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NAVAL AIR MISSILE TEST CENTER

POINT MUGU, CALIFORNIA

Decoys On Or Near A Target

TECHNICAL REPORT NO. 33

11 OCTOBER 1948

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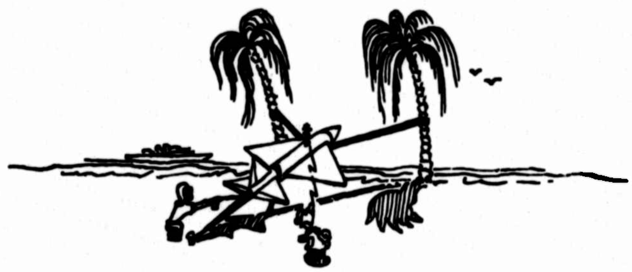
UNITED STATES

NAVAL AIR MISSILE TEST CENTER

POINT MUGU, CALIFORNIA



TECHNICAL REPORT NO. 33

Decoys On Or Near A Target

11 OCTOBER 1948

CONFIDENTIAL

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BUREAU OF AERONAUTICS

Foreword

The U. S. Naval Air Missile Test Center was established at Point Mugu, California, by the Secretary of the Navy (SecNav ltr. Op-24/mad Serial 1873P24), effective 1 October 1946. It is an activity of the ELEVENTH Naval District. The Bureau of Aeronautics exercises management and technical control over this activity.

The mission of the Naval Air Missile Test Center is the testing and evaluation of guided missiles and their components. NAMTC is assigned cognizance over all facilities at Point Mugu, and on San Nicolas Island.

Capt. R. S. Hatcher, USN, is Commanding Officer, Capt. E. W. Parrish, Jr., USN, is Deputy Commanding Officer, Comdr. E. R. Eastwold, USN, is Executive Officer and Comdr. Grayson Merrill, USN, is Technical Director.

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DECOYS ON OR NEAR A TARGET

SUMMARY

In addition to other means, it is possible to frustrate the attack of a radar homing missile by using the poor angular discriminating ability of the radar homing device and the property of the control device of the missile to aim at a predicted collision point. Such possibilities should be considered as means for the defense against enemy missiles and in our development program of guidance systems.

INTRODUCTION

Attempts will be made to camouflage targets against homing missiles by distributing decoys around the target. This is a short study concerning decoys on a target or in the immediate vicinity of a target. It will deal primarily with decoys against radar homing missiles and especially with "active" systems; i.e., systems in which the missile itself transmits the radio waves. However, some of the study may also be of interest for other systems.

A radar homing device is able to discern differences in reflectivity; after it has approached close to the target it will no longer approach the center of reflection of the whole target but the most concentrated area of reflectivity. If the most concentrated reflectivities of the target are attenuated or screened by removing 90° angles of inside corners, (*Figure 1*) the missile will attack a decoy, placed for instance near but outside of the bounds of the target, even if this decoy has a

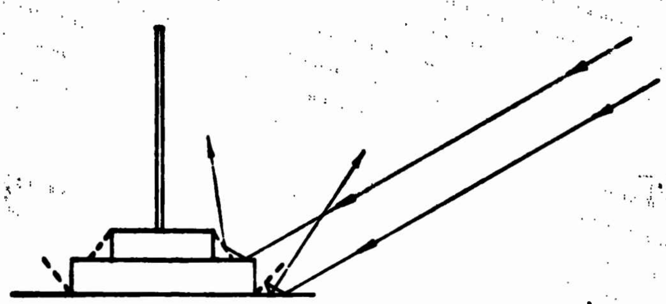

Fig. 1. The change of inside corners to angles other than 90° will remove concentrated reflectivities of a target.

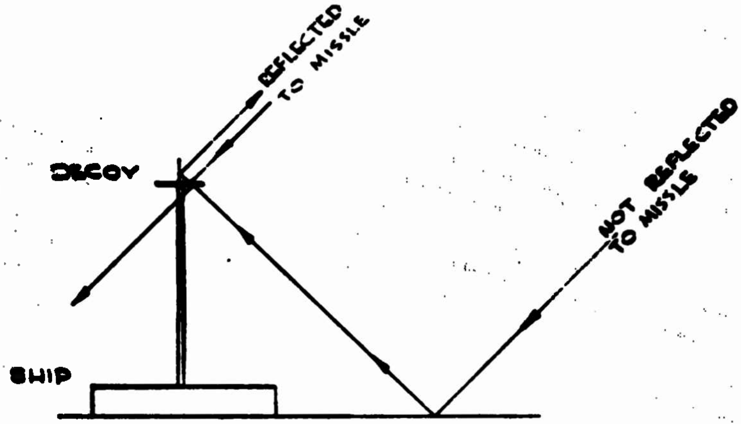

Fig. 2. Decoy which reflects only directly to the missile.

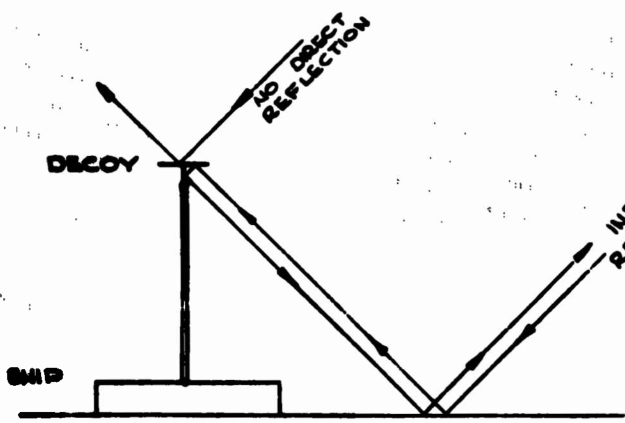

Fig. 3. Decoy which returns only the reflection of the water surfaces.

weaker reflection than the whole target. This characteristic may be especially helpful to groups of aircraft. It should be possible to devise formations for such a group which includes some decoys towed by some of these aircraft so that a decoy will be attacked instead of the aircraft. It should be possible to cover most of the directions from which a missile may attack.

A ship may use the reflection of a decoy (*Figure 3*) on the surface of the water to deflect a missile's path.

To enable a missile to hit a moving target, the control device of the missile aims at the "predicted collision point." If the center of reflection is moved on the target, the missile can be forced to pursue a path that bypasses the target. Such a shift of the center of reflection occurs even during the approach of the missile onto a target with a stationary decoy at its end; however, to achieve a considerable miss, the reflectivity of this decoy must be increased during the attack, or a moving decoy on the target must be used. This method will give reliable results only if the missile's navigation ratio *(See Footnote 2 and* [*B-3*]) is about 2 or greater. Such high navigation ratios are necessary to missiles in order to attack aircraft. It is probably quite possible to use smaller navigation ratios against ships; *(see Appendix B, Figure B-3)*.

By using decoys oscillating in effect, it is possible to exhaust the linear properties of the missile at a considerable distance from the target. It should be investigated to what extent this may interfere with the path of the missile.

This study is incomplete inasmuch as an evenly reflecting line has been the only target considered. Neither oblique directions of that line to the radar line of sight of the missile (i.e., the flight path of the missile) nor two- or three-dimensional targets have been investigated.

DECOY CHARACTERISTICS

Corner reflectors are used as decoys for radar homing missiles. If used on a ship, the reflection on the surface of the water must be taken into consideration. The arrangement in Figure 2 will give only direct reflection from the reflector to the missile but no indirect reflection on the water. The arrangement in Figure 3 will give no direct reflection but only indirect

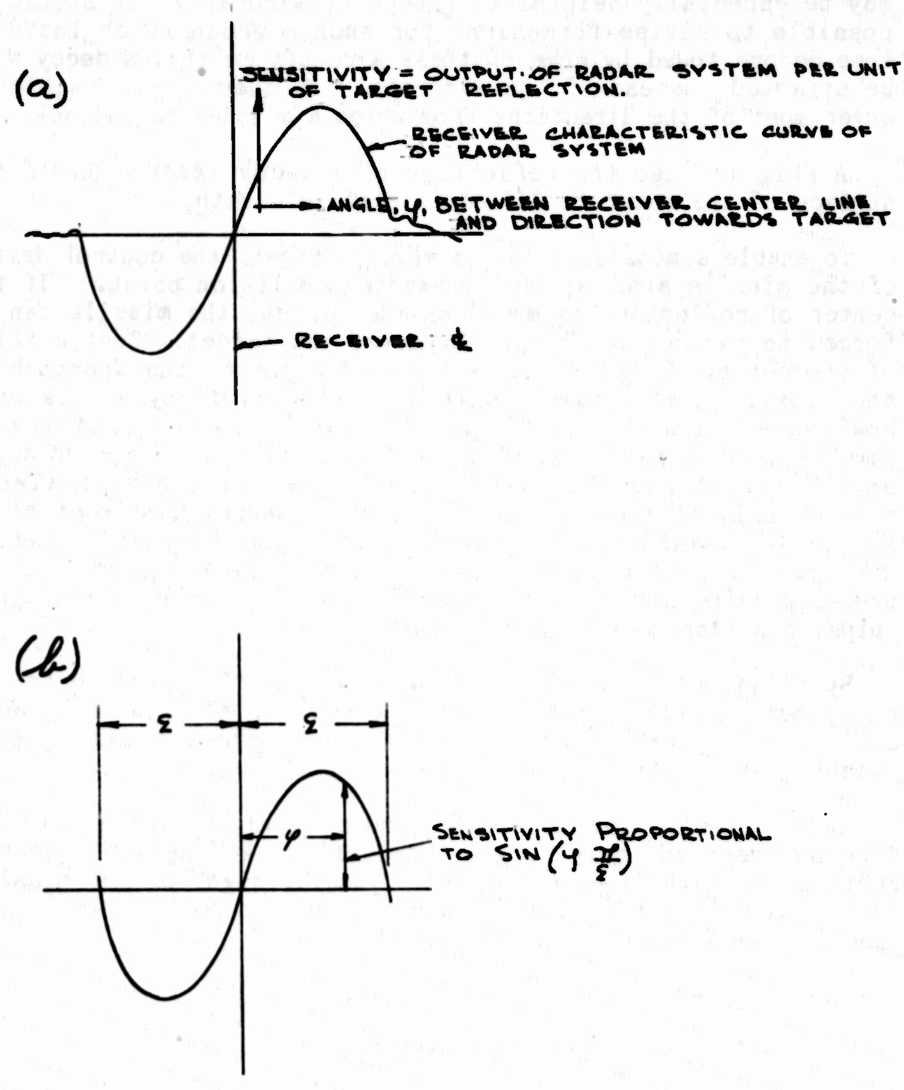

Fig. 4. Receiver characteristics of a homing antenna system; actual characteristics (a), and assumption for this report (b).

reflection; if the decoy is large enough it may deflect the missile onto the water.[1]

The reflectivity of a corner reflector decoy may be diminished by warping one or more of its three surfaces, or by deflecting its surfaces from their accurate 90° position. The effect of this method is dependent on the wave length of the radar guidance system. If these angles are greater or less than 90° by a large amount, for example 30°, the reflectivity of the corner reflector almost vanishes, except for the directions which are about normal to its surfaces.

I. RECEIVER CHARACTERISTICS

The receiver characteristic curve of a radar homing device (*Figure 4 (a)*) has a middle portion where the output distribution is approximately linear. This is designated later as the linear range of the radar system. Outside of this portion the sensitivity attains a maximum and then at an angle $\varphi = \pm\epsilon$ drops down to about zero. In this study a sine-shaped sensitivity distribution (between $\pm\epsilon$) is assumed. (*Figure 4(b).*)

In most radar homing missiles the receiver output is used to keep the center line of the receiver antenna system, i.e., the center line of the scanning motion, automatically on the target. The direction of this receiver center line and its angular velocity are used to control the missile. This report deals principally with such arrangements.

2. MISSILE DISTANT FROM TARGET

As long as the missile is distant from the target, the reflections from all parts of the target are received within the linear range of the receiver. In this case the center line of the receiver is directed toward the center of reflection of the target.

[1]This is the case where the water is calm. If there are waves, the reflection will not be sufficiently concentrated.

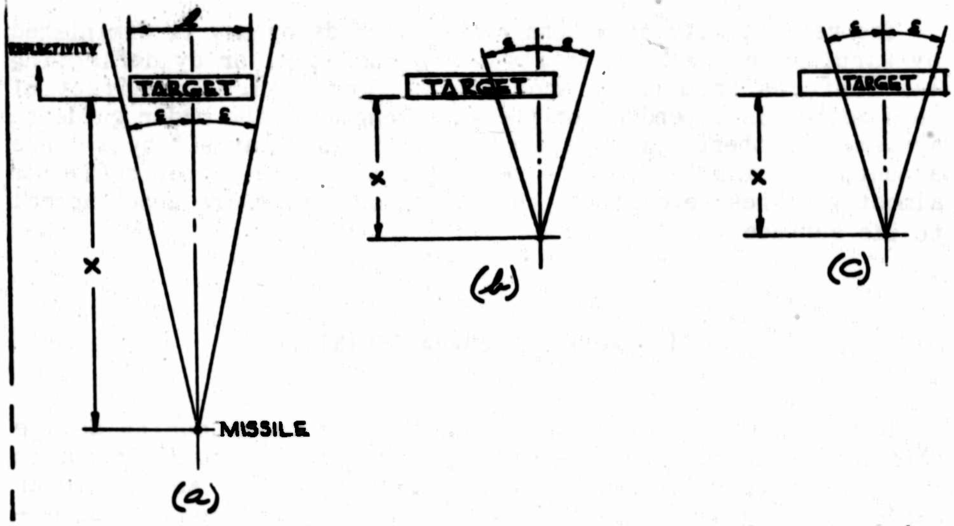

Fig. 5. Effect of the distance from missile to target on the output of the homing system.

