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Interaction Between the Laminar Boundary Layer over a Plane Surface and an Incident Oblique Shock Wave

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The problem of the interaction between the laminar boundary layer over a plane surface and an oblique shock wave incident on the surface was treated by separating the flow in the immediate vicinity of the shock from the rest of the flow in the viscous gas layer adjacent to the surface. Except for very weak incident shock waves the pressure rise along the surface causes the laminar boundary layer to separate upstream of the shock, which determines the shock reflection condition. When the laminar separation upstream of the shock is studied, it is found that the ratio of the pressure at the separation point to the pressure far upstream of the shock depends only upon the Reynolds number, the Mach number, and the ratio of the specific heats of the gas.

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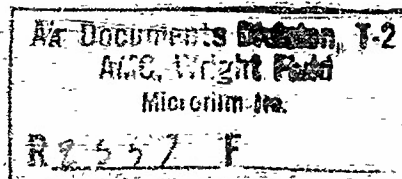
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INTERACTION BETWEEN
THE LAMINAR BOUNDARY LAYER
OVER A PLANE SURFACE
AND AN
INCIDENT OBLIQUE SHOCK WAVE

By

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INTERACTION BETWEEN THE LAMINAR BOUNDARY LAYER OVER
A PLANE SURFACE AND AN INCIDENT OBLIQUE SHOCK WAVE

by

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SUMMARY

The problem of the interaction between the laminar boundary layer over a plane surface and an oblique shock wave incident on the surface is treated by separating the flow in the immediate vicinity of the "shock" from the rest of the flow in the viscous gas layer adjacent to the surface. Away from the shock wave the ordinary boundary layer theory remains valid in this case, and the properties of the "main" supersonic flow are combined with a modified Pöhlhausen method for the calculation of the boundary layer flow. It is found that the pressure rise communicated upstream of the shock wave through the subsonic portion of the boundary layer decays exponentially with distance from the shock. Expressed as a fraction of the distance "X" from the leading edge, the relaxation distance, " ℓ/X ", or distance required for the pressure disturbance to decay to $1/e$ of its original value, varies with Reynolds number Re_x like $\frac{1}{(Re_x)^{1/4}}$, is large near $M = 1$, falls to a minimum at $M \cong 1.6$ (for air), and thereafter increases rapidly with Mach number.

Except for very weak incident shock waves the pressure rise along the surface causes the laminar boundary layer to separate upstream of the shock. This flow separation determines the shock reflection condition. The subsonic "reversed" flow near the surface downstream of separation cannot support a pressure discontinuity and the incident shock wave is reflected as an expansion "fan" which, just cancels the pressure rise across the shock. The flow is deflected back toward the surface and reattaches itself to the surface some distance downstream of the shock.

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When the laminar separation upstream of the shock is studied in more detail, it is found that the ratio of the pressure at the separation point, p_s , to the pressure far upstream of the shock, p_1 , depends only on the Reynolds number, the Mach number, and the ratio of the specific heats of the gas. For incident shock pressure ratios $p_2/p_1 < p_s/p_1$ (approx.), separation does not occur. For $p_2/p_1 \geq p_s/p_1$ the boundary layer separates, and the separation point moves rapidly upstream with increase in incident shock pressure ratio. In some cases the influence of the incident shock extends upstream for a distance of 100-150 boundary layer displacement thicknesses. When the distance between separation point and shock is large enough, transition to turbulent flow occurs in the separated outer "jet". This situation arises when $p_2/p_1 \approx 2 p_s/p_1$.

Certain general conclusions are obtained concerning the effect of Reynolds number and Mach number on the extent of upstream influence of the incident shock wave.

(i) Near sonic velocity ($M \approx 1$) boundary layer separation does not occur and the extent of upstream influence l/x is measured by the "relaxation distance", $l/x \approx \frac{1}{(Re_x)^{1/4}} \frac{1}{(M^2-1)^{1/4}}$

(ii) For $M_1 > 1.25$, and for $Re_x \approx 10^6$, roughly, all possible incident oblique shock waves, except those of the so-called "weak" shock family with very small flow deflections ($\omega \leq 1^\circ$), will have shock pressure ratios greater than $2 p_s/p_1$, and will involve transition to turbulent flow in the separated jet upstream of the shock. The extent of upstream influence in this case is measured by $\frac{l}{x} + \frac{Re_{x_{tr}}}{Re_x}$, where $Re_{x_{tr}}$ is the transition

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Reynolds number for the separated outer jet), and increases rapidly with decreasing Reynolds number, and with increasing Mach number for $M > 1.6$ (air). At very high Reynolds numbers, provided the boundary layer over the surface is still laminar, the distance between separation and transition is small,

$$\text{and } \frac{\delta}{x} \approx \frac{1}{x}$$

(iii) For incident shocks for which $\frac{p_2}{p_1} < \frac{p}{p_1} < 2 \frac{p_2}{p_1}$ the situation is as follows: For $M_1 < 1.15$ (approx.), an increase in Mach number increases ξ_s and $\frac{\delta}{x}$, while for $M_1 > 1.15$ (approx.) an increase in Mach number decreases ξ_s and $\frac{\delta}{x}$. The separation point moves closer to the incident shock wave rapidly with decrease in Reynolds number, and the overall extent of the upstream influence also decreases markedly. If the Reynolds number is reduced sufficiently, boundary layer separation does not occur and $\frac{\delta}{x} \approx \frac{1}{x}$. Such a sequence of events might be observed if an oblique shock of given strength is made to strike a plane surface closer and closer to the leading edge.

The methods utilized in the present study give only approximate solutions to the boundary layer-shock wave interaction problem. Carefully planned experiments are now needed to test the validity of these approximations and to help determine the proper direction for future theoretical work.

Some of the methods of the present paper can be generalized to treat laminar boundary layer-shock wave interactions for supersonic flow in a corner and for the flow near the trailing edge of a supersonic airfoil. It is also possible that interactions between shock waves and a turbulent boundary layer can be studied by similar methods. These questions are now under investigation.

LIST OF SYMBOLS

The subscript "1" denotes physical quantities in the free stream far upstream of the incident shock; the subscript " δ " denotes quantities at the "edge" of the boundary layer; the subscript "s" denotes quantities at the separation point; while "R" denotes the reattachment point. The subscript "0" denotes quantities at some point on surface far upstream of shock.

$\bar{x}$	coordinate parallel to plane surface, measured from point of incidence of oblique shock
x	distance along surface, measured from leading edge
y	coordinate normal to plane surface
p	pressure
ρ	density
T	absolute temperature
μ	ordinary coefficient of viscosity
m	exponent in relation $\mu \sim T^m$
γ	ratio of specific heats of gas, $\frac{c_p}{c_v}$
c	local speed of sound
$u, \text{ or } v$	component of gas velocity parallel to surface
M	Mach number $\frac{u}{c}$
δ	boundary layer thickness

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 δ^*

boundary layer displacement thickness,

$$\int_0^{\infty} \left(1 - \frac{\rho}{\rho_0} \frac{u}{u_0}\right) dy$$

 L characteristic length, later taken as δ_0^* $\bar{\xi}$

$$\bar{\xi}/\delta_0^*$$

 $Re_{\delta_0^*}$

Reynolds number,

$$\frac{\rho_0 u_0 \delta_0^*}{\mu}$$

 Re_x

Reynolds number,

$$\frac{\rho_0 u_0 x}{\mu}$$

 $\bar{L}$ "relaxation distance", or distance upstream of shock required for disturbance to decay to $1/e$ of original value $\bar{L}$

overall extent of upstream influence of incident shock wave

 p

$$\frac{p}{p_0} - 1$$

 $\bar{p}/p_0$

pressure ratio across incident shock

 ω'

angle of flow deflection across incident shock

 θ

angle of inclination of flow,

$$\frac{d\delta_0^*}{d\xi}$$

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1. Introduction

Experimental investigations of transonic flow over airfoils and curved surfaces have shown that the interaction between shock waves and the boundary layer is instrumental in determining the flow pattern and pressure distribution over the surface (references 1-3). Boundary layer-shock wave interaction must also play an important role in supersonic flow at the rear of airfoils and bodies of revolution, and in supersonic nozzles and diffusers. From the results of earlier work on this problem it appears that the interplay between the boundary layer and an "outer" or "main" supersonic flow differs in at least two important respects from the more familiar case of subsonic flow.

(1) In supersonic flow the development of the boundary layer along the surface produces pressure changes that are propagated outward into the main stream along Mach lines (or within Mach cones). These pressure changes, in turn, determine the subsequent development of the boundary layer. In the present paper it will be shown that this "feedback" phenomenon, which has no counterpart in subsonic flow, is particularly important when the static pressure is increasing in the flow direction.

(2) In supersonic flow, rapid pressure changes near the surface, generated by incident shock waves or sudden flow deflections at the surface, are communicated upstream with diminishing intensity through the subsonic portion of the boundary layer. Unlike subsonic flow, the downstream conditions affect the upstream boundary layer flow directly.

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In subsonic flow the boundary layer is determined completely by the "external flow", and the effect of the boundary layer growth on the pressure distribution is negligible at high Reynolds numbers, so long as "separation" does not occur. For certain supersonic flows in which viscous forces are important, the boundary layer and the so-called "external flow" will have to be determined simultaneously as part of the same problem.

The present paper is particularly concerned with the part played by the subsonic portion of the boundary layer in the interaction between the boundary layer flow and an "external" supersonic stream. An attempt is made to go beyond the qualitative discussion in reference 4, by combining the boundary layer equations with the properties of the main supersonic flow, at least approximately. In order to concentrate on the important physical aspects of the problem, and reduce the mathematical complications as much as possible, the study deals with the interaction between the laminar boundary layer over a plane surface and an oblique incident shock wave in an otherwise uniform, plane supersonic flow. Even in this "simplified" case the physical situation is fairly complicated, involving boundary layer separation, for example, under certain flow conditions, so that one is not yet at the point where exact theoretical solutions are possible. Nevertheless, the theoretical analysis can be expected to furnish an approximation to the surface pressure distribution and the extent of upstream influence of the incident shock wave, as determined by the Reynolds number, the Mach number, and the incident shock strength. General results obtained in this relatively simple case can perhaps open the way to a consideration of more involved boundary layer-shock wave interactions.

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Although the boundary layer-shock wave interaction problem has not yet been attacked directly, the general question of viscous effects in supersonic flow is receiving considerable attention at present. For example, Lagerstrom (reference 5) and his co-workers are engaged in a fundamental study of the part played by viscosity in supersonic flow, including the question of the applicability of the usual Prandtl boundary layer equations. Liepmann (reference 2) and Oswatitsch and Wieghardt (reference 6) have dealt briefly with the question of the "stability" of the interaction between the boundary layer over a surface and the "main" supersonic stream. When the static pressure increases in the flow direction, the boundary layer thickens rapidly along the surface. If the pressure gradient is large enough so that the effective streamline curvature $\frac{d^2 \delta^*}{dx^2}$ becomes positive, then the boundary layer growth contributes still further to the adverse pressure gradient, according to the Prandtl-Meyer relation $\frac{1}{\rho} \frac{d\rho}{dx} = \frac{\gamma M^2}{\sqrt{M^2 - 1}} \frac{d^2 \delta^*}{dx^2}$.

Thus, there is the possibility of an "unstable" growth of the boundary layer in a decelerating supersonic flow. With the aid of the von Karman momentum integral equation for the boundary layer, Liepmann finds that $\frac{d\delta^*}{dx} \sim \frac{du}{dx}$ if the curvature is sufficiently large and the change in shape of the velocity "profile" is neglected. By combining this result with the Prandtl-Meyer relation he shows that the boundary layer grows exponentially with distance along the surface. Similarly, an exponential decrease of the boundary layer thickness can occur in the supersonic flow around a sharp corner, where the static pressure decreases extremely rapidly in the flow direction. Of course, as Liepmann points out, one must make certain that the boundary layer equations are valid in such cases before drawing definite conclusions.

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Oswatitsch and Wieghardt examine the stability of the interaction between the boundary layer and the main flow by studying the growth or decay of a small, steady disturbance u' superposed on the supersonic stream near the surface of a flat plate. From the von Kármán momentum equation, by taking the change of velocity "profile" into account, they find that

$$\frac{d\delta^*}{dx} \sim Re_{\delta^*} \frac{d^2 u'}{dx^2}$$

for the laminar boundary layer in the limit of very small pressure gradients.

Since $\frac{1}{u} \frac{du'}{dx} = \frac{1}{\sqrt{M^2-1}} \frac{d^2 \delta^*}{dx^2}$, the small disturbance grows exponentially with distance along the surface, i.e., $u' = u_0' e^{kx}$.

(Of course the linearized theory cannot follow the subsequent history of the disturbance.) While this result is undoubtedly correct, the "initial" magnitude u_0' of the small disturbance is of practical importance, just as in the case of the amplification of unstable (Tollmien) disturbances within the laminar boundary layer above the minimum critical Reynolds number. The point of view of the present paper is that the magnitude of the disturbance must be linked to a definite physical situation for a meaningful result -- such as the disturbance communicated upstream of a shock wave incident on the surface. In this case, it is the growth of the disturbance as the shock is approached that turns out to be important. In other words, the physical origin of the steady disturbance must be carefully specified.

