

# THE EFFECT OF VARIATION OF COLLISIONAL GAS PARAMETERS ON THEORETICAL ELECTROMAGNETIC TRANSIENT BREAKDOWNS

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## Abstract

Prediction of the performance of intense electromagnetic transient, gaseous breakdown depends on various parameters used for the description of the gaseous media. Of these parameters, some describe the impact ionization cross section for stable atoms and molecules, and are specific for each species. These parameters are: the first ionization energy,  $E_i$ , a scaling parameter denoted as  $\alpha$  which characterizes the collision cross section per electron volt of the species, and a parameter which represents the incident electron energy at which the peak impact ionization cross section occurs, denoted as  $\beta$ . This paper discusses the effect that these parameters have upon electrical breakdown performance in an intense electromagnetic transient environment.

## I. INTRODUCTION

In the performance of breakdown experiments in gases subjected to intense electromagnetic transients the impact ionization cross section determines the conditions in which the ionization rate,  $\nu_i$ , exceeds the attachment rate,  $\nu_a$ , and breakdown may be expected to occur. Lupan<sup>1</sup> has derived the functional dependence of the ionization rate on energy as

$$\left\langle \frac{\nu_i}{P} \right\rangle = \frac{N}{P} \left\langle \sigma(E) \sqrt{\frac{2E}{m_e}} \right\rangle \quad (1)$$

where:  $P$  is the gas pressure,  $N$  is the gas density,  $E$  is the energy,  $m_e$  is the electron mass, and  $\sigma(E)$  is the impact ionization cross section of the gas molecule. This impact ionization cross section can be approximated as<sup>1</sup>

$$\sigma(E) = \alpha(E - E_i) e^{-\left\{ \frac{E - E_i}{\beta} \right\}} \quad (2)$$

where:  $E_i$  is the first ionization energy,  $\alpha$  is a scaling parameter which characterizes the collision cross section per electron volt of the species, and  $\beta$  represents the incident electron energy at which the peak impact ionization cross section occurs. To better illustrate the definitions of these parameters, a plot of the impact ionization cross section is shown in Figure 1.

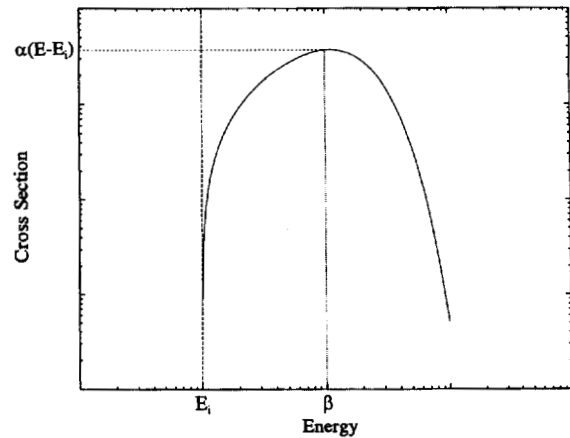


Figure 1. Cross Section vs. Energy

Prediction of the performance of intense electromagnetic transient, gaseous breakdown depends on these parameters used for the description of the gaseous media. Of these parameters, some describe the impact ionization cross section for stable atoms and molecules, and are specific for each species. Extending the method of Lupan<sup>1</sup> this paper discusses the effect that these parameters have upon electrical breakdown performance in an intense electromagnetic transient environment.