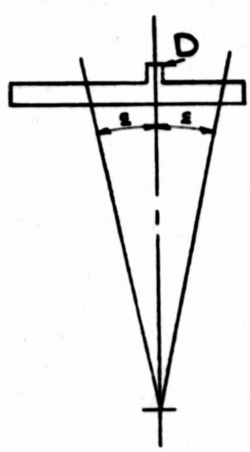

Fig. 6. If the missile is close to an evenly reflecting target, it will only be affected by an additional concentrated reflection.

3. MISSILE CLOSE TO TARGET

When the missile has approached the target, reflections from parts of the target will be received outside of the linear range and will finally lie outside of the sensitive range of the radar system, i.e., outside of $\pm\epsilon$; in this final situation reflections from these parts of the target will have no effect on the missile.

For example, let it be assumed that the target is a straight line with an even reflectivity *(Figure 5)*. At long distances *(Figure 5 (a))* the target is within $\pm\epsilon$ and the missile will attack the center of the target. At short distances, and in case the rim of the sensitive range extends beyond the end of the target line *(Figure 5 (b))*, the missile does derive a directive force from the target reflection. However, no directive force is derived as long as the whole target is covered by the sensitive range *(Figure 5 (c))*. Any small local reflectivity D *(Figure 6)*, placed additionally on the target will attract the missile. In case there are several such localized reflections, the final choice of aim will be dependent upon the distribution and size of these reflections, the special properties of the missile, and upon random conditions.

The case of one single local reflection (that is, one decoy) placed at the end of the target line, has been investigated. *(Appendix A.)* The result is shown in Figure 7 and on a larger scale in Figure 8. For some ratios of

$$\frac{D}{T} = \frac{\text{reflectivity of the decoy localized at the end of the target}}{\text{total reflectivity of the target}}$$

the location Y on the target of the "Effective Center of Reflection" EC (i.e., the point at which the receiver center line is directed) is plotted over the distance x from the missile to the target. In addition to dimensionless coordinates, the distances are given in feet for the following example:

$$\epsilon = \frac{\pi}{20} = 9^\circ$$

and the length of the target line = l = 600 feet.

Figure 7 shows that at great distances x, the weaker the local reflection at the end of the target, i.e., the smaller D/T, the more the missile aims at the center of the target. However, at the end of its flight the missile will always aim at the end of the target line.

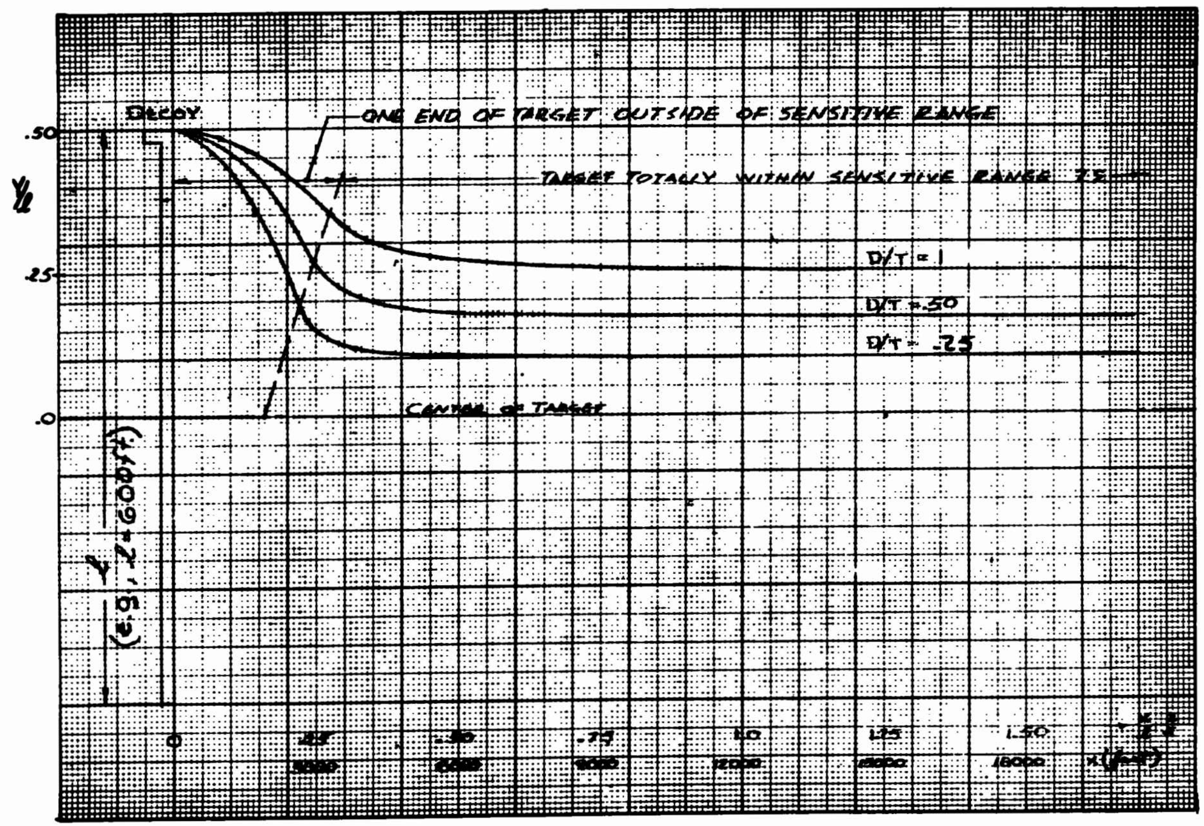

Fig. 7. Effective center of reflection is shown as a function of the distance from the missile to the target for different reflectivities of the decoy (D/T). As long as the missile is farther away from the target than the dashed line, the whole target is within the sensitive range of the receiver; after the missile has passed the dotted line only a part of the target is within this range. For missiles with infinite navigation ratio the curves also show the missile's path.

4. THE PREDICTED COLLISION POINT

Stationary Decoy

In order to avoid extreme acceleration of the missile at the end of its attack on a moving target, the computing device of the missile takes into account the motion of the target. The point in space at which the missile would hit the target if the target were to maintain a constant speed (i.e., the predicted collision point) plays an important role in this automatic computation.

The arrangement in Figure 7 (i.e., a local reflectivity which does not change its size or its location at the end of the target line) gives an effective center of reflection which moves on the target as the missile approaches the target. The missile thereby receives a false impression of a moving target. The locations of the predicted collision point, shown in Figure 9 (*see Appendix A* [*A 6*] *and* [*A 8*]) is independent of the speed of the missile.

For the last part of the path of the missile, the predicted collision point is outside of the target but finally again approaches the end of the target. When a smaller D/T is selected, the predicted collision point moves at a greater rate in the last part of the flight of the missile.

For a missile the path of which is always directed toward the predicted collision point,[a] Figure 7 and Figure 8 also show the path of the missile. This path is curved and the curvature depends on the length of the target line and on D/T. In practical cases (*see Appendix A*[*A 11*]) this curvature will be within the

the range of $\frac{1}{30{,}000\ \text{ft}}$ to $\frac{1}{5{,}000\ \text{ft}}$ and any missile should be

able to follow the proper course. (A missile with a navigation ratio N of the usual value may pass beyond the target but only by a very small fraction of the length of the target line. (*See Appendix B Figure B-3.*)

[a]This means that the navigation ratio

$$N = \frac{\text{rate of change of the missile's course}}{\text{rate of change of the direction from missile to "target"}}$$

is infinitely large. This is identical with the case in which the direction of missile to target is constant. (In this use "target" signifies the effective center of reflection.) Therefore the sidewise motion of the missile is the same as that of the effective center of reflection.

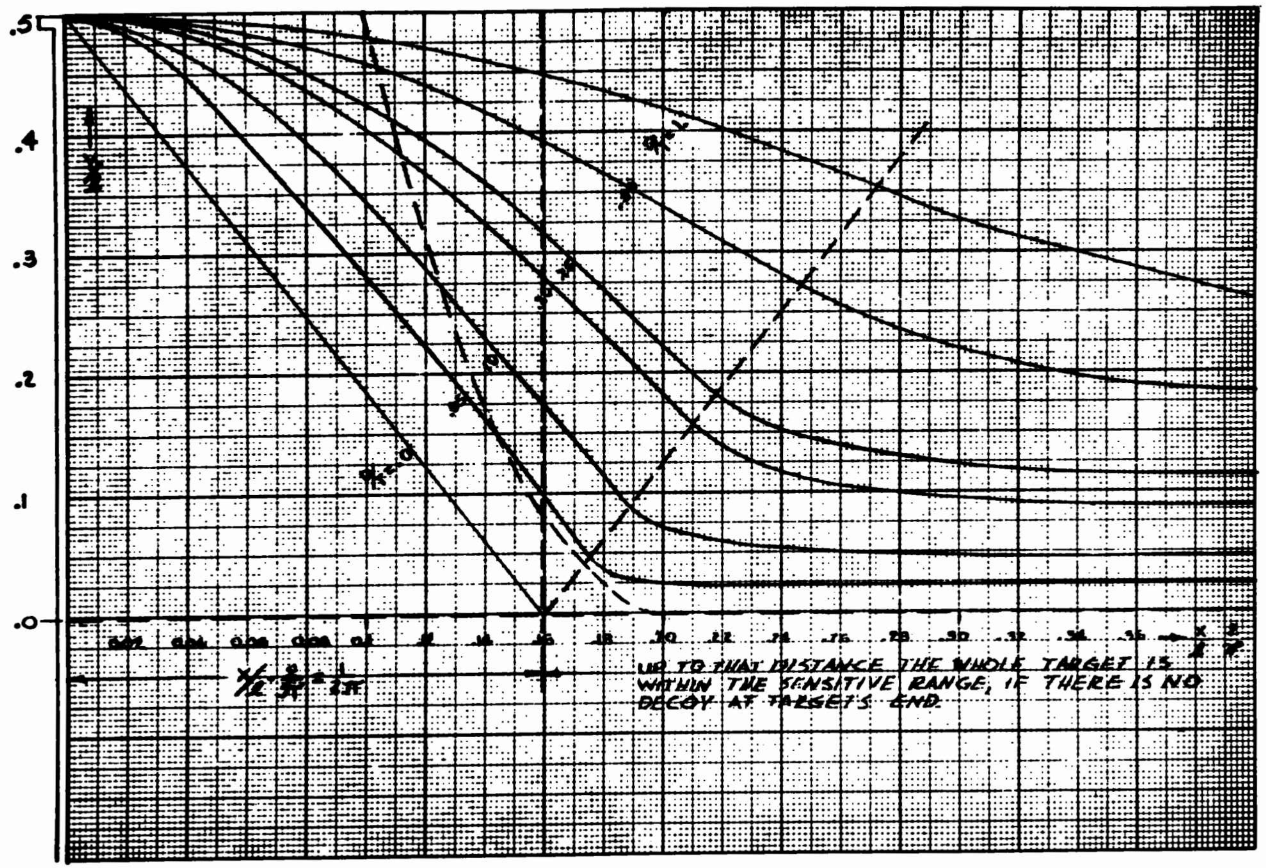

Fig. 8. Same as Figure 7. The missile can be forced to follow a certain path in which y is a function of x (e.g., the dashed parabola drawn into this Figure) by changing the reflectivity of the decoy. See Appendix D, and the example Figure D 1.

