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PRIORITY RANKING OF PROJECTS HAVING MULTIPLE ATTRIBUTES

BY

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OTTAWA, ONTARIO

NOVEMBER 1981

ABSTRACT

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PRIORITY RANKING OF PROJECTS HAVING MULTIPLE ATTRIBUTES

INTRODUCTION

In reponse to a request from the Director of Land Requirements (DLR), a literature search was undertaken for existing methods that could be used to construct a priority ranking of projects having multiple attributes. This note discusses the use of several such methods which may be relevant to DLR's request, and could be considered worthy of use on a trial basis. Readers interested in detailed specifics of any of the methods should refer to the original papers.

THE PROBLEM

A set of objects (or alternatives, or projects) are to be ranked, taking into account their desirability with respect to a number of different attributes. The attributes do not necessarily have equal importance. For instance, the objects to be ranked could be possible acquisitions such as new vehicles, additional tanks, improved ammunition or a command and control system. Attributes to be considered in forming a ranking could be characteristics such as acquisition expense, operating expense, safety of personnel operating the equipment, effectiveness of the equipment in its role, and requirement for the equipment.

In the case of the Directorate of Land Requirements, they have a number of projects (new acquisitions, upgrades of existing equipment, more items of existing equipment, etc.) for which they would like to get approval, and allocation of funds. Whether or not funds are allocated to a project depends on how well it is justified. However, DLR has limited manpower with which to staff project proposals. A major project may require several man-years, or more, of highly qualified technical personnel in order to be adequately justified. Not all of the projects can be properly staffed, and a ranking of the projects (indicating overall order of preference) would provide guidance with regard to the allocation of manpower to them.

Several methods that could be considered for the construction of priority rankings are presented. The methods are based on Multiple Objective Optimization Theory, Relative Position Estimate, fuzzy set theory, and conjoint scaling.

AN APPROACH BASED ON MULTIPLE OBJECTIVE OPTIMIZATION THEORY

This approach is taken from Reference 1. In the example which is worked out below, the approach alternates between two processes: first, an examination of alternatives to see which ones clearly dominate others, and second a "trade-off" of some attributes according to weighting factors chosen by the decision maker. (These processes will be described in detail shortly.) As the decision maker alternately applies these processes, one after the other, a priority ranking emerges in finer and finer detail. Such a ranking of alternatives (which, for the sponsor, would correspond to projects or acquisitions) can be used to assist in deciding which ones are most desirable.

Consider a set of 8 possible equipment acquisitions, each one of which is rated according to 5 attributes ('a', 'b', 'c', 'd' and 'e') on a scale of 0 to 10 as represented in Table I. In each case, an attribute rating of 10 is most desirable, and a rating of 0 least desirable. So, for attribute 'a', a rating of 10 indicates a most desirable (low) acquisition cost, while for attribute 'c' it would indicate a highest safety level. The attributes are assumed to be independent of one another. So, a high or low rating on one attribute for a given project should not automatically imply the necessity of a high or low rating for another attribute of the same project.

All 8 types of equipment are considered desirable, but it is not possible to acquire all of them. So, a priority ranking (overall most desirable equipment down to

TABLE I

	a	b	c	d	e
Alternative 1	9	10	10	8	7
2	3	9	4	9	8
3	3	8	0	2	5
4	8	7	8	4	0
5	5	1	3	10	2
6	5	4	2	9	1
7	0	8	3	5	5
8	2	0	2	9	8

a: acquisition expense rating
b: operating expense rating
c: safety rating
d: effectiveness rating
e: requirement rating

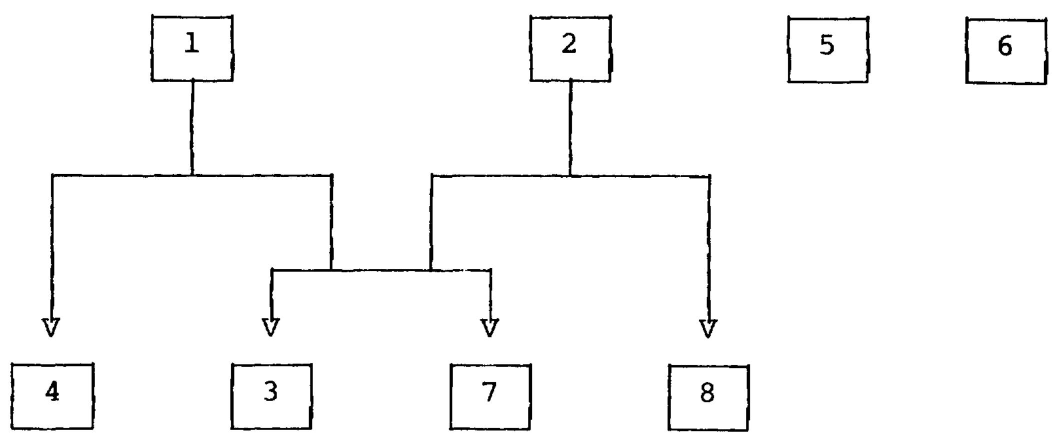

Figure 1

overall least desirable equipment) would help in deciding which ones are most worthy to obtain. The difficulty is, of course, that such a ranking cannot immediately be constructed from the information in Table I.

The first step in the Multiple Objective Optimization Theory (MOOT) based approach is to examine the eight alternatives for any clearly dominating relationships. For instance, alternative 2 dominates alternative 3 since each of its five attribute values is, respectively, bigger than or equal to alternative 3's attribute values. With regard to alternative 1 versus alternative 2, neither of them dominates the other since alternative 1 has the higher values for attributes 'a', 'b', and 'c' while alternative 2 has the higher values for attributes 'd' and 'e'. The domination pattern corresponding to Table I is presented in Figure 1. A path downwards from one alternative to another indicates a clear domination of the upper alternative over the lower one. This pattern gives a small amount of information with regard to a priority ranking. For instance, the highest ranking alternative must be one of 1, 2, 5 or 6 since all the rest are dominated by alternative 1 and/or 2. Alternative 1 is clearly preferable to alternatives 3, 4, and 7, while alternative 2 dominates 3, 7, and 8.

