).

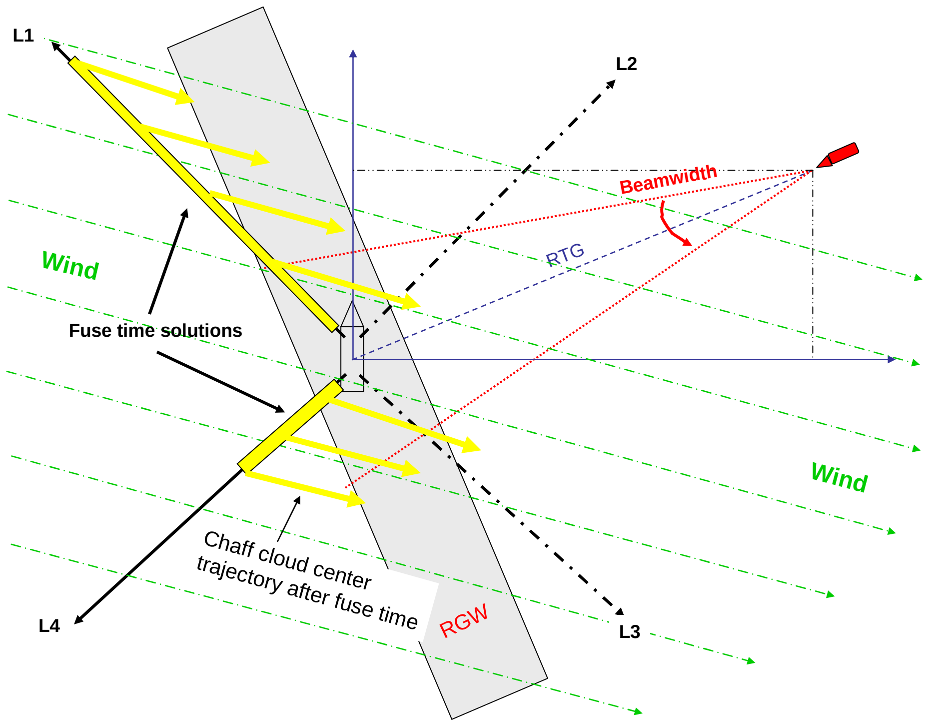


Figure 9: Ship, missile, wind engagement geometry.

2.4.1 Cloud extent in down range

The chaff cloud is first centered on the round position at fuse time and then every part of the cloud is drifted by the wind in the same direction and speed. The extent of the chaff cloud relative to the chaff center is given by equations (14), (15) and (16).

$$x_{chaff\_extent} = \left| \frac{L_{chaff}}{2} * \sin(\theta_{az\_Launcher}) + \frac{W_{chaff}}{2} * \cos(\theta_{az\_Launcher}) \right| \tag{14}$$

$$y_{chaff\_extent} = \left| \frac{L_{chaff}}{2} * \cos(\theta_{az\_Launcher}) + \frac{W_{chaff}}{2} * \sin(\theta_{az\_Launcher}) \right| \tag{15}$$

$$z_{chaff\_extent} = \frac{H_{chaff}}{2} \tag{16}$$

The cross range and down range extent of the chaff cloud in the RGW coordinate system is obtained through the equations of coordinates transformation, (17) and (18).

$$c_{chaff\_extent} = -x_{chaff\_extent} \cos(\theta_{Missile}) + y_{chaff\_extent} \sin(\theta_{Missile}) \quad (17)$$

$$d_{chaff\_extent} = -y_{chaff\_extent} \cos(\theta_{Missile}) - xchaff\_extent \sin(\theta_{Missile}) \quad (18)$$

The down range values of the chaff cloud extremities, $d_{chaff\,+}$ and $d_{chaff\,-}$, in the RGW coordinate system are given by equations (19) and (20).

$$d_{chaff\,dt+} = d_{chaff\,dt} + d_{chaff\_extent} \quad (19)$$

$$d_{chaff\,dt-} = d_{chaff\,dt} - d_{chaff\_extent} \quad (20)$$

2.4.2 Intersection with the RGW

There are four cases where the cloud of chaff can intersect the RGW as shown in Figure 10. Using equations (19) and (20), we can determine if one of these intersects cases occurs. The amount of chaff in down range that intersects with the RGW can then calculated using the equation associated to that intersects case, (21), (22), (23) or (24).

Case 1: $d_{chaff\ dt+}$ > RGW/2 and $d_{chaff\ dt-}$ is inside RGW

$$I_{chaff\,dt}(L\#, T_f, dt_{case1}) = \frac{RGW}{2} - d_{chaff\,dt-} \quad (21)$$

Case 2: $d_{chaff\ dt-}$ < - RGW/2 and $d_{chaff\ dt+}$ is inside RGW

$$I_{chaff\,dt}(L\#, T_f, dt_{case2}) = d_{chaff\,dt+} + \frac{RGW}{2} \quad (22)$$

Case 3: $d_{chaff\ dt-}$ and $d_{chaff\ dt+}$ are inside RGW

$$I_{chaff\,dt}(L\#, T_f, dt_{case3}) = d_{chaff\,dt+} - d_{chaff\,dt-} \quad (23)$$

Case 4: $d_{chaff\ dt-} < -RGW/2$ and $d_{chaff\ dt+} > RGW/2$

$$I_{chaff\ dt}(L\#, T_f, dt_{case4}) = RGW \tag{24}$$

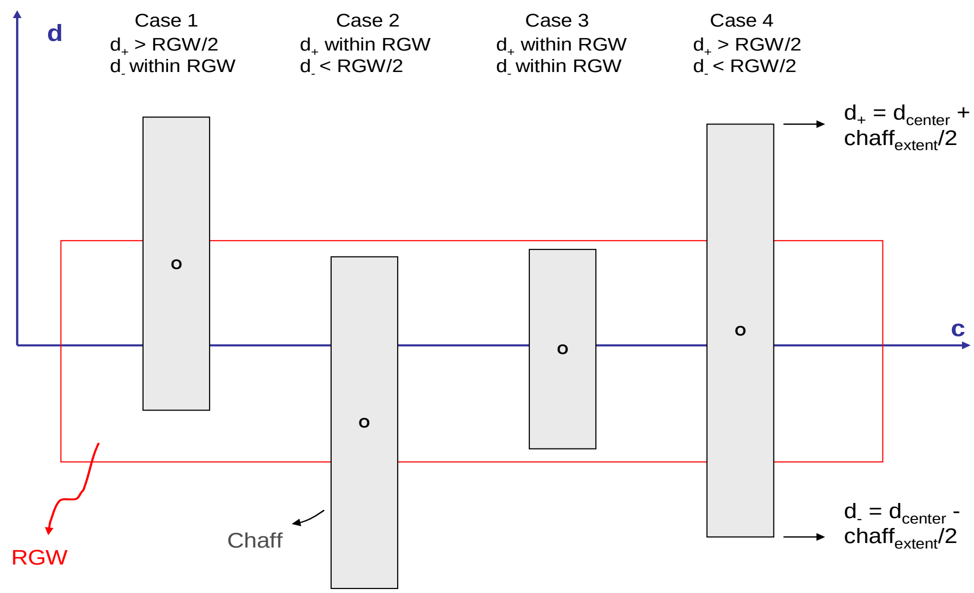


Figure 10: Chaff cloud interseciont cases with RGW

The ratio (or fraction) of chaff intersects the RGW is given in equations (25) and (26).

$$I_{ratio\ dt}(L\#, T_f, dt) = \frac{I_{chaff\ dt}(L\#, T_f, dt)}{d_{chaff\_extent}} \tag{25}$$

$$I_{ratio\ dBdt}(L\#, T_f, dt) = 10\log_{10}(I_{ratio\ dBdt}(L\#, T_f, dt)) \tag{26}$$

Using these equations, the fuse time solutions that result in the chaff trajectory intersecting the RGW can be extracted and plotted.

2.5 Minimum miss distance

The minimum value of miss distance in cross range is a function of the physical extent of the ship and a minimum safe distance threshold set by the user. Figure 11 shows the physical extent of the ship relative to the RGW coordinate system. It is found that the minimum value of miss distance is given by equations (27) through (29).

$$c_{miss\_\min} = \max\left(c_{miss\_\min 1}, c_{miss\_\min 2}\right) \tag{27}$$

where

$$c_{miss\_\min 1} = \left| \frac{W_{ship}}{2} * \cos(\theta_{missile}) + \frac{L_{ship}}{2} * \sin(\theta_{missile}) \right| + Thres_{\min\_miss} \tag{28}$$

$$c_{miss\_\min 2} = \left| -\frac{W_{ship}}{2} * \cos(\theta_{missile}) + \frac{L_{ship}}{2} * \sin(\theta_{missile}) \right| + Thres\min\_{}_{miss} \tag{29}$$

and $Thres_{\min\_miss}$ is the minimum cross range distance between the missile and the ship for the ship to be safe. Solutions, where the chaff miss distance in cross range is smaller than the user minimum value of miss distance, are dismissed.