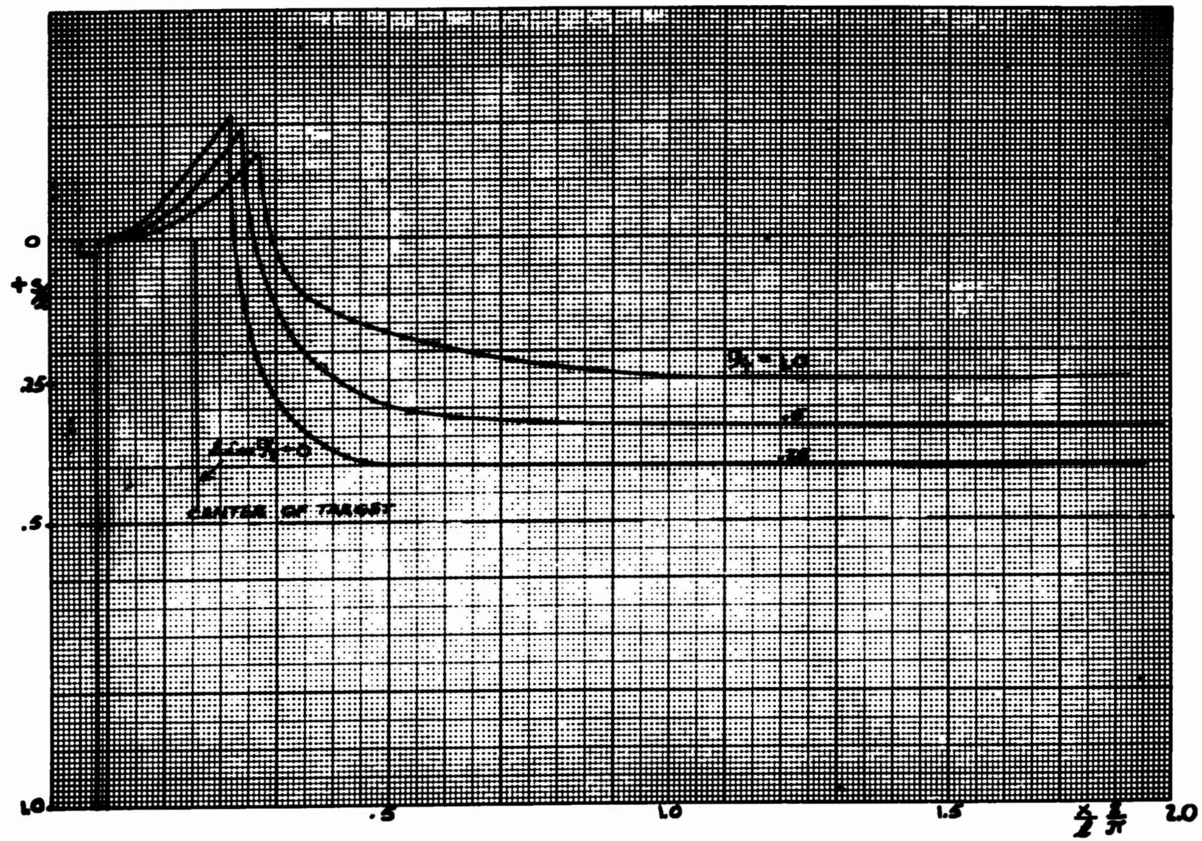

Fig. 9. For an evenly reflecting target with a decoy at one end, the location, S, of the predicted collision point is shown as a function of the distance from target to missile. (The distance, S, is measured from the target's end.)

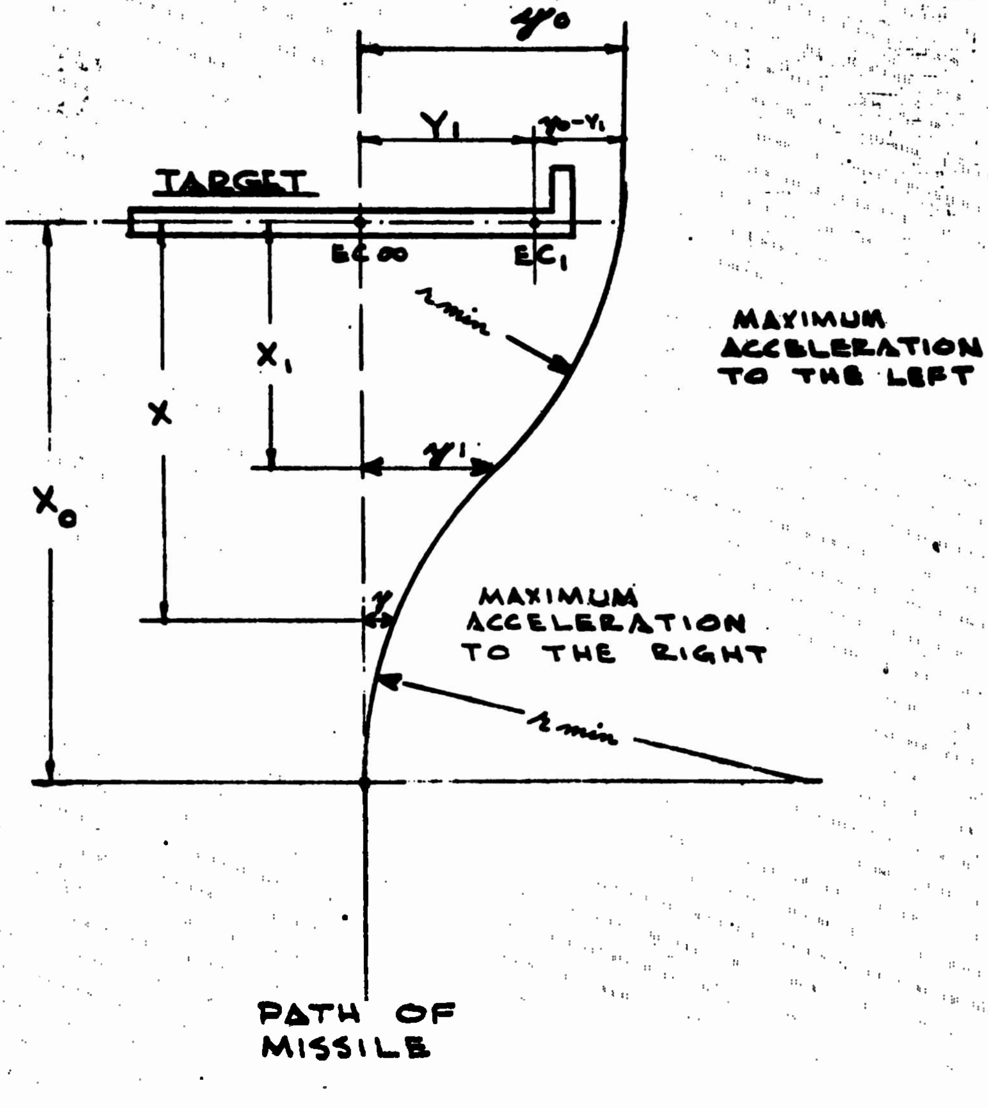

Fig. 10. Deflecting the missile's path beyond the target by changing the reflectivity of a decoy at the target's end.

5. DEFLECTION OF THE PATH OF THE MISSILE OUTSIDE OF THE TARGET
(See Figure 10)

Assume the target has such a reflectivity distribution (e.g., a symmetrical distribution) that its center of reflection, EC_∞ does not change with the missile's approach. Therefore, the target point EC_∞ is at rest until the missile reaches the distance x_0 from the target. After that, assume that by some artificial means, the center of reflection is shifted to the right from EC_∞ to EC_1, (abscissa Y_1) in such a way that the missile is accelerated to the right with the maximum acceleration a_{max} of which it is capable. The missile will then follow a circle. When it has reached the distance x_1 it will have been deflected to the right by the distance y_1. If y_1 is not too small compared to Y_1 and if x_1 is not too large (or too small) the missile will be unable to hit EC_1; using its maximum acceleration a_{max} to the left, it will pass the target at the distance $y_0 - Y_1$ outside of EC_1. This distance has been computed for different navigation ratios (*Appendix C*); it is plotted in Figure 11 as a function of $\frac{x_1}{x_0}$. Furthermore, the maximum accelerations pertaining to these paths of the missile are plotted in Figure 12. The smaller the navigation ratio of the missile, the smaller the range of $\frac{x_1}{x_0}$ for which positive misses $y_0 - Y_1$ are obtained, and the smaller x_0 must be chosen so that the missile is forced to use its maximum acceleration. (*See Figure 12.*) The method is applicable even if a considerable error is made in judging the maximum acceleration of the missile (*see Appendix C, Figures C-3 to C-5*). However, if the navigation ratio is below 2, the method may not be effective enough to be feasible.

6. DECOY ARRANGEMENT TO ACHIEVE THIS DEFLECTION

Such a deflection of the course of the missile may be achieved with one decoy at the end of the target line by a suitable change of the reflectivity of this decoy (*see Appendix D*). However, the use of this arrangement is limited, because the deflection has to begin when the missile is still remote enough from the target so that the decoy at the end of the target is still within the sensitive range of the receiver when the receiver center line is directed to the middle of the target. (*See Figure 5;* $x_0 > \frac{l}{2\epsilon}$.)

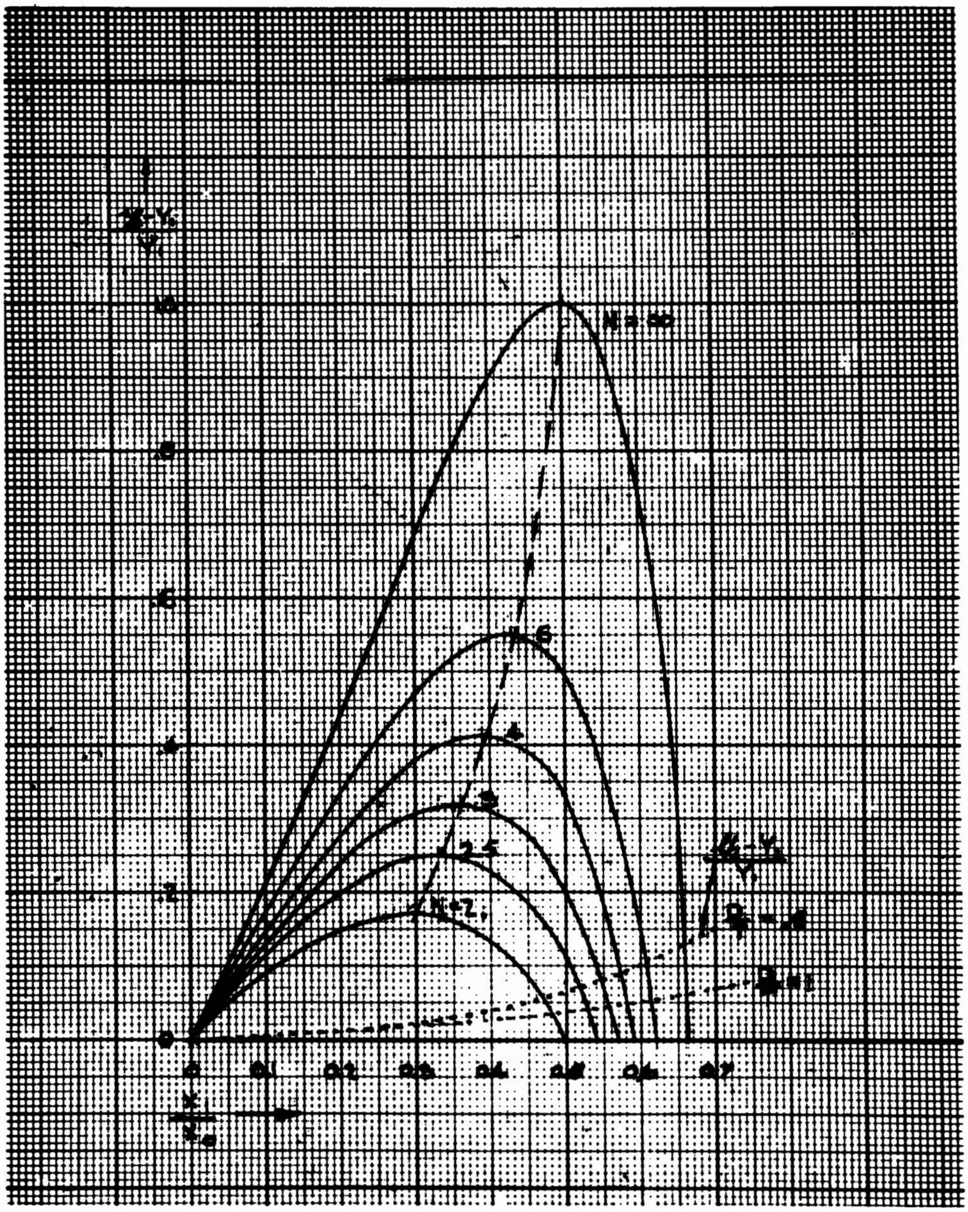

Fig. 11. Misses attainable by the method shown in Figure 10. y_o-Y_1 is the distance the missile passes outside of the effective center of reflection; the dotted line, $l/2$-Y_1, shows the distance the target extends outside of the effective center of reflection; the difference between these two curves is the miss. $l/2$-Y_1 is shown for the example $x_o = 1.2 \frac{l}{2\epsilon}$ and for the two values 0.5 and 1.0 of D/T. The dashed line connects the maximum values of y_o-Y_1; values coordinated to that dashed line are also dashed in the following graphs.