The next step is to choose suitable attributes (or groups of attributes) to "trade-off" weights between them. Ideal attributes to be considered would be ones that are a) the least critical, or b) components of some overall aspect, so that the choosing of relative weights is easier. By doing the easier or less critical trade-offs first, the problem structure is simplified by the time the more difficult trade-offs must be made.

In the example, attributes 'd' and 'e' are chosen to be combined. (Effectiveness and requirement attributes together can be considered to be the aspect of operational importance). The decision maker gives attribute 'd' a weight factor of .4, and attribute 'e' a weight factor of .6. On alternative 1, for example, the combining of 'd' and 'e' results in the value (.4)(8) + (.6)(7) or 7.4. Table II gives the ratings of the 8 alternatives after performing the trade-off. The steps of 1) analyzing the domination relationships and 2) performing trade-offs can be repeated until a final answer emerges. One of the big advantages of the MOOT method is that the implications of trade-offs can be seen as the method progresses, so the decision maker has an awareness of the effect of his choices for relative weights. Figure 2 shows the domination relationships derived from Table II. All of the dominating relationships from Figure 1 still hold (as they must) but new dominations are introduced by the combining of attributes 'd' and 'e'. It is now clear, for instance that the most preferred alternative is either 1 or 2.

TABLE II

	a	b	c	d-e
Alternative 1	9	10	10	7.4
2	3	9	4	8.4
3	3	8	0	3.8
4	8	7	8	1.6
5	5	1	3	5.2
6	5	4	2	4.2
7	0	8	3	5
8	2	0	2	8.4

a: acquisition expense rating
b: operating expense rating
c: safety rating
d-e: combined effectiveness and requirement rating

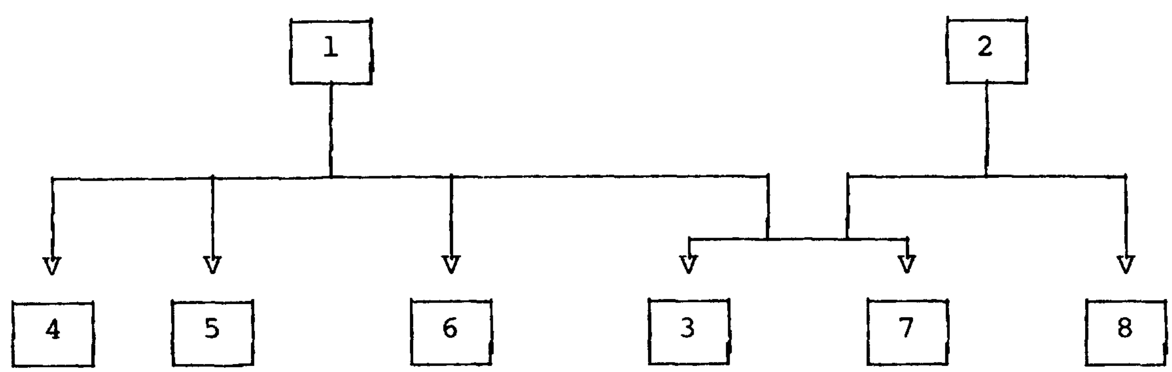

Figure 2

The decision maker next combines attribute 'c' (with a weight of .5) with the 'd-e' attribute (again with a weight of .5) from Table II to form Table III. Figure 3 shows the resulting domination relationships. From Figure 3 it is seen that alternative 1 is, as a result of the trade-offs that were made, ranked first. Both alternatives 2 and 4 are candidates for second. If the decision maker felt he could do so, he might want to choose elements from Figure 3 one by one, using his judgement in relation to the information in Table III, in order to construct a final ranking. Alternative 1 is ranked first. Say, using Table III he decides alternative 2 is ranked second. Then, alternatives 3, 4, 7 and 8 would have to be compared in order to choose one of them as third ranked.

If the decision maker is not prepared to compare alternatives directly from Table III, then he will have to consider another trade-off. Since the final ranking may be very sensitive to his choice of trade-off weights, it is reasonable for him to try out different ranges and combinations of weights, to see how greatly changes in weights affect the final ranking. If the answer does not change readily in response to moderate weight alterations, then the decision maker may more readily put faith in the results. On the other hand, if small weight changes were to make drastic differences in the final ranking, then the decision maker should consider carefully possible ramifications of the weights he chooses.

TABLE III

	a	b	c-d-e
Alternative 1	9	10	8.7
2	3	9	6.2
3	3	8	1.9
4	8	7	4.8
5	5	1	4.1
6	5	4	3.1
7	0	8	4.0
8	2	0	5.2

a: acquisition expense rating
b: operating expense rating
c-d-e: combined safety, effectiveness, and requirement rating

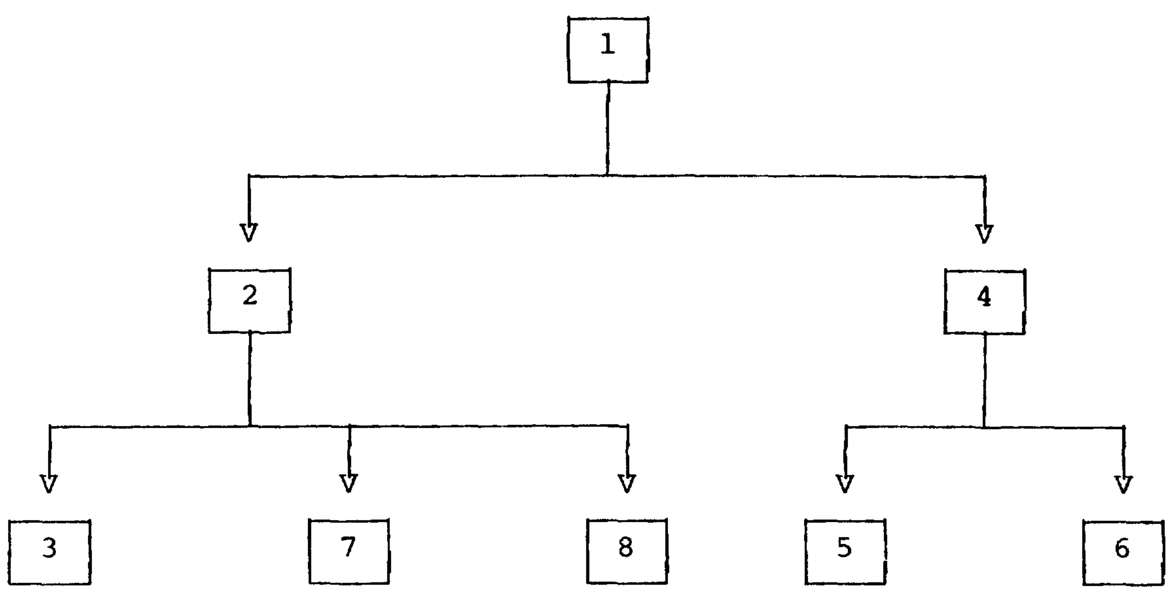

Figure 3

TABLE IV

	Attribute a-b-c-d-e
Alternative 1	9.18
2	6.08
3	4.06
4	6.42
5	3.44
6	3.94
7	4.00
8	2.68