Several investigators have studied the problem of the reflection of steady disturbances incident on the boundary layer over a surface, but without bringing in the viscous effects directly. In an unpublished paper, Marble considers the refraction and reflection of an incident Mach wave as it passes through the outer, supersonic portion of the boundary layer. The

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boundary layer is replaced by a series of vortex layers and reflection conditions at each interface are investigated by means of the linearized theory. As anticipated from the shock-polar of pressure vs. deflection angle, the incident wave is reflected as a wave of the same sign for local $M < \sqrt{2}$, and of opposite sign for local $M > \sqrt{2}$. While a linearized treatment of the shock reflection from the outer, supersonic portion of the boundary layer furnish useful results, it is not capable of describing the observable shock wave-boundary layer interaction for a laminar boundary layer. Howarth (reference 7) has attempted to take the subsonic portion of the boundary layer into account, at least approximately. He deals with the behavior of small, steady disturbances in an infinite supersonic stream bounded on one side by an infinite parallel subsonic stream. An important result is the upstream spreading of these disturbances through the subsonic region. Pressure disturbances incident on the interface between the supersonic and subsonic streams are reflected as disturbances of opposite sign, because the pressure is continuous in the steady subsonic flow; thus an incident compression is reflected as an expansion "fan". Tsien has extended Howarth's study to the case of an infinite supersonic stream bounded on one side by a parallel subsonic stream flowing over a plane surface.* The results of these investigations are clear proof of the importance of the subsonic region of the boundary layer. The present study seeks, in addition, to bring in the viscous effects, by investigating the behavior of the boundary layer in response to the pressure rise communicated upstream of an incident shock wave through the subsonic region near the surface.

* Paper to be presented at 17th Annual Meeting of Institute of the Aeronautical Sciences, January, 1949.

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2. Statement of the Problem

An oblique shock wave in a steady, plane supersonic flow is incident upon the laminar boundary layer over a plane surface (figure 1). Far away from the surface the reflected wave emerges as an oblique shock which returns the flow to its original direction; the pressure rise across this shock and the shock angle can of course be computed from non-viscous flow theory.* Near the surface the phenomenon is not quite so simple, because the incident shock wave cannot penetrate the subsonic portion of the boundary layer and the pressure distribution on the surface must be continuous. The pressure "jumps" across the incident and reflected shocks are effectively diffused through the subsonic portion of the boundary layer in both directions away from the point at which the incident shock first strikes the boundary layer. The total increase in pressure on the surface, $p_3 - p_1$, must be equal to the pressure increase across the incident and reflected shocks far from the surface.

Two principal flow regions can be distinguished; (1), the flow in the immediate vicinity of the point at which the oblique shock strikes the boundary layer, where viscous forces are presumably of secondary importance, and (2), the flow in the laminar boundary layer upstream and downstream of the shock reflection region, where viscous forces are dominant. Although one flow region must pass continuously into the other, the approximation will be made that the two regions are distinct. In other words, the flow inclination $\frac{dy}{dx}$ and Mach number just upstream of the incident shock are calculated from viscous flow theory. Using this information, the reflection

* For simplicity it is assumed that any other physical boundaries in the flow are sufficiently far away so that they do not influence the interaction directly.

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of the shock wave and the inclination of the flow immediately downstream of the reflected disturbance are calculated on the basis of non-viscous flow theory, once the proper reflection conditions are established. The problem is to determine these reflection conditions and to relate the behavior of the boundary layer upstream and downstream of the shock to the incident shock strength, the Reynolds number, and the Mach number.

3. Surface Pressure Distribution and Development of Laminar Boundary Layer Upstream of Incident Shock Before Flow Separation Occurs.

Upstream of the incident shock the form of the pressure distribution $p(x)$ along the surface is specified by the "compatibility" or equilibrium condition between the boundary layer and the main supersonic stream. As a first approximation the pressure changes in the flow produced by the growth of the boundary layer are calculated by replacing the boundary layer by a streamline of slope $\frac{d\delta^*}{dx}$ at the surface. The magnitude of the local pressure gradient is then given by the Prandtl-Meyer relation

$$(1) \quad \frac{1}{\rho} \frac{dp}{dx} = \frac{\gamma M^2}{\sqrt{M^2 - 1}} \frac{d^2 \delta^*}{dx^2}$$

On the other hand, the development of the boundary layer depends on the pressure gradient through a relation of the form

$$(2) \quad \frac{d\delta^*}{dx} = f\left(\delta^*, \frac{d\delta^*}{dx}, \frac{dp}{dx}, Re_{\delta^*}, M\right)$$

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These two relations lead to a second-order differential equation for $\frac{dp}{dx}$. One arbitrary constant in the solution of this equation is determined by the condition that $\frac{dp}{dx} \rightarrow 0$ as $x \rightarrow \infty$, while the other constant must be related to the strength of the incident oblique shock. The pressure distribution $p(x)$ is of course determined from the pressure gradient, with the condition that $p(-\infty) \rightarrow p_1$. Once the pressure distribution is obtained, the development of the boundary layer can be calculated in terms of a single parameter related to the pressure ratio across the incident shock.

In calculating the development of the laminar boundary layer in response to the pressure gradient communicated upstream of the incident shock, the usual assumptions of the boundary layer theory are to be employed, so long as the flow remains attached to the surface. This approximate treatment of the problem will be justified a posteriori, by showing that the pressure gradient $\frac{1}{\rho} \frac{dp}{dx}$ is of the order of $\frac{1}{\sqrt{Re_{\delta_0^*}}}$, so that terms in the viscous flow equations of the type $\frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial x} \right)$ are of the order of

$$\frac{1}{(Re_{\delta_0^*})^{3/2}}$$

as compared with the ordinary viscous shear term

$$\frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right)^*$$

Since $\frac{1}{\rho} \frac{\partial p}{\partial y}$, or $\frac{\delta_0^*}{\rho} \frac{dp}{dy}$, is also of the order of $\frac{1}{\sqrt{Re_{\delta_0^*}}}$ in this case, the pressure is approximately uniform

* Except in the immediate region of the incident shock wave.

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across the boundary layer for large Reynolds numbers. In fact, the radius of curvature $\frac{R}{\delta^*}$ of the streamline near the surface that replaces the boundary layer flow is of the order of $\sqrt{Re_{\delta^*}}$.

(a) Modified Pohlhausen Method for Calculation of Laminar Boundary Layer

Because of the approximations introduced in the present study, a highly refined treatment of the boundary layer equations is hardly justified. The development of the boundary layer is to be calculated by the Pohlhausen method as modified by Dorodnitsyn for compressible flow over an insulated plane surface with the Prandtl number equal to 1.0 (reference 8). The starting point of the Pohlhausen method is the von Kármán integral equation for the momentum balance in the boundary layer (reference 9):

$$(3) \quad \frac{\partial}{\partial x} \int_0^{\delta} \rho u^2 dy - \rho U \frac{\partial}{\partial x} \int_0^{\delta} \rho u dy = \delta \rho U \frac{dU}{dx} - \mu \left(\frac{\partial u}{\partial y} \right)_w$$

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Dorodnitsyn noticed that this equation is reduced to a form similar to that for isothermal, low-speed flow if the coordinates parallel and normal to the surface are modified as follows:

$$(4a) \quad ds = \frac{1}{L Re_0} \frac{p_0}{p} dx$$

$$(4b) \quad dt = \frac{1}{L} \frac{\rho_0}{\rho} dy$$

(Here the subscript "0" denotes "stagnation" values, and $Re_0 = \frac{\rho_0 U_{MAX} L}{\mu_0}$, where $U_{MAX} = \sqrt{2c_p T_0}$).

With the introduction of these new variables s and t , equation (3) becomes:

$$(5) \quad \bar{\sigma} \frac{d}{ds} \left\{ \bar{v}^2 \bar{\sigma} \int_0^1 \omega^2 d\tau \right\} - \bar{v} \bar{\sigma} \frac{d}{ds} \left\{ \bar{v} \bar{\sigma} \int_0^1 \omega d\tau \right\} \\ = \bar{v} f(\tau) \bar{\sigma}^2 \int_0^1 (1 - \bar{v}^2 \omega^2) d\tau - \bar{v} \left(\frac{d\omega}{d\tau} \right)_{\tau=1}$$

where $\bar{v} = \frac{U}{U_{MAX}}$, $\bar{\sigma} = L \bar{\sigma} \left(1 + \frac{\gamma-1}{2} M_\infty^2 \right)^{\frac{\gamma}{\gamma-1}} \int_0^1 (1 - \bar{v}^2 \omega^2) d\tau$,
 $\tau = \frac{t}{\bar{\sigma}}$, $f(\tau) = \frac{1}{1 - \bar{v}^2} \frac{d\bar{v}}{ds}$
 $\omega = \frac{u}{U_\infty}$

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From the boundary condition for $\frac{\partial^2 w}{\partial y^2}$ at the surface one finds that $\frac{d^2 w}{dy^2} = -\bar{\sigma}^2 f(\lambda)$. If $\bar{\sigma}^2 f(\lambda)$ is now selected as the modified Pohlhausen parameter $\lambda(\lambda)$, then the velocity distribution across the boundary layer can be approximated by a fourth degree polynomial, exactly as in the case of "incompressible" flow:

$$(6) \quad w = A \eta + B \eta^2 + C \eta^3 + D \eta^4$$

$$\text{Here, } A = 2 + \frac{\lambda}{6}, \quad B = -\frac{\lambda}{2}, \quad C = \frac{\lambda}{2} - 2, \quad D = 1 - \frac{\lambda}{6},$$

in order that the boundary conditions

$$w(0) = 0, \quad \left(\frac{d^2 w}{dy^2}\right)_{y=0} = -\lambda, \quad w(1) = 1, \quad \left(\frac{dw}{dy}\right)_{y=1} = 0,$$

shall be satisfied.

By substituting the approximation for the velocity profile (equation 6) into equation (5), a first-order differential equation for $\lambda(\lambda)$ of the type $\frac{d\lambda}{d\lambda} = f(\lambda; \frac{d\lambda}{d\lambda}, \frac{d^2\lambda}{d\lambda^2})$ is obtained. For the purposes of the present investigation it is more convenient to rewrite this equation in the following form:

$$(7) \quad \frac{d\lambda}{d\lambda} = S_1(\lambda) N_1(\lambda) + S_2(\lambda) N_2(\lambda)$$

$$\text{where (7a)} \quad S_1(\lambda) = \frac{2.13.12 - 1.92\lambda - 0.2\lambda^2}{2.13.12 - 5.76\lambda - \lambda^2} \quad *$$

$$(7b) \quad S_2(\lambda) = \frac{7257.6 - 1336.32\lambda + 37.92\lambda^2 + 0.8\lambda^3}{2.13.12 - 5.76\lambda - \lambda^2} \quad **$$

*These functions are identical with those utilized by Pohlhausen.

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$$(7c) \quad N_1(\xi) = \left[\frac{\left(\frac{d^2 p}{d\xi^2} \right)}{\left(\frac{dp}{d\xi} \right)} - \frac{1 + \frac{\gamma-1}{2} M_0^2}{\gamma M_0^2} \frac{1}{\rho} \frac{dp}{d\xi} \right]$$

$$(7d) \quad N_2(\xi) = - \frac{1}{\gamma M_0^2} \frac{1}{\rho} \frac{dp}{d\xi} \quad ** \quad \text{where } M_0^2 = \frac{M_\infty^2}{1 + \frac{\gamma-1}{2} M_\infty^2}$$

At some point ξ_0 on the surface sufficiently far upstream of the incident shock the pressure gradient is very small, and the boundary layer thickness and velocity "profile" are very nearly the same as for uniform flow. At this point the Reynolds number and Mach number are also known and the value of $\lambda = \lambda_0$ is calculated from the defining relation $\lambda = \delta^2 f(\lambda)$. Integration of equation (7) furnishes the desired relation between $\lambda(\xi)$ and the pressure distribution. The growth of the boundary layer displacement thickness, on which the pressure gradient in supersonic flow depends in turn, is given by the following expression:

$$(8) \quad \frac{\delta^*}{\delta_0} = \frac{M_0 \sqrt{\gamma}}{\sqrt{R_{\delta_0} + (1 + \frac{\gamma-1}{2} M_0^2)^{1/2}}} \frac{I \sqrt{-\lambda}}{\sqrt{\frac{1}{\rho} \frac{dp}{d\xi}}}$$

where $I(\lambda) = \int_0^\lambda \{1 + \frac{\gamma-1}{2} M_0^2 (1-w) - w\} d\lambda = b_0 + b_1 \lambda + b_2 \lambda^2$

$$\text{with } b_0 = 0.30 + 0.4175 \frac{\gamma-1}{2} M_0^2$$

$$b_1 = - [0.0083 + 0.0094 \frac{\gamma-1}{2} M_0^2]$$

$$b_2 = - 0.0001 \frac{\gamma-1}{2} M_0^2$$

For air, $\gamma = 1.4$, $\alpha = 0.76$, and the relation (8) is

$$\frac{\delta^*}{\delta_0} = \frac{1.185 M_0}{\sqrt{R_{\delta_0} + (1 + 1.20 M_0^2)^{1/2}}} \frac{I \sqrt{-\lambda}}{\sqrt{\frac{1}{\rho} \frac{dp}{d\xi}}}$$

** At $M_0 \rightarrow \infty$, $N_1(\xi) \rightarrow \frac{d^2 U}{d\xi^2} \frac{(\eta/\xi)}{\frac{d\eta}{d\xi}}$, and $N_2(\xi) \rightarrow \frac{1}{4} \frac{dU}{d\xi}$, which agrees with Fohlnhausen's formulation for $M \ll 1$.