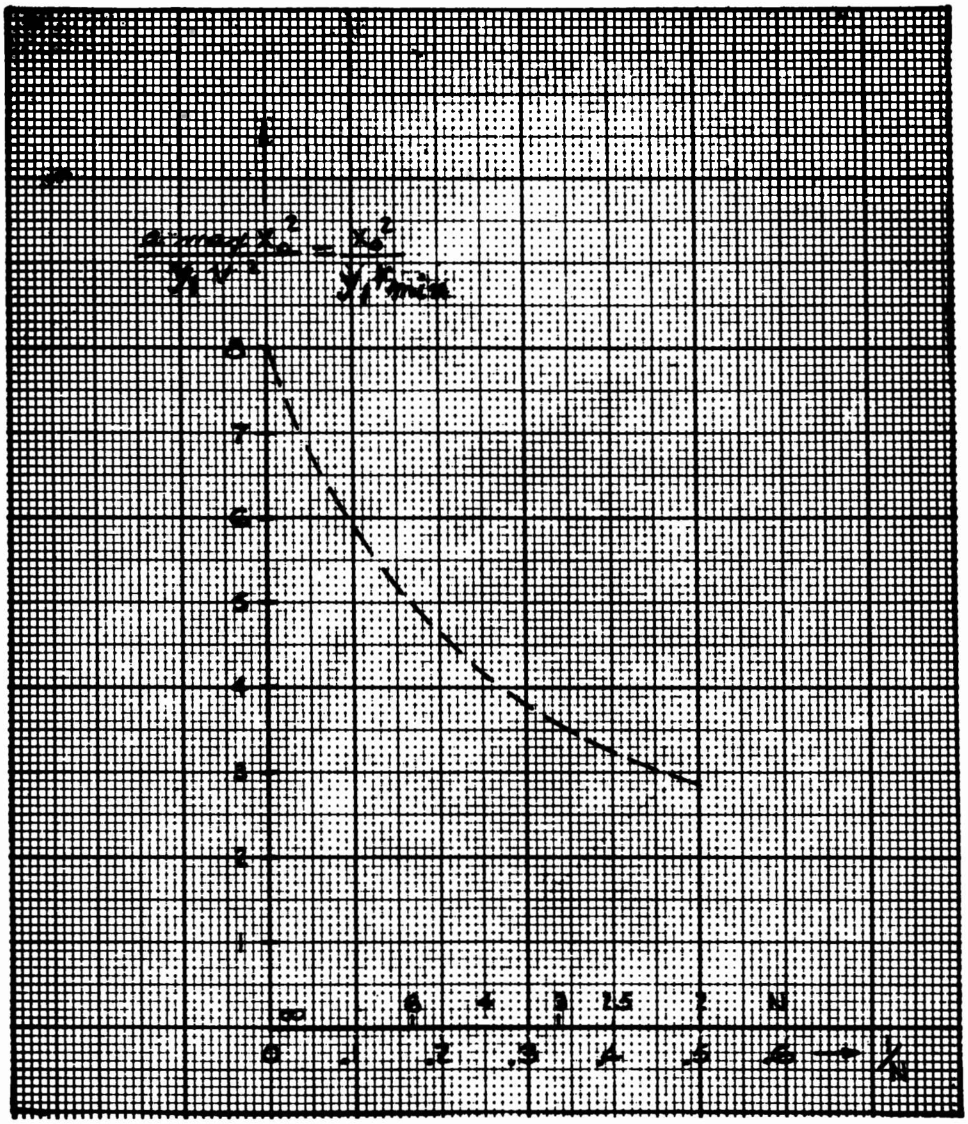

Fig. 12. Values of the maximum acceleration of the missile, a_{max}, coordinated to the dashed line in Figure 11. $\frac{1}{r_{min}} = \frac{a_{max}}{V^2}$ is the maximum curvature of path which the missile is capable to perform; V is the missile's velocity.

Therefore *(compare Figure 12 with* $x_0 = \frac{l}{2\epsilon}$ *and* N = 2.5) the maximum curvature of path, $\frac{1}{r_{min}}$, of which the missile is capable must not be larger than

$$\frac{1}{r_{min}} \cong 3\left(\frac{y_1}{l/2}\right)\frac{\epsilon^2}{l/2} \cong 3\frac{\epsilon^2}{l/2} \qquad [1]$$

(since $\frac{y_1}{l/2} \cong 1$ *, see Figure 10.)*

(Compare with Figure C 5.) The value $\frac{1}{r_{min}}$ may vary for different missiles within the range of*

$$\frac{1}{r_{min}} = \frac{1}{2{,}000\ ft} \text{ to } \frac{1}{5{,}000\ ft}.$$

For high altitudes this curvature decreases with the decreasing density of the air. For ϵ a value of about $\frac{\pi}{20} = 9°$ seems to be preferred. According to equation [1], this would correspond to target dimensions l of 300 feet to 700 feet, or less; at high altitudes about two to three times these values, or less. Therefore, aircraft (or small groups of aircraft) or a ship's mast would have suitable dimensions, but the length of the ship may be too large. In such a case two decoys changing in size or one movable decoy would have to be chosen.

The period of time the decoy has to act to deflect the missile's path is approximately

$$t = \frac{x_0 - x_1}{V} \qquad (V = \text{velocity of the missile})$$

and is within the range from 0.5 seconds (for small target lengths and missiles with high acceleration) to approximately 2 seconds. This method of deflecting the path of the missile fails if several missiles are attacking within this period of time.

The reflectivity of the decoy could be changed automatically by using radar to measure the distance of the missile from the target.

*If not otherwise limited: e.g., by the control system or by the strength of the airframe, the maximum curvature is determined by the wing load and the maximum lift coefficient.

7. PERIODICALLY CHANGING DECOYS

Consider the case in which two decoys are used, one at each end of the target operating alternately on-off in a periodic manner. Then the center of reflection of the target would oscillate. The closer the missile approaches the target the more these deflection angles increase and finally (at a distance x_{lim} from the target) the missile will exhaust its acceleration in following these oscillations. The distance x_{lim} depends on the frequency used and on the properties of the missile and its control device. This distance may be within the range of 10 to 20 times the target's length (*see Appendix E*); for a ship this would be about 10,000 feet. It is hard to predict how much such a jamming of the missile's linear properties would affect the accuracy of the hit, but it might be a successful method of defense.

CONCLUSIONS

It appears to be possible to defend against a missile which is homing by radar on ships or aircraft by the following means:

a. By the use of a decoy on an extremity of the target, especially such a decoy whose reflectivity increases with the approach of the missile.

b. If the target consists of a group of units by the choice of a formation suitably equipped for the defense against radar homing and/or the use of a decoy within the group.

c. By decoys with oscillating reflectivity which exhaust the missile's controllability.

APPENDICES

SYMBOLS

- t time
- g gravitational acceleration
- V velocity of missile
- a sidewise acceleration
- a_{max} maximum sidewise acceleration of the missile which is possible
- r radius of the missile's path
- $r_{min} = \frac{V^2}{a_{max}}$ narrowest radius that the missile is able to fly
- N navigation ratio of the homing system of the missile
- ϵ sensitive angular range of the homing device of the missile
- φ angle measured from the center line of the missile's antenna system
- l target dimension (length or height)
- T total reflectivity of target
- D reflectivity of a decoy
- x (coordinate) distance from missile to target (measured in the direction of the original flight path of the missile)
- y coordinate (normal to x) of the missile's location
- y_o distance at which the missile departs from the straight approach onto the target
- x_o, $y = 0$ beginning, x_1, y_1 end } of a circular path forced onto the missile
- Y distance of effective center of reflection from the center of target
- Y_1 the Y coordinate of the effective center of reflection when the missile is in position x_1, y_1

In all appendices the sine of small angles is assumed to be equal to the angle; the cosine to one.

APPENDIX A

The effective center of reflection for an evenly reflecting target with a decoy at its end.

SYMBOLS:

φ_1, φ_0 — angle coordinates (seen from the missile) for the left and the right end of the target. (The decoy is on the right end.) See Figures A 1, A 2.

$\rho = \rho(\varphi)$ — reflectivity of the target per unit of angle.

Z — distance of the effective center of reflection measured from the target's end (Z > 0 if on target).
$Z = \frac{l}{2} - Y$

S — distance of the predicted collision point from the target's end (S > 0 on target, S < 0 outside of target).

The radar system shall have the characteristic shown in Figure 4. Then

[A 1] $$\int_{-\epsilon}^{+\epsilon} \rho \sin \varphi \frac{\pi}{\epsilon} d\varphi = 0$$

is the equation which establishes the condition that the output of the receiver is equal to zero.

For an evenly reflecting target line of the length l and at the distance x from the missile

[A 2] $$\rho = T \frac{x}{l} \quad \text{(T = reflectivity of the total line)}$$

Case 1: Target line partly outside of the sensitive range.

Equation [A 1] (*Figure A 1*) gives

$$\int_{-\epsilon}^{+\varphi_0} T \frac{x}{l} \sin \varphi \frac{\pi}{\epsilon} d\varphi + D \sin \varphi_0 \frac{\pi}{\epsilon} = 0$$

This yields

[A 3] $$\frac{x}{l} \frac{\epsilon}{\pi} = \frac{D}{T} \frac{\sin \varphi_0 \frac{\pi}{\epsilon}}{1 + \cos \varphi_0 \frac{\pi}{\epsilon}}$$

Figure A 1 shows that the location Ξ of the effective center of reflection is given by

[A 4] $\Xi = \varphi_o x$

For small values of φ_o (small distances x) [A 3] and [A 4] yield

[A 5] $$\frac{\Xi}{l} = 2\frac{x^2}{l^2}\left(\frac{\epsilon}{\pi}\right)^2 \frac{D}{T}$$

The coordinate S of the predicted collision point is

[A 6] $$S = \Xi + \frac{d\Xi}{dT}\,\frac{x}{V}$$

$\frac{x}{V}$ is the period of time that the missile still has to fly. This yields with [A 4] and $\frac{dx}{dt} = -V$:

[A 7] $$S = -x^2 \frac{d\varphi_o}{dx} ,$$

and with [A 3]:

[A 8] $$S = -x\frac{\epsilon}{\pi} \sin \varphi_o \frac{\pi}{\epsilon}$$

For small values of φ_o (compare [A 8] with [A 4]).

[A 9] $S = -\Xi$

With infinite navigation ratio Ξ, also signifies the lateral position of the missile; [A 5] shows that the end of the path of the missile is circular; the radius of the curvature is

[A 10] $$r = \frac{1}{4}\left(\frac{\pi}{\epsilon}\right)^2 \frac{D}{T}\, l .$$

NUMERICAL EXAMPLE:

[A 11]

D/T	ϵ	l	r
0.5	$\frac{\pi}{20}$	600'	30,000'
0.5	$\frac{\pi}{20}$	100'	5,000'

Case 2: Target line covered by the sensitive range: (Figure A 2)

[A 12] $Z = \varphi_0 x$ (see [A 4])

[A 13] $l = (\varphi_0 - \varphi_1)x$

[A 1] gives:

$$\int_{\varphi_1}^{\varphi_0} \frac{T}{\varphi_0 - \varphi_1} \sin \varphi \frac{\pi}{\epsilon} d\varphi + D \sin \varphi_0 \frac{\pi}{\epsilon} = 0$$

This yields

[A 14] $$\frac{D}{T} \sin \varphi_0 \frac{\pi}{\epsilon} = \frac{\cos \varphi_0 \frac{\pi}{\epsilon} - \cos \varphi_1 \frac{\pi}{\epsilon}}{\frac{\pi}{\epsilon} (\varphi_0 - \varphi_1)}$$

[A 14] can be solved numerically for special values of D/T and then Z and l can be computed according to [A 12] and [A 13]. The results are shown in Figures 7 to 9.

The instant when one end of the target is just at the rim of the sensitive range is given by [A 14] with $\varphi_1 = -\varepsilon$.

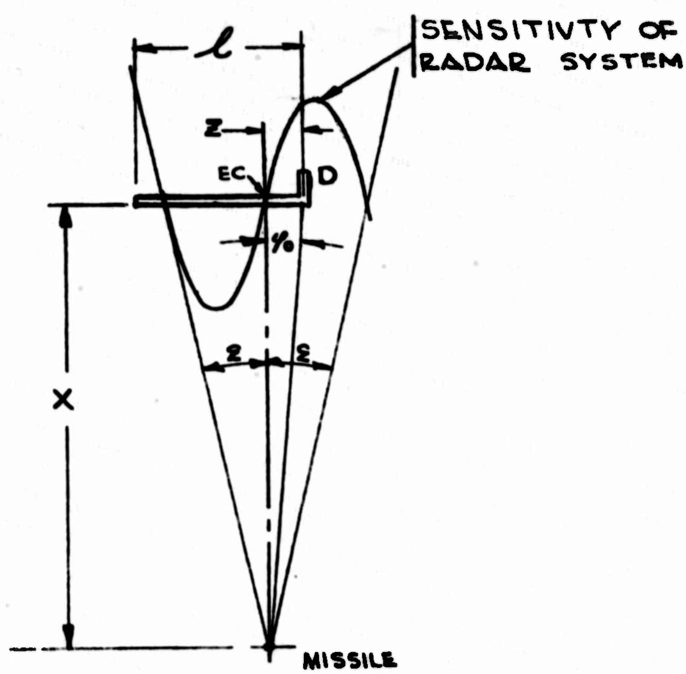

Fig. A 1. The axis of the radar system is directed toward the Effective Center of Reflection, EC. Part of the target extends outside of the sensitive range.