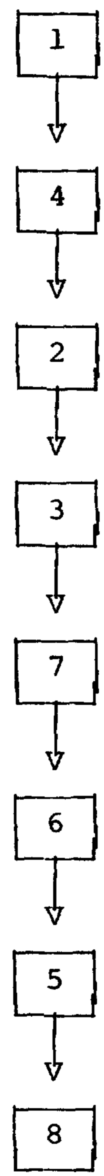

Figure 4

The decision maker specifies a weight of .3 for attribute 'a', .3 for attribute 'b', and .4 for the combined 'c-d-e' attributes. The trade-off results are shown in Table IV and the final overall ranking is shown in the domination relationships of Figure 4.

In order to get a feeling for how sensitive the ranking is to these weights, two comparison trade-offs are generated. The first has weights of .2 for 'a', .2 for 'b', and .6 for the combined 'c-d-e' attributes. The results are shown in Table V and Figure 5. The second has weights of .33 for 'a', .33 for 'b', and .34 for 'c-d-e'. The results are shown in Table VI and Figure 6.

Figures 4 and 6 are almost identical, so the different weightings for Tables IV and VI have little bearing on the final ranking.

Figures 4 and 5, on the other hand, do have some considerable differences. The first three alternatives in each ranking are the same, but not in an identical ordering. If the decision maker were interested in the first five alternatives, then the difference between the rankings is much more significant. So, if the decision maker were not totally confident in the weightings of Table IV being correct as opposed to the weightings of Table V, he should analyze the problem more carefully.

TABLE V

	Combined Attribute Rating
Alternative 1	9.02
2	6.12
3	3.34
4	5.88
5	3.66
6	3.66
7	4.00
8	3.52

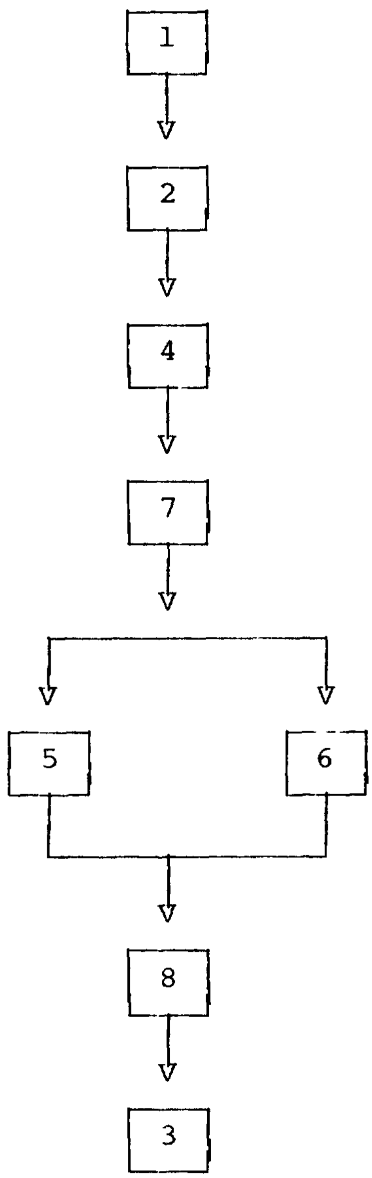

Figure 5

TABLE VI

	Combined Attribute Rating
Alternative 1	9.23
2	6.07
3	4.28
4	6.58
5	3.37
6	4.02
7	4.00
8	2.43

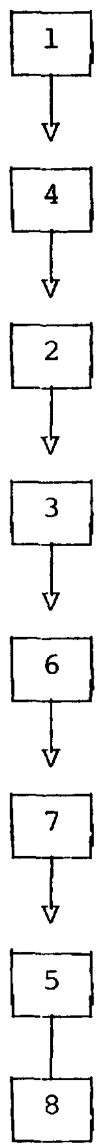

Figure 6

The final equivalent weight of each attribute as it is taken into account in Table IV is:

Attribute a: .3

b: .3

c: (.4)x(.5) = .2

d: (.4)x(.5)x(.4) = .08

e: (.4)x(.5)x(.6) = .12

A big advantage of this method over the simple assigning of all weights at one step is the ability to see the development of a ranking step by step as only some attributes are traded off, so the decision maker can better understand the effect of his chosen weights. Also, at critical steps the decision maker can try several different weight combinations to investigate how robust the ranking is. Application of the method is is a fairly straightforward process.

A disadvantage is that if there are several decision makers, consensus agreement on attribute values and relative weights may be difficult to obtain. Ideally, attribute ratings would be consistent so that a rating of 8 would be as much better than 6 as 6 is better than 4.

A METHOD BASED ON RELATIVE POSITION ESTIMATE

The relative position estimate method is taken from Reference 2 and does not require accurate information concerning attribute ratings as does the MOOT method, but it may produce a less refined ranking. For each attribute, the alternatives are judged relative to each other in a pairwise fashion, and the results are shown in a preference matrix. If alternative i is judged to be better than or equivalent to alternative j, then row i column j of the matrix is a 1. Otherwise, if alternative i is judged to be worse than alternative j, then row i column j of the matrix is a 0. So, the sum of elements across row k, gives the number of alternatives that are no better than k, while the sum of elements down column k gives the number of alternatives that are no worse than k. No estimates of attribute ratings need be made, only relative "better or worse" comparisons. However, the attributes are assumed to be independent of one another.

The preference matrices used to illustrate the method are constructed so as to be consistent with Table I, to facilitate comparison with the MOOT method. They are shown in Tables VII to XI, with Table VII being the preference matrix for attribute 'a', Table VIII being the preference matrix for attribute 'b' and so on.