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with

$$b_0 = 0.30 + 0.0835 M_\infty^2$$

$$b_1 = -[0.0083 + 0.00188 M_\infty^2]$$

$$b_2 = -0.00002 M_\infty^2 \quad (\text{negligibly small})$$

(b) Surface Pressure Distribution Upstream of Flow Separation

The boundary layer equations (7) and (8), together with the Prandtl-Meyer relation (1), furnish the conditions for equilibrium between the boundary layer on the plane surface and the "external" supersonic flow. In principle these non-linear differential equations determine the surface pressure distribution. Fortunately it turns out that for $(-\lambda) < 6$ (approx.), equation (7) for $\lambda(\xi)$ can be linearized and thus considerably simplified, and the approximation to $f(\xi)$ so determined satisfies the conditions for equilibrium very closely up to the immediate vicinity of the separation point (if any)*.

For $(-\lambda) < 6$ (approx), $S_1(\lambda) \approx \lambda$, and

$S_2(\lambda) \approx 34.1 - 5.34\lambda$, so that the differential equation (7) is approximated by

$$(9) \quad \frac{d\lambda}{d\xi} - \lambda \left[\frac{P''}{P'} + \frac{5.34 - [1 + \frac{5}{2}(K-1)M_\infty^2]}{\gamma M_\infty^2} \frac{P'}{1+P} \right] = -\frac{34.1}{\gamma M_\infty^2} \frac{P'}{1+P}$$

where $P = \frac{p}{p_\infty} - 1$ and the primes denote differentiation with respect to ξ .

The solution of equation (9) is

$$(10) \quad \lambda(\xi) = P' \int \frac{\lambda}{P'} + \frac{34.1}{\gamma M_\infty^2} (\xi - \xi_0) \left[1 + K P f_1(\xi) + \frac{K(K-1)}{2!} P^2 f_2(\xi) + \dots \right]$$

* It is also consistent with this approximation to linearize the Prandtl-Meyer relation, in which case the analysis applies to incident shocks of only moderate strength. However, it will be shown later that the boundary layer remains completely laminar, and therefore the present theory remains applicable, only for incident shock waves of moderate strength.

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where $K = \frac{5.34 - \frac{1}{2}(6-M_0^2)}{M_0^2}$

Here λ_0 and P_0' are taken at some upstream location ξ_0 , where P' is very small.

For incident shock waves of moderate strength the higher order terms in equation (10) do not contribute appreciably to the final result when $-\lambda < 6$, so that

$$(10a) \quad \lambda(\xi) \approx P' \left[\frac{\lambda_0}{P_0'} + \frac{34.1}{8M_0^2} (\xi_0 - \xi) \right]$$

Utilizing this approximation for $\lambda(\xi)$ and equation (9) for $\frac{d\lambda}{d\xi}$, the following expression for $\frac{d\delta^*/\delta_0^*}{d\xi}$ is obtained by differentiating equation (8):

$$(11) \quad \frac{d\delta^*/\delta_0^*}{d\xi} = \left[-\frac{\beta_1}{\beta_0^3} \frac{(1 + \frac{1}{2} M_1^2)^{2-m}}{8M_1^2} Re_{\delta_0^*} \right] P'' + g(M_1) P' + \frac{17.05 \beta_0^2}{(1 + \frac{1}{2} M_1^2)^{1-m}} \frac{1}{Re_{\delta_0^*}} + O(P^2)$$

For large Reynolds numbers and moderate pressure rise upstream of the incident shock wave, a first approximation to the differential equation for $p(\xi)$ is obtained from equation (11) and the Prandtl-Meyer relation:

$$(12) \quad P''' - a^2 P' = 0,$$

where $a = \frac{(M_1^2 - 1)^{1/4}}{\sqrt{Re_{\delta_0^*}}} \frac{1}{f(M_1)}$

and $f(M_1) = \frac{(0.0083 + \frac{1}{2}(0.3094)M_1^2)^{1/2}}{(0.30 + 0.41\pi \frac{1}{2} M_1^2)^{3/2}} \cdot (1 + \frac{1}{2} M_1^2)^{1-\frac{m}{2}}$

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The solution of equation (12) is

$$(12a) \quad \frac{p}{p_0} - 1 = \text{Const.} e^{-x/\ell}$$

thus, the pressure rise communicated through the boundary layer decays exponentially with distance upstream of the incident shock wave. The constant in (12a) must be related to the pressure ratio across the incident shock, and it is the object of the succeeding analysis to determine this relation.

If " ℓ " is the "relaxation distance", or distance required for the disturbance represented by equation (12a) to decay to $1/e$ of its original value, then

$$(13) \quad \frac{\ell}{\delta_0^*} = \frac{\sqrt{Re_{\delta_0^*}}}{(M_1^2 - 1)^{1/4}} f(M_1)$$

The quantity $\frac{\ell}{\delta_0^*} \frac{1}{\sqrt{Re_{\delta_0^*}}}$ is plotted as a function of Mach number in figure 2. The relaxation distance is also expressible in terms of the distance " x " from the leading edge of a flat plate, as follows:

$$(14) \quad \frac{\ell}{x} = \frac{\ell}{\delta_0^*} \frac{\delta_0^*}{x} = \frac{1}{(Re_x)^{1/4}} \frac{f(M_1)}{(M_1^2 - 1)^{1/4}} (1.13 + 2.508 M_1^2)^{3/2}$$

(Here, the approximation $\frac{\delta_0^*}{x} \sqrt{Re_x} = 1.13 + 2.508 M_1^2$ is employed).

The quantity $\frac{\ell}{x} (Re_x)^{1/4}$ is also plotted as a function of Mach number in figure 3.

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Near $M = 1$ the disturbance extends far upstream because the streamline curvature is extremely small for a given pressure rise, or flow deflection, $\delta \delta^{*1/2}$. At higher Mach numbers the streamline curvature increases and $\frac{\delta}{x}$ decreases at first. This effect is eventually outweighed by the fact that the pressure gradient $\frac{dp}{dx}$ required to produce a given rate of boundary layer growth $\frac{d\delta^*}{dx}$ decreases rapidly with increasing Mach number. Consequently $\frac{\delta}{x} (Re_x)^{1/4}$ eventually increases rapidly with Mach number. (In spite of this fact the quantity $\frac{\delta}{\delta^*} * \frac{1}{\sqrt{Re_{\delta^*}}}$ decreases steadily with increasing Mach number because of the rapid increase of δ^* with Mach number.)

The approximation for the growth of the boundary layer displacement thickness, or streamline deflection, given by equation (11) illustrates the dominant influence of viscosity in the diffusion of the pressure gradient upstream through the boundary layer. The physical situation here is actually quite different than in the more familiar applications of the boundary layer theory. In the case of the boundary layer over a flat plate in uniform flow, vorticity is continually diffused into the main stream, and the boundary layer growth is parabolic, as indicated by the third term on the right-hand side of equation (11). When the external flow is non-uniform, as in flow over a curved surface, the pressure gradient $\frac{dp}{dx}$ is generally of the order of $\frac{p}{L}$, where L is a characteristic body dimension in the flight direction. Consequently $\frac{dp}{dx} = \delta^* \frac{dp}{dx} \sim \frac{1}{\sqrt{Re_x}}$; similarly, $Re_{\delta^*} p'' \sim \frac{1}{\sqrt{Re_x}}$, and the first three (dominant) terms in the expression for $\frac{d\delta^*}{dx}$ in equation (11) are all of the same order of magnitude. In the case of the supersonic laminar boundary layer over a plane surface upstream of an incident oblique shock.

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wave, the pressure gradient $\frac{dP}{d\xi} \sim \frac{1}{\sqrt{Re_{\delta_0^*}}} \sim (Re_{\delta_0^*})^{-1/4}$, while $Re_{\delta_0^*} \frac{d^2P}{d\xi^2} \sim 1$, so that $\frac{d\delta_0^*}{d\xi} \sim Re_{\delta_0^*} \frac{d^2P}{d\xi^2}$, and all the other terms on the right hand side of equation (11) are negligible compared with the term involving P'' .

So far as the surface pressure distribution is concerned the boundary layer is apparently replaced by a thin gas film of thickness δ^* , within which inertia forces are negligible. In fact the relation $\frac{d\delta_0^*}{d\xi} \sim Re_{\delta_0^*} \frac{d^2P}{d\xi^2}$ is similar to the relation $\frac{dh}{dx} \sim Re_h \frac{d^2P}{dx^2}$ for a thin oil film of thickness " h " in a slightly inclined slipper bearing (reference 10), although the phenomena are naturally quite different otherwise.

In these terms the increase of the relaxation distance $\frac{\delta_0^*}{x}$ with Reynolds number is perhaps more understandable. At high Reynolds numbers the viscous forces are small relative to the pressure gradients and the boundary layer is less able to sustain large pressure gradients than at low Reynolds numbers. For a given pressure rise the relaxation distance therefore increases with increasing Reynolds number. Measured in absolute terms, e.g., $\frac{\delta_0^*}{x}$, the influence distance decreases with increasing Reynolds number because

$$\frac{\delta_0^*}{x} \sim \frac{1}{\sqrt{Re_x}} \sim \frac{1}{Re_{\delta_0^*}}$$

(c) Calculation of Laminar Boundary Layer With "Equilibrium" Surface Pressure Distribution: Accuracy of Exponential Approximation

As stated in the Introduction, a result similar to equation (12a) was obtained by Oswatitsch and Wieshardt for the growth of a small disturbance superposed on the "main" supersonic stream near the surface of a flat plate.

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(reference 6). However they put $\lambda = 0$ immediately without further study, while the method of analysis of the present paper suggests that the exponential function in equation (12a) may be a good approximation to the "influence" function up to the immediate vicinity of the "separation point". To investigate this possibility, and to determine the behavior of the boundary layer upstream of the incident shock wave, the boundary layer development was calculated from the exact equations (7) and (8) for a series of representative cases. (Values of "a" in each case are obtained from equation (12a).) The rate of boundary layer growth $\frac{d\delta^*/\delta_0^*}{d\xi}$ calculated from the boundary layer equations with the exponential pressure distribution is then compared with the rate of growth obtained by integrating the Prandtl-Mayer relation for the required flow deflection,

$$(15) \quad \frac{d\delta^*/\delta_0^*}{d\xi} - \left(\frac{d\delta^*/\delta_0^*}{d\xi} \right)_{F=\xi_0} = \frac{\sqrt{M_1^2 - 1}}{\gamma M^2} (p - p_0)$$

with $p = (p_0/p_1)^{1/\gamma} e^{a\xi}$

Results obtained are shown in figures 3(a)-3(d): (Incident shock pressure ratios determined later are indicated in the figure legends.) Evidently the exponential approximation to $p(\xi)$ is a good one up to the immediate vicinity of the separation point.

A striking fact about the results of the calculations is that boundary layer separation almost always occurs, except for very weak incident shocks. Of course the criterion for separation employed, namely $\lambda_c = -10$ is arbitrary. (Pohlhausen method gives $\lambda_c = -12$, strictly speaking), and it is also well known from previous experience with subsonic boundary layer

studies that the Pohlhausen method does not predict separation soon enough. However, it is believed that the results obtained give the correct physical process occurring in the laminar boundary layer upstream of the incident shock wave. Later (Section 5) it will be shown how the incident shock pressure ratio, the Reynolds number and Mach number affect the location of the separation point ξ_s , which is a good measure of the extent of the upstream influence of the incident shock wave.

4. Laminar Flow Downstream of Separation

(a) Shock Reflection Conditions

If the boundary layer flow were always completely attached to the surface the boundary layer equations could be applied up to the immediate vicinity of the incident shock wave. Because of the pressure gradient along the surface the boundary layer flow "separates" before the flow reaches the shock, unless the shock is very weak. A region of reversed flow develops near the surface (figure 1), and the boundary layer equations are of course inapplicable downstream of the separation point. Boundary layer separation makes it difficult to determine the surface pressure distribution analytically, but it does at least fix the shock reflection conditions. Since the "inner" reversed flow near the surface is subsonic, it cannot support a pressure discontinuity; when the incident shock strikes the separated outer "jet", it is reflected as an expansion "fan" which just cancels the pressure rise across the shock. The separated flow is deflected back toward the surface, the flow reattaches itself to the surface some distance downstream, and the region of reversed flow adjacent to the surface is erased.

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As the flow returns to its original direction approximately parallel to the surface the pressure increases to the final value p_2^* .

(b) Relation Between Surface Pressure Distribution and Pressure Ratio Across Incident Shock Wave.