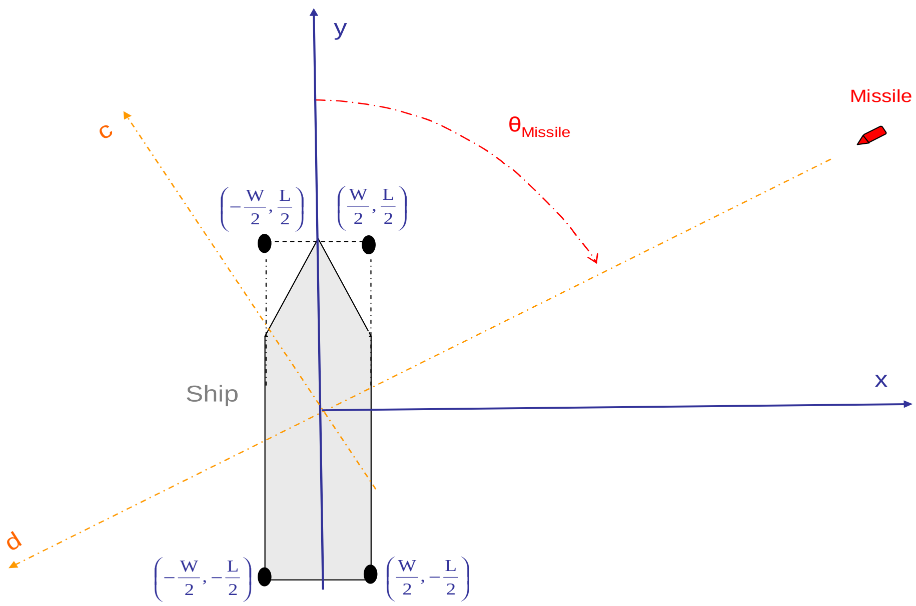


Figure 11: Ship dimension in x, y coordinates

2.6 Chaff RCS Models

The model for the chaff RCS as a function of elapsed time since fuse time is taken from reference [1]. In this model, the chaff RCS as a function of time is given by equation (30).

$$RCS_{chaff\ dt} = RCS_{chaff\ \max} * \left[1 - e^{-\left(\frac{dt}{K_{bloom}}\right)}\right] \tag{30}$$

where K_{bloom} is a blooming constant which is set as 2.2, and $RCS_{chaff\ dt}$ is in dB. Figure 12 shows the value of RCS as a function of elapsed time after fusing. The chaff takes about 10 seconds to reach its maximum value, which is relatively long considering that a missile is coming toward the ship. The value of the chaff RCS, relative to its maximum value, is given by equation (31).

$$RCS_{chaff\ dt\,rel} = RCS_{chaff\ dt} - RCS_{chaff\ \max} \tag{31}$$

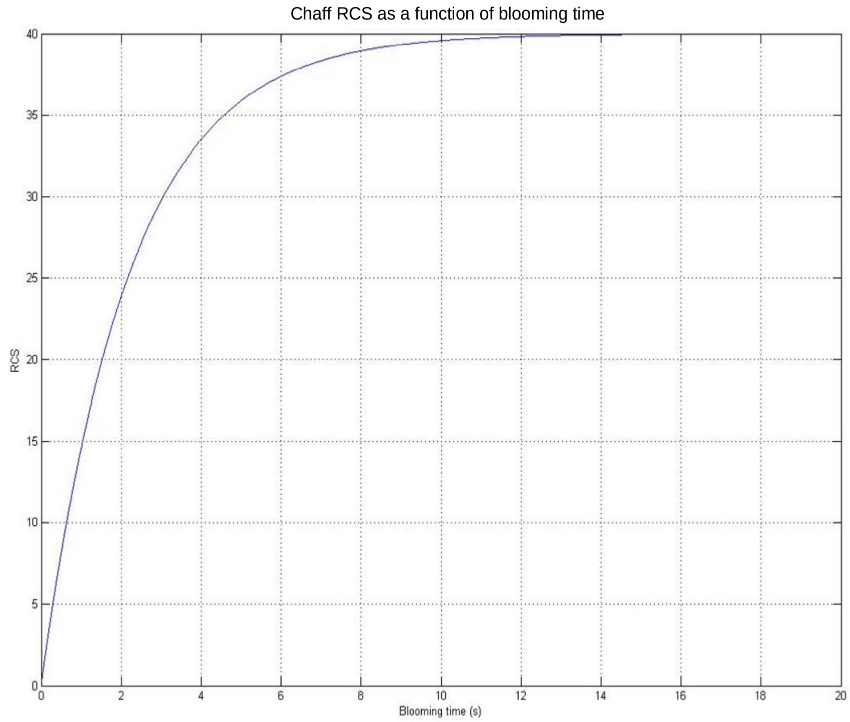


Figure 12: RCS of blooming chaff as a function of time

2.7 Seeker Antenna model

An ASM seeker typically uses a monopulse antenna system to track the targeted ship. Monopulse radar consists of two antenna beams offset by the 3dB antenna beamwidth [3]. The modeling of the whole tracking process would be relatively complex and would require a lot of processing time. A simple approach is used in this document to model the probability of the seeker detecting the chaff and being captured by it. This subsection starts by modeling an antenna beam pattern followed by the monopulse antenna system.

2.7.1 Antenna beam pattern

In the far-field the electric-field pattern of an aperture antenna is the Fourier transform of the electric field illuminating the aperture [4] to [6]. For uniformly illuminated one-dimensional aperture, the antenna pattern is proportional to the square of a scaled sinc function [4]. The sinc function is given by equation (32) and is plotted in Figure 13.

$$\mathrm{sinc}(\varphi) = \begin{cases} 1, & \varphi = 0 \\ \dfrac{\sin(\pi\varphi)}{\pi\varphi}, & \varphi \neq 0 \end{cases} \tag{32}$$

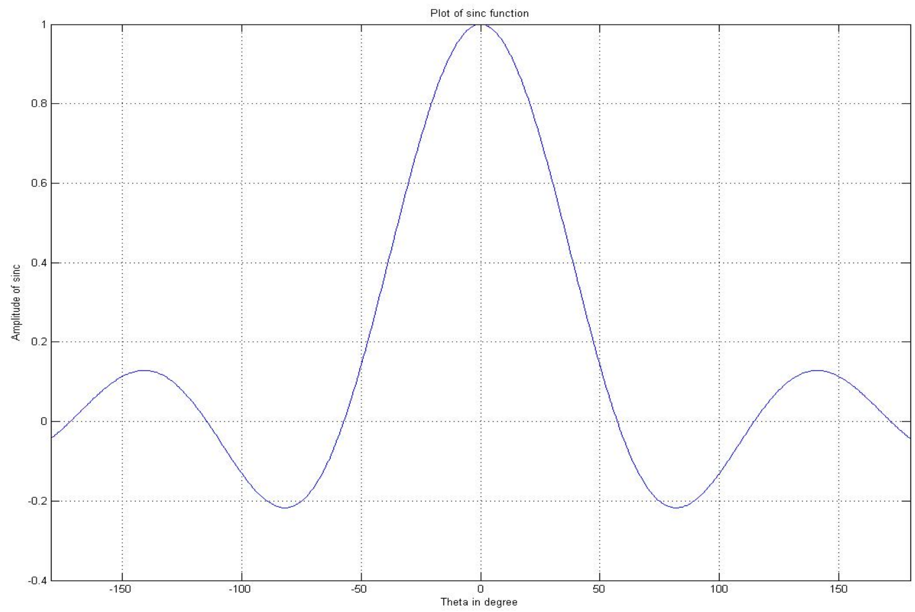
where φ is in radian. The 3dB beamwidth (half value on linear scale) of the sinc function is equal to 25.379 degrees. The sinc function can be scaled to other beamwidths by multiplying the φ argument in radians with the ratio of the 3dB beamwidth of the sinc function on the new beamwidth as shown in equation (33).

$$\varphi_{scaled} = \varphi * \frac{\varphi_{3dB\sin c}}{\varphi_{3dBbeam}} \tag{33}$$

The normalized antenna gain at angle φ, relative to boresight, is given by equation (34).

$$G_{rel} = \left[\mathrm{sinc}\left(\varphi * \frac{2\,\varphi_{3dB\sin c}}{\varphi_{3dBbeam}} \right) \right]^2 \tag{34}$$

where $\varphi_{3dB\sin c}$ is the 3dB beamwidth of the non-scaled sinc function and is equal to 25.379 in degrees. $\varphi_{3dBbeam}$ is the 3dB beamwidth of the antenna beam to be modeled. Figure 14 shows the antenna pattern using the square of the scaled sinc function for a 3dB beamwidth of 5^0. Annex A shows values of the arc sustained by a beam of 4 degrees in function of range. The length of the arc sustains by the seeker main beam having a 4 degrees beam will be about 700m.