TABLE VII

	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	0	1	1	0	0	0	1	1
3	0	1	1	0	0	0	1	1
4	0	1	1	1	1	1	1	1
5	0	1	1	0	1	1	1	1
6	0	1	1	0	1	1	1	1
7	0	0	0	0	0	0	1	0
8	0	0	0	0	0	0	1	1

Attribute 'a' (Acquisition expense)
Preference Matrix

Weight .3

TABLE VIII

	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	0	1	1	1	1	1	1	1
3	0	0	1	1	1	1	1	1
4	0	0	0	1	1	1	0	1
5	0	0	0	0	1	0	0	1
6	0	0	0	0	1	1	0	1
7	0	0	1	1	1	1	1	1
8	0	0	0	0	0	0	0	1

Attribute 'b' (Operating expense)
Preference Matrix

Weight .3

TABLE IX

	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	0	1	1	0	1	1	1	1
3	0	0	1	0	0	0	0	0
4	0	1	1	1	1	1	1	1
5	0	0	1	0	1	1	1	1
6	0	0	1	0	0	1	0	1
7	0	0	1	0	1	1	1	1
8	0	0	1	0	0	1	0	1

Attribute 'c' (Safety)
Preference Matrix

Weight .2

TABLE X

	1	2	3	4	5	6	7	8
1	1	0	1	1	0	0	1	0
2	1	1	1	1	0	1	1	1
3	0	0	1	0	0	0	0	0
4	0	0	1	1	0	0	0	0
5	1	1	1	1	1	1	1	1
6	1	1	1	1	0	1	1	1
7	0	0	1	1	0	0	1	0
8	1	1	1	1	0	1	1	1

Attribute 'd' (Effectiveness)
Preference Matrix

Weight .08

TABLE XI

	1	2	3	4	5	6	7	8
1	1	0	1	1	1	1	1	0
2	1	1	1	1	1	1	1	1
3	0	0	1	1	1	1	1	0
4	0	0	0	1	0	0	0	0
5	0	0	0	1	1	1	0	0
6	0	0	0	1	0	1	0	0
7	0	0	1	1	1	1	1	0
8	1	1	1	1	1	1	1	1

Attribute 'e' (Requirement)
Preference Matrix

Weight .12

The decision maker must choose weights for the attributes, and using these weights calculations are performed with elements of the preference matrices to generate an aggregated preference matrix. Some simple calculations on entries of the aggregated preference matrix produces a preference ranking of the alternatives.

In this example the weights are chosen (to be consistent with the resultant weights in the MOOT example) as .3 for attribute 'a', .3 for attribute 'b', .2 for attribute 'c', .08 for attribute 'd', and .12 for attribute 'e'.

Not all of the generality possible in the method is illustrated here. Rather, one specific implementation is shown in order to give an idea of how it works.

Let $w(i > j)$ be the sum of weights of attributes whose preference matrix has row i column j equal to 1, and row j column i equal to 0 (i.e. where alternative i is clearly preferred to element j), and similarly let $w(i < j)$ be the sum of weights of attributes whose preference matrix has row i column j equal to 0 and row j column i equal to 1 (i.e. alternative j is clearly preferred to alternative i). Also, let $w(i = j)$ be the sum of weights of attributes whose preference matrix has both row i column j and row j column i equal to 1 (i.e. no clear preference).

An indicator of aggregate preference that can be used is:

$$A(i > j) = \frac{w(i > j) + w(i = j)}{w(i < j) + w(i = j)}$$

One property of this indicator is that

$$A(i > j) = 1/A(j < i)$$

If $A(i > j) = 1$ (or very close to 1) then it can be considered that, considering weighted values of attribute preferences, alternatives i and j are equally preferable. If $A(i > j) > 1$, then i is considered preferable to j (disregarding all other alternatives) and if $A(i > j) < 1$, then j is considered preferable to i (disregarding all other alternatives).

By calculating all such $A(i > j)$ values, the information needed to construct an aggregate preference matrix is obtained. As an example, consider the calculation of $A(2 > 6)$.

$$w(2 > 6) = .3 + .2 + .12 = .62$$

$$w(2 < 6) = .3$$

$$w(2 = 6) = .08$$

and $$A(2 > 6) = \frac{.62 + .08}{.3 + .08} = 1.84$$

Since $A(2 > 6) > 1$, then 2 is clearly preferred to 6 (as a pairwise comparison) and row 2 column 6 of the aggregate preference matrix is a 1, and row 6 column 2 of the aggregate preference matrix is a 0. The other calculations are not shown here, but the resulting aggregate preference matrix is shown as Table XII. In working out the values, if $A(i > j)$ ranged from .9 to 1.11, then $A(i > j)$ was considered (for practical purposes) to be 1.

TABLE XII

	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	0	1	1	1	1	1	1	1
3	0	0	1	0	0	0	1	1
4	0	1	1	1	1	1	1	1
5	0	0	1	0	1	1	1	1
6	0	0	1	0	0	1	0	1
7	0	0	1	1	1	1	1	1
8	0	0	0	0	0	0	1	1

A function 'v' which is an index of relative position can be defined in terms of two other functions, 'p' and 'q'. Let p(k) be the sum of the numbers in row k of the aggregate preference matrix and let q(k) be the sum of the numbers in column k of the aggregate preference matrix. p(k) is the number of alternatives which are no better than alternative k, while q(k) is the number of alternatives which are no worse than k. The index v(k) of alternative k is defined as:

$$v(k) = p(k) - q(k)$$

So, for the aggregate preference matrix of Table XII, the indices are calculated as follows:

$$v(1) = 8 - 1 = 7$$

$$v(2) = 7 - 3 = 4$$

$$v(3) = 3 - 7 = -4$$

$$v(4) = 7 - 4 = 3$$

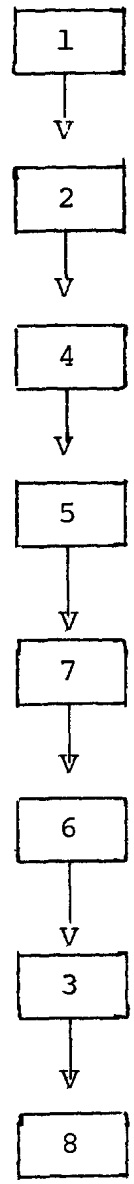

Figure 7

v(5) = 5 - 5 = 0

v(6) = 3 - 6 = -3

v(7) = 6 - 7 = -1

v(8) = 2 - 8 = -6

The corresponding aggregate preference ranking is shown in Figure 7. In comparison to the rankings produced by the MOOT method (Figures 4, 5, 6) it is most similar to

Figure 5 even though the attribute weights correspond to those used for Figure 4.