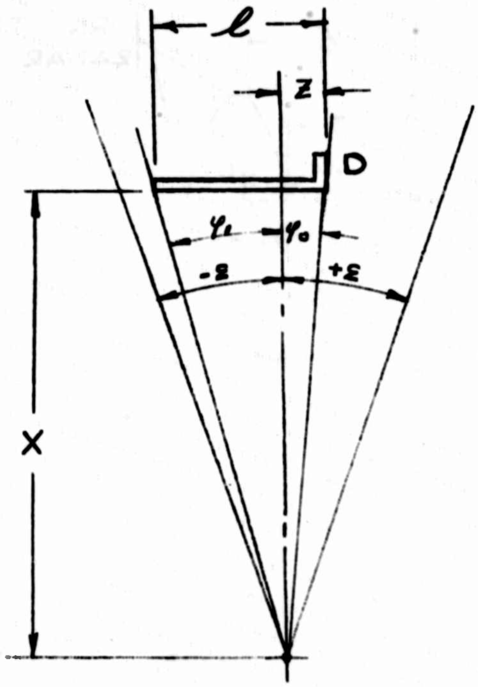

Fig. A 2. The axis of the radar system is directed toward the Effective Center of Reflection, EC. The entire target is covered by the sensitive range.

APPENDIX B

The effect of an accelerated sidewise motion of the target on the path of a missile; finite navigation ratio.

SYMBOLS:

λ	line of sight angle
$\varkappa$	course angle
A	sidewise acceleration of target
M	miss
Y	(sidewise) position of target

Figure B 1 shows:

[B 1] $$\lambda = \frac{Y - y}{x}$$

[B 2] $$\dot{y} = \varkappa V$$

The definition equation for the navigation ratio N is:

[B 3] $$N = \frac{\dot{\varkappa}}{\dot{\lambda}}$$

If y is eliminated and $\varkappa$, λ are contracted to $\varkappa - \lambda$, [B 1] to [B 3] yield:

[B 4) $$\dot{\varkappa} - \dot{\lambda} = \frac{N - 1}{x} [\dot{Y} - V (\varkappa - \lambda)]$$

In the special case in which the target is at rest (or has a constant velocity) until the missile reaches the distance x_o from the target (time t_o), and then the target assumes an additional constant sidewise acceleration A ($\ddot{Y} = A$, $\dot{Y} = A(t - t_o)$), the equation

[B 5] $$\dot{\varkappa} - \dot{\lambda} = \frac{N - 1}{Vt} [V(\varkappa - \lambda) - A(t - t_o)]$$

is derived from [B 4]. (The time of hitting is made equal to zero, therefore $x = -Vt$; t and t_0 have negative values.) The equation [B 5] has the solution:

$$[B\ 6] \qquad \kappa - \lambda = -\frac{At_0}{V}\left[1 - \frac{N-1}{N-2}\, t/t_0 + \frac{1}{N-2}\,(t/t_0)^{N-1}\right];$$

$$[B\ 7] \qquad \dot{\kappa} - \dot{\lambda} = \frac{N-1}{N-2}\,\frac{A}{V}\left[1 - (t/t_0)^{N-2}\right]$$

The acceleration $\ddot{y}$ of the missile

$$\ddot{y} = V\frac{N}{N-1}(\dot{\kappa} - \dot{\lambda})$$

is derived from [B 2] and [B 3] ; this with [B 7] yields:

$$[B\ 8] \qquad \frac{\ddot{y}}{A} = \frac{N}{N-2}\left[1 - (t/t_0)^{N-2}\right]$$

This acceleration of the missile increases as the missile approaches the target. If, after t_1, it exceeds the maximum acceleration a_{max} of the missile, the miss M will be *(see Figure B 2)*:

$$M = (\lambda - \kappa)x_1 - \dot{Y}t_1 + (\ddot{Y} - a_{max})\frac{t_1^2}{2}$$

with

$$\ddot{Y} = A, \quad \dot{Y} = A(t_1 - t_0) :$$

$$[B\ 9] \qquad M = (\lambda - \kappa)x_1 - At_1(t_1 - t_0) + (A - a_{max})\frac{t_1^2}{2}$$

with $t_1 = t$ according to [B 8] with $\ddot{y} = a_{max}$. If we introduce into [B 9] the equations [B 6] to [B 8], we derive

$$[B\ 10] \qquad \frac{M}{Y_A} = \left(\frac{t_1}{t_0}\right)^N = \left(1 - \frac{N-2}{N}\,\frac{a_{max}}{A}\right)^{\frac{N}{N-2}} \quad \text{with } Y_A = \frac{At_0^2}{2}$$

Y_A is the total (additional) sidewise path that the target assumes during the period of time $-t_0$ because of the acceleration A. This value $\frac{M}{Y_A}$ is plotted over $\frac{a_{max}}{A}$ in Figure B 3.

[B 8] shows that for navigation ratios larger than 2, the missile will actually hit the target if its maximum acceleration is

$$a_{max} \geq \frac{N}{N-2}A$$

otherwise it will pass the target by the distance M, given by [B 10].

Numerical Example:

Assume that the time the missile still has to fly when the target assumes an acceleration A = 1g is 4 seconds. Therefore, Y_A = 242 ft. The maximum acceleration of the missile shall be a_{max} = 2g. For this and for N = 2.5, Figure B 3 gives $\frac{M}{Y_A} = 0.08$. The miss M would be about 20 feet.

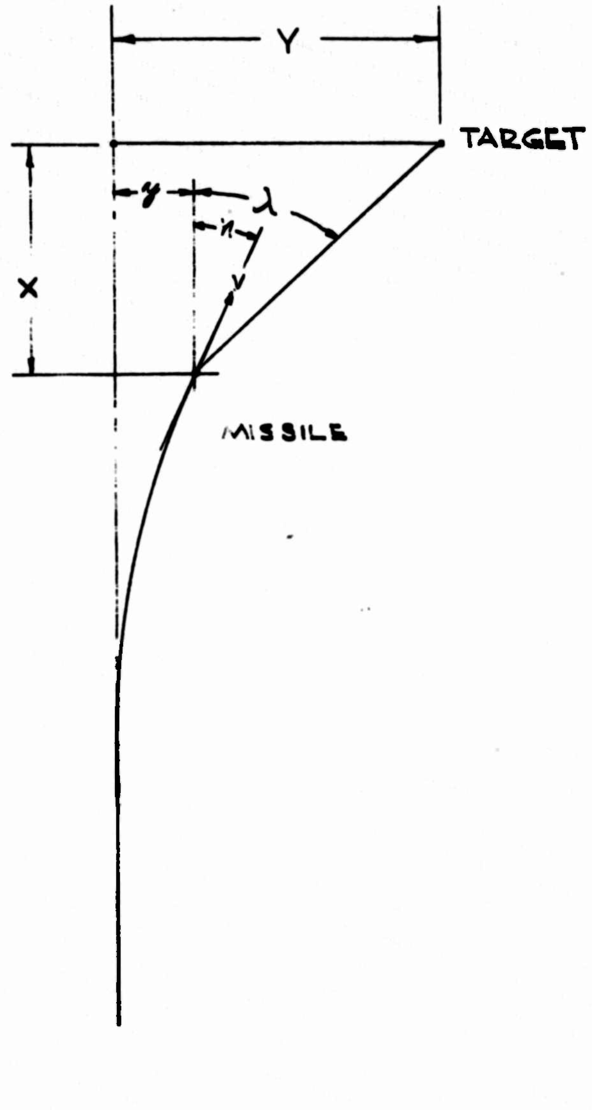

Fig. B 1. Diagram showing coordinates and angles relative to the homing problem.

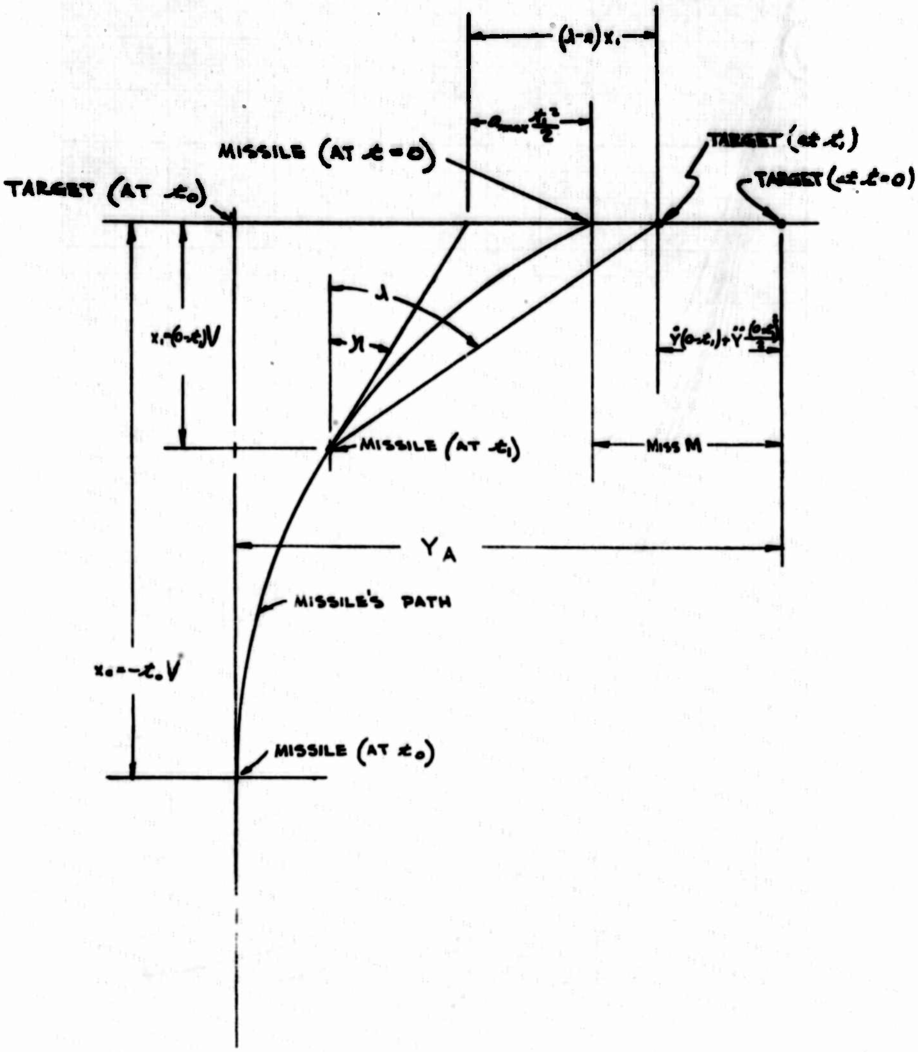

Fig. B 2. Chart showing flight path of missile when target has a constant acceleration in a direction perpendicular to the original flight path of the missile. Relative locations are shown at $t=t_0$, $t=t_1$, and $t=0$.

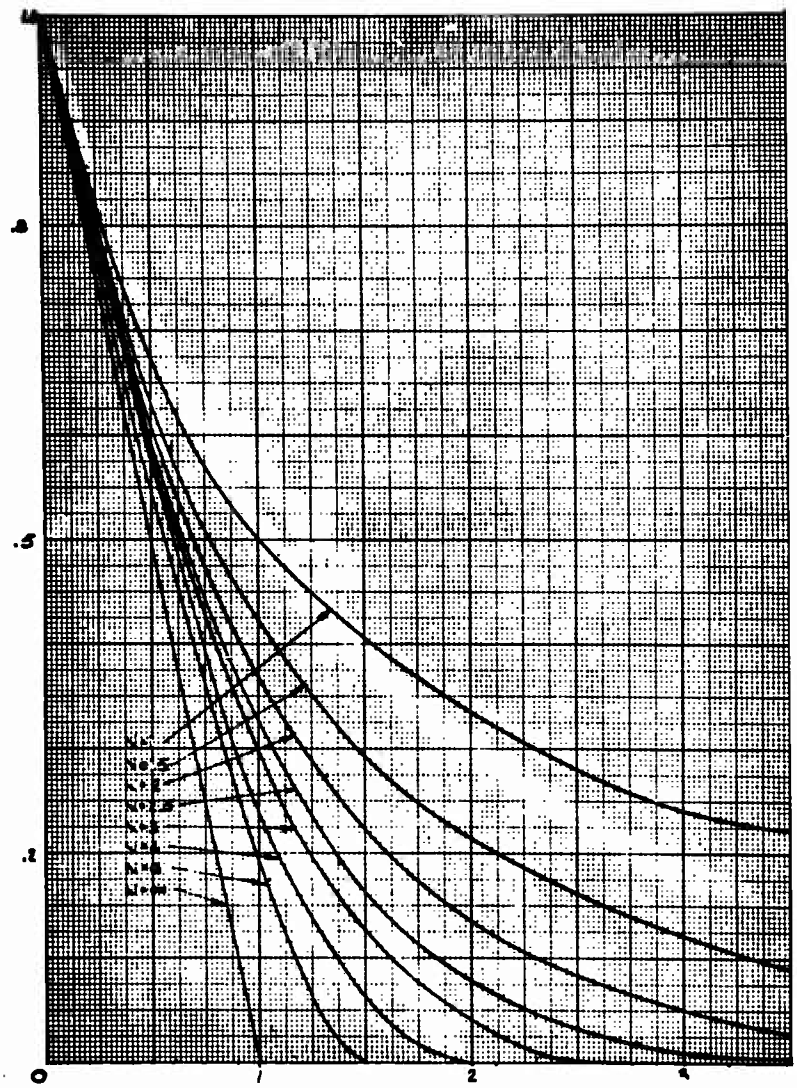

Fig. B 3. The target shall have a constant (sidewise) velocity until the time, t_o, before the missile passes the target. Then the target should assume a constant acceleration A. The missile will then miss the target by M. For different navigation ratios N the value $\frac{M}{Y_A}$ (with $Y_A = \frac{1}{2} A t_o^2$) is plotted as a function of the maximum acceleration of the missile.