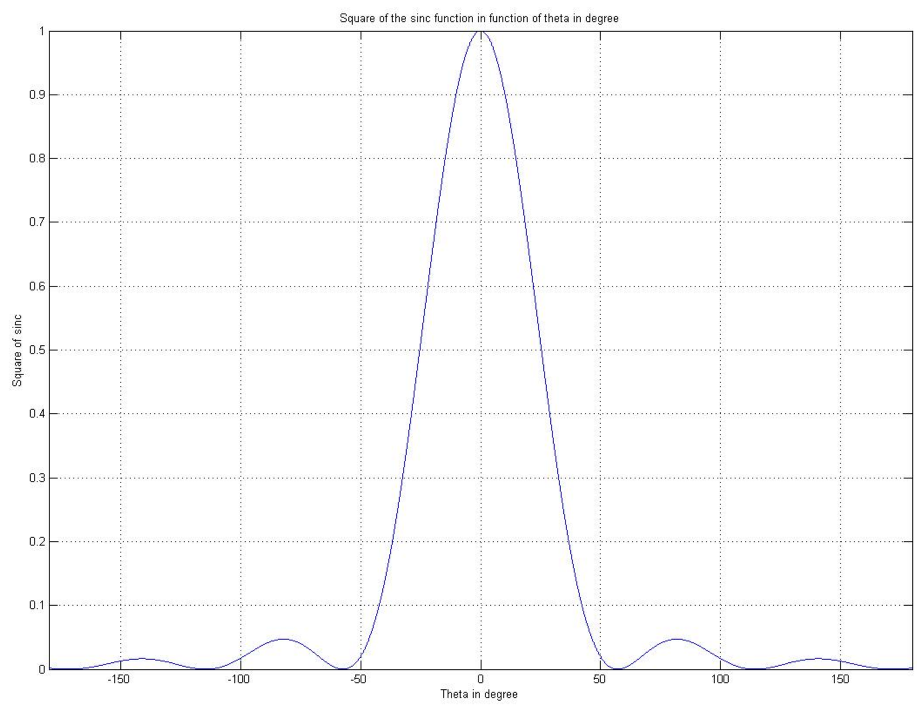


Figure 13: Plot of the sinc function and its squaret

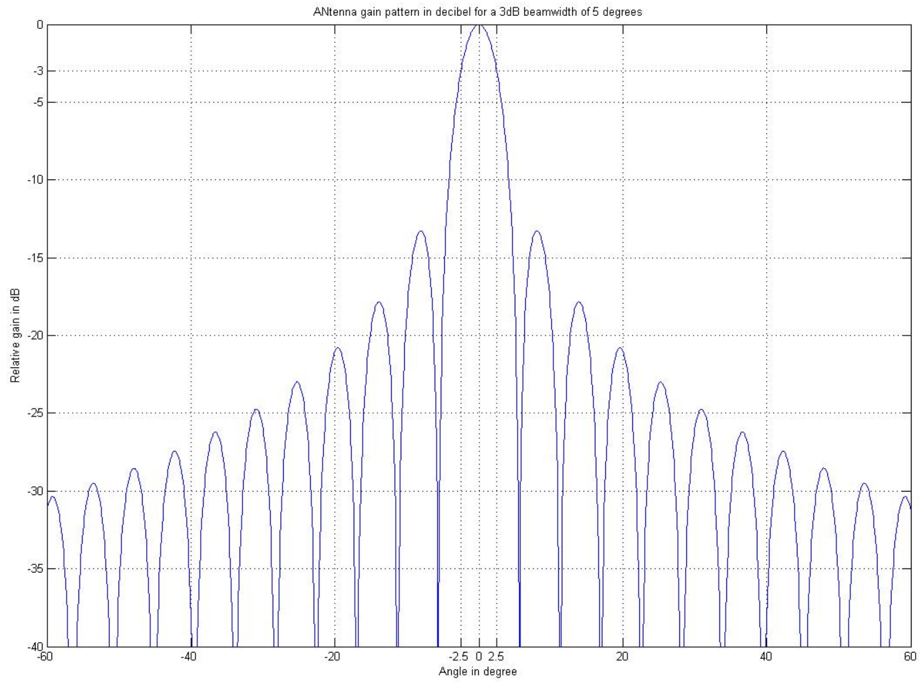


Figure 14: An example of antenna gain pattern in decibels for a 3dB beamwidth of 5 degrees

2.7.2 Monopulse antenna system

A monopulse antenna system uses two identical antenna beams offset by an angle equal to the 3dB of the antenna beam as shown in Figure 15. From the figure, we have equations (35) and (36) .

$$\theta_A = \theta_M - \frac{\theta_{3dB}}{2} \tag{35}$$

$$\theta_B = \theta_M + \frac{\theta_{3dB}}{2} \tag{36}$$

where θ_A is the boresight direction of antenna A; θ_B is the boresight direction of antenna B; θ_M is the boresight direction of the monopulse antenna system; and θ_{3dB} is the 3dB antenna beamwidth. The chaff angle relative to the boresight of antenna A and B are given by equations (37) and (38).

$$\theta_{chaff\_A} = \theta_{chaff} - \theta_A \tag{37}$$

$$\theta_{chaff\_B} = \theta_{chaff} - \theta_B \tag{38}$$

The antenna gain at the chaff position for each antenna, relative to their boresight, is given by equations (39) and (40).

$$G_{chaff\_A} = \left[\mathrm{sinc}\left(\theta_{chaff\_A} * \frac{2\,\theta_{3dB\sin c}}{\theta_{3dBbeam}} \right) \right]^2 \tag{39}$$

$$G_{chaff\_B} = \left[\mathrm{sinc}\left(\theta_{chaff\_B} * \frac{2\,\theta_{3dB\sin c}}{\theta_{3dBbeam}} \right) \right]^2 \tag{40}$$

where G_{chaff_A} and G_{chaff_B} are the antenna gain for the chaff located at θ_{chaff} from antenna A and B relative to their boresight directions.

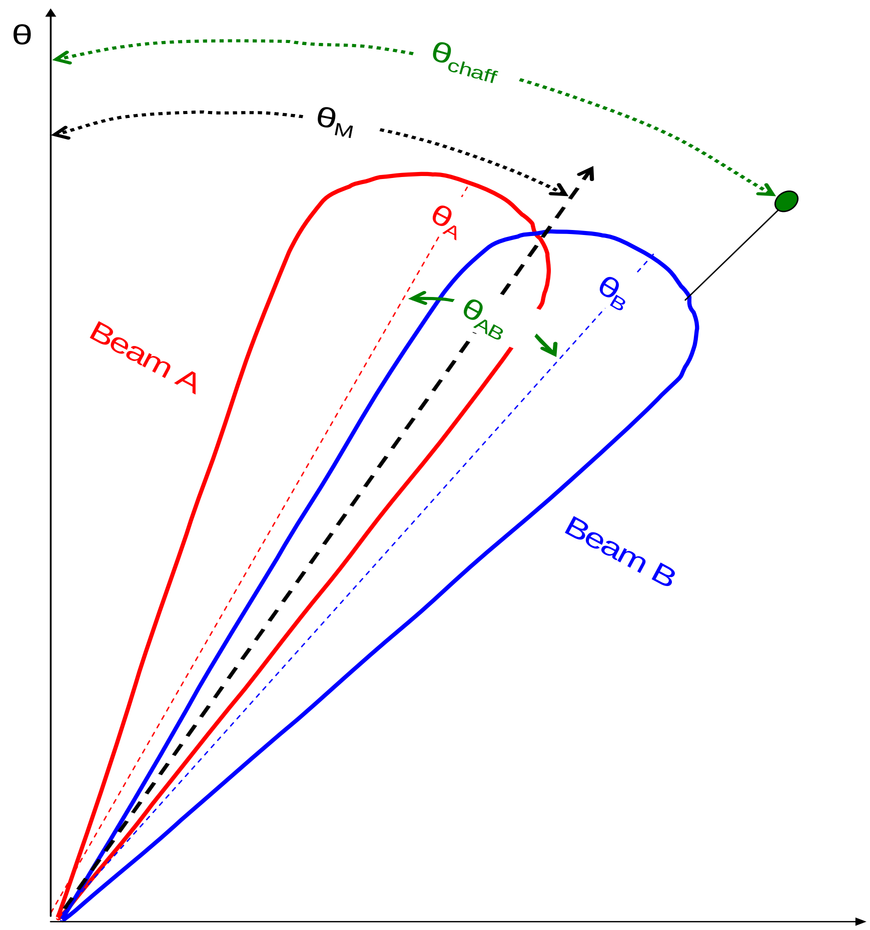


Figure 15: Monopulse antenna system

2.7.3 Probability of detection

The probability of detecting the chaff is a function of the chaff position relative to the monopulse boresight. The probability of detecting a target for each antenna channel can be calculated using the equation of Neuvy [7], (41).