It is very possible that in large problems the relative position estimate may generate an answer having many ties. For instance, a ranking of 100 alternatives might generate only 60 distinct index values V(k), so that 40 alternatives would have to be tied in rank with other alternatives.

One of the big advantages of this method is that it can be used when the decision maker is unsure of placing attribute ratings on various alternatives, as long as he is willing to make pairwise comparisons of the alternatives with respect to the various attributes. Also, application of the method is a simple and straightforward matter.

A big disadvantage is that the task of making pairwise comparisons would be extremely tedious if a large number of projects were being compared. If 100 projects were being ranked over 6 attributes, then someone would have to prepare 6 preference matrices, each having 10,000 elements. So, decisions on 60,000 pairwise comparisons would have to be made and recorded. This workload could be alleviated substantially by ranking the projects within 5 groups of 20, say, and then taking the highest ranking projects within each group together for a final overall comparison. However, the pairwise comparison process would still be somewhat tedious. As with the MOOT method, the difficulty of obtaining consensus agreement on weights may be a significant problem when there are several decision makers.

A FUZZY SET METHOD

The fuzzy set method is taken from Reference 3 and is a method which can accomodate (to a degree) different individuals' opinions about attribute rating levels and attribute weights. Both weighting and rating levels can be obtained by surveying members of a committee, and combining the opinions into one overall "fuzzy" representation. The combined "fuzzy" representation for each alternative is compared with those for other alternatives, and a priority ranking is determined. Again, attributes are assumed to be independent of one another.

Let us first consider the fuzzy representation of an attribute rating level. On a two member committee, say that one member would be 100% satisfied with a rating level (for some arbitrary attribute) of 6 to 8, and at least 50% satisfied with any rating level of 4 to 9, and not at all satisfied with any other rating. A convenient way of writing this would be that he has a 100% interval of (6,8), and a 50% interval of (4,9) and his opinion can be pictured as in Figure 8. As well, say the second member has a 100% interval of (5,7) and a 50% interval of (2,7) as

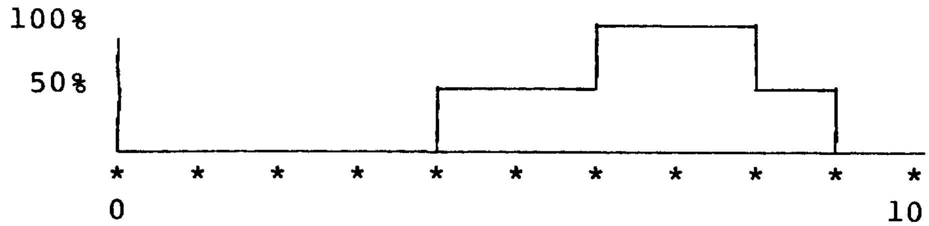

Figure 8 Member 1 - Attribute rating

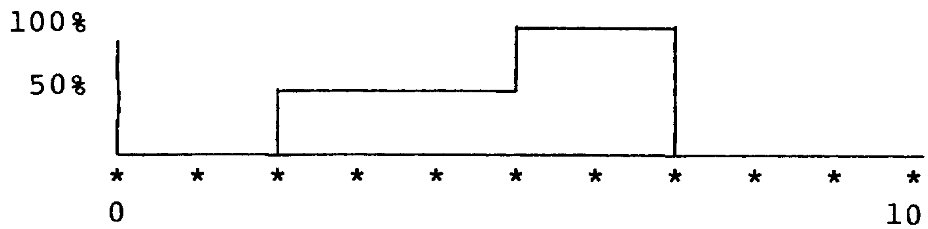

Figure 9 Member 2 - attribute rating

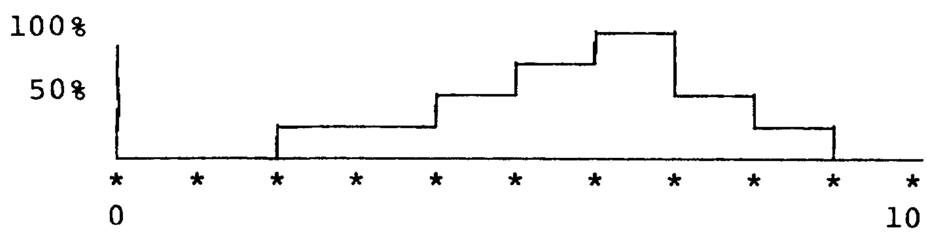

Figure 10 Consensus - attribute rating

pictured in Figure 9. One method of combining these "fuzzy" opinions would be to proportionally "add" individual representations together, and normalize the results to 100%. So, combining Figures 8 and 9, the resulting consensus representation would be Figure 10, and could be described as a 100% interval of (6,7), a 75% interval of (5,7), a 50% interval of (4,8), and a 25% interval of (2,9).

Committee members opinions on attribute weights would be combined in a similar manner in order to generate consensus weight representations.

A definite number of distinct levels is used to represent ratings (instead of smooth curves or an indefinite number of levels), and some representations might have to be portrayed as approximations. In practice, fuzzy representations must not have multiple "peaks", or computational problems will result.

As an example, Table XIII shows weight and attribute rating representations (based loosely on Table I, to facilitate comparison with the MOOT method), at levels of 100% and 50%, for eight alternatives having five attributes.