Possibly the separated laminar flow might be amenable to treatment by the methods developed by Kirchhoff and Levi-Cevita for flows involving "free jets", but so far only very limited progress can be reported in this direction. However, the relation (11) $\frac{d\delta^*/\delta_0^*}{d\xi} = \frac{[f(M)]^2}{\gamma M^2} Re_{\delta_0^*} \frac{d^2 p}{d\xi^2}$ obtained for the attached boundary layer shows the dominance of the viscous forces in determining the surface pressure distribution, and suggests the assumption that the mechanism of diffusion of the pressure gradient through the boundary layer away from the incident shock is essentially the same for the entire "inner" gas layer adjacent to the surface, whether the flow near the surface is "reversed" or not. In other words, it is assumed that the surface pressure distribution satisfies the differential equation (12)

$$p''' - a^2 p' = 0,$$

or

that a relation of the form (11a) $\frac{d\delta^*/\delta_0^*}{d\xi} = \text{const.} + \frac{[f(M)]^2}{\gamma M^2} Re_{\delta_0^*} \frac{d^2 p}{d\xi^2}$,

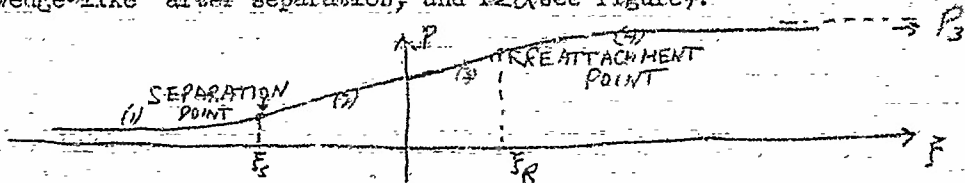
holds in each of the three distinct flow regions downstream of separation.

* So far it is assumed that transition to turbulent flow does not occur. Experimental results obtained by Liepmann (reference 3) and Ackert (reference 4) show that laminar flow persists even at Reynolds numbers $Re_x \approx 10^6$. However, transition eventually occurs in the separated outer jet (see Section 7).

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In the intermediate regions 2 and 3, where the flow is "separated", the general solution for the surface pressure distribution obtained from (12) is of the form $a_1 + a_2 e^{a\xi} + a_3 e^{-a\xi}$.

In region 2 upstream of the incident shock wave the flow soon becomes "wedge-like" after separation, and $P'' < 0$ (see figure).



In region 3, on the other hand, the thickness of the layer of reversed flow near the surface is steadily reduced as the flow approaches the "reattachment point", and $P'' > 0$. This fact suggests imposing the condition $P'' = 0$ at $\xi = 0$. Actually, only the $e^{-a\xi}$ solution was utilized in region 2 and only the $e^{a\xi}$ solution in region 3, but the discontinuity in curvature at $\xi = 0$ is extremely small and the effect on the surface pressure distribution is also small. In region 4, downstream of the reattachment point, $p \rightarrow \text{const.}$ (p_3 , say) as $\xi \rightarrow \infty$, and the applicable solution for $p(\xi)$ is of the form $a_4 + a_5 e^{-a\xi}$, with $P'' < 0$.

The pairs of arbitrary constants in the expressions for the surface pressure distribution are obtained in terms of the single parameter

$\frac{p''}{p}$ in region (1) by imposing the condition that both p and $\frac{dp}{d\xi}$ be continuous on the surface at the junction of any two flow regions. Analytical representations of $p(\xi)$ in each of the flow regions are given in the Appendix. In view of the assumptions introduced, these expressions should not be taken too literally, but it is believed that they do give the correct general behavior of the surface pressure distribution in each of the flow regions.

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The relation between the surface pressure distribution and the incident shock pressure ratio is now determined by means of an inverse procedure, which amounts to matching the pressure on the surface far downstream of the incident shock with the value p_3 behind the reflected shock wave far from the surface. Suppose the (as yet) undetermined constant $\frac{p_3}{p_1} - 1$ in the expression for $p(F)$ is selected as a parameter. Then, for any arbitrary value of this parameter, the development of the attached boundary layer upstream of the shock is calculated, the separation point F_s determined with the aid of the criterion $\lambda_s = -10$, and $p(F)$ in region 2 downstream of separation calculated by matching p and $\frac{dp}{dF}$ at the separation point. The growth of the boundary layer displacement thickness, or the streamline deflection, $\frac{d\delta^*/\delta_0^*}{dF}$, is obtained approximately from the Prandtl-Meyer relation,

$$\frac{d\delta^*/\delta_0^*}{dF} - \left(\frac{d\delta^*/\delta_0^*}{dF} \right)_{F=F_s} = \frac{\sqrt{M^2-1}}{M^2} (p-p_0),$$

so $\theta = \theta_2$ is known just upstream of the incident shock. Select for "trial" an incident shock wave with flow deflection angle ω' somewhat less than θ_2 (say); the condition of constant pressure for the reflection means that θ_3 is given very closely by $\theta_2 - 2\theta_1^{wl}$. By utilizing the initial condition $\theta = \theta_3$, and the fact that p and $\frac{dp}{dF}$ are both continuous on the surface, the surface pressure distribution and boundary layer displacement thickness in region 3 can be calculated. The reattachment point is taken (crudely) at the location at which $\frac{d\delta^*}{dF}$ has the same value as at the separation point upstream. Once the reattachment point is fixed the surface pressure distribution in region 4 is determined, and the value approached by $p(F)$ as $F \rightarrow \infty$ is compared with p_3 behind the reflected shock. If $p_4 > p_3$

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for example, a new "trial" incident shock with slightly larger ω' is chosen and the above procedure repeated. The process is found to be rapidly convergent in most cases. Again it must be emphasized that no great accuracy can be claimed for such a process, but the results probably describe the physical phenomenon correctly.

Calculations of the surface pressure distribution and the location of the separation point were carried out for several incident shock pressure ratios at $M_1 = 1.235$ and $M_1 = 3.0$. The results are presented in figures 5a and 5b. In some cases the influence of the incident shock wave extends upstream for a distance of the order of 100 displacement thicknesses.

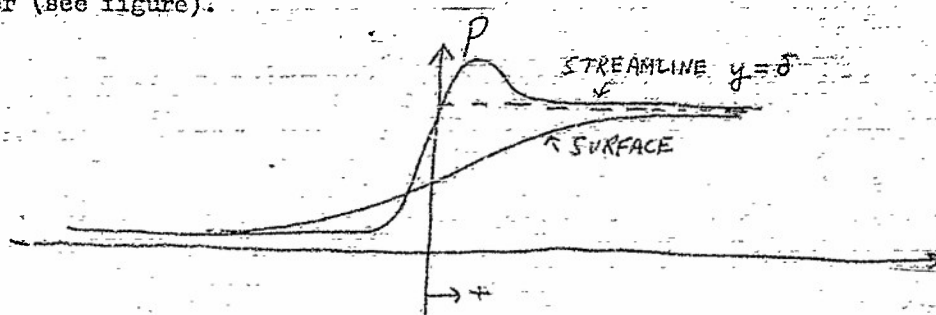
(c) Relation Between Surface Pressure Distribution and Shock Pressure Ratio for Weak Incident Shock Wave.

For sufficiently "weak" incident shock waves the pressure rise communicated upstream is small enough so that the laminar boundary layer does not separate (for example, $\frac{p}{p_\infty} = 1.01$ in figure 4). In that case the pressure increases exponentially along the surface in accordance with equation (12a), at least to within a distance from the shock of the order of a few boundary layer displacement thicknesses.

As the incident shock wave penetrates the outer supersonic region of the boundary layer it is continuously refracted and reflected, and this process must culminate in a fairly complicated shock-flow pattern near the line of sonic velocity. At any rate the flow in the supersonic portion of the boundary layer experiences a pressure rise that is not shared by the subsonic flow. A readjustment of pressures takes place downstream of the

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incident shock and it is doubtful that the usual assumptions of the boundary layer theory hold, because of the pressure difference across the boundary layer (see figure).



So far as the surface pressure distribution downstream of the incident shock is concerned, the assumption could be made that the mechanism of diffusion of pressure is essentially the same all along the surface (i.e., relation (11) holds). Suppose that this assumption is adopted tentatively until further study shall clarify the problem. Then, downstream of the shock,

$$(12b) \quad \frac{p}{p_1} - 1 = \left(\frac{p_2}{p_1} - 1 \right) \left[2 - e^{-a\xi} \right]$$

on the surface, since p and $\frac{dp}{d\xi}$ are continuous on the surface at $\xi = 0$.

From (12b), $\left(\frac{p_2}{p_1} - 1 \right) = \left(\frac{p_3}{p_1} - 1 \right) = 2 \left(\frac{p_2}{p_1} - 1 \right)$, where p_3 is the pressure behind the reflected shock wave far from the surface; thus half the pressure rise occurs upstream and the remainder downstream of the shock. In this case the relaxation distance $\frac{1}{2}\lambda^*$, or $\frac{\lambda^*}{2}$ (figure 2) is a good measure of the extent of upstream influence of the incident shock wave.

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5. Extent of Upstream Influence of Incident Shock Wave as a Function of Shock Pressure Ratio, Reynolds Number and Mach Number.

The considerable extent of the upstream influence of the incident shock wave is a direct consequence of the limited ability of the laminar boundary layer to support a static pressure rise in the flow direction without "separating". By introducing the condition $\lambda_s = -10$ as a criterion for separation, a quantitative estimate is obtained of the maximum permissible value of the pressure gradient for the attached boundary layer. By equation

$$(12), \quad \left(\frac{1}{\rho} \frac{dp}{dx} \right)_{\xi} = a \left(\frac{p'}{p} - 1 \right) e^{a\xi} = a \left(\frac{p_s}{p_1} - 1 \right), \quad \text{and}$$

this limitation on the pressure gradient fixes a definite value of the surface pressure ratio p_s/p_1 at separation for each "initial" boundary layer Reynolds number Re_{ξ_0} and "initial" Mach number, M_1 . For incident shock pressure ratios less than or equal to a certain critical value, such that

$$\frac{p_s}{p_1} - 1 \leq \left(\frac{p_s}{p_1} - 1 \right), \quad \text{boundary layer separation does not}$$

occur. For incident shock waves with pressure ratios greater than this critical value, such that $\left(\frac{p_s}{p_1} - 1 \right) > \left(\frac{p_s}{p_1} - 1 \right)$, the boundary layer separates, and the location of the separation point ξ_s is given by

$$e^{a\xi_s} = \frac{\frac{p_s}{p_1} - 1}{\frac{p_s}{p_1} - 1}.$$

The separation point moves farther and farther upstream with increasing

incident shock strength, and the surface pressure ratio $p_s/p_1 - 1$

just upstream of the incident shock approaches the limiting value of

$$2 \left(\frac{p_s}{p_1} - 1 \right) \quad (\text{see Appendix}). \quad \text{Of course, before this limit}$$

is reached the distance ξ_s grows sufficiently large so that the separated outer laminar "jet" becomes unstable before it is struck by the incident shock. Transition to turbulent flow occurs in the "jet" and the surface

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pressure rise between the transition "point" and the incident shock is governed by the (as yet) unknown relations for turbulent flow-shock wave interaction (see, for example, figure 34 of reference 2).

With the aid of the approximate criterion for separation, $\lambda_r = -10$, a relation between the location of the separation point and the incident shock pressure ratio, the Reynolds number and the Mach number is obtained from equation (8):

$$(16) \quad \left(\frac{1}{\rho} \frac{d\rho}{d\xi} \right)_{\xi_s} = \frac{\gamma M_1^2}{(1 + \frac{\gamma-1}{2} M_1^2)^{2-\gamma}} \frac{10 I_s^2}{Re_{\delta^*} \left(\frac{\delta_s^*}{\delta^*} \right)^2}$$

where $I_s = I(\cdot) = 0.383 + 0.5015 \frac{\gamma-1}{2} M_1^2$

Therefore,

$$\frac{p_2}{p_1} - 1 = 10 \frac{\gamma M_1^2}{(M_1^2 - 1)^{1/2}} \frac{1}{Re_{\delta^*} \left(\frac{\delta_s^*}{\delta^*} \right)^2} \frac{(0.0083 + 0.1099 \frac{\gamma-1}{2} M_1^2)^{1/2} (0.383 + 0.5015 \frac{\gamma-1}{2} M_1^2)^{1/2}}{(1 + \frac{\gamma-1}{2} M_1^2)^{1-\gamma/2} (0.30 + 0.4175 \frac{\gamma-1}{2} M_1^2)^{1/2}}$$

Now $\frac{p_2}{p_1} - 1$ depends only on the pressure ratio at the separation point.

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Therefore the value of $(\frac{p_2}{p_1} - 1)$ for a given Reynolds number and Mach number is obtained by calculating the boundary layer development from equations (7) and (8) for one arbitrary value of the parameter $(\frac{p_2'}{p_1} - 1)$, and determining the separation point location ξ_s . The separation point location for any other value of $(\frac{p_2'}{p_1} - 1)$ at the given values of Re_{δ^*} and M_1 is obtained from the relation

$$\frac{e^{a\xi_2}}{e^{a\xi_1}} = \frac{(\frac{p_2'}{p_1} - 1)_1}{(\frac{p_2'}{p_1} - 1)_2}$$

Once ξ_s is known the calculation of the corresponding incident shock pressure ratio is carried out by the method described in Section 4b. Figure 6a illustrates the effect of Mach number on the variation of ξ_s with incident shock pressure ratio at a given Reynolds number; figure 6b shows the effect of Reynolds number at a given Mach number. In figures 6c and 6d the "separation distance", l_s , is plotted in terms of the distance from the leading edge of a flat plate.