## II. THEORY

For a maxwellian distribution function the expected value for the ionization rate becomes

$$\left\langle \frac{\nu_i}{P} \right\rangle = \int_{E_i}^{\infty} \frac{N}{P} \sigma(E) \sqrt{\frac{2E}{m_e}} \frac{2}{\sqrt{\pi}} \sqrt{\frac{E}{\frac{2}{3}\epsilon}} e^{\left(\frac{-3E}{2\epsilon}\right)} \frac{dE}{\left(\frac{2}{3}\epsilon\right)} \quad (3)$$

which may be rewritten as

$$\left\langle \frac{\nu_i}{P} \right\rangle = \frac{3}{\sqrt{\pi}} \sqrt{\frac{3}{m_e}} \int_{E_i}^{\infty} \frac{N}{P} \sigma(E) \frac{E}{\epsilon^2} e^{\left(\frac{-3E}{2\epsilon}\right)} dE \quad (4)$$

where  $\epsilon$  is the free electron energy, which is the sum of the thermal energy and the kinetic energy due to the electric field

$$\epsilon = \frac{3}{2} k_B T + \frac{\frac{1}{2} m_e \langle v(\tau)^2 \rangle}{\delta_{eff}} \quad (5)$$

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14. ABSTRACT <b>Prediction of the performance of intense electromagnetic transient, gaseous breakdown depends on various parameters used for the description of the gaseous media. Of these parameters, some describe the impact ionization cross section for stable atoms and molecules, and are specific for each species. These parameters are: the first ionization energy, <math>\sim i</math>, a scaling parameter denoted as <math>C_t</math> which characterizes the collision cross section per electron volt of the species, and a parameter which represents the incident electron energy at which the peak impact ionization cross section occurs, denoted as <math>\sim</math>. This paper discusses the effect that these parameters have upon electrical breakdown performance in an intense electromagnetic transient environment.</b>			
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and:  $k_B$  is Boltzmann's constant,  $T$  is temperature,  $\langle v(\tau)^2 \rangle$  is the time average of the velocity squared over a period of  $\tau$ , and  $\delta_{eff}$  is the effective fraction of energy transferred from an electron to a molecule in a collision. For an electron in an electric field the free electron energy,  $\varepsilon$ , becomes<sup>2</sup>

$$\varepsilon = \frac{3}{2}k_B T + \frac{q^2 E_{eff}^2}{m_e \delta_{eff} v_{eff}^2} \quad (6)$$

where:  $q$  is the electron charge,  $v_{eff}$  is the effective collision frequency, and  $E_{eff}$  is the effective electric field. Combining equations and utilizing the ideal gas law produces

$$\left\langle \frac{v_i}{P} \right\rangle = \frac{3}{\sqrt{\pi}} \sqrt{\frac{3}{m_e}} \left( \frac{N}{P} \right) \int_{E_i}^{\infty} \frac{\alpha E (E - E_i) e^{-\left\{ \frac{E - E_i}{\beta} \right\}} e^{-\left( \frac{-3E}{2\varepsilon} \right)}}{\varepsilon^{\frac{3}{2}}} dE \quad (7)$$

where  $R$  is the ideal gas law constant,  $T$  is temperature, and the effective electric field is a function of the applied electric field. For an electron at room temperature  $\frac{3}{2}k_B T \approx \frac{1}{40}$  eV. Likewise the coefficients preceding the integral reduce to a value of

$$\left\langle \frac{v_i}{P} \right\rangle = \frac{3}{\sqrt{\pi}} \sqrt{\frac{3}{m_e}} \left( \frac{6.02 \times 10^{23}}{RT} \right) \int_{E_i}^{\infty} \frac{\alpha E (E - E_i) e^{-\left\{ \frac{E - E_i}{\beta} \right\}}}{\left[ \frac{3}{2}k_B T + \frac{q^2 E_{eff}^2}{m_e \delta_{eff} v_{eff}^2} \right]^{\frac{3}{2}}} dE$$

Assuming that ionization is a single event process the functional dependence of the first three terms contained in the integral are estimated<sup>1</sup> to be, for  $E < 400$  eV,

$$\frac{N}{P} \sigma(E) \sqrt{\frac{2E}{m_e}} = 6 \times 10^7 \alpha \sqrt{E} (E - E_i) \exp\left(\frac{E_i - E}{\beta_{N2}}\right) \quad (9)$$

and for  $E > 400$  eV,

$$\frac{N}{P} \sigma(E) \sqrt{\frac{2E}{m_e}} = \frac{6 \times 10^7 \sqrt{E}}{\sqrt{E}} 10^2 = 6 \times 10^9 \quad (10)$$