For each alternative the consensus attribute ratings are compared with the corresponding weight representation,

TABLE XIII

	Interval Level(%)	Attribute a	b	c	d	e
Weight	100	(.2,.4)	(.2,.4)	(.1,.3)	(0,.1)	(.1,.2)
	50	(.1,.5)	(.1,.5)	(0,.4)	(0,.2)	(0,.3)
Alternative 1	100	(8,10)	(9,10)	(9,10)	(7,9)	(6,8)
	50	(7,10)	(8,10)	(8,10)	(6,10)	(5,9)
2	100	(2,4)	(8,10)	(3,5)	(8,10)	(7,9)
	50	(1,5)	(7,10)	(2,6)	(7,10)	(6,10)
3	100	(2,4)	(7,9)	(0,1)	(1,3)	(4,6)
	50	(1,5)	(6,10)	(0,2)	(0,4)	(3,7)
4	100	(7,9)	(6,8)	(7,9)	(3,5)	(0,1)
	50	(6,10)	(5,9)	(6,10)	(2,6)	(0,2)
5	100	(4,6)	(0,2)	(2,4)	(9,10)	(1,3)
	50	(3,7)	(0,3)	(1,5)	(8,10)	(0,4)
6	100	(4,6)	(3,5)	(1,3)	(8,10)	(0,2)
	50	(3,7)	(2,6)	(0,4)	(7,10)	(0,3)
7	100	(0,1)	(7,9)	(2,4)	(4,6)	(4,6)
	50	(0,2)	(6,10)	(1,5)	(3,7)	(3,7)
8	100	(1,3)	(0,1)	(1,3)	(8,10)	(7,9)
	50	(0,4)	(0,2)	(0,4)	(7,10)	(6,10)

FUZZY REPRESENTATIONS OF PARAMETER VALUES

Note: Attribute weight intervals are between 0 and 1, while attribute rating intervals are between 0 and 10.

and an overall rating representation is created. At the 100% level, the maximum and minimum endpoints of the weighting representations are considered in combination with the maximum and minimum endpoints of the corresponding attribute representations, in order to produce upper and lower end-points of an overall rating at the 100% level. Then, a similiar process is carried out to produce upper and lower endpoints of the overall rating at the 50% level. Details of the process are described in Annex A. The overall ratings are listed in Table XIV, and pictured in Figure 11. A final priority ranking is obtained by computing the centroid of each overall rating representation (Table XIV), and ordering the alternatives from most desirable to least desirable by decreasing value of centroids (Figure 12). This ranking is almost identical to the one produced by the MOOT method (Figure 4).

TABLE XIV

	Interval Level	Overall Rating	Centroid
Alternative 1	100	(7.8,9.8)	8.3
	50	(6,10)	
2	100	(4,8.1)	5.9
	50	(2,9.5)	
3	100	(2.5,6.4)	4.8
	50	(0.9,9.2)	
4	100	(4.5,8)	6.1
	50	(2.1,9.9)	
5	100	(1.3,5)	3.7
	50	(.3,7.5)	
6	100	(1.9,5.4)	4.2
	50	(.6,8.3)	
7	100	(2.4,6)	4.6
	50	(1,8.7)	
8	100	(1.2,4.9)	3.7
	50	(0,8)	