Certain general observations can now be made concerning the upstream influence of an oblique shock wave incident on a laminar boundary layer over a plane surface:

Effect of Incident Shock Pressure Ratio

When the incident shock pressure ratio $\frac{p_2}{p_1}$ is less than $\frac{p_2'}{p_1}$ (approximately), the boundary layer does not separate and the upstream influence

is measured by the "relaxation distance" $\frac{l}{x}$ or $\frac{l}{\xi_s}$ plotted in figure 2:

For incident shock pressure ratios greater than $\frac{p_2'}{p_1}$ (approx.), boundary layer separation occurs, and $l = l_s + l$. Therefore $\frac{l}{x} = \frac{l_s}{x} + \frac{l}{x}$.

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$$\frac{\bar{L}}{x} \approx \frac{L}{x} + \frac{1.73 + 2.50 \sqrt{Re_x}}{Re_x}, \text{ provided that } Re_{\bar{L}} = \frac{\bar{L}}{x} Re_{\bar{L}}^* \leq Re_{x_{tr}}$$

where $Re_{x_{tr}}$ is the transition Reynolds number for the "outer" separated supersonic jet.* In this range the extent of upstream influence increases rapidly with increase in incident shock pressure ratio, as indicated in figures 6a and 6d.

For incident shock pressure ratios large enough so that transition occurs in the "jet" the present treatment of the separated laminar flow region naturally no longer applies. This situation arises when $\frac{\bar{L}}{x} \approx \frac{2L}{x}$ (approx.) at $Re_x \approx 10^6$. For lower Reynolds numbers $\left(\frac{L}{x}\right)$ is somewhat greater than $\frac{2L}{x}$ (figures 6a-6d). An estimate of the extent of upstream influence can be made when more information is available as to the magnitude of $Re_{x_{tr}}$. When $\frac{\bar{L}}{x} \geq \frac{2L}{x}$, $\frac{\bar{L}}{x} \approx \frac{L}{x} + \frac{x_{tr}}{x}$, or, $\frac{\bar{L}}{x} \approx \frac{L}{x} + \frac{Re_{x_{tr}}}{Re_x}$, and $\frac{\bar{L}}{\bar{\sigma}_0^*} \approx \frac{L}{\bar{\sigma}_0^*} + \frac{Re_{x_{tr}}}{Re_{\bar{\sigma}_0^*}}$.

(The pressure rises very rapidly in the turbulent flow region just upstream of the shock and this region is of limited extent.)

Effect of Mach Number (Fixed Reynolds number, Re_x)

(i) Near sonic velocity ($M \approx 1$) the pressure ratio $\left(\frac{p_2}{p_1} - 1\right)$ for separation is high (equation (17)), while the maximum possible pressure ratio across an incident shock wave is not much greater than unity. Separation

* In the experiments reported in reference 1, $Re_{x_{tr}}$ appears to be of the order of 2×10^6 , but very little information on this phenomenon is available at present.

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generally does not occur and the extent of upstream influence is measured

by $\frac{\ell}{\lambda} \sim \frac{1}{(M^2 - 1)^{1/2}}$

For a plane surface with a definite leading edge, when $M \approx 1$ the disturbance communicated upstream of the incident shock may interfere with the flow in the neighborhood of the leading edge. In that case, a true separation of these two flows is impossible and the problem is much more difficult.

(ii) The value of the pressure ratio at separation drops very rapidly with increase in Mach number from unity, until it reaches a minimum value at $M \approx 1.15$, and then increases steadily with Mach number thereafter. For Reynolds numbers (Re_x) of the order of 10^6 , $(\frac{p_2}{p_1} - 1)$ is quite small; for example (for air), $(\frac{p_2}{p_1} - 1) = 0.015$ at $M = 1.135$, as compared with $(\frac{p_2}{p_1} - 1) = 0.926$ for $w' = w'_{max}$ at $M = 3.0$, $(\frac{p_2}{p_1} - 1) = 0.056$.

corresponding to an incident shock with $w' \approx 0.70^\circ$. For $M_{1.715}$ (any) all possible incident oblique shock waves, except those of the so-called "weak" shock family with very small flow deflections ($w' \approx 1^\circ$), will have shock pressure ratios greater than $2 \frac{p_2}{p_1}$. Thus, for $M_{1.715}$, the extent of upstream influence $\frac{\ell}{\lambda}$ is generally measured by $\frac{\ell}{\lambda} \sim \frac{Re_{x_{max}}}{Re_x}$, and increases rapidly with Mach number for $M_{1.715}$ (air) (figure 2).

(iii) For incident shocks for which $\frac{p_2}{p_1} < \frac{p_2}{p_1} < 2 \frac{p_2}{p_1}$ the situation is as follows: for $M_{1.15}$ (approx.) an increase in Mach number increases $\frac{p_2}{p_1}$ and $\frac{\ell}{\lambda}$, while for $M_{1.715}$ (approx.), an increase in Mach number decreases $\frac{p_2}{p_1}$ and $\frac{\ell}{\lambda}$. For a thorough treatment of this problem it would be necessary to prepare curves such as those presented in figure 6 for a whole series of Mach numbers. In the absence of experimental data such calculations are largely justified at this time.

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Effect of Reynolds number (Fixed Mach number)

(i) Incident shock waves for which $\frac{p_2}{p_1} \approx 2 \frac{p_2}{p_1}$ (approx.) will involve transition to turbulent flow in the separated jet upstream of the shock. The effect of a decrease in Reynolds number is to increase the distance between separation and transition, and the extent of upstream influence increases rapidly with decreasing Reynolds number in accordance with $\frac{l}{x} \approx \frac{l}{x} + \frac{Re_{x_{tr}}}{Re_x}$. For $M_1 > 1.25$ and for $Re_x \approx 10^6$, these remarks apply to all incident shocks except those involving very small deflections. At very high Reynolds numbers, provided the boundary layer over the surface is still laminar, the distance between separation and transition is small and $\frac{l}{x} \approx \frac{l}{x} \sim \frac{1}{(Re_x)^{1/4}}$.

(ii) For incident shock waves with pressure ratios in the range $\frac{p_2}{p_1} < \frac{p_2}{p_1} < 2 \frac{p_2}{p_1}$, the separation point rapidly moves closer to the incident shock wave with decrease in Reynolds number, and the overall extent of the upstream influence also decreases markedly. If the Reynolds number is reduced sufficiently, boundary layer separation does not occur and

$$\frac{l}{x} \approx \frac{l}{x} \sim \frac{1}{(Re_x)^{1/4}}$$

This sequence of events might be observed when an oblique shock with given pressure ratio is allowed to strike a plane surface closer and closer to the leading edge. When the pressure disturbance communicated upstream of the shock begins to interfere with the flow in the neighborhood of the leading edge, the present analysis is inapplicable.

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6. Some Problems for Future Investigation(a) Boundary Layer-Shock Wave Interaction for Supersonic Flow in a Corner.

The pressure rise across an oblique shock generated by the sudden deflection of a supersonic stream at a solid surface is diffused through the boundary layer upstream and downstream of the corner. It seems reasonable to expect that the behavior of a laminar boundary layer on a plane surface upstream of the corner is qualitatively similar to the behavior of the laminar boundary layer upstream of an incident oblique shock wave. For sufficiently small pressure rise, or angle of flow deflection, the boundary layer does not separate and the pressure on the surface must grow exponentially as the corner is approached, in accordance with the relation $\frac{p}{p_0} - 1 = \text{const.} \cdot x^{\frac{1}{2}}$. For flow deflections larger than some critical angle, depending on the Reynolds number and Mach number, boundary layer separation occurs upstream of the corner. The location of the separation point moves rapidly upstream with increase in the angle of turn of the flow above the critical value. Eventually, with further increase in deflection angle, transition to turbulence occurs in the separated outer "jet". Qualitatively the extent of the upstream influence of the oblique shock emerging from the corner must depend upon Reynolds number and Mach number in a manner similar to that already described for the latter shock wave incident on a plane surface.

The main questions to be settled by further study are: (1), what is the relation of the surface pressure distribution to the pressure ratio across the oblique shock emerging from the corner; and (2) utilizing the results

of (1), what is the quantitative relation between the upstream location of the separation point, F , and the shock pressure ratio, the Reynolds number and the Mach number. A preliminary investigation indicates that the surface pressure distribution may even be quantitatively similar to the pressure distribution for an oblique shock incident on a plane surface, with the pressure rise for the corner flow of course replacing the pressure ratio $\frac{p_2}{p_1}$ for the flow behind the reflected shock.

(b) Boundary Layer-Shock Wave Interaction for Flow at the Rear of a Supersonic Airfoil

The pressure rise across the oblique shock at the trailing edge of a supersonic airfoil is diffused upstream through a laminar boundary layer, and this phenomenon modifies the pressure distribution over the surface and the aerodynamic characteristics computed on the basis of non-viscous flow theory. Boundary layer-shock wave interaction in this case is somewhat different than in the supersonic flow in a corner, because of the (negative) pressure gradient over the rear of the airfoil, and the fact that the layer of gas near the surface becomes a "free jet" on leaving the trailing edge of the airfoil. Of these two factors the negative pressure gradient is probably more important. This gradient must be overcome by the pressure rise diffused upstream through the boundary layer before separation can occur. For a given shock pressure ratio the distance between the separation point and the airfoil trailing edge should be much smaller than the distance between separation point and "vertex" for a flow in a corner.

As in other boundary layer-shock wave interaction problems, the separation point should move steadily upstream with increase in the strength of the trailing-edge shock wave. For a given airfoil geometry and angle of attack the pressure ratio across the trailing edge shock decreases as the local Mach number increases for $M < \sqrt{2}$, and increases rapidly with Mach number for $M > \sqrt{2}$. In other words the extent of upstream influence of the trailing edge shock should grow rapidly with Mach number for $M > \sqrt{2}$.

The effect of Reynolds number on the extent of upstream influence should be qualitatively similar to the Reynolds number effect for the case of a shock wave incident on a plane surface.

(c) Turbulent Boundary Layer-Shock Wave Interaction.

In many supersonic flows at high Reynolds numbers the boundary layer on the surface upstream of an incident or emergent shock wave is turbulent, particularly with a (positive) pressure gradient in the flow direction. The boundary layer may also be turbulent in uniform flow at low supersonic flight Mach numbers and low flight altitudes, where the radiative-heat conductive effects that stabilize the laminar boundary layer are inoperative. Two main facts are responsible for the basic differences between laminar and turbulent boundary layer shock wave interaction (see reference 1, for example): (1), the fact that in uniform flow the subsonic region in a turbulent boundary layer is confined to an extremely thin layer near the surface; and (2), the fact that to produce a given rate of growth of the boundary layer a much larger positive pressure gradient is required for turbulent flow

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than for laminar flow. In particular, a much larger pressure gradient is required to produce turbulent separation than laminar separation. Both of these facts indicate that the curvature $\frac{d^2 \delta^*}{dx^2}$ of the turbulent boundary layer will increase sharply in the immediate vicinity of the incident (or emergent) shock wave. Recent experiments on the reflection of an incident shock wave from a plane surface with a turbulent boundary layer show that the reflected shock actually begins in the boundary layer upstream of the incident shock, apparently because of the strong compression waves generated by the large curvature of the turbulent boundary layer.

The question arises as to whether the boundary layer flow and the flow in the immediate vicinity of the shock wave can be considered separately, as in the case of the laminar boundary layer. A first step in this study is to determine the relation between the turbulent boundary layer growth $\frac{d\delta^*}{dx}$ and the pressure gradient. Oswatitsch and Wieghardt (reference 6) have indicated one method by which this may be done in their study of the "stability" of the interaction between the turbulent boundary layer on a flat plate and the "external" supersonic stream. Here, the methods are necessarily semi-empirical, because of the present state of knowledge of turbulent boundary layer flow.

7. Conclusions

1. When an oblique shock wave strikes the laminar boundary layer over a plane surface, the pressure rise communicated upstream through the subsonic portion of the boundary layer decays exponentially with distance from the incident shock wave. The "relaxation distance", L , or the distance

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required for the pressure disturbance to decay to $\frac{1}{e}$ of its original value, is given by $\frac{x}{\delta_o^*} = \frac{1}{\sqrt{Re_{\delta_o^*}}} \frac{f(M_1)}{(M_1^2 - 1)^{1/4}}$, or

$$\frac{x}{\delta_o^*} = \frac{1}{(Re_{\delta_o^*})^{1/4}} \frac{f(M_1)}{(M_1^2 - 1)^{1/4}} (1.73 + 2.50 \frac{\delta_o^*}{x} M_1^2)^{3/2}$$

where $f(M_1) = \frac{(0.0083 + 0.0099 \frac{\delta_o^*}{x} M_1^2)^{1/2}}{(0.30 + 0.417 \frac{\delta_o^*}{x} M_1^2)^{1/4}} (1 + \frac{\gamma-1}{2} M_1^2)^{1-\frac{\gamma}{2}}$

The ratio $\frac{x}{\delta_o^*}$ is large near $M = 1$, falls to a minimum at $M \cong 1.6$ (for air), and thereafter increases rapidly with Mach number. (See figure 2).

