COMPUTER MODELING OF MEDIUM COUPLED RESONANT AIR CORE TRANSFORMERS INCLUDING RESISTIVE LOSSES

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SUMMARY

Transient computer modeling has been applied to medium coupled resonant air core transformers with resistive losses. The completed program was used to study the effects that various parameters (circuit Q and coupling coefficient,k) have on the primary and secondary voltage waveforms. Results showed that even for low Q's and k's there is considerable voltage transfer. The waveforms of voltage and current versus time were in good agreement with experimental measurements. The results of this paper provide design information needed for constructing resonant air core transformers circuits.

INTRODUCTION

The study of medium coupled (k= 10% to 60%) resonant air core transformers on a digital computer has led to a better understanding of the effects that resistive losses and coefficient of coupling have on the primary and secondary voltage and current waveforms.

Closed form solutions for resonant air core transformer circuits with resistive losses do not exist.[1,2,3] By applying a finite element transient circuit analysis technique[4,5] results were obtained for the resonant air core transformer circuit including both primary and secondary resistive losses. The circuit analysis program was run on an HP-85 desk top computer.

COMPUTER CIRCUIT MODEL AND PREDICTED RESULTS OF RESISTIVE LOSSES

Resonant air core transformer circuits allow energy transfer efficiencies to reach 100% in the ideal (lossless) case of 60% primary to secondary transformer coupling. Realistically resistive losses prevent achieving 100% energy transfer though transfer values of 90%-95% have routinely been achieved. The model of the circuit shown in Figure 1 contains lumped capacitors, inductors and resistors as the circuit elements though, the model and the computer program are not restricted to just these elements but could include both the primary and secondary closing switches and distributed effects such as capacitance to ground and stray inductances. The range of values chosen for this study for k were 10 through 60 percent and for Q they were 5 through 100 where Q and k are:

$$Q_1 = \omega_1' \times L_1/R_1 \approx (L_1/C_1)^{\frac{1}{2}} / R_1 \tag{1}$$

$$Q_2 = \omega_2' \times L_2/R_2 \approx (L_2/C_2)^{\frac{1}{2}} / R_2 \tag{2}$$

$$k = M / (L_1 \times L_2)^{\frac{1}{2}} \tag{3}$$

The defined Q's and values of angular frequencies, ω_1 and ω_2, are the values obtained when there is no mutual coupling between the circuits.

The definitions of other quantities used to compare the results are:

$$Q_m = \sqrt{Q_1 \times Q_2} \tag{4}$$

$$E_T = \tfrac{1}{2} \times C_2 \times (V_{2max})^2 \times 100\% \tag{5}$$

The geometrical mean of Q_1 and Q_2, Q_m, is a useful parameter whose purpose will be shown later. Additionally the energy transfer efficiency, E_T, provides a comparison between variation of parameters Q_m and k.

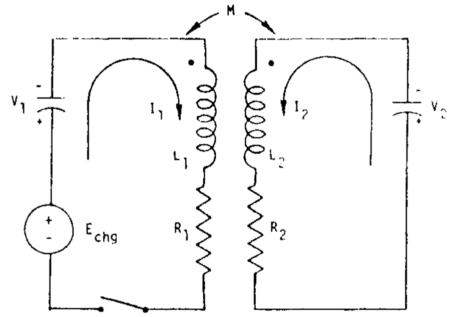

Figure 1. Basic Circuit Model.

The objective of the computer analysis is to predict the instantaneous maximum energy that is transferred from C_1, and stored in C_2 where C_2 represents the load for this model. Referencing Figure 1, at time t = t_0 switch S_1 is closed and discharges capacitor C_1 (charged to dc voltage E_{chg}). Energy is coupled into the secondary load through the mutual inductance of the transformer. The energy transferred from the primary storage element to the secondary load may be determined by the maximum voltages obtained across C_1 and C_2. Resistors R_1 and R_2 represent energy losses in the

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primary and secondary circuits respectively.

The computer program was used to calculate voltage waveforms across the load C_2 for a given set of component values to evaluate the effects of resistive losses.