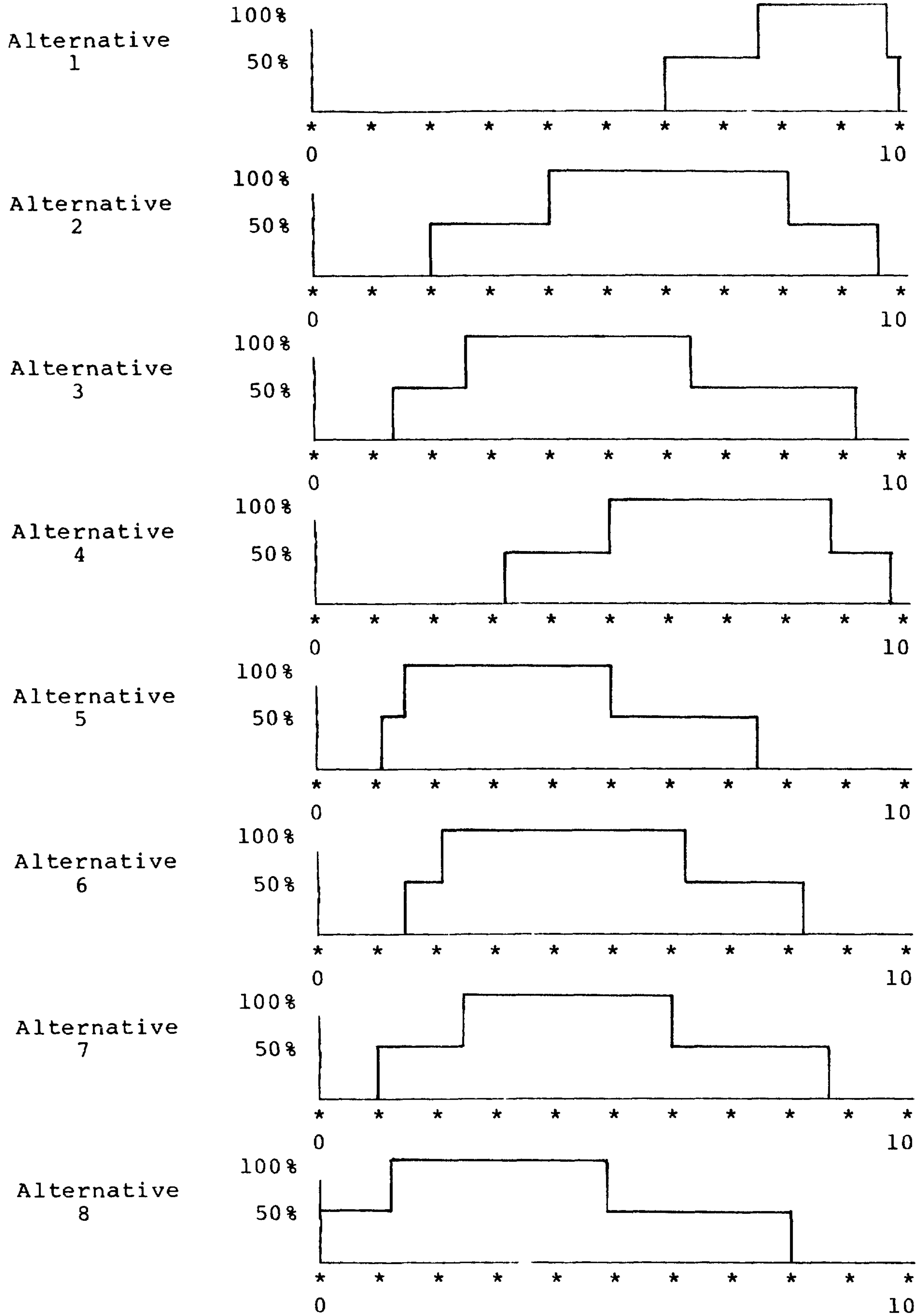

Figure 11 Overall Rating Representations

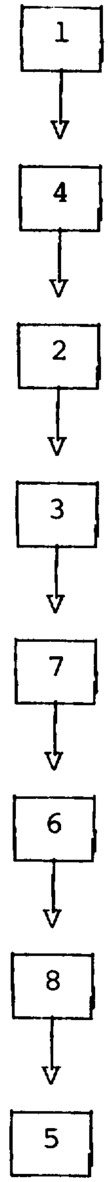

Figure 12 Priority Ranking

Perhaps the biggest advantage of using this method is that it can avoid problems associated with obtaining consensus agreement on attribute ratings and relative weights.

A disadvantage with large problems could be the number of intervals that must be elicited and recorded, before computation can proceed. On a problem with 100 alternatives, each having 10 attributes to be rated by 5 decision makers, with rating intervals specified at 4 levels (25%, 50%, 75%, 100%), a total of 20,000 rating intervals would have to be elicited (4,000 from each decision maker). Then, they would have to be typed as input data to a computer program and checked for errors. The effort required to generate this input data might in such a case be considered prohibitive. A similar problem with rating intervals specified at only two levels by two (instead of five) decision makers, would be much more manageable.

ASSESSMENT OF AN ADDITIVE VALUE FUNCTION USING CONJOINT SCALING

Consider again the example shown in Table 1. For some arbitrary alternative, let the values of attributes 'a', 'b' ..., 'e' be represented by X_a, X_b, ..., X_e. Then, under certain conditions a value function 'v' which signifies the desirability of an alternative can be constructed in terms of separate functions 'v_a' through 'v_e', such that

$$v(X_a, X_b, \ldots, X_e) = V_a(X_a) + V_b(X_b) + \ldots + V_e(X_e)$$

This form of the function, which is represented by the summation of functions of individual parameters, is referred to as additive. So once the functions 'v_a', through 'v_e' are determined, the relative value of any alternative can quite easily be calculated, and these values can be used to construct a priority ranking. The difficult part is the effort that must expended in order to obtain, assimilate, and verify the information needed to form 'v_a'. through 'v_e'.

Some conditions must be satisfied before constructing an additive value function. First, any pair of attributes must be "preferentially independent" of the others. Second, relative trade-offs over the entire possible ranges of the five attributes must be determined. These conditions are now discussed in further detail.

Let the attribute ratings of an alternative be represented by an ordered quintuple ('a' is the first element, 'b' is the second, and so on). Say that one attribute having ratings (5, 7, X, Y, Z) is preferred over another having ratings (3, 9, X, Y, Z) for <u>some specific</u> X, Y, Z. If the first pair of attributes is preferentially independent of the rest, then for <u>any</u> X, Y, Z, an alternative having ratings (5, 7, X, Y, Z) is preferred over one with ratings (3, 9, X, Y, Z). An explanation of how the preferential independence property is intrinsically related to an additive value function is given in Reference 4.

The specification of trades-offs over the ranges of the attributes involves extensive and detailed examination of attribute interrelationships. One approach in doing this is to determine the relative value of one attribute at certain ratings, with respect to another attribute at specific ratings. For instance, if one alternative had an 'a' value of 5, and a 'b' value of 7 and a second alternative had an 'a' value of 4, the decision maker would have to determine which value for 'b' would, with all other attributes the same, make the second alternative just as desirable as the first. Another way of looking at this would be to say, "If I devalued the 'a' attribute from 5 to 4, how much would I have to increase 'b' from 7 in order to compensate?" Such trade-offs must be done for all pairs of attributes. Then, checks for consistency must be made. One form of consistency check is to ensure that for any pair of attributes no trade-off specified at certain ratings is in contradiction with any other specified trade-offs. Another form of consistency check is to ensure that larger groups of attributes are well determined. For instance, the trade-offs for attributes 'a' and 'b', as well as the trade-offs for 'b' and 'c', should not contain any contradictions with respect to the trade-offs between 'a' and 'c'.

In order for this method to be useful, the parameter values specified for each alternative should be known to be accurate, and the decision maker must have faith in the trade-offs that are determined. Since the essence of this method is in the compiling of trade-offs, it is not so readily illustrated as the other methods, and the interested reader should consult Reference 4. This source also illustrates the construction of the additive value function after completing the trade-off elicitation process.

An advantage of conjoint scaling is that it will, with accurate trade-off and rating values, provide an accurate ranking.

A disadvantage is that the extensive trade-off determination and checking of responses for consistency would be time consuming, and would probably require a very talented analyst. If several decision makers were to be involved, there would likely be even more difficulty in obtaining consensus agreement than there would be with the other ranking methods.

SUMMARY OF METHODS

Each one of the four methods described in this note may, under certain conditions, be more desirable than any of the others.

If attribute levels for each alternative can be specified with accuracy, and if trade-offs can be ascertained accurately, then the construction of an additive value function using conjoint scaling would give the most precise priority ranking.

On the other hand, if attribute levels are reasonably exact, and the decision maker is only moderately confident in assigning weights to attributes, the MOOT method is then preferable. It more readily demonstrates the ramifications of assigned weights, and allows some sensitivity analysis.

If the decision maker is only moderately confident in both attribute levels and attribute weights, the fuzzy set method described here would be most appropriate.

However, if attribute levels cannot be confidently assigned, then the relative position estimate method can be of use provided the appropriate pairwise comparisons can be made, and reasonable weights can be assigned to the attributes. For the relative position estimate, and possibly for the MOOT method, the Analytic Hierarchy Process described in Reference 5 provides a procedure which is well worth considering for determining attribute trade-off weights.

Accuracy requirements and implementation effort are summarized for the four methods in Tables XV and XVI. For repeated applications of any of the methods, computer assistance would be desirable for sizable rankings, while it may be feasible to do one-time applications of some methods with a calculator.