$$P_d = 10^{-u} \qquad \textit{where} \quad u = \left[\frac{\alpha \log\left(\frac{\ln(2)}{P_{fa}} \right)}{n_{nc}^{2/3}\ SNR} \right]^{1/\beta} \tag{41}$$

The constant α and β are functions of the Swelling type of target detected. For a chaff target, β would be equal to 1. In our case, the Signal to Noise Ratio (SNR) cannot be calculated since the seeker characteristics are not known. However, the antenna seeker is assumed to be locked onto the center of the ship. Hence, the probability of detection should be equal to one when reflected power is greater or equal to the power received from the ship. As the ratio of chaff to ship power decreases below unity, the probability of detecting the chaff will decrease until it approaches zero. Using the same form of equation as Neuvy, the probability of detecting the chaff can be expressed as in equation (42).

$$P_{d\ chaff} = \begin{cases} 10^{1-\frac{1}{P_{chaff\ /\ ship}}} & ; P_{chaff\ /\ ship} < 0 \\ 1 & ; P_{chaff\ /\ ship} \geq 0 \end{cases} \tag{42}$$

where $P_{chaff\ /\ ship}$ is the maximum ratio of received power from the chaff and the ship. The 1 in the exponent was added to get a probability of detection of unity when the power ratio is 1. Figure 16 shows the modeled probability of chaff detection as a function of the received power ratio, in dB, between the chaff and the ship. The relative antenna gain at the ship location is 3dB lower than on the boresight of each antenna. Hence, the probability of chaff detection decreased rapidly beyond the 3dB beamwidth of both antennas. This is a realistic feature of a monopulse system.

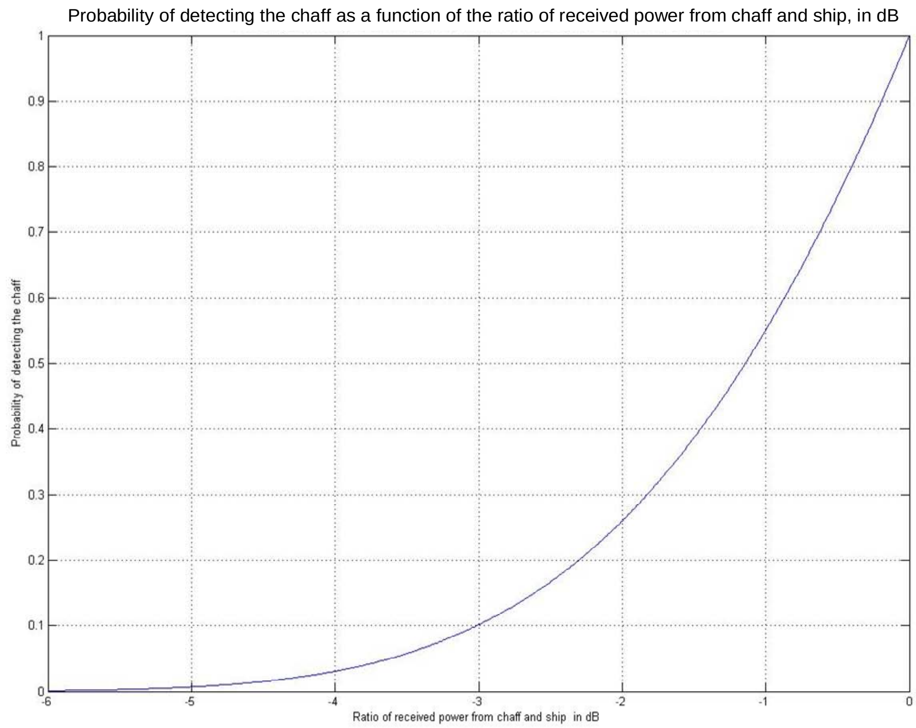


Figure 16: Probability of detecting the chaff when the monopulse system is locked onto the ship

The power reflected back from the ship and the chaff into each antenna channel is proportional to the RCS, propagation factor, antenna gain relative to boresight and intersect ratio, as shown in equations (43) and (44).

$$P_{chaff}(t) \propto \sigma_{chaff}(t)\, F_{chaff}(t)\, G_{rel\,chaff\,\max}(t)\, I_{chaff}(t) \tag{43}$$

$$P_{ship}(t) \propto \sigma_{ship}(t)\, F_{ship}(t)\, G_{rel\_ship}(t)\, I_{ship}(t) \tag{44}$$

where σ is RCS, F is the propagation factor, G_{rel} is the antenna gain at chaff position relative to boresight, and I is the intersect ratio with the missile RGW. The ratio between the received power from the chaff and the ship is then given by equation (45).

$$P_{chaff\,/\,ship}(t) = \frac{\sigma_{chaff}(t)\, F_{chaff}(t)\, G_{rel\ chaff\ \max}(t)\, I_{chaff}(t)}{\sigma_{ship}(t)\, F_{ship}(t)\, G_{rel\_ship}(t)\, I_{ship}(t)} \quad (45)$$

The ship is located on the monopulse antenna system boresight and centered on the RGW. Hence, the relative antenna gain is 0.5 and the intersect ratio is 1.0. Ignoring propagation effects such as multipath we have equation (46).

$$P_{chaff\,/\,ship}(t) = \frac{c_{\ chaff}(t)\ \ G_{rel\ chaff\ \max}(t)\, I_{chaff}(t)}{0.5\, \sigma_{ship}(t)} \quad (46)$$

This generation of chaff solutions does not consider missile height. This is not too important since the seeker is locked onto the ship, i.e., it is within the seeker beamwidth. The missile height will become important when the propagation factor is included in the model.

2.7.4 Probability of capture

An analysis of the monopulse response ($\Delta/\sum$ channel), as the chaff moves away from the ship, was done using the output of the monopulse antenna channels (see Annex B). It was found that the equivalent RCS center will be tracked until the monopulse system switches to one of the targets. The monopulse tracker will come back to the ship whenever its RCS is greater or equal to the chaff RCS. Also, the probability of recapture by the ship decreases as the chaff RCS exceeds the RCS of the ship. Based on previous findings, a simple model for the probability that the chaff will stay captured until the end is given by equation (47).

$$P_{capt} = 1 - e^{-\sigma_{chaff\ ship\ dB}}; \quad and\ \sigma_{chaff\ ship\ dB} > 0 \quad (47)$$

When the chaff RCS exceeds the ship RCS by 3dB, the probability of chaff staying captured until the end is close to 95%, which seems realistic [1]. Figure 17 shows the probability of chaff capture as a function of reflected power relative difference based on equation (47).

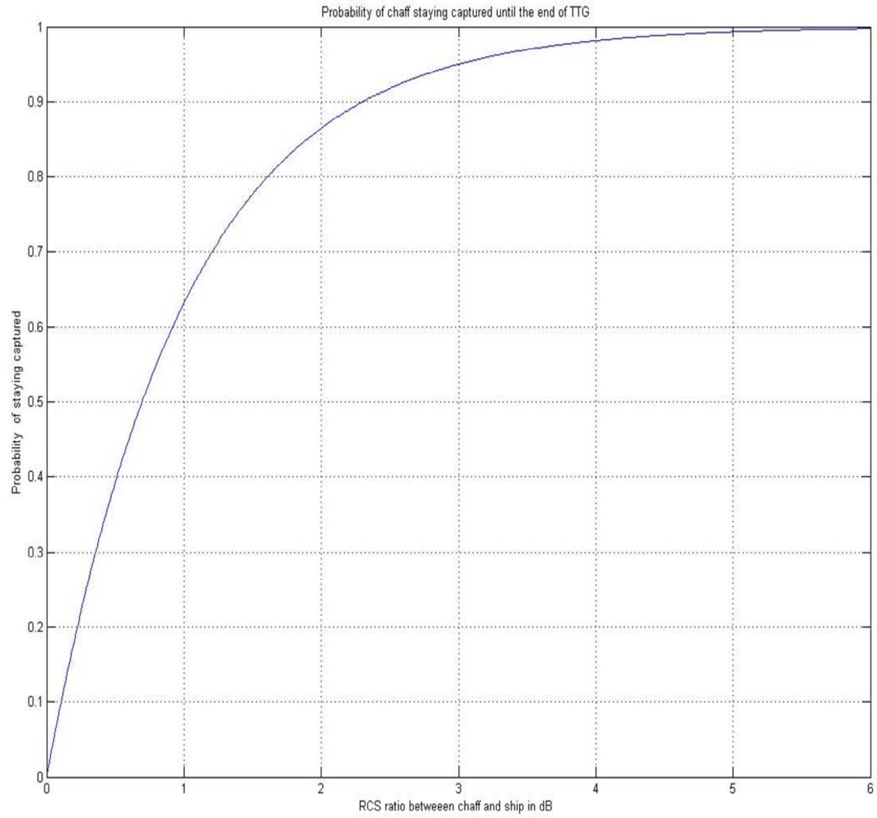


Figure 17: Probability of chaff capture in function of reflected power relative difference