The following component values were chosen for the computer analysis:

$$C_1 = 1F \quad L_1 = 1H \quad E_{chg} = 1V$$

$$C_2 = 1F \quad L_2 = 1H$$

R_1 and R_2 were varied by varying Q_1 and Q_2 respectively.

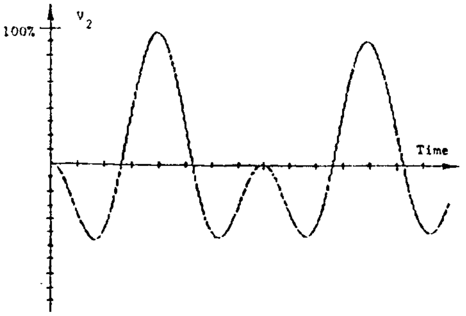

Figure 2. Dual Resonant Charging.

Figure 2 shows the load voltage, V_2, versus time for the ideal (lossless) case when k, the transformer coupling coefficient, is equal to 60 percent. In this figure, the first positive peak is the time where all the primary energy stored in C_1 has been transferred to the load, C_2. This is the condition called "dual resonance", which requires $L_1 x C_1 = L_2 x C_2$ and the load voltage in time has reached a maximum on the second excursion of load voltage. For this paper, the output voltage at the dual resonant condition will be referred to as V_2*. Dual resonant circuits have applications in charging water pulse forming lines to megavolt levels[6].

Figure 3a and 3b are computer solutions which examine the sensitivity of V_2* to variations in Q_1 and Q_2. Both Figures 3a and 3b examine the variation in V_2*, with variations in the ratio of Q_1 to Q_2 holding Q_m constant. Figure 3a is for a coupling coefficient of 30 percent and 3b the coupling coefficient is at the "magic" value of 60 percent. For either k = 30 or 60 percent the dual resonant output voltage V_2* is relatively insensitive to the ratio of Q's but is more dependent on the geometrical mean of the Q's, especially at low values of Q_m where circuit losses have reached approximately 5 percent of the energy stored per cycle. The interpretation of Figure 3 is that value of Q_m is a better representation of the losses for the entire circuit, both primary and secondary portions.

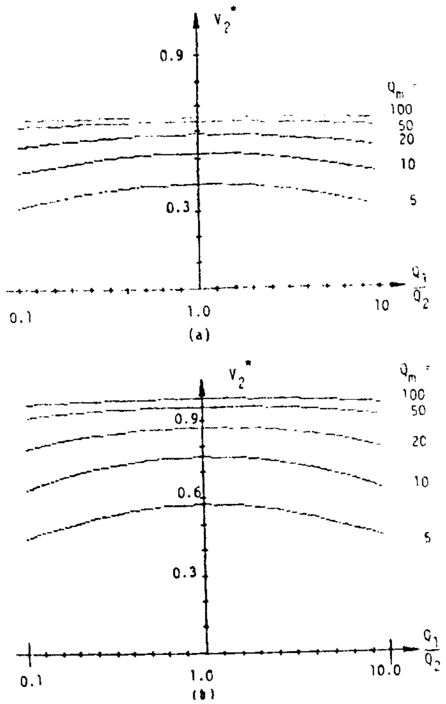

Figure 3. Variation in Dual Resonant Output With Ratio of Primary and Secondary Q's for Constant Q_m's.

For further treatment of the computer model where variations in V_2* are sought, the losses of the circuit can be accounted for using Q_m rather than having to consider both the primary and secondary losses separately.

The energy at dual resonance that has been transferred to the secondary in percent of the initial input energy will be referred to as E_2*.

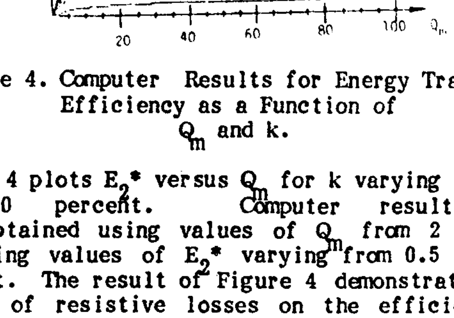

Figure 4. Computer Results for Energy Transfer Efficiency as a Function of Q_m and k.