Substituting the above estimates into the integral yields,

$$\left\langle \frac{v_i}{P} \right\rangle = 6 \times 10^7 \sqrt{\frac{27}{2\pi}} \varepsilon^{-3/2} \left[ \alpha \int_{E_i}^{400} E (E - E_i) \exp\left(\frac{E_i - E}{\beta_{N2}}\right) \exp\left(\frac{-3E}{2\varepsilon}\right) dE + \right.$$

$$\left. 10^2 \int_{400}^{\infty} \sqrt{E} \exp\left(\frac{-3E}{2\varepsilon}\right) dE \right] \quad (12)$$

The left hand side of the above equation may be estimated by noting that gaseous electrical breakdown is governed by the time rate of growth of free electrons.

$$\frac{\partial n}{\partial t} = v_i n - v_a n + \nabla^2 (Dn) \quad (13)$$

where  $n$  is the electron density,  $v_i$  is the ionization rate,  $v_a$  is the attachment rate, and  $D$  is electron diffusion coefficient. When an electromagnetic pulse is short compared to the diffusion time, the electron diffusion coefficient may be neglected<sup>2</sup>. This allows the previous

equation to be rewritten as  $\frac{\partial n}{\partial t} = \Delta v n$  where  $Dv = v_i - v_a$ .

Solving this equation, assuming  $Dv$  is not a function of time, yields

$$\ln \frac{n_f}{n_o} = \Delta v \tau_p \quad (14)$$

where  $n_f$  is the final electron density,  $n_o$  is the initial electron density, and  $\tau_p$  is the pulse length. Gould and Roberts<sup>3</sup> established the generally recognized breakdown ratio of  $n_f/n_o$  as  $10^8$ .

$$\ln 10^8 = 18.4 \approx \Delta v \tau_p \quad (15)$$

For breakdown to occur it is required that  $v_i \gg v_a$ . Thus

$$\frac{v_i}{P} = \frac{18.4}{P \tau_p} \quad (16)$$

where  $P$  is the gas pressure. Substituting the above equation into the integral, performing the integration<sup>4</sup>, and simplifying yields

$$\frac{18.4}{P \tau_p} = 6 \times 10^7 \sqrt{\frac{27}{2\pi}} \varepsilon^{-3/2} \exp\left(\frac{-3E_i}{2\varepsilon}\right) \left\{ \frac{16\alpha(\beta_{N2}\varepsilon)^3}{(2\varepsilon + 3\beta_{N2})^3} + \frac{4\alpha(\beta_{N2}\varepsilon)^2 E_i}{(2\varepsilon + 3\beta_{N2})^2} + 10^2 \sqrt{\frac{2\pi\varepsilon}{3}} \exp\left(\frac{3E_i - 1200}{2\varepsilon}\right) \left(\frac{2\varepsilon}{3} + 800\right) \right\} \quad (17)$$

Thus, given a knowledge of the first ionization energy  $E_i$ , the collision cross section per electron volt of the species  $\alpha$ , and the energy at which the peak impact ionization cross section occurs  $\beta$ , one is able to generate an estimate of the Paschen Curve of a given species of

gas. A tabulation of these parameters for various gases, based on the effort of McDaniel<sup>5</sup>, is listed in Table 1.