For each intersection point within the RGW, the probability that the chaff will be detected and stay captured is given by equation (48).

$$P_{d\ capt}(t) = P_d(t) * P_{capt} \tag{48}$$

The total probability of chaff being detected and captured within the RGW is given by equation (49).

$$P_{tot\_d\_capt} = 1 - \prod_{t} (1 - P_{d\_capt}(t)) \tag{49}$$

2.8 Choice of best solution

Chaff can be launched immediately at initial Time To Go (TTG) or can be delayed by steps. Chaff solutions having the larger number of time delays before launch, provide more flexibility to the operator. The best solution is then chosen among the group of solutions (Tf, L#) having the greater number of possible time delays. The best solution from this group is the one that has greatest weight for miss distance, detection and capture defined in equations (50) and (51):

$$w_{tot} = 0.75\, w_{d\_capt} + 0.25\, w_{Rmiss} \qquad (50)$$

$$\begin{aligned} w_{d\_capt} &= P_{t\_d\_capt}; \\ w_{Rmiss} &= 1 - e^{3\left(1 - \frac{R_{miss}}{Min\_Rmiss}\right)} \end{aligned} \qquad (51)$$

where w_{d_capt} is the weight associated with the total probability of being detected and staying captured, and w_{Rmiss} is the weight associated with the miss distance. A greater weight was given to detection and capture since it must occur to be successful. The weight for miss distance is also lower since only the solutions with miss distance above safe thresholds are kept.

The weight associated with the miss distance was defined assuming that beyond a certain distance in miss distance there is not much difference in the outcome. Figure 18 shows an example of weight associated with the miss distance. The minimum miss distance was set to 100m in this case. As it is seen, the weight associated with the miss distance decreases rapidly when the miss distance exceeds the minimum value and then approaches unity around 300 meters.

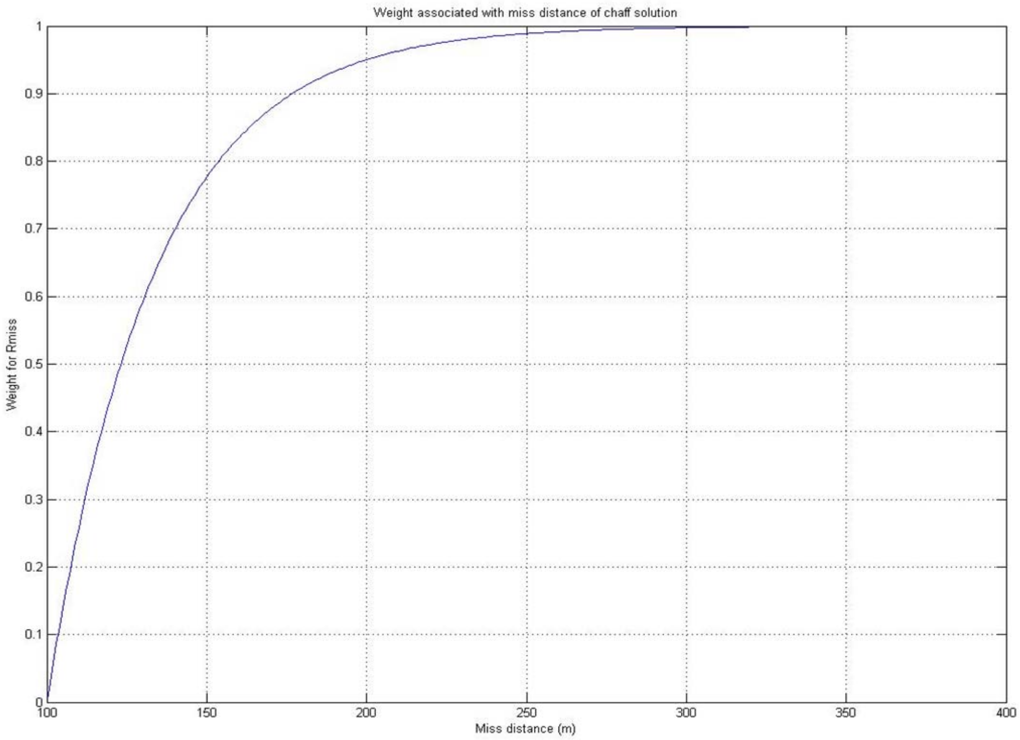


Figure 18: Weight associated with miss distance

3 Improved algorithm

3.1 Algorithm

Table 3 shows the improved chaff solution algorithm, at a high level. It can be divided into four parts: possible solutions, geometric criteria, seeker criteria ,and selection of best solution.

This algorithm starts by generating trajectories of all possible chaff solutions including a set of discrete time delays before launch of chaff round. The chaff round, as seen in section 2, will fall back at launcher level after about 19.5 seconds. This maximum value of fuse time for chaff solutions is then set to 19.5s (or initial Time To Go (TTG) if smaller than that). The minimum value of Tf was set to 0.42s to avoid exploding the chaff round over the ship. Indeed, the chaff round travels 50 meters during that time, which corresponds to half the length of the exploded chaff cloud. In the past, only small subsets of candidate solutions for fuse time were considered.

After that, geometric considerations are used to filter out chaff solutions where chaff clouds do not intersect with the seeker range gate and where miss distance is below a given threshold. The range gate is here assumed to be infinite in length, perpendicular to the missile trajectory. Equations developed in section 2 are used to filter out unwanted solutions.

After that seeker considerations are used to filter out chaff solutions, which the total probability of detection and capture is below a minimum threshold. This step takes into account the difference in RCS and received power between the chaff and the ship. The corresponding equations developed in section 2 are used to filter out unwanted solutions.

The next step consists of selecting the best solutions from among these final solutions. The group of solutions (Tf, L#) having the greater number of possible time delays before launching is first selected. The best solution of this group is then selected based on a weighted sum for total probability of detection and capture, and also for the miss distance.

Table 3: High-level chaff solution algorithm

Possible solutions

1. Calculate trajectories of all possible chaff solutions

Geometric criteria

2. Keep solutions that intersect with RGW
3. Keep solutions that exceed minimum miss distance

Seeker criteria

4. Calculate relative difference in reflected power
5. Keep solutions that exceed minimum probability of capture

Select optimal solution

6. Select solution that optimize probability of soft-kill

3.2 Results example

The scenario considered in this subsection includes a missile flying at 300m/s coming from the head of the ship at an initial range of 12,000m. The antenna beams of the seeker have a 3dB beamwidth of 4^0 and the seeker RGW is equal to 100m. The wind speed is 20 knots away from ship toward, 190^0 clockwise from ship heading. The minimum miss distance is set to 50m. The minimum probability for detection and capture is set to 60%. The maximum RCS of chaff is set to 40dB and the average ship RCS is set to 37dB. Figure 19 shows the trajectories of all possible chaff solutions for this scenario. The grey circle indicates the position of the chaff round at fuse time. The chaff then drifts along the wind direction until the total elapsed time (Tf plus drift time) is equal to the initial missile TTG. The chaff trajectories are not symmetrical on each side of the ship. This is due to the relative wind direction heading of 190^0. Figure 20 and Figure 21 show the solutions from previous figures where the chaff clouds intersect with the missile range gate and the miss distance above a minimum value. Chaff center positions within the minimum miss distance are shown in red to enhance awareness. As can be seen, the number of solutions has been significantly reduced. Figure 22 shows the chaff solutions where the total probability of chaff detection and capture is above the minimum value set by the user. Once again the number of chaff solutions was reduced. All solutions from launchers 3 and 4 were discarded. This makes sense since their trajectories intersect very little with the RGW. The best solution is shown in green and the probability of soft-kill is one in this case. The black dots on the chaff trajectories are the final positions when chaff launches are delayed until the missile reaches initial Range To Go (RTG) minus 0, 1.5, 3, 4.5 and 6km. The more delays you can have, the more robust the solution. For example, the best solution is still successful even if the chaff is launched when the missile RTG is only 6km rather than 12km.