Figure 4 plots E_2* versus Q_m for k varying from 10 to 60 percent. Computer results, $\omega_1 = \omega_2$ were obtained using values of Q_m from 2 to 100 producing values of E_2* varying from 0.5 to 97.3 percent. The result of Figure 4 demonstrates the effect of resistive losses on the efficiency of energy transfer at the dual resonance condition. Above Q_m of 20 the percent of energy transferred to the load is practically independent of Q_m for all values of coupling. Below Q_m of 20 resistive losses strongly affect E_2*.

EXPERIMENTAL VERIFICATION OF COMPUTER RESULTS

A resonant air-core transformer was constructed according to the dimensions shown in Figure 5.

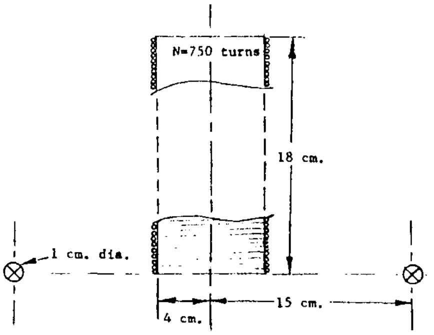

Figure 5. Dimensions of Air Core Transformer Used to Verify Computer Results.

The transformer that was built used 30 gauge enameled wire for the secondary winding and the primary consisted of a single turn of 3/8 inch diameter copper tubing. Since the vertical dimension of the secondary coil was more than a factor of 2 greater than the coil diameter the lumped parameter assumption was investigated. A study of the distributed nature of the coil showed that the resonant frequencies associated with its turn-to-turn capacitance and capacitance to ground, were much higher than the lumped parameter resonant frequency and were in fact, above the combined bandwidth of the voltage divider and digital storage oscilloscope. The transformer was inserted in a circuit similar to that shown in Figure 1 with the following values listed in Table 1:

TABLE 1.
Measured Values used in Experimental Verification Model.

PARAMETER	MEASURED VALUE
C_1	0.14 µF
C_2	5.2 pF
L_1	0.48 µH
L_2	15 mH
M	10.6 µH
K	11.7%
R_1	0.1 ohm
R_2	63. ohm

The input capacitor C_1 was charged to 50 volts and then a mechanical switch was used to initiate the primary discharge. Both the primary and secondary voltage waveforms were monitored with calibrated voltage dividers connected to a Nicolet digital storage oscilloscope. The resulting waveforms are shown in Figure 6 alongside the corresponding computer results.

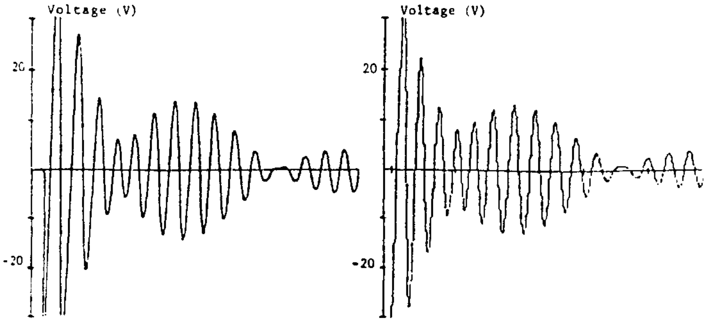

Figure 6a. Comparison of Experimental and Computer Solutions for the Primary Voltage Waveform.

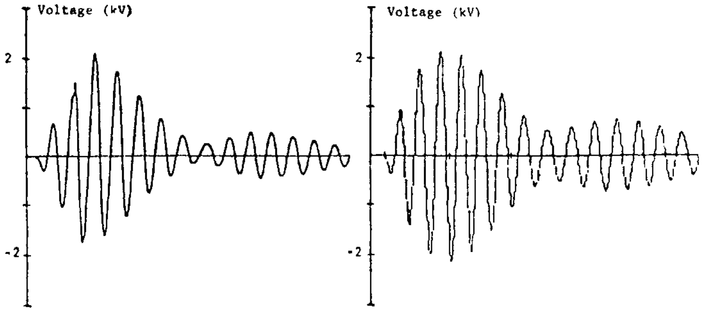

Figure 6b. Comparison of Experimental and Computer Solutions for the Secondary Voltage Waveform.