Table I. Gaseous Breakdown Parameters

Gas	$E_i$	$\alpha$	$\beta$
H <sub>2</sub>	13.59	2.98	70.145
He	24.59	0.029	115.8
N <sub>2</sub>	14.50	0.5706	108.0
O <sub>2</sub>	13.60	0.2090	135.33
Ne	21.56	0.0310	170.2
Ar	15.76	1.128	97.4
Atomic Hydrogen	15.40	0.034	65.5
Atomic Nitrogen	15.60	0.0955	106.7
Atomic Oxygen	12.06	0.053	90.0

Figure 2 is a graph of the Paschen Curves for H<sub>2</sub>, He, N<sub>2</sub>, O<sub>2</sub>, Neon, and Argon based on the values of the parameters listed in Table 1. Note that the positions of the various gases strongly correlate with the valence position of the element on the periodic chart. This positioning is to be expected since the valence position on the periodic chart is a function of the first ionization energy of an element. The nose structure occurring at approximately 10<sup>-9</sup> Torr-seconds and 10<sup>3</sup> Volts per centimeter per Torr has been previously predicted by Graham and Roussel-Dupre<sup>6</sup> and has been experimentally observed by two of the authors<sup>4</sup>.

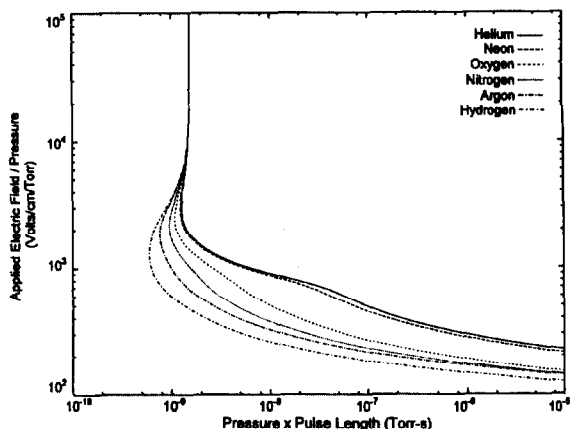


Figure 2. Paschen Curves for Various Gases

Figure 3 is a graph of the Paschen Curves for H<sub>2</sub>, N<sub>2</sub>, O<sub>2</sub>, Atomic Hydrogen, Atomic Nitrogen, and Atomic Oxygen based on the values of the parameters listed in Table 1. Note the separation between the pairs of: Atomic Hydrogen and Molecular Hydrogen, Atomic Nitrogen and Molecular Nitrogen, and Atomic Oxygen and Molecular Oxygen.

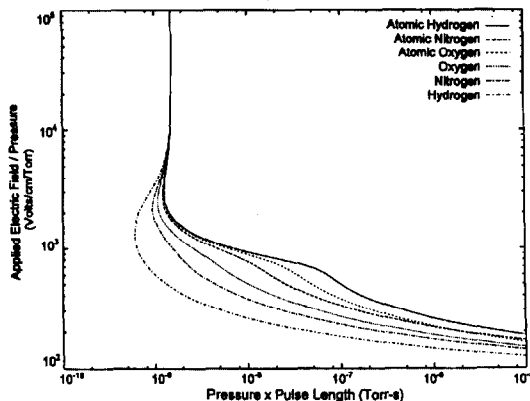


Figure 3. Paschen Curves for Molecular and Atomic Gases

Figure 4 is a graph of the Paschen Curve of a hypothetical gas with values of  $\alpha = 1.0$ ,  $\beta = 100.0$  eV, and  $E_i$  varied from 10 eV to 80 eV. As observed in Figure 2, increased ionization energy has the effect of increasing the insulating property of the gas, producing an upward shift in the Paschen Curve.