2.

Because of the pressure rise along the surface, the boundary layer separates upstream of the shock, except for very weak incident shock waves. Since the region of subsonic "reversed flow" near the surface cannot support a pressure discontinuity, the shock wave is reflected from the separated "outer jet" as an expansion "fan" which just cancels the pressure rise across the incident shock. The flow is deflected back toward the surface and "reattaches" itself downstream of the reflected expansion "fan". As the flow returns to its original direction approximately parallel to the surface, compression waves are propagated into the main stream, and the pressure along the surface rises steadily to a value equal to the pressure behind the reflected shock wave far from the surface.

3.

For each Reynolds number, $Re_{\delta_o^*}$, and Mach number M_1 , there is a definite value of the pressure ratio $\frac{p}{p_o}$ at separation. For incident shock waves with shock pressure ratio $\frac{p_2}{p_1} < \frac{p}{p_o}$ (approx.), boundary layer separation does not occur and the extent of upstream influence $\frac{x}{\delta_o^*}$ is measured

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is measured by the relaxation distance, $\frac{\bar{l}}{x}$. When $\frac{\bar{l}}{x} > \frac{p_2}{p_1}$ (approx.), the boundary layer separates upstream of the incident shock wave, and the separation point location $\bar{x}_s$ moves rapidly upstream with increase in shock pressure ratio. Here, $\frac{\bar{l}}{x} \cong \frac{l}{x} + \frac{\bar{x}_s}{x} \cdot \frac{1.73 + 2.50 M^2}{\sqrt{Re_x}}$

For $\frac{\bar{l}}{x} \cong \frac{2p_2}{p_1}$, the distance between the separation point and the incident shock is great enough so that transition to turbulent flow occurs in the separated "jet". When $\frac{\bar{l}}{x} \cong \frac{2p_2}{p_1}$ (approx.), $\frac{\bar{l}}{x} \cong \frac{l}{x} + \frac{Re_{x_{tr}}}{Re_x}$,

where $Re_{x_{tr}}$ is the transition Reynolds number for the separated jet. (Only meagre information is available as to the magnitude of $Re_{x_{tr}}$, but from some recent experiments it appears to be of the order of 2×10^5).

4. Certain general conclusions can be drawn concerning the effect of incident shock pressure ratio, Reynolds number and Mach number on the extent of upstream influence of the incident shock wave:

(i) Near sonic velocity ($M \cong 1$) the pressure ratio $\frac{p_2}{p_1} - 1$ for separation is high, while the maximum possible pressure ratio across an incident shock wave is not much greater than unity. Separation generally does not occur and the extent of upstream influence is measured by $\frac{l}{x} \sim \frac{1}{(M^2-1)^{1/2}}$

(ii) For $M_{1.25}$, and for $Re_x \cong 10^6$, roughly, all possible incident oblique shock waves, except those of the so-called "weak" shock family with very small flow deflections ($\alpha' \leq 1^\circ$), will have shock pressure ratios greater than $\frac{2p_2}{p_1}$, and will involve transition to turbulent flow in the separated jet upstream of the shock. The extent of upstream influence

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in this case is measured by $\frac{l}{x} + \frac{Re_{x+1}}{Re_x}$, and increases rapidly with decreasing Reynolds number and with increasing Mach number for $M > 1.6$ (air).

At very high Reynolds numbers, provided the boundary layer over the surface is still laminar, the distance between separation and transition is small,

$$\text{and } \frac{l}{x} \approx \frac{l}{x}$$

(iii) For incident shocks for which $\frac{p_2}{p_1} < \frac{p}{p_1} < 2\frac{p_2}{p_1}$, the situation is as follows: For $M_1 < 1.5$ (approx.), an increase in Mach number increases $\bar{x}_s$ and $\frac{l}{x}$, while for $M_1 > 1.5$, (approx.), an increase in Mach number decreases $\bar{x}_s$ and $\frac{l}{x}$. The separation point moves closer to the incident shock wave rapidly with decrease in Reynolds number, and the overall extent of the upstream influence also decreases markedly. If the Reynolds number is reduced sufficiently, boundary layer separation does not occur and

$$\frac{l}{x} \approx \frac{l}{x}$$

Such a sequence of events might be observed if an oblique shock of given strength is made to strike a plane surface closer and closer to the leading edge.

5.

Exact theoretical solutions for the boundary layer-shock wave interaction problem have not been found, and the methods utilized in the present study give only an approximate solution of the problem. Carefully planned experiments are now needed to test the validity of these approximations and to help determine the proper direction for future theoretical work.

6. It is already known that the subsonic region of the boundary layer plays an important role in the boundary layer-shock wave interaction problem. The present study indicates that consideration of the viscous effects in the boundary layer is also essential for an understanding of the phenomenon.

7. Some of the methods of the present study can be generalized to treat laminar boundary layer-shock wave interaction for supersonic flow in a corner, and for the flow near the trailing edge of a supersonic airfoil. In these cases it is already clear that for a given geometry, i.e., given flow deflection, the extent of upstream influence of the oblique shock increases rapidly with Mach number for local $M > \sqrt{2}$. Thus, it is not safe to generalize conclusions obtained from data at low supersonic Mach numbers to high supersonic Mach numbers.

It is also possible that turbulent boundary layer-shock wave interactions can be treated by methods similar to those of the present paper, but there is some question as to whether the boundary layer flow can be considered separately from the flow in the immediate vicinity of the shock wave.

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APPENDIX

Analytical Representation of Surface Pressure Distribution, $p(\xi)$

Upstream of Incident Shock Wave

Region 1

Upstream of separation point

$$-\infty < \xi \leq \xi_s$$

$$p/p_1 - 1 = (p_2'/p_1 - 1) e^{a\xi}$$

"0"

given by equation (12).

Region 2

Downstream of separation point

$$\xi_s \leq \xi \leq 0$$

$$p/p_1 - 1 = (p_2'/p_1 - 1) [2 - e^{a(\xi - \xi_s)}]$$

$$\text{at } \xi = 0, \quad p/p_1 - 1 = (p_2'/p_1 - 1) [2 - e^{a\xi_s}]$$

$$\delta^*/\delta_0^* = 1 + \left[\left(\frac{d\delta^*/\delta_0^*}{d\xi} \right)_{\xi_s} - \frac{\sqrt{M_1^2 - 1}}{\gamma M_1^2} (p_2'/p_1 - 1) e^{a\xi_s} \right] (\xi - \xi_s)$$

$$+ \frac{\sqrt{M_1^2 - 1}}{\gamma M_1^2} (p_2'/p_1 - 1) \left[2(\xi - \xi_s) + \frac{1}{a} e^{a(\xi - \xi_s)} \right]$$

$$- \frac{\sqrt{M_1^2 - 1}}{\gamma M_1^2} \frac{(p_2'/p_1 - 1)}{a} e^{a\xi_s}$$

$$\text{at } \xi = 0, \quad \theta = \theta_2 = \frac{\sqrt{M_1^2 - 1}}{\gamma M_1^2} (p_2 - p_0) + \left(\frac{d\delta^*/\delta_0^*}{d\xi} \right)_{\xi=0}$$

Downstream of Incident Shock Wave

Region 3

Upstream of reattachment point

$$0 \leq \xi \leq \xi_R$$

$$(p/p_1 - 1) = (p_2'/p_1 - 1) \left[e^{a(\xi + \xi_s)} + 2(1 - e^{a\xi_s}) \right]$$

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$$\frac{\delta^*}{\delta_0^*} = \left(\frac{\delta^*}{\delta_0^*} \right)_{\xi=0} + \xi \left[\frac{\phi}{3} - \left(\frac{\phi}{\phi_1} - 1 \right) e^{a \xi} \frac{\sqrt{M^2 - 1}}{\gamma M^2} \right] + \frac{\sqrt{M^2 - 1}}{\gamma M^2} \frac{(\phi/\phi_1 - 1)}{a} \left[e^{a(\xi + \xi_R)} - e^{a \xi} \right]$$

$$\theta_3 = \theta_2 - 2\omega', \quad \text{very nearly.}$$

$$\xi_R \text{ determined by condition } \left(\frac{\delta^*}{\delta_0^*} \right)_{\xi_R} = \left(\frac{\delta^*}{\delta_0^*} \right)_{\xi_R}$$

Region 4 Downstream of reattachment point

$$\xi_R \leq \xi < \infty$$

$$\frac{\phi}{\phi_1} - 1 = \left(\frac{\phi}{\phi_1} - 1 \right) \left[2 \left\{ e^{a(\xi + \xi_R)} + (1 - e^{a \xi_R}) \right\} - e^{a(2\xi_R + \xi - \xi_R)} \right]$$

$$\text{As } \xi \rightarrow \infty, \quad \left(\frac{\phi}{\phi_1} - 1 \right) \rightarrow 2 \left(\frac{\phi}{\phi_1} - 1 \right) \left[e^{a(\xi_R + \xi_R)} + (1 - e^{a \xi_R}) \right]$$

and this pressure must equal $\left(\frac{\phi}{\phi_1} - 1 \right)$

$$\frac{\delta^*}{\delta_0^*} = \left(\frac{\delta^*}{\delta_0^*} \right)_{\xi=\xi_R} + \left[\left(\frac{\delta^*}{\delta_0^*} \right)_{\xi=\xi_R} + \frac{\sqrt{M^2 - 1}}{\gamma M^2} \left(\frac{\phi}{\phi_1} - 1 \right) e^{a(\xi_R + \xi)} \right] (\xi - \xi_R) + \frac{1}{a} \frac{\sqrt{M^2 - 1}}{\gamma M^2} \left(\frac{\phi}{\phi_1} - 1 \right) \left[e^{a(2\xi_R + \xi - \xi_R)} - e^{a(\xi_R + \xi)} \right]$$

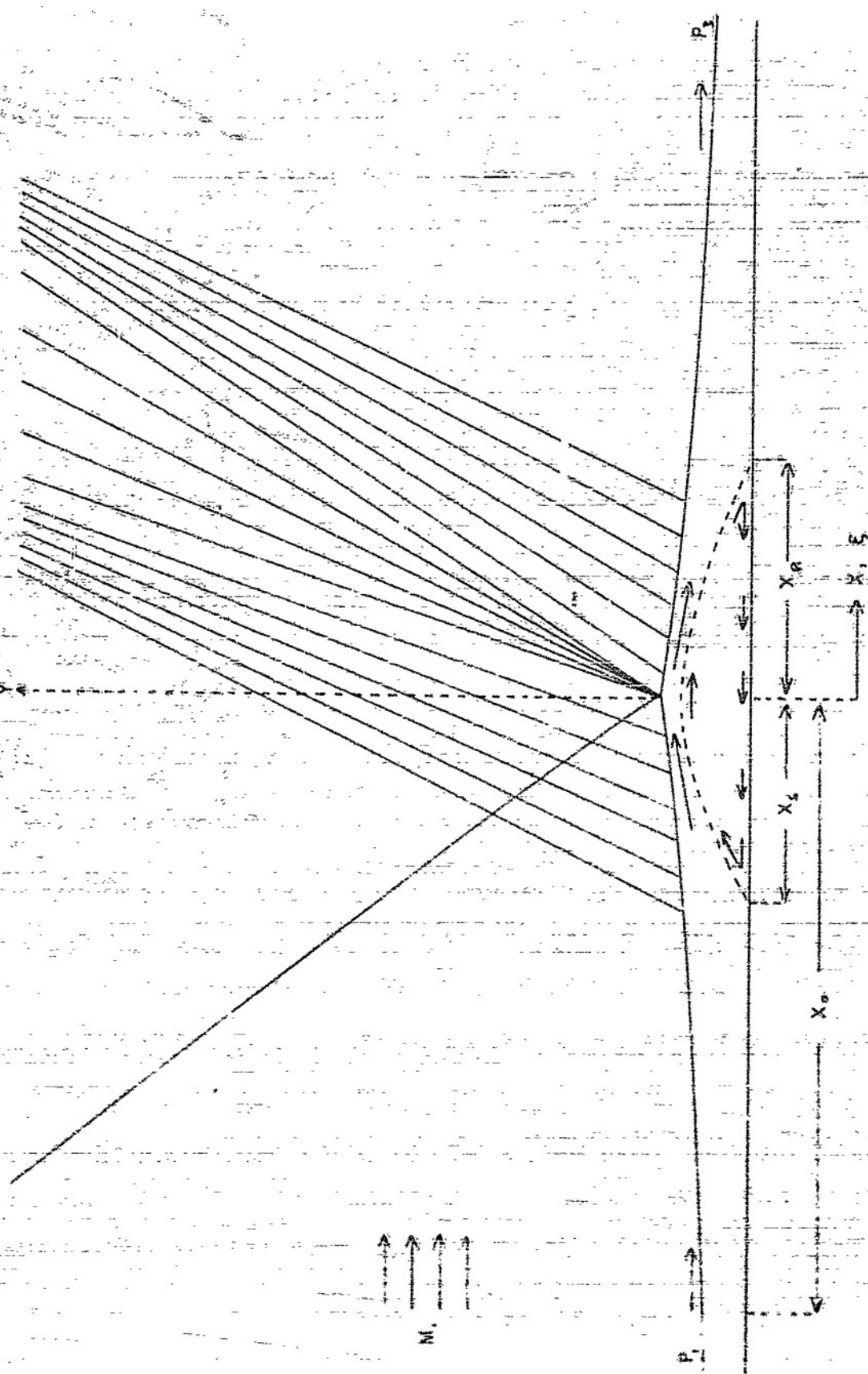


FIGURE 1.