APPENDIX C

Shift of the effective center of reflection necessary to deflect the missile; formulas for the miss. The missile shall be forced to assume a circular path (*see Figure C 1*).

[C 1] $$y = y_1\left(\frac{x_0 - x}{x_0 - x_1}\right)^2$$

with the constant cross acceleration

[C 2] $$a_1 = V^2\frac{2y_1}{(x_0 - x_1)^2}$$

To determine what positions Y of the effective center of reflection are coordinated to that path, the following proof is is developed.

Definition equation [B 3] yields with $\lambda = \kappa = 0$ for $x = x_0$:

$$\lambda = \frac{\kappa}{N}$$

therefore (*see Figure C 1*)

[C 3] $$Y = y + x\frac{\kappa}{N} = y + \frac{x}{N}\,\frac{dy}{d(x_0-x)}$$

This yields with [C 1] for the extreme value Y_1 of Y which is coordinated to the end x_1, y_1 of the circular path:

[C 4] $$Y_1 = y_1\left(1 + \frac{2}{N(\frac{x_0}{x_1} - 1)}\right)$$

This value is plotted in Figure C 2.

Let a circle with the maximum acceleration a_{max} (to the left) of the missile be assumed for the remainder of the missile's path. (*See Figure 10.*) Then the missile will pass the target at the point y_0:

[C 5] $$y_0 = \frac{a_1}{V^2}\frac{x_0^2}{2} - \frac{a_{max} + a_1}{V^2}\frac{x_1^2}{2}$$

Three cases are now to be considered:

Case 1: $a_{max} = a_1$

[C 4], [C 5] and [C 2] yield:

[C 6] $$\frac{y_0}{Y_1} = \frac{x_0^2 - 2x_1^2}{(x_0 - x_1)^2 \left[1 + \frac{2}{N\left(\frac{x_0}{x_1} - 1\right)}\right]}$$

Figure 11 shows the values $y_0 - Y_1$, relative to Y_1, by which the missile passes outside of Y_1. The maximum values for the different navigation ratios are connected by a dotted line. The coordinated accelerations are shown in Figure 12. (The scale for the abscissa is $\frac{1}{N}$.)

Case 2: $y_0 = Y_1$

The larger the maximum acceleration of the missile a_{max} (compared with a_1) the smaller the miss will be. The smallest value of a_{max}/a_1 that enables the missile to hit is shown in Figure C 3. (The equations [C 5] and [C 2] yield with $y_0 = Y_1$:

[C 7] $$\frac{a_{max}}{a_1} = \frac{x_0 - x_1}{x_1^2} \left[(x_0 + x_1) - (x_0 - x_1)\frac{Y_1}{y_1}\right] ;$$

$\frac{Y_1}{y_1}$ is given by [C 4].)

Case 3:

The following questions should be considered: After the missile has reached x_0 it shall be given maximum acceleration to the right as long as possible by an effective center of reflection located in Y_1. Then it shall accelerate to the left also using its maximum acceleration. Then consider how large this maximum acceleration must be so that Y_1 is hit.

There are two accelerations that satisfy this condition (*see Figure C 4*). One is

[C 8] $$\frac{(a_{max})_{min}}{V^2} \frac{x_0^2}{Y_1} = 2$$

the other one is given by [C 7] with $a_1 = a_{max}$; it is of the magnitude

$$\text{[C 9]} \qquad \frac{(a_{max})_{max}}{V^2} \frac{x_0^2}{Y_1} = \frac{1}{\frac{1}{2} - \left(\frac{2N - 2}{3N - 2}\right)^2}$$

The ratio of these two accelerations is shown in Figure C 5. If the maximum acceleration that the missile is able to perform is between these two values [C 8] and [C 9], it will hit outside of Y_1.

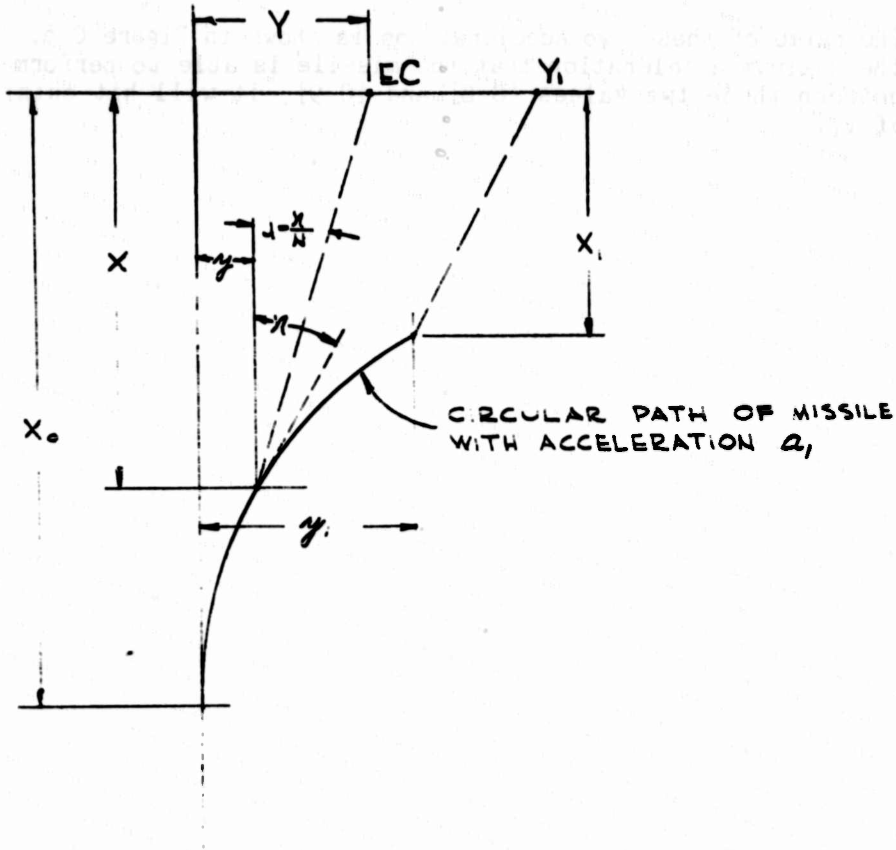

Fig. C 1. Relationship between the shift of EC and the path of the missile.

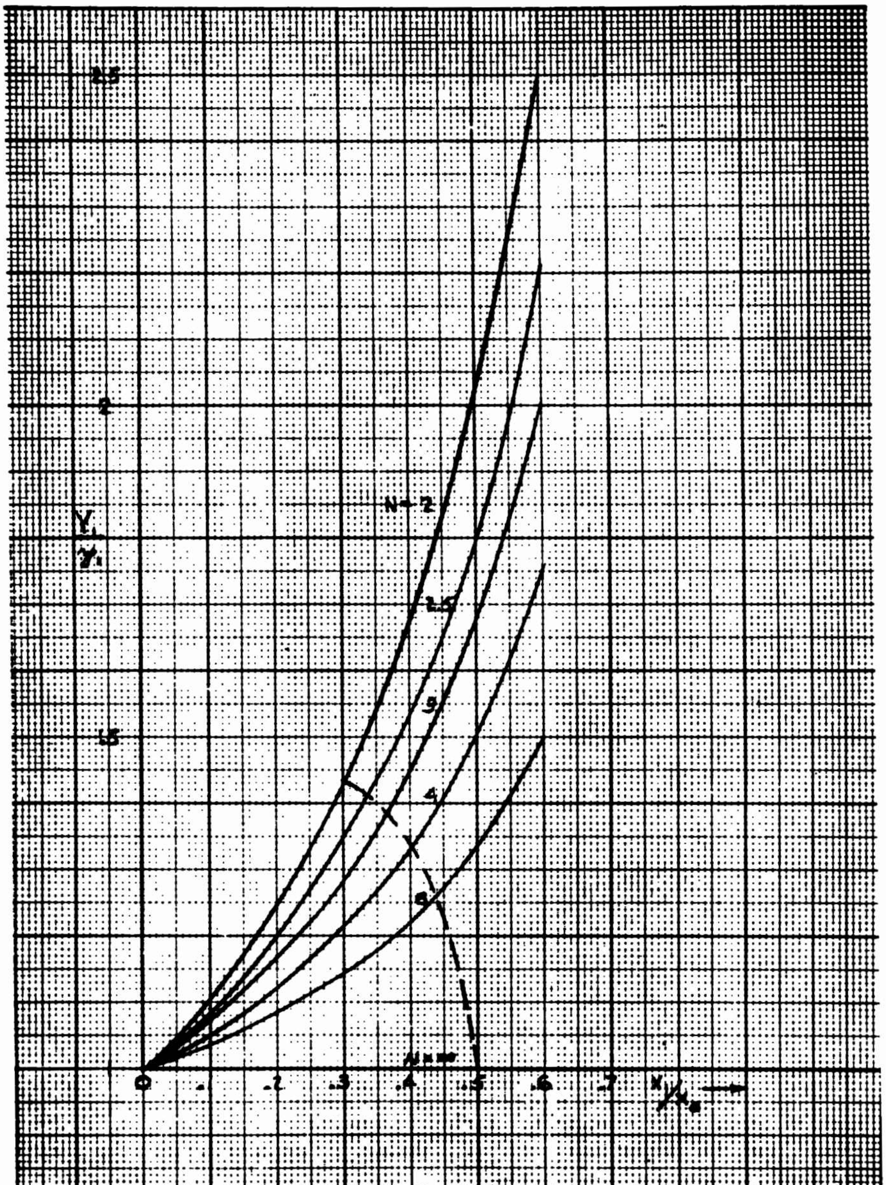

Fig. C 2. See [C 4]. The values on the dashed line are coordinated to the dashed line in Figure 11.

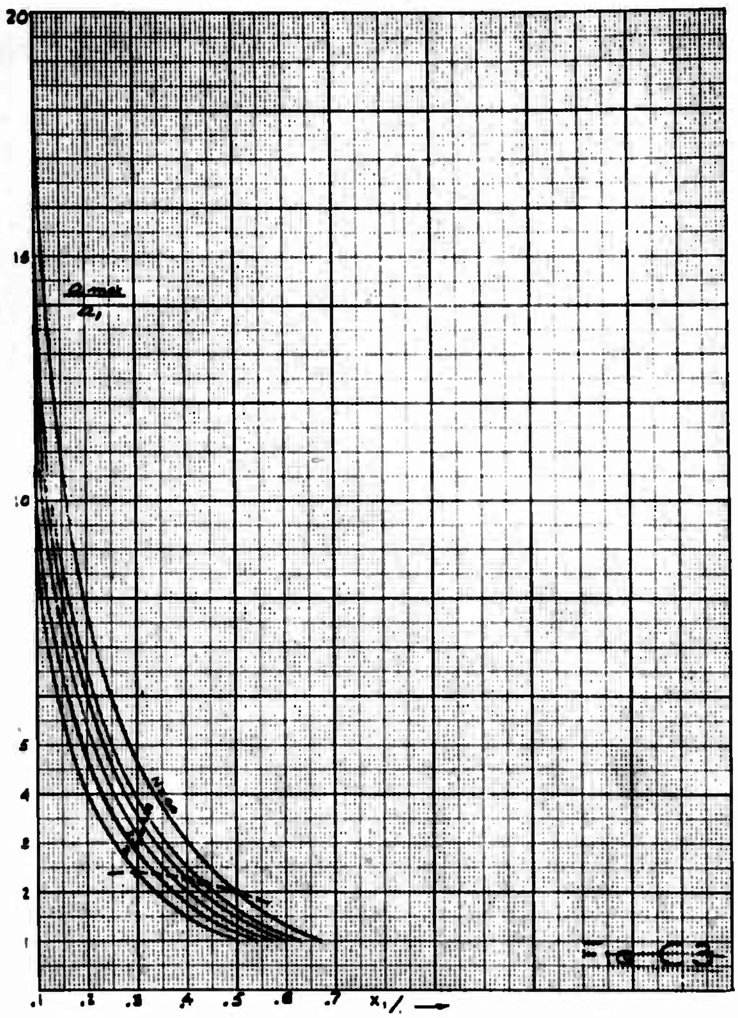

Fig. C 3. This graph gives a measure for the sensitivity of the method of deflecting a missile's path beyond a target by forcing the missile to follow a circular path (acceleration a_1) before it reaches the target. (x_0 is the distance from the target where this path begins, x_1 where it ends.) The graph shows that the maximum acceleration, a_{max}, of the missile (e.g., for $N=2.5$, $\frac{x_0}{x_1}=0.3$) must be about 3 times as high as a_1 or higher, if the missile is to hit the target.