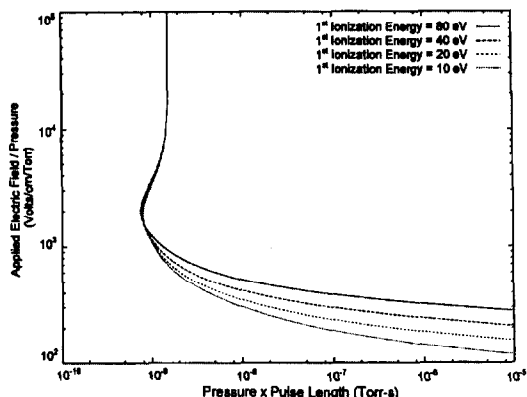


Figure 4. Hypothetical Gas, 1<sup>st</sup> Ionization Constant Varied

Figure 5 is a graph of the Paschen Curve of a hypothetical gas with values of  $\alpha$  varied from 0.01 to 10,  $\beta = 100.0$  eV, and  $E_i = 10$  eV. The  $\alpha$  parameter adjusts the scale of the collisional cross section. Small values are indicative of relatively small collisional cross sections while large values are indicative of relatively large collisional cross sections. Larger collisional cross sections produce curves exhibiting less insulating ability while smaller collisional cross sections produce curves exhibiting more insulating ability. This is again a reasonable result since a larger collisional cross section is

easier for an electron to impact while a smaller collisional cross section is harder for an electron to impact.

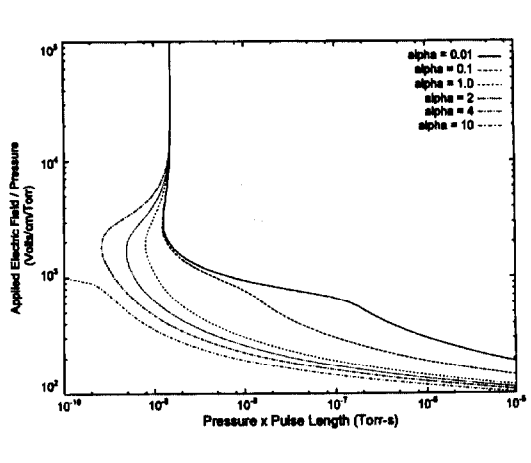


Figure 5. Hypothetical Gas,  $\alpha$  Varied.

Figure 6 is a graph of the Paschen Curve of a hypothetical gas with values of  $\alpha = 1.0$ ,  $\beta$  varied from 10 eV to 400 eV, and  $E_i = 10$  eV. Note that  $\beta$ , the electron energy producing the peak collisional cross section, cannot be less than the first ionization constant for breakdown to occur. Note that as  $\beta$  increases in magnitude the nose structure of the Paschen Curve becomes more pronounced.

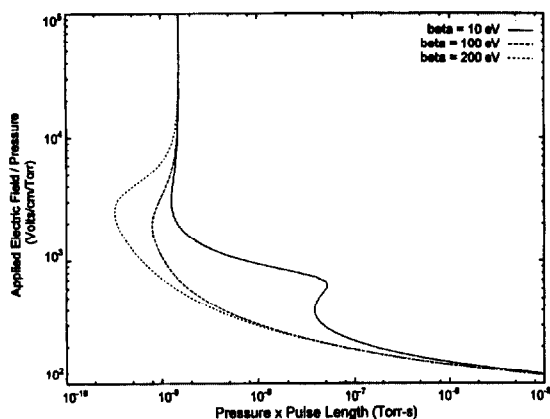


Figure 6. Hypothetical Gas,  $\beta$  Varied

### III. CONCLUSION

This paper discusses the effect that these parameters have upon electrical breakdown performance in an intense electromagnetic transient environment. Small values of  $\alpha$  yield relatively small collisional cross sections while large values of  $\alpha$  yield relatively large collisional cross sections. Larger collisional cross sections produce curves exhibiting less insulating ability while smaller collisional

cross sections produce curves exhibiting more insulating ability. Large values of  $\beta$ , the electron energy regulating placement of the peak collisional cross section, yield more pronounced nose structures within the Paschen Curve. Finally, increased ionization energy,  $E_i$ , has the effect of increasing the insulating property of the gas, producing an upward shift in the Paschen Curve.

Given that the first ionization energy of a gas is a parameter which has been measured to a high degree of accuracy, it should be possible to determine the remaining parameters,  $\alpha$  and  $\beta$ , by examination of the Paschen Curve of that gas.

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