The computer model results are in good agreement with those measured under medium coupling conditions (k from 10% to 60%).

An accepted method of accounting for the resistive circuit losses incurred during the operation of a resonant air core transformer is to multiply[7] the closed form solution of the lossless case by an exponential decay term[5].

The resulting equation is:

$$V_2(t) = \frac{E_{chg}}{2} \times \sqrt{\frac{L_2}{L_1}} \times e^{-t/T} \times \left[\cos\frac{\omega t}{\sqrt{1-k}} - \cos\frac{\omega t}{\sqrt{1+k}}\right] \qquad (6)$$

where:

$$T = \frac{4L_1L_2}{R_2L_1 + R_1L_2}\,(1 - k^2) \qquad (7)$$

$$\omega_1 = \omega_2 = \omega \qquad (8)$$

The validity of the exponential decay term as an approximation for the resistive losses was examined with the computer model. The output waveform results of the comparison for k = 60% and a Q of 10 are shown in Figure 7 where,

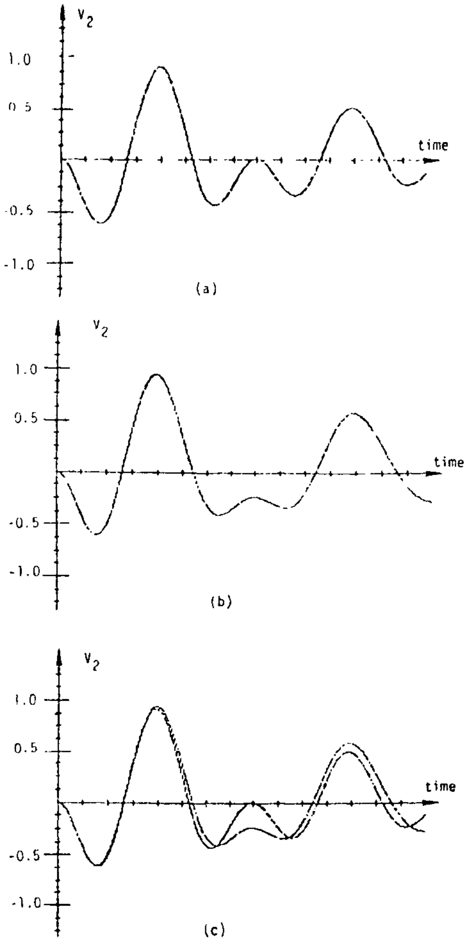

Figure 7. Comparison of the Closed Form Approximation and Computer Solution for the Output Voltage Waveform Where Q_m = 10 and k = 60%.

7a gives the results using the above Equation, 7b is the computer waveform and 7c the overlay comparison.

Comparison of computer results versus equation waveforms for a range of Q (5 to 100) and k's (10% to 60%) showed the most significant variations occuring for the low values of Q and higher values of k. The results of Figure 7 are representative of the larger deviations. The comparison in Figure 7c shows an excellent match for the first and second voltage excursions with increasing variance with time. Fortunately practical applications utilize either the first or second peak where the exponential decay approximation is valid.

CONCLUSION

Results of the computer analysis showed that the effects of resistive losses on the secondary voltage waveform are not a strong function of the ratio of the primary and secondary Q's for $Q_m > 20$. An 80% energy transfer is possible for equal Q's and dual resonant charging with Q's as low as 24 where the coupling of coefficient, k, is at its "magic" value of 60 percent.

The closed form solution of the ideal case with a decaying exponential to account for losses is in good agreement with the computer calculations for high Q's ($Q_m > 25$). Additionally, the waveforms are in good agreement during energy transfer to the secondary for low Q's.

Finally, a comparison between experimental and the computer waveforms which includes resistive losses verify the accuracy of this method in the modeling of this type of transformer.

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