TABLE XV - Accuracy Requirement

	Attribute Rating Levels			Attribute Weight or Tradeoff		
	Accurate Specification	Moderately Accurate	Vague	Accurate Specification	Moderately Accurate	Vague
MOOT	X				X	
Relative Postion Estimate			X	X		
Fuzzy Set Method		X			X	
Conjoint Scaling	X			X		

TABLE XVI - Implementation Effort
(Ranking 80 Alternatives with 5 Attributes)

	Preliminary Data Set - Up (Tradeoffs, Comparisions, Consensus)			Subsequent Computation	
	Tedius	Moderate	Low	Computer	Calculator
MOOT			X		X
Relative Postion Estimate	X			X	
Fuzzy Set Method		X		X	
Conjoint Scaling	X				X

REFERENCES

1. C.C. White and A.P. Sage, "A Multiple Objective Optimization - Based Approach to Choicemaking", IEEE Transactions on Systems, Man, and Cybernetics, vol. SMC-10, No. 6, pp. 315-326, June 1980.

2. N. Navarrete, Jr., M. Fukushima, and H. Mine, "A New Ranking Method Based on Relative Position Estimate and its Extensions", IEEE Transactions on Systems, Man, and Cybernetics, vol. SMC-9, No. 11, pp. 681-689, November 1979.

3. Y.M. Cheng and B. McInnis, "An Algorithm for Multiple Attribute, Multiple Alternative Decision Problems Based on Fuzzy Sets with Application to Medical Diagnosis", IEEE Transactions on Systems, Man, and Cybernetics vol. SMC-10, No. 10, pp. 645-650, October 1980.

4. R.L. Keeney and H. Raiffa, "Decisions with Multiple Objectives: Preferences and Value Tradeoffs", Wiley, New York, 1976.

5. T.L. Saaty, "The Analytic Hierarchy Process", McGraw-Hill, New York, 1980.

ANNEX A

CONSTRUCTION OF OVERALL RATING REPRESENTATIONS IN THE FUZZY SET METHOD

The construction of an overall rating representation is explained here by means of an example. The calculation of the overall 100% level upper and lower endpoints for Alternative 2 (taken from the example in the main text) is shown. Calculation of the 50% level endpoints would be done in similar fashion.

The 100% level intervals for attribute weights and Alternative 2 ratings are reproduced below:

TABLE I

	Interval Level	Attribute a	b	c	d	e
Weight	100%	(.2,.4)	(.2,.4)	(.1,.3)	(0,.1)	(.1,.2)
Alternative 2	100%	(2,4)	(8,10)	(3,5)	(8,10)	(7,9)

The upper endpoint of the overall rating at the 100% level is an expression of the form

$$\bar{r} = \frac{\sum_{j=1}^{5} w_j r_j}{\sum_{j=1}^{5} w_j}$$

where r_j is the upper endpoint of the jth attribute rating at the 100% level, and w_j is either the lower or upper endpoint of the jth weight at the 100% level. The w_j values are chosen so as to maximize $\bar{r}$, and are determined with a set of computations which are explained presently.

ANNEX A

A series of five calculations (equal to the number of attributes) is performed. In the first one, $\bar{r}$ is calculated with w_k being the upper endpoint for k corresponding to a highest rating value r_k, and all other w_j values ($j \neq k$) as lower endpoints. In the second calculation, $\bar{r}$ is worked out with w_k and w_l as upper endpoints for those values of k and l which correspond to the two highest rating values r_k and r_l, and all other w_j values ($j \neq k$ or l) are lower endpoints. This pattern continues, with successive w_j values for the third-highest, fourth-highest, etc. r_j values being switched from the lower to the higher endpoint. It can be shown that the maximum $\bar{r}$ value calculated amongst such successive trials is indeed the maximum $\bar{r}$ value that can be generated by choosing any combination of w_j and r_j values which are within the specified intervals. The calculation results are shown in Table II.

TABLE II

MAXIMUM $\bar{r}$ CALCULATIONS

TRIAL	w_1	r_1	w_2	r_2	w_3	r_3	w_4	r_4	w_5	r_5	$\bar{r}$
1	.2	4	.4	10	.1	5	0	10	.1	9	7.8
2	.2	4	.4	10	.1	5	.1	10	.1	9	8.0
3	.2	4	.4	10	.1	5	.1	10	.2	9	8.1
4	.2	4	.4	10	.3	5	.1	10	.2	9	7.6
5	.4	4	.4	10	.3	5	.1	10	.2	9	7.1

Maximum $\bar{r}$ value is 8.1

TABLE III

MINIMUM $\bar{r}$ CALCULATIONS

TRIAL	w_1	r_1	w_2	r_2	w_3	r_3	w_4	r_4	w_5	r_5	$\bar{r}$
1	.4	2	.2	8	.1	3	0	8	.1	7	4.3
2	.4	2	.2	8	.3	3	0	8	.1	7	4.0
3	.4	2	.2	8	.3	3	0	8	.2	7	4.3
4	.4	2	.4	8	.3	3	0	8	.2	7	4.8
5	.4	2	.4	8	.3	3	.1	8	.2	7	5.1

Minimum $\bar{r}$ value is 4.0

.../A-4

In calculating the lower endpoint of the overall rating at the 100% level, the same expression for $\bar{r}$ is used, but the r_j values are the lower endpoints of attribute ratings, and w_j values are chosen so as to minimize $\bar{r}$.

Again, a number of calculations equal to the number of attributes are performed. In the first one, $\bar{r}$ is calculated with w_k being the upper endpoint for k corresponding to a lowest rating value r_k, and all other w_j values ($j \neq k$) as lower endpoints. In the second calculation, $\bar{r}$ is worked out with w_k and w_l as upper endpoints for those values of k and l which correspond to the two lowest rating values r_k and r_l, and all other w_j values ($j \neq k$ or l) are lower endpoints. This pattern continues, with successive w_j values for third-lowest, fourth-lowest, etc. r_j values being switched from the lower to the higher endpoint. It can be shown that the minimum $\bar{r}$ value calculated amongst such successive trials is indeed the minimum $\bar{r}$ value that can be generated by choosing any combination of w_j and r_j values within the specified intervals. The calculation results are shown in Table III.

So, the overall rating of Alternative 2 at the 100% level can be represented by the interval (4.0, 8.1). Interval calculations at the 50% level are done in the same way as the preceding presentation, using the appropriate weight and rating upper and lower endpoints.

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