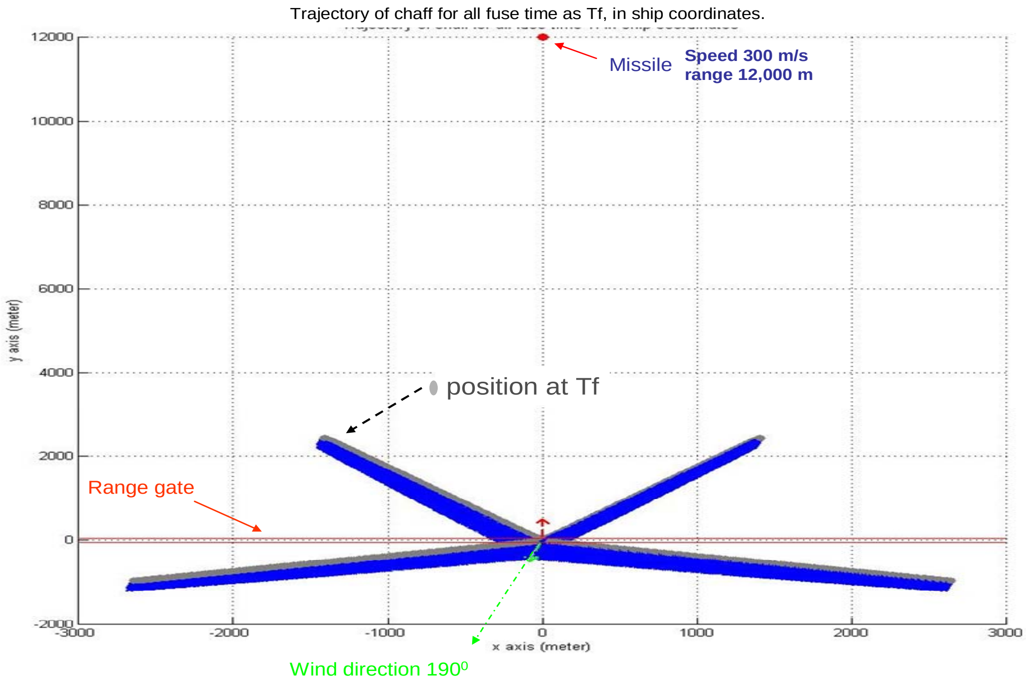


Figure 19: Trajectories of all possible solutions for a given scenario

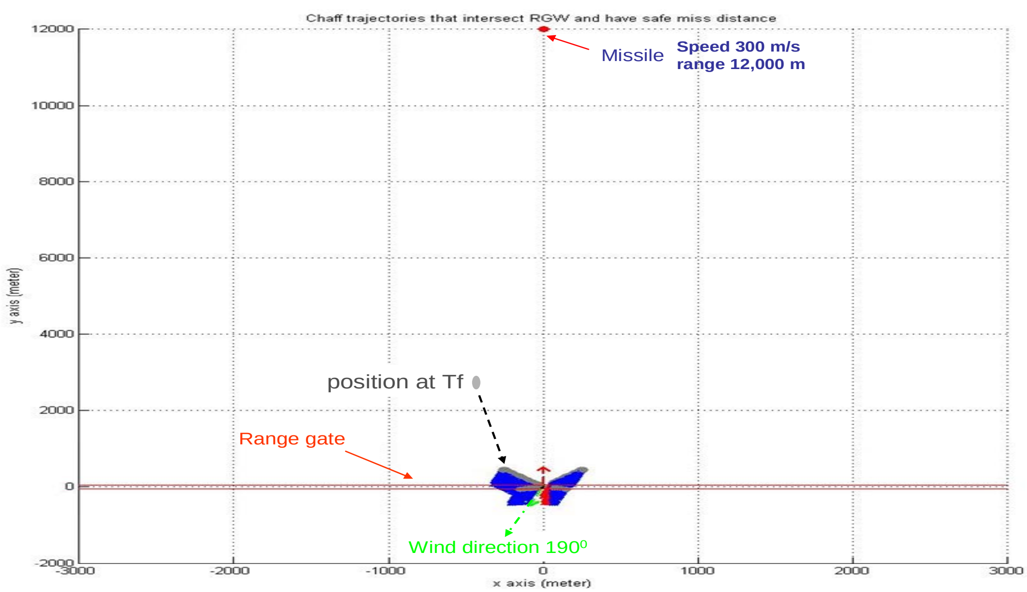


Figure 20: Trajectories of all solutions that intersect RGW for a given scenario

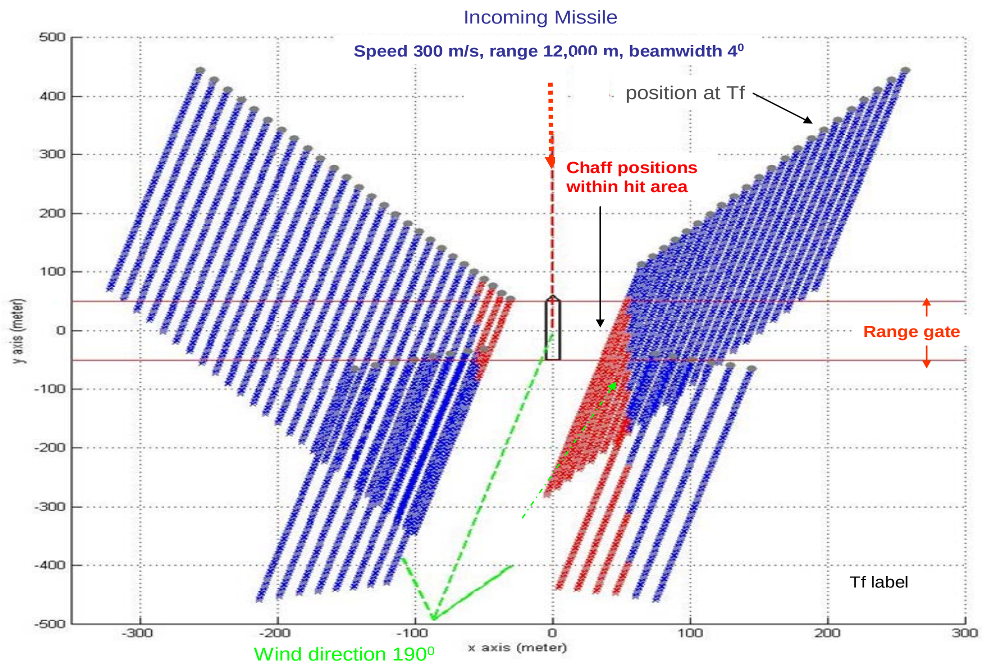


Figure 21: Close up of the trajectories of all solutions that intersect RGW for a given scenario

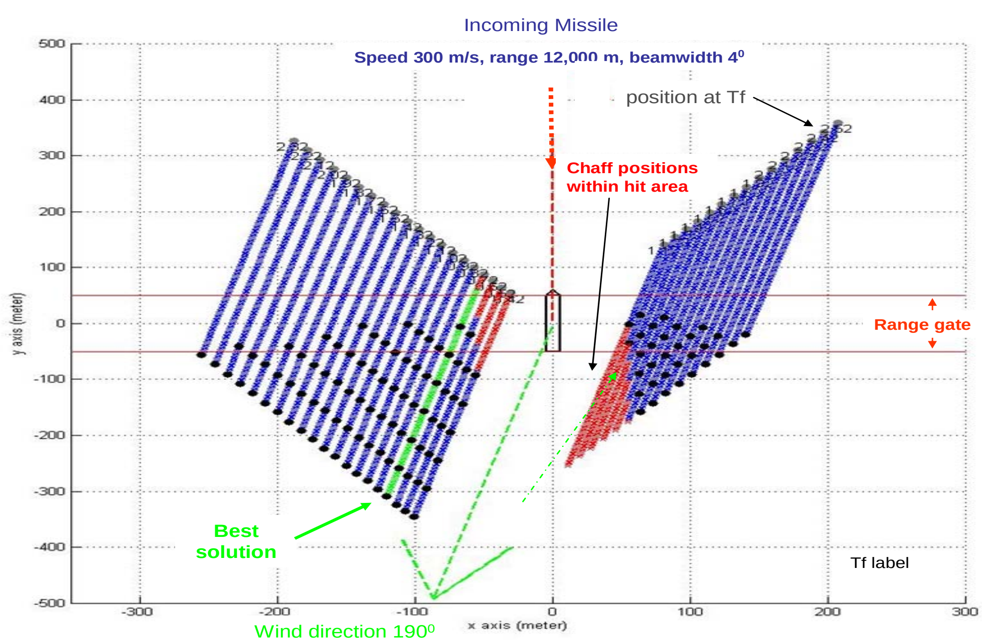


Figure 22: Trajectories of all solutions that exceed minimum probability of soft-kill

Another scenario considered here is a missile flying at 300m/s coming from the head of the ship and initially at a range of 4,700m. The antenna beams of the seeker have a 3dB beamwidth of 4^0 and the seeker RGW is equal to 100m. The wind speed is 20 knots and is varied from 0 to 360 degrees going away from ship clockwise from ship's heading. The minimum miss distance was set to 50m. The minimum probability for detection and capture is set to 60%. The maximum RCS of chaff is set to 40dB, and the average ship RCS is set to 37dB. Figure 23 shows the probability of soft-kill for this stressing scenario. It is seen that despite the short range where the missile appears, chaff solutions would still protect the ship relatively well for most wind directions.