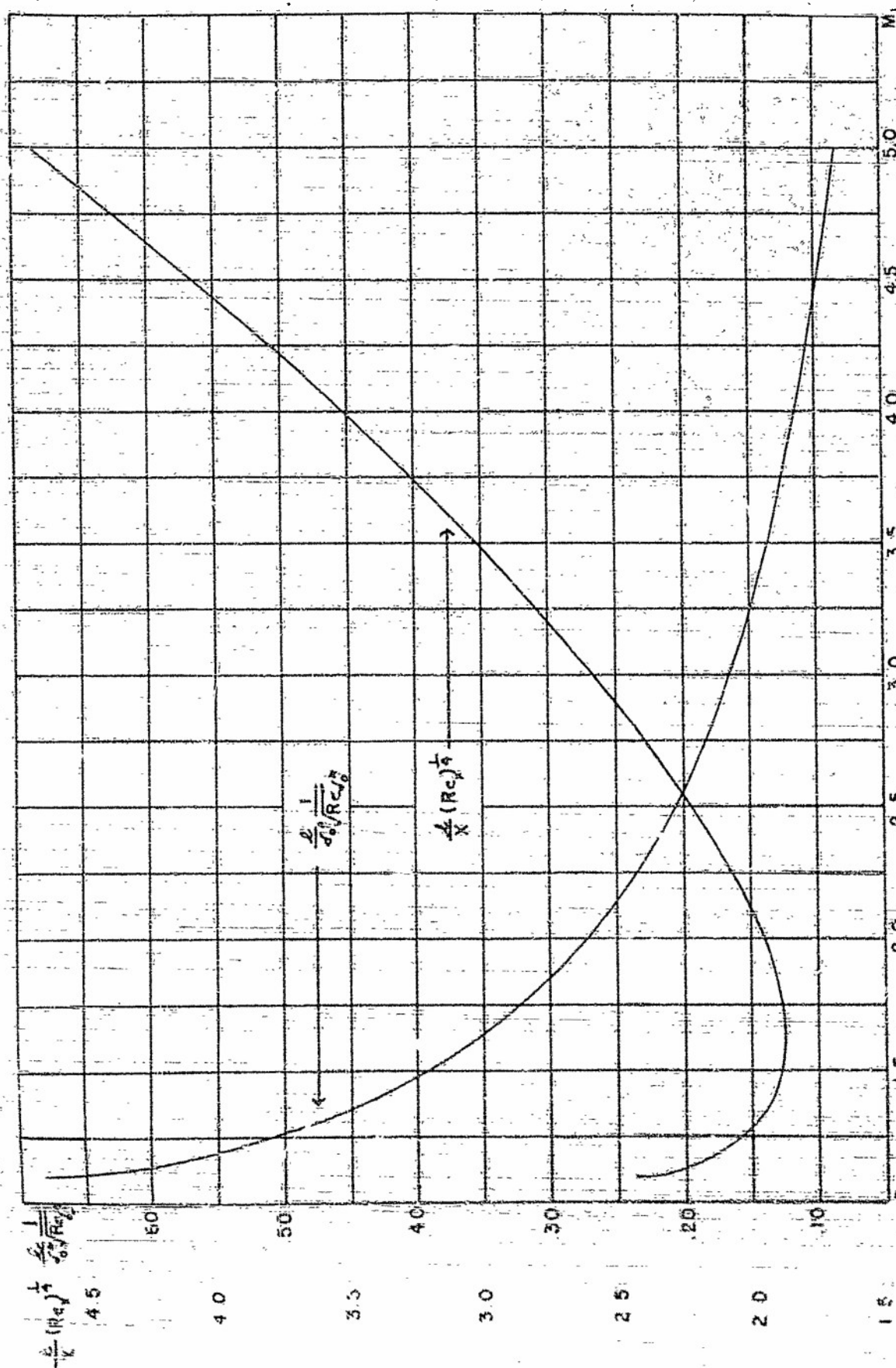


FIGURE 2. RELAXATION DISTANCE, L , OR DISTANCE REQUIRED FOR PRESSURE DISTURBANCE TO DECAY TO $1/4$ th OF ORIGINAL VALUE

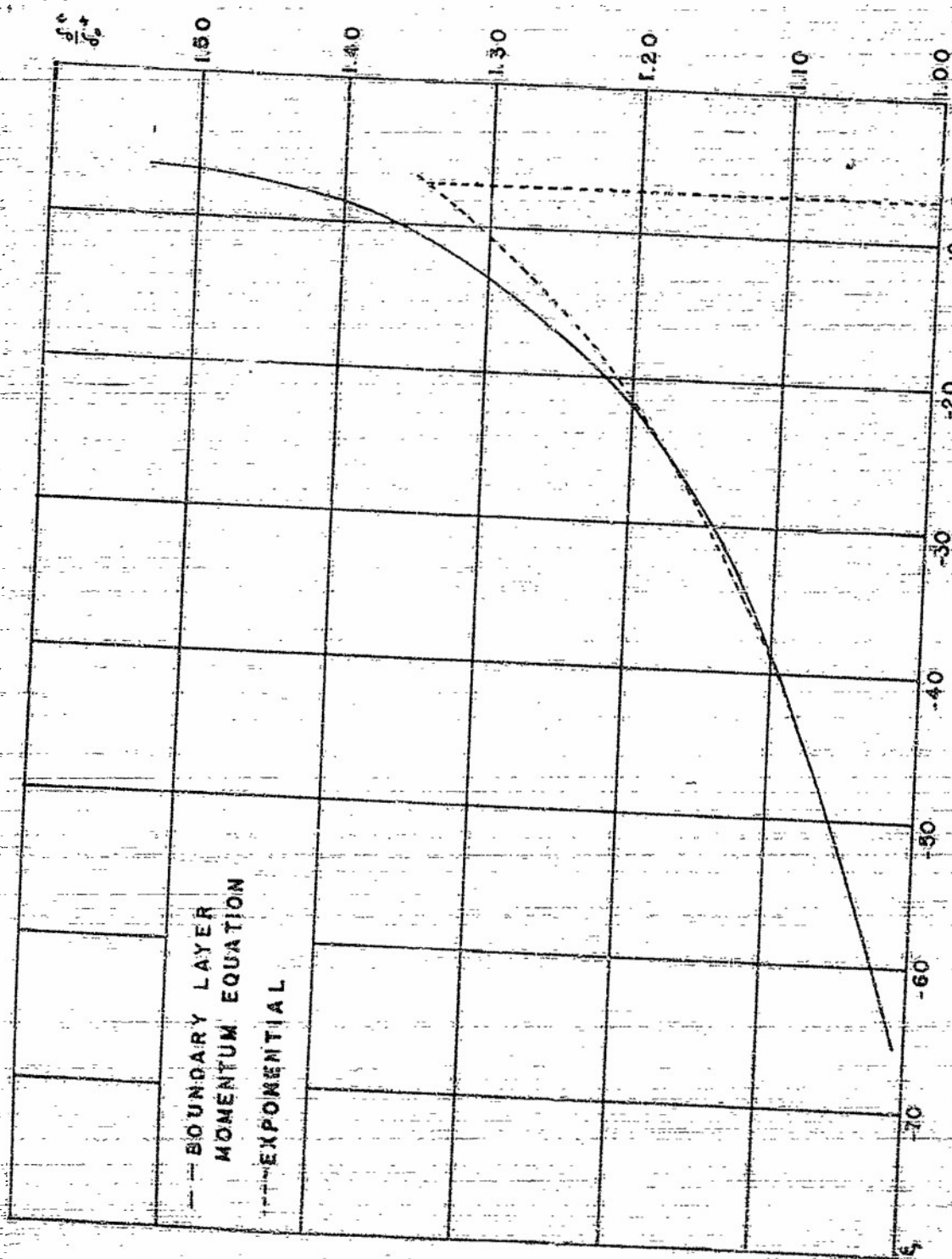


FIGURE 3. BOUNDARY LAYER GROWTH UPSTREAM OF INCIDENT SHOCK WAVE

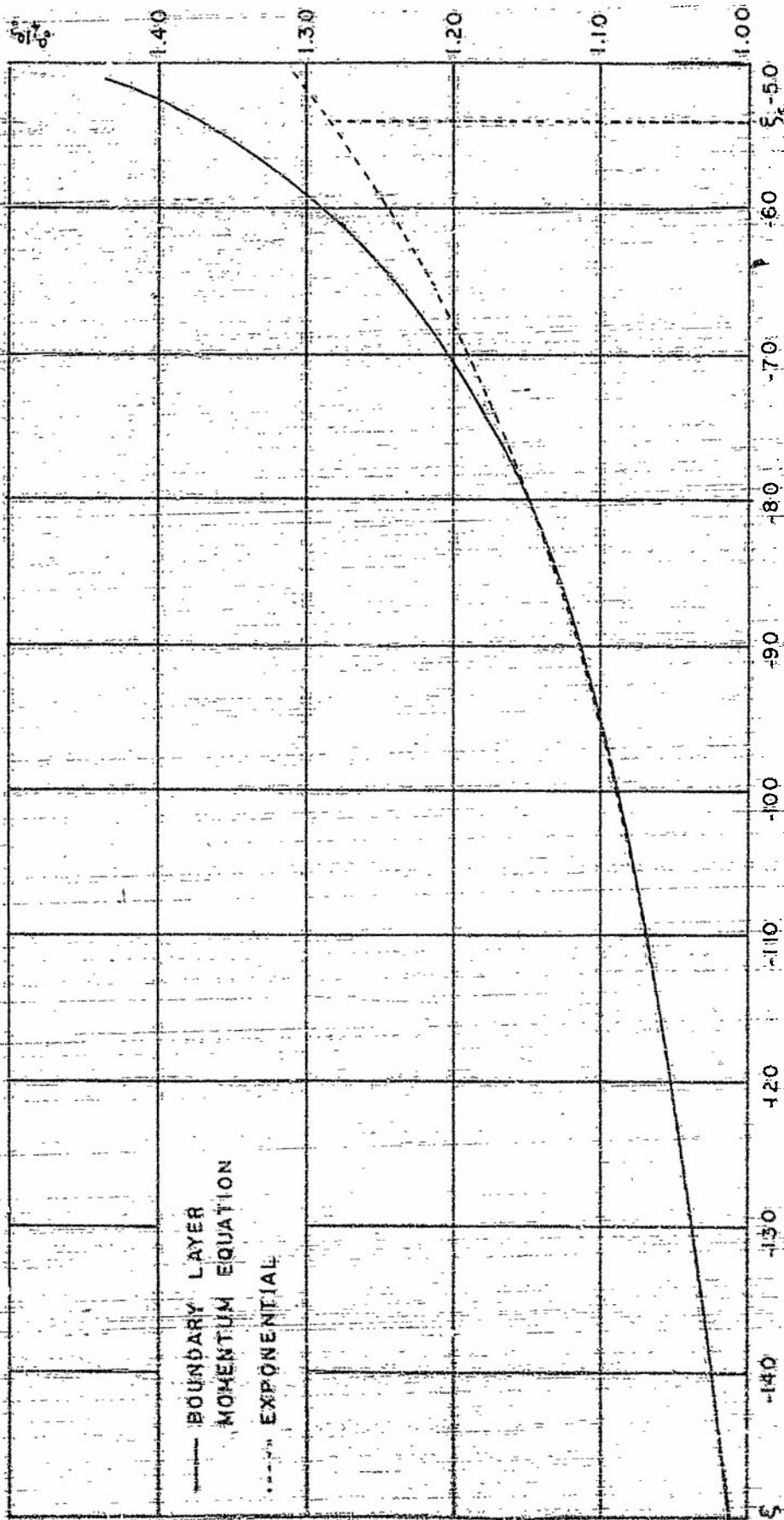


FIGURE 3b. $Re_{\delta^*} = 3972$; $M = 1.235$; $\bar{P} = 1.154$