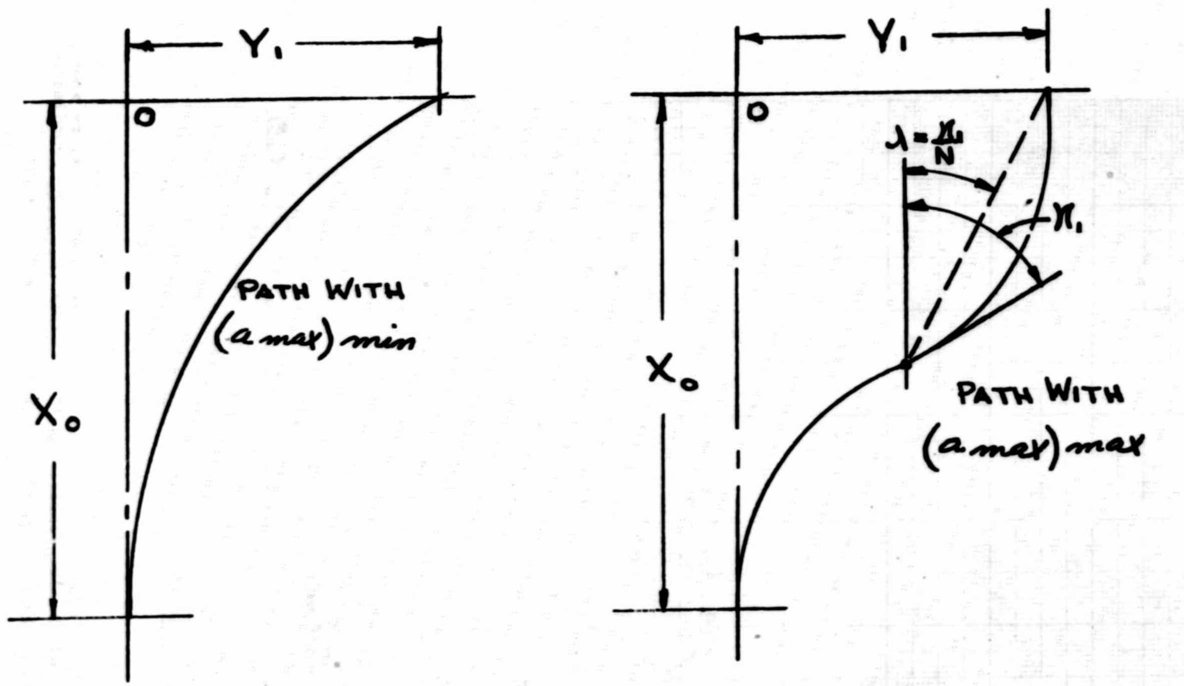

Fig. C 4. After the missile has reached the distance x_0 from the target, the missile should be given maximum acceleration to the right as long as possible by a shift of the effective center of reflection from Y 0 to $Y=Y_1$. After that the missile should produce its maximum acceleration to the left. There are two accelerations which fulfill the conditions which will enable the missile to hit Y_1, $(a_{max})_{min}$ and $(a_{max})_{max}$. If the actual acceleration of the missile is between these two values, the missile will pass beyond Y_1.

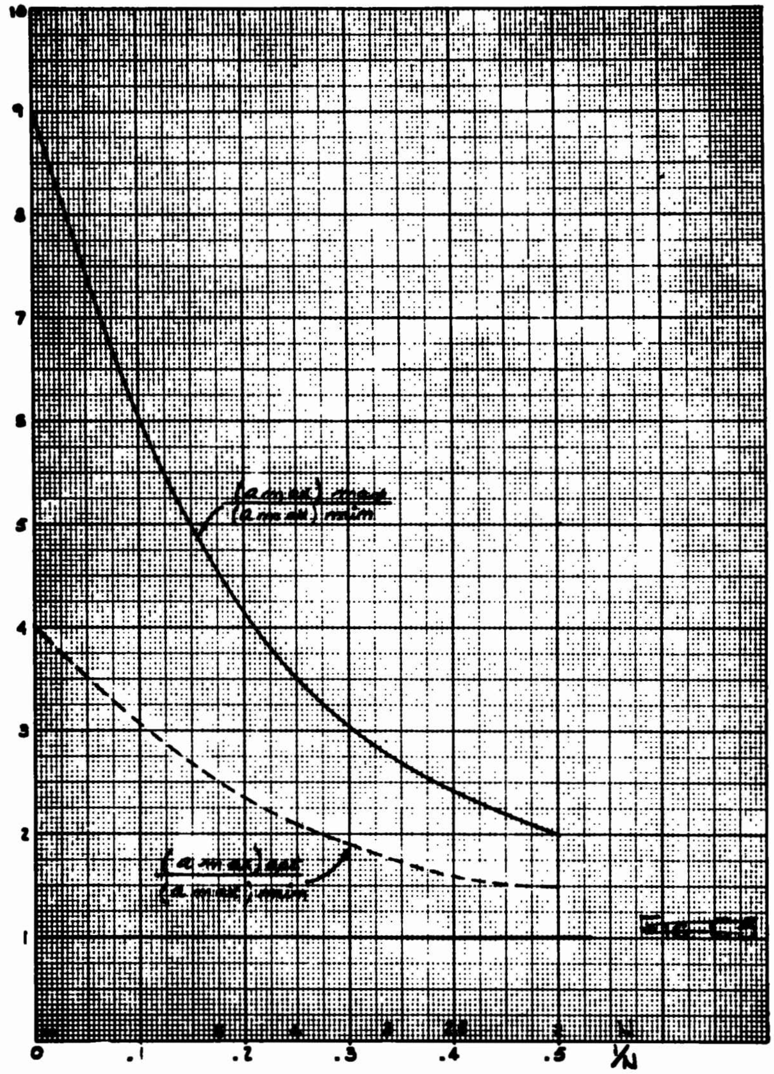

Fig. C 5. The ratio of the two extreme accelerations (Figure C 4) is shown as a function of 1/N and is also compared with the value $(a_{max})_{opt}$ coordinated to the largest missile deflections (dashed line in Figures 11 and 12).

APPENDIX D

Determination of the size of a decoy at the end of the target for a given center of reflection.

To determine the changing reflectivity of a decoy coordinated to a parabolic path of the missile, Figure 8 may be used. If, e.g., the navigation ratio is infinitely large, one has only to draft the parabolic path into Figure 8 and to depict the values of D/T for the points of the parabola. For the result of the example shown in Figure 8 see Figure D 1. For other values of the navigation ratio the value Y (*given by* [C 3]) has to be drafted into Figure 8.

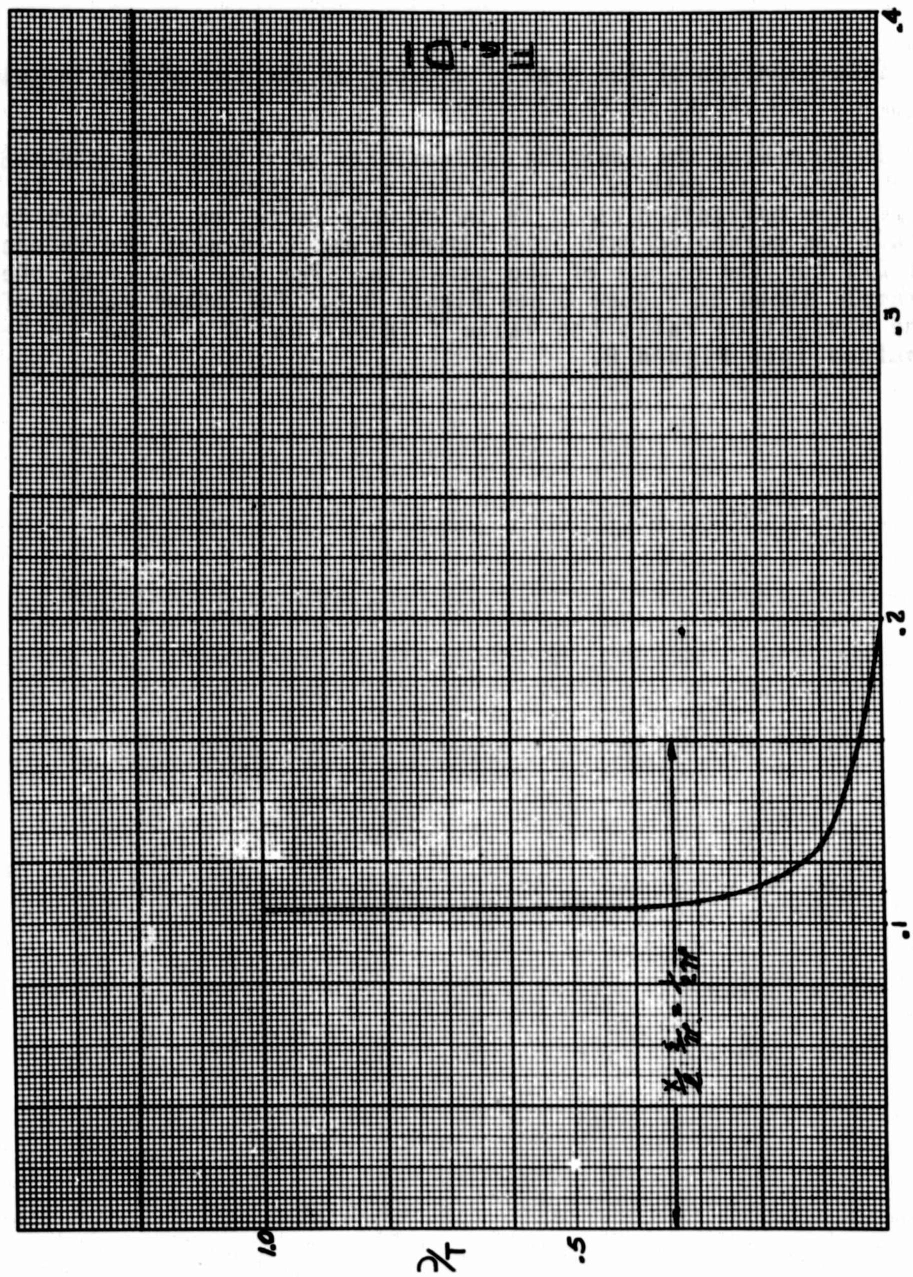

Fig. D 1. A missile can be made to follow a parabolic flight path during the last period of its approach by increasing the reflectivity D, of the decoy. This reflectivity is increased as a function of a decreasing distance, x, between target and missile.

APPENDIX E

Oscillating center of reflection caused by changing decoys. Oscillation forced by a disturbance, the amplitude of which changes according to $\frac{1}{t}$.

SYMBOLS:

ϑ	time of a full sine period of the critical frequency of the missile
τ	time constant of damping of the free oscillation of the missile
Y	amplitude of the oscillating motion of the effective center of reflection which causes the missile to oscillate
f	frequency of this oscillation (for resonance $f = \frac{1}{\vartheta}$)

For a sine-shaped motion (*Figure E 1*)

[E 1] $$y = B \sin \frac{2\pi (x_s - x)}{L}$$

(x_s and B are constants; L is the length of a full sine period) [C 3] gives for the nodes

[E 2] $$Y = 2\pi \frac{xB}{NL}$$

We introduce into [E 2] the maximum curvature

$$\frac{1}{r_{min}} = B \frac{4\pi^2}{L^2} \qquad \text{(from [E 1])}$$

of the path of the missile and the frequency

$$f = \frac{V}{L}$$

of the sine motion:

[E 3] $$x_{lim} = 2\pi \frac{Y N f r_{min}}{V} .$$

[E 3] gives the distance x_{lim} from the target at which an oscillating motion will exhaust the maximum acceleration of the missile.

Even larger amplitudes will occur if Y oscillates with the critical frequency of the missile. Also to be considered is the fact that the angular disturbance $\frac{Y}{x}$ also changes with the missile's distance x. This influence is investigated at the end of this appendix. The result is that by resonance the distance x_{lim} will be augmented by a factor of about 5 (*see [E 12] and the last paragraph of this appendix*).

NUMERICAL EXAMPLE

Properties of the target:

Target length = l.

Decoy at each end of the target.

$\frac{D}{T} = \frac{1}{2}$, therefore $Y = \frac{l/2}{1 + T/D} = \frac{l}{6}$.

Frequency of decoy shift 1/2 cps.

Properties of the missile:

R_{min} = 3,000 feet.

v = 600 feet/sec.

Navigation ratio N = 3.

[E 3] gives $E/l \cong 8$. Resonance will increase this value.

OSCILLATION FORCED BY A DISTURBANCE WITH A CHANGING AMPLITUDE $\left(\frac{1}{t}\right)$

As an example for a damped oscillation consider the differential equation

[E 4] $$c_1\ddot{\gamma} + c_2\dot{\gamma} + \gamma = 0$$

which has the solution

[E 5] $$\gamma = C e^{-t/\tau} \sin\frac{2\pi t}{\vartheta} = \Gamma \sin\frac{2\pi t}{\vartheta};$$

τ is the time constant of damping, ϑ is the time of one full oscillation; Γ is the variable amplitude.