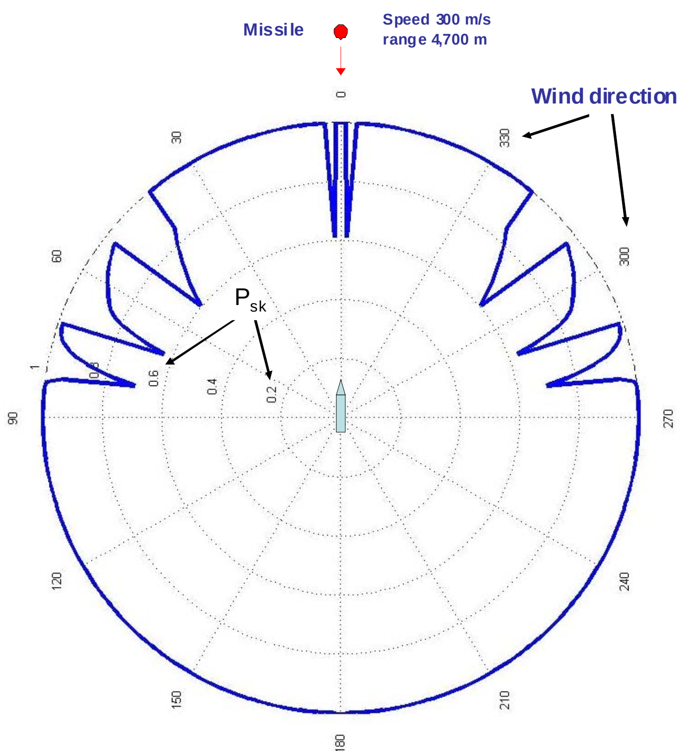


Figure 23: Trajectories of all solutions that exceed minimum probability of soft-kill

4 Future Work

The SISWS chaff solution algorithm was improved as described, in this document, by adding a monopulse antenna model to the seeker, and by using a more physical approach to the selection of solutions. The generated chaff solutions should provide more accurate responses for optimal survivability.

The next step would consist of analysing the efficiency of the chaff solutions in SADM and also comparing the results with the SISWS chaff solution algorithm. After that, the algorithm can be modified further to include the tactic of combining Continuous Wave (CW) jamming with chaff solutions.

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Annex A Distance traveled in function of wind speed

Distance traveled in function of wind speed			
Time (sec)	10 knots	20 knots	30 knots
10	5.1	10.3	15.4
20	10.3	20.6	30.9
30	15.4	30.9	46.3
40	20.6	41.2	61.7
50	25.7	51.4	77.2
60	30.9	61.7	92.6

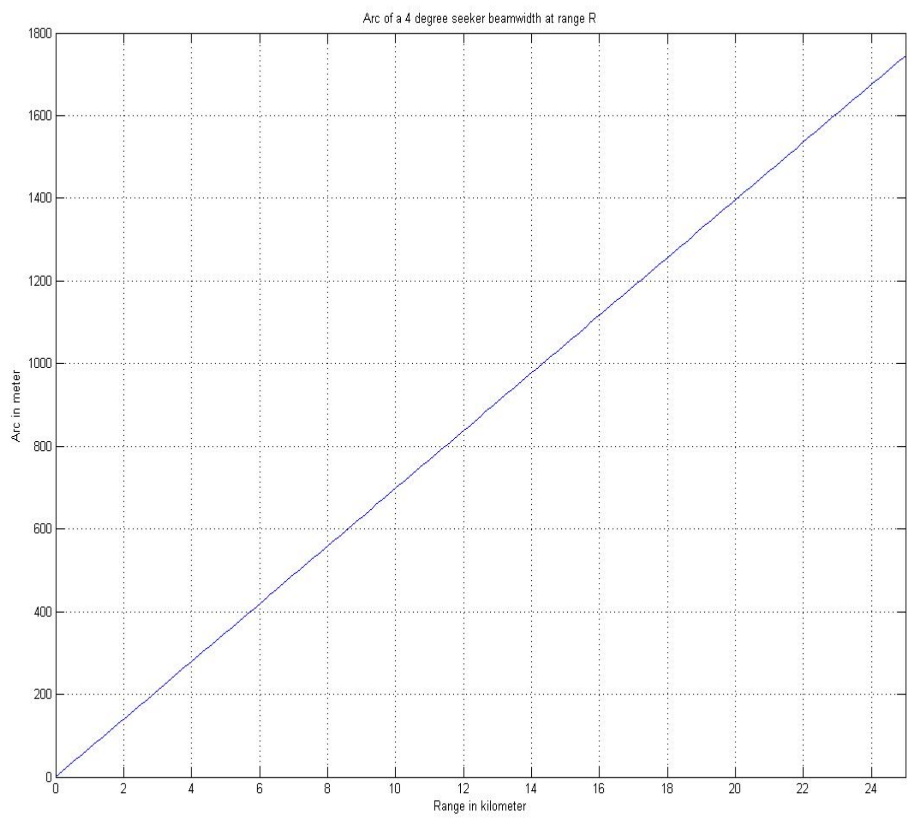

Figure 24: Arc of a 4^0 beamwidth at range R.

Annex B Monopulse tracking for two RCS targets

In this annex, the monopulse response (Δ/∑ channel) is analyzed to determine how well a target moving away from another stationary target will be tracked. Figure 25 though Figure 27 shows the delta sum ratio of the monopulse tracker when two targets of same RCS values are separating in time. Target 1 is fixed and is located at 0^0. Target 2 is moving away from target 1 in discrete steps from one figure to the next. Red dots represent the position of the two targets at each time snapshot. On Figure 25a, the monopulse response will cause the tracker to be centered on the equivalent RCS center. As target 2 moves away from target 1, the monopulse response will cause the tracker to follow the equivalent RCS center, as shown on Figure 25b. Around the last track position, the monopulse response will become flat, as shown on Figure 26a. As a result, the monopulse tracker will stay at the last track position. As target 2 continues to move away, the monopulse response will revert in polarity. This means that the tracker will now track the fix target, as shown on Figure 26b and Figure 27, causing the monopulse to track the fix target. Looking at other values of RCS for target 2, it was found that the equivalent RCS center will be tracked until the monopulse system switches to one of the targets. The monopulse tracker will come back to the ship whenever the ship RCS is greater or equal to the chaff RCS. Also, the probability of recapture by the ship decreases as the chaff RCS exceeds the RCS of the ship.

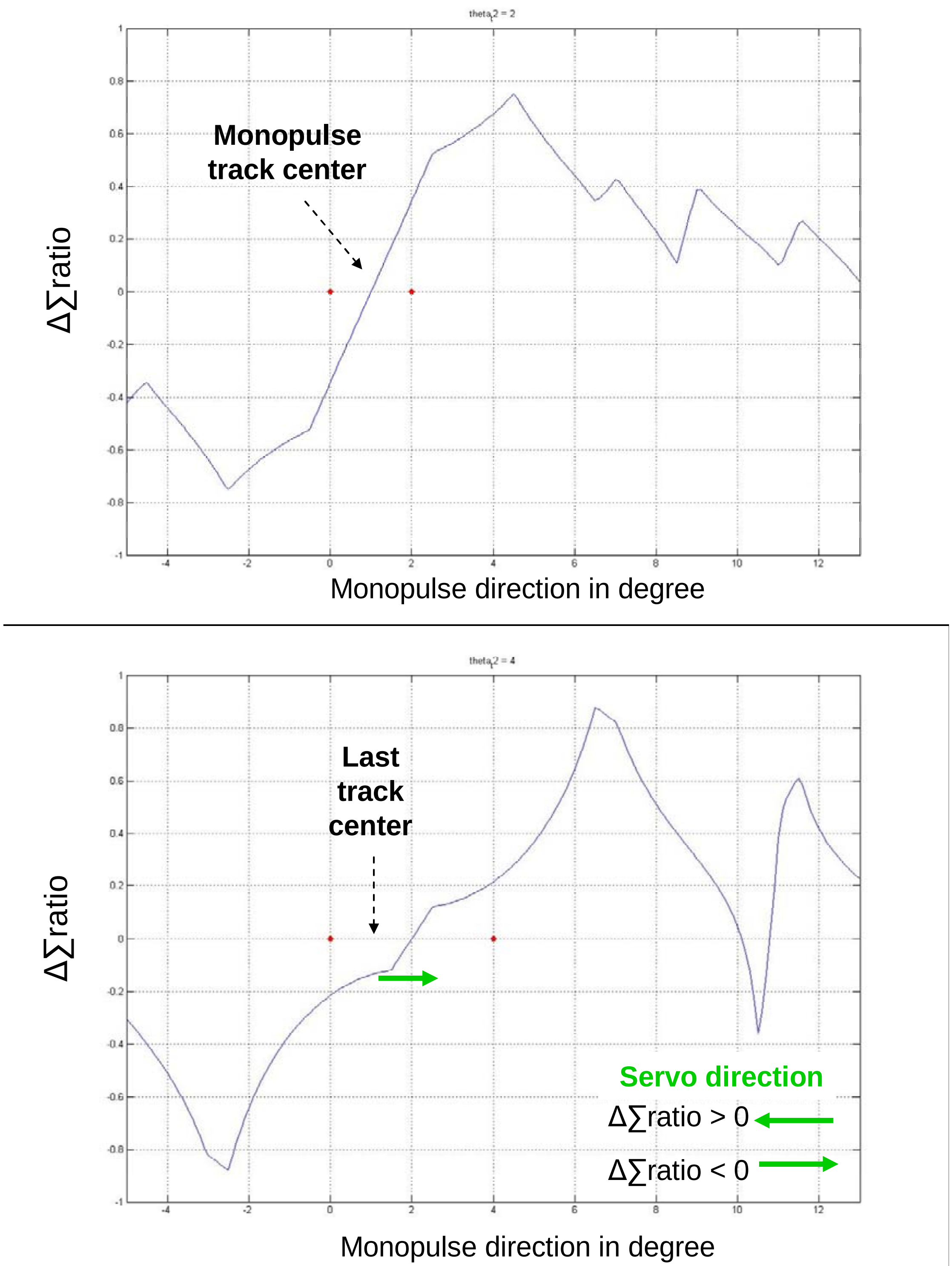

Figure 25: Monopulse delta sum ratio for 2 targets as a function of monopulse direction