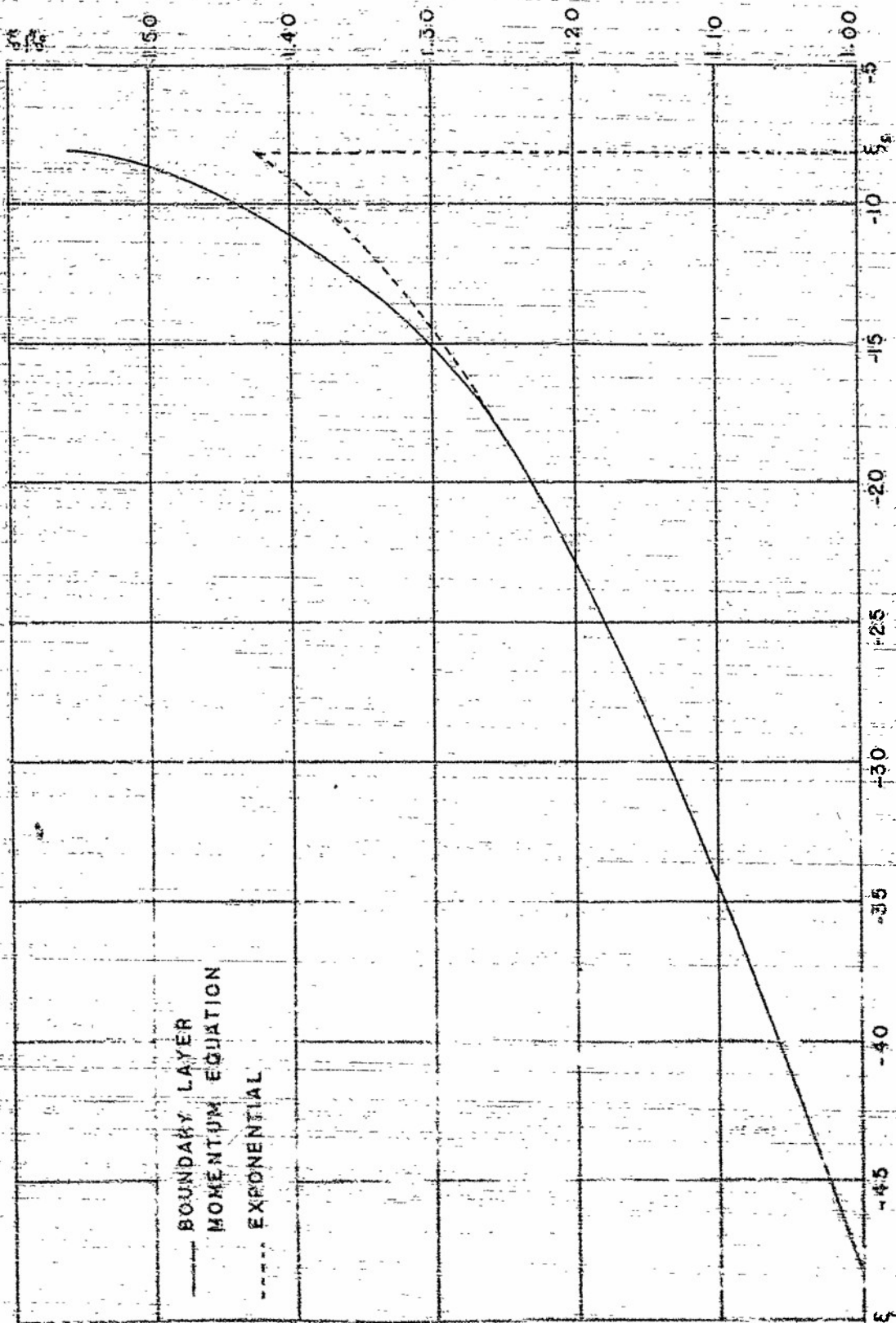


FIGURE 3-2. $Re_{\delta} = 2754$; $M = 3.00$; $\bar{P} = 1.154$

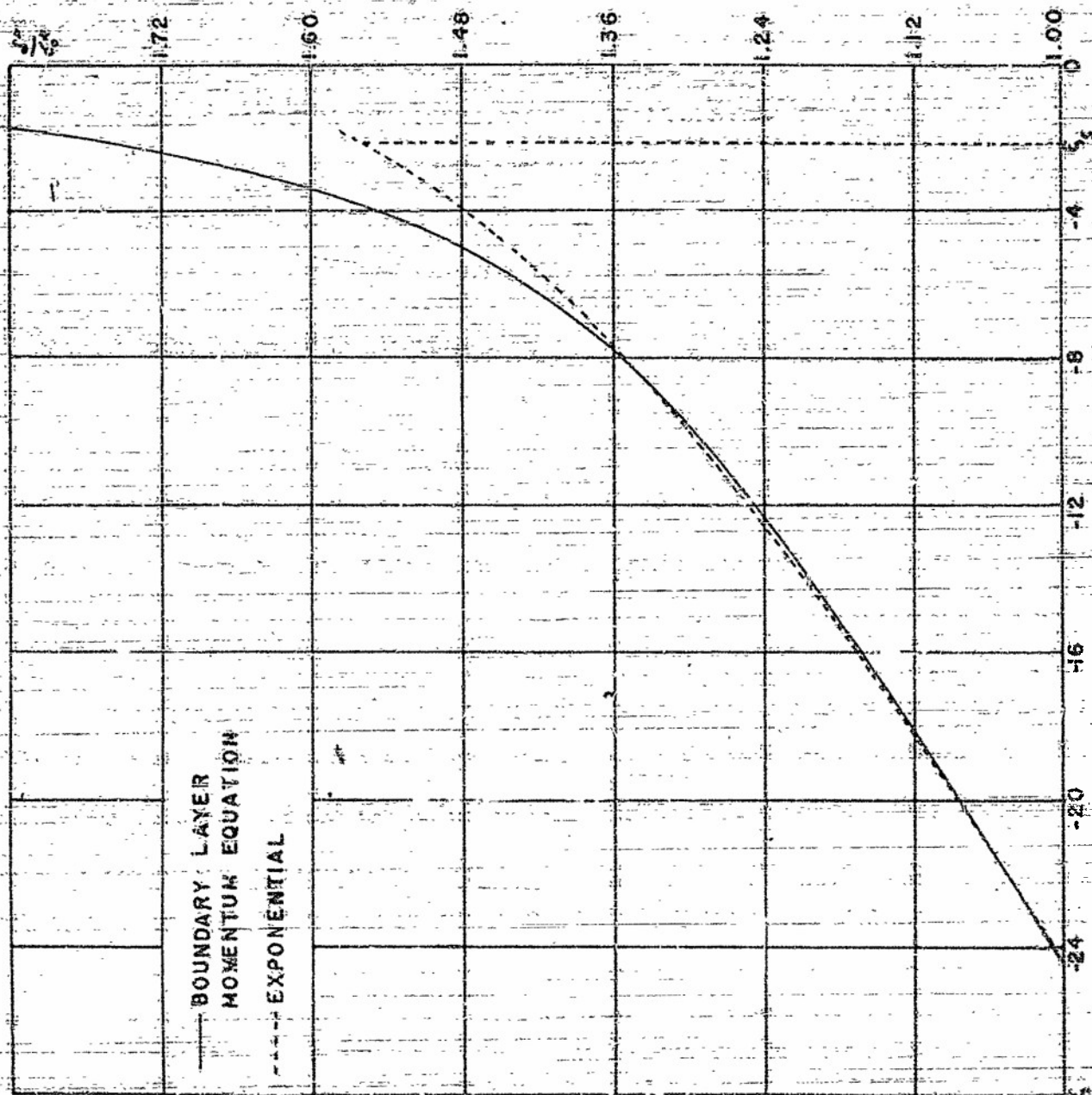


FIGURE 3d. $R_{\delta^*} = 688$; $M = 3.00$; $\bar{P} = 1.233$

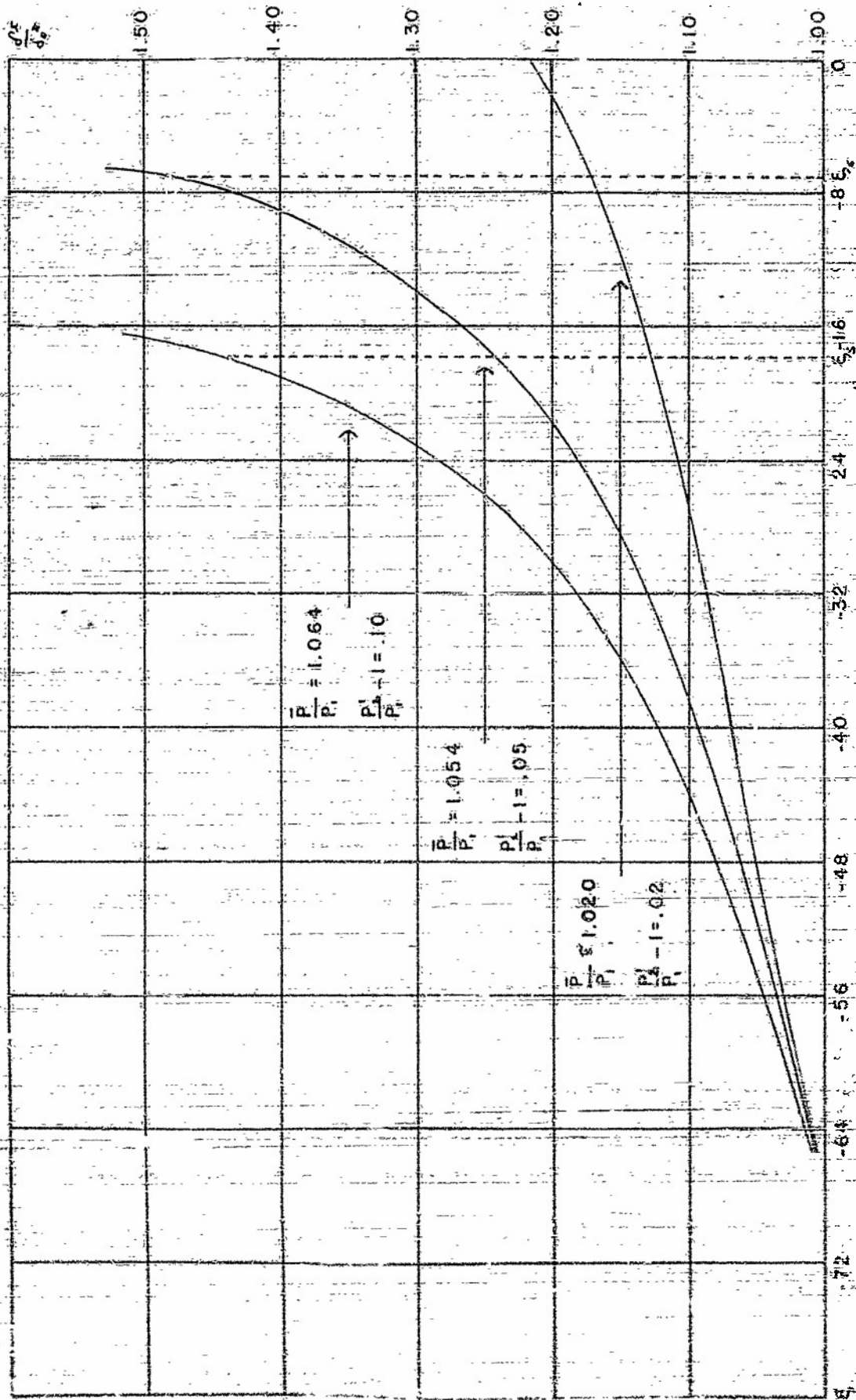


FIGURE 4. EFFECT OF INCIDENT SHOCK STRENGTH ON UPSTREAM BOUNDARY LAYER GROWTH

$R_{0.5} = 993$ $N = 1.235$

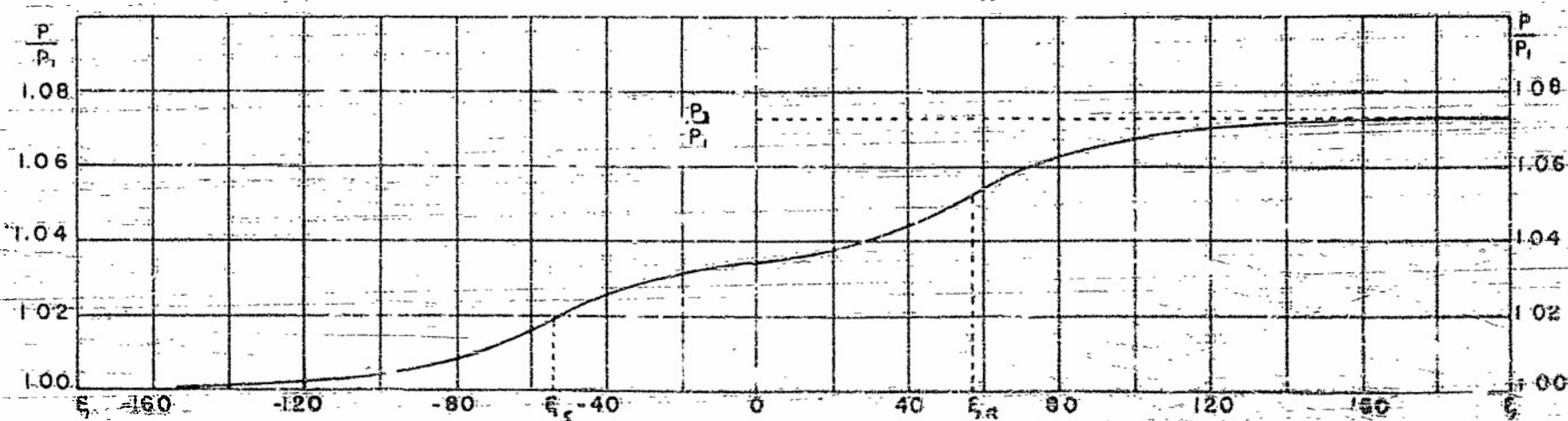