For an oscillation forced by a disturbance of constant amplitude β and with resonance frequency, namely for

$$c_1\ddot{\gamma} + c_2\dot{\gamma} + \gamma = \beta \sin \frac{2\pi t}{\vartheta}$$

the (constant) amplitude is

$$\Gamma \cong \beta\, \pi \frac{\tau}{\vartheta}. \qquad \text{[E 6]}$$

If the disturbance ceases at the time t_1, then at a later time t_o the (variable) amplitude will be (*compare* [*E* 5]):

$$\Gamma = \beta\, \pi \frac{\tau}{\vartheta} e^{-\frac{t_o - t_1}{\tau}}$$

If now, β is variable in time, e.g.,

$$\beta = \beta_o \frac{t_o}{t} \qquad \text{[E 7]}$$

(*see Figure E 2; t and t_o have negative values*)

the amplitude of the forced oscillation may be computed by integrating over all partial disturbances $d\beta$, e.g.,

$$\Gamma = \beta_o\, \pi \frac{\tau}{\vartheta} - \int_0^{\beta_o} d\beta\, \pi \frac{\tau}{\vartheta} \cdot e^{-\frac{t_o - t_1}{\tau}} \qquad \text{[E 8]}$$

If, according to [E 7],

$$d\beta = - \beta_o \frac{t_o}{t^2} dt$$

is introduced into [E 8], we derive

$$\Gamma = \beta_o\, \pi \frac{\tau}{\vartheta} 1 + \frac{t_o}{\tau} e^{-\frac{t_o}{\tau}} \int_{-\infty}^{t_o} \left(\frac{t}{\tau}\right)^{-2} e^{t/\tau} d\frac{t}{\tau} \qquad \text{[E 9]}$$

Now, referring to the missile, we introduce

$$x = - V\, t_o$$

and derive

[E 10] $$\Gamma = \beta_o \, \pi \frac{\tau}{\vartheta} K$$

with

[E 11] $$K = 1 - \frac{x}{V\tau} e^{\frac{x}{V\tau}} \int_{\xi = \frac{x}{V\tau}}^{\infty} \xi^{-2} e^{-\xi} d\xi$$

The factor K (smaller than 1) gives the effect of the increase of

$$\beta_o = \frac{Y_1}{x}$$

as the missile approaches the target (as x decreases). K is plotted in Figure E 3. The factor

[E 12] $$\frac{\Gamma}{\beta_o} = \pi \frac{\tau}{\vartheta} K$$

gives the whole influence of resonance, in comparison with an infinitely slow frequency of the oscillation. By the same factor x_{lim} would have to be multiplied if resonance is used. (Because [E 4] is not the true differential equation of the missile's motion, this is only a rough approximation.)

In practical cases $\frac{\tau}{\vartheta}$ will be about 2 (or at least not much smaller) and at rather large distances K will not be much smaller than 1; therefore $\frac{\Gamma}{\beta_o}$ may be about 5. In practical application it will not be possible to hit the resonance frequency accurately and $\frac{\Gamma}{\beta_o}$ will be smaller.

If the amplitude Y_1 is worked in an on-off manner, the first harmonic has the amplitude $\frac{\pi}{2} Y_1$, and therefore the factor $\frac{\pi}{2}$ had to be added to [E 10].

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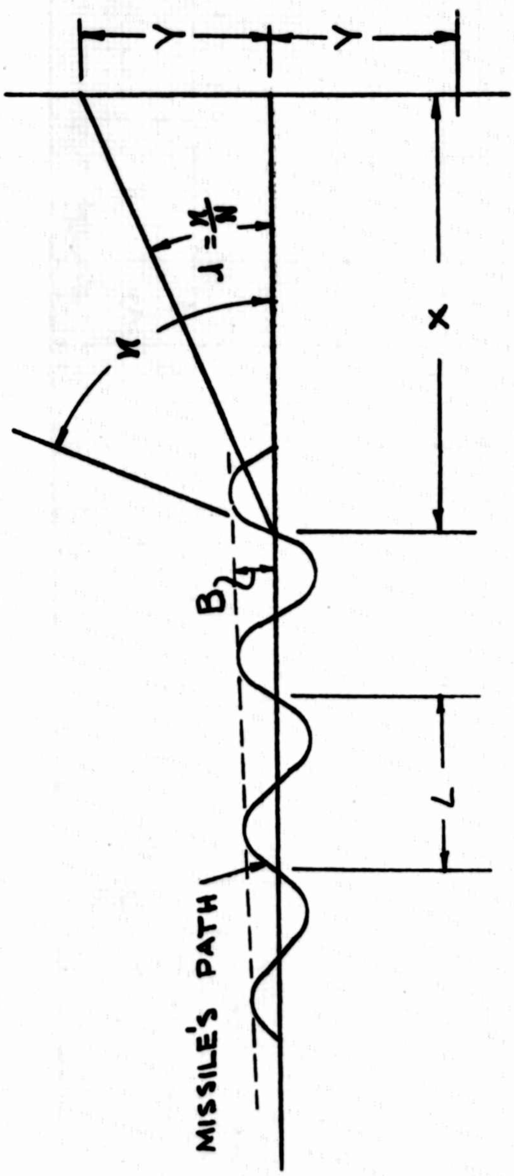

Fig. E 1. Two decoys, one at each end of the target, the reflectivities of which are intermittently changed, create an oscillating path of the missile.

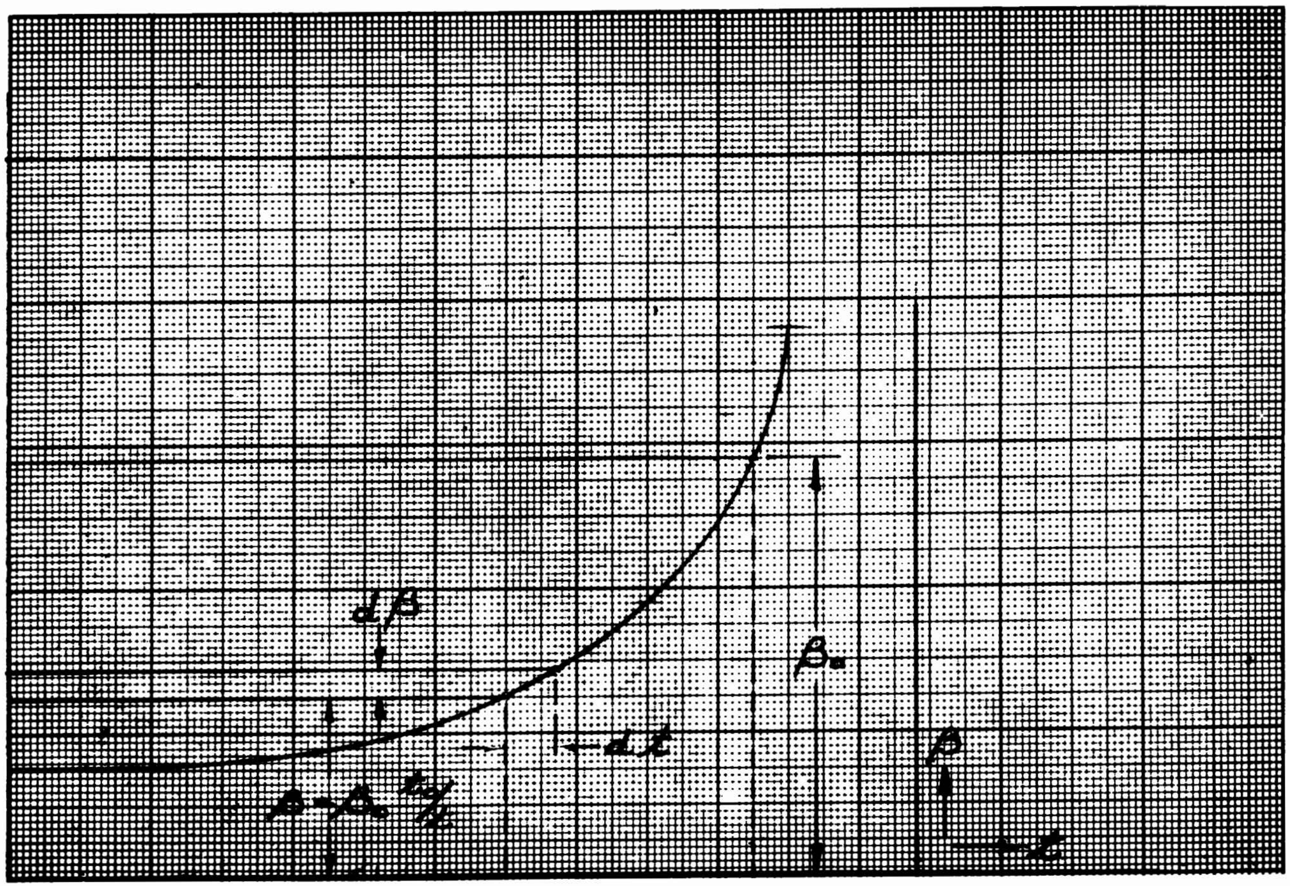

Fig. E 2. Definition of symbols concerning the angular amplitude, β, of the disturbance increasing with the approach of the missile.

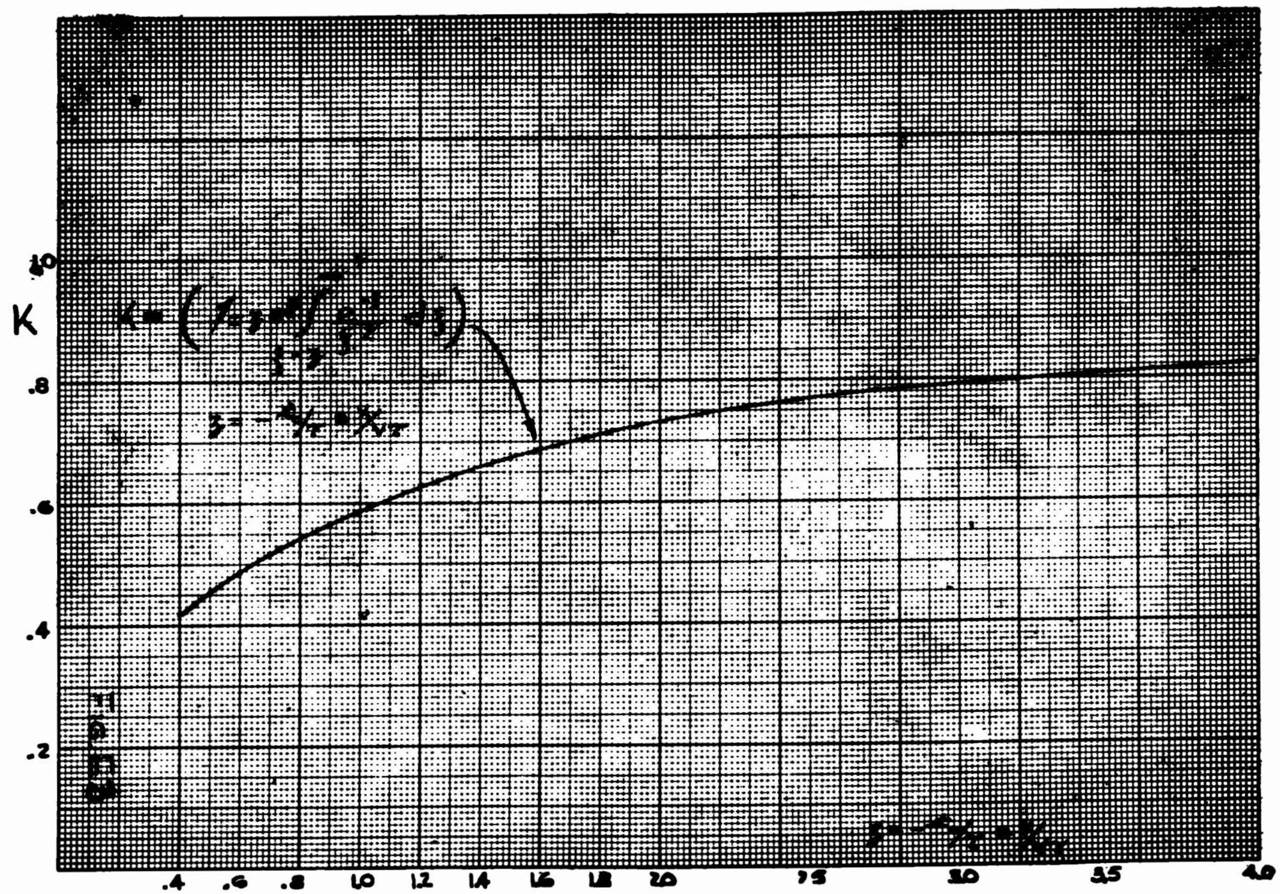

Fig. E 3. The graph shows the resonance amplitude of an oscillation forced by a disturbance the amplitude of which increases according to 1/t, compared with the resonance amplitude caused by a disturbance with a constant amplitude. t_o is the period of time before t=0 is reached. τ is the time constant of damping of the free oscillation.

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ABSTRACT:

Attempts were made to camouflage targets against homing missiles by distributing decoys on the target or in the immediate vicinity of a target. It dealt primarily with decoys against radar homing missiles and especially with systems in which the missile itself transmits the radio waves. It appears possible to defend against a missile which is homing on ships or aircraft by the use of a decoy on an extremity of the target, especially one whose reflectivity increases with the approach of the missile, by the choice of a formation suitably equipped for the defense against radar homing and/or the use of a decoy within the group, by decoys with oscillating reflectivity which exhaust the missile' s controllability.

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