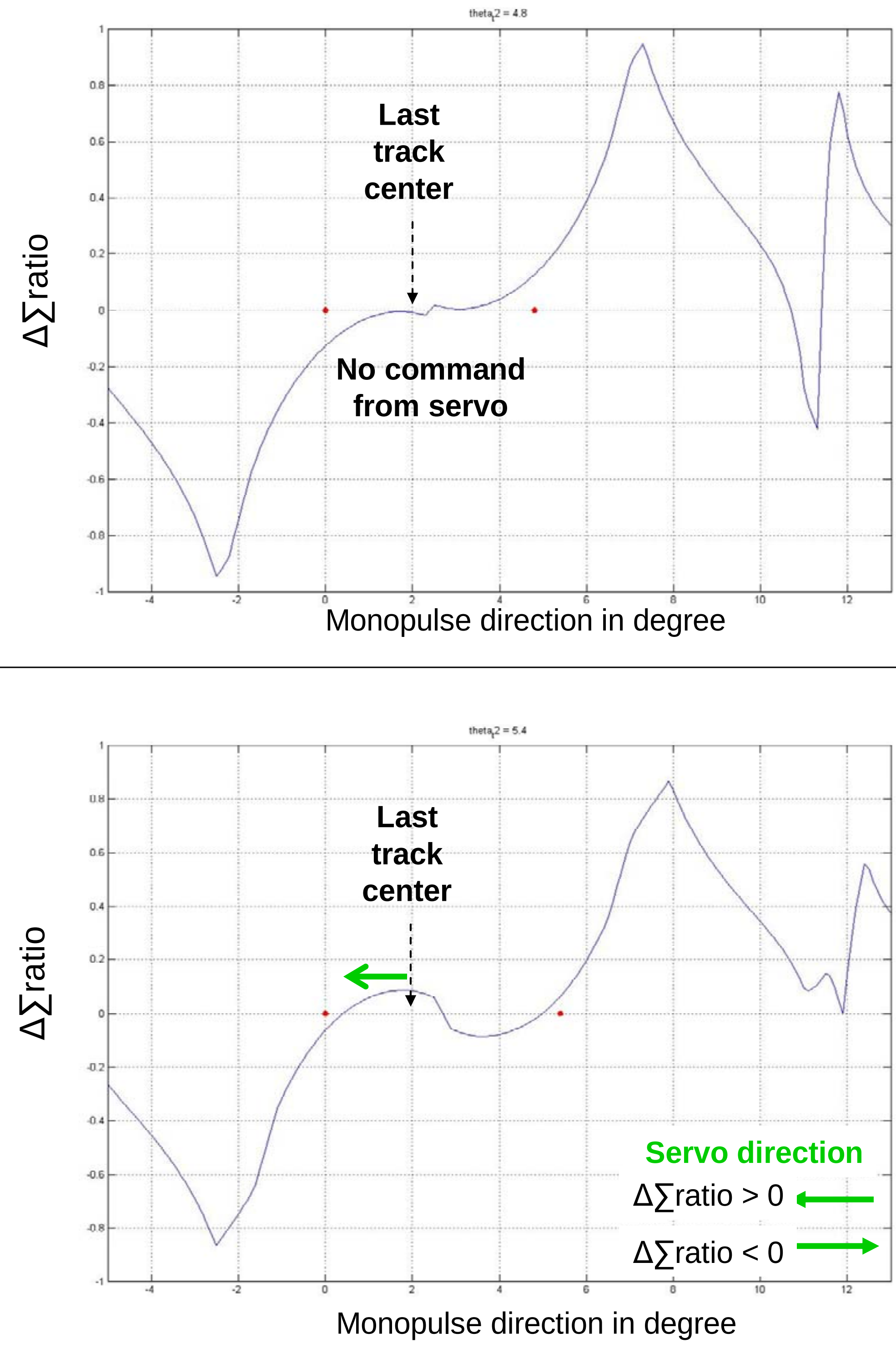

Figure 26: Monopulse delta sum ratio for 2 targets as a function of monopulse direction

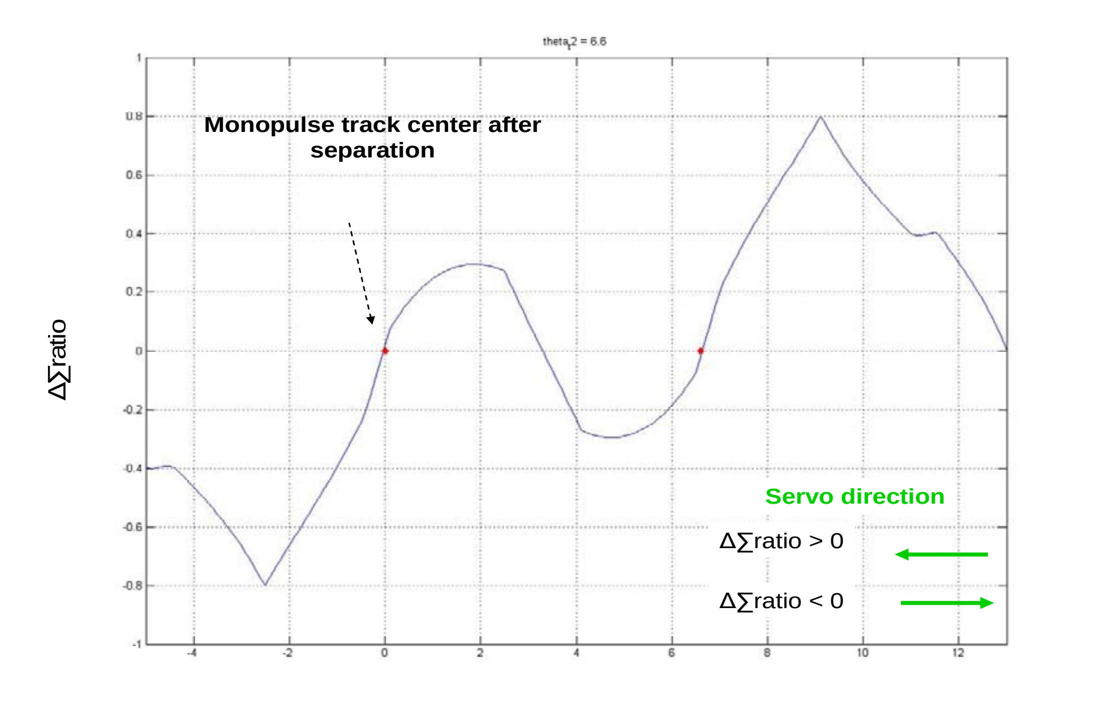

Figure 27: Monopulse delta sum ratio for 2 targets as a function of monopulse direction

List of symbols/abbreviations/acronyms/initialisms

ASM	Antiship Missile
CW	Continuous Wave
DND	Department of National Defence
DRDC	Defence Research & Development Canada
DRDKIM	Director Research and Development Knowledge and Information Management
RCS	Radar Cross Section
RGW	Range Gate Width
RTG	Range To Go
R&D	Research & Development
SADM	Ship Air Defence Missile
SISWS	Shipboard Integration of Sensor and Weapons Systems
SNR	Signal to Noise Ratio (SNR)
TDP	Technology Demonstration Project
TTG	Time To Go

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During the Shipboard Integration of Sensor and Weapons Systems (SISWS) Technology Demonstration Project (TDP), an algorithm was developed to automatically generate the optimal chaff solution as a function of missile initial range and velocity, wind velocity, and engagement geometry. The chaff solution algorithm was developed using customized score functions and simple models. The SISWS chaff seduction algorithm was improved by taking a more physical approach in the selection of chaff solutions, including a monopulse antenna model. The document starts with the scenario being modeled, followed by models for chaff, intersect ratio, monopulse seeker, and probability of soft-kill.

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Chaffs, antiship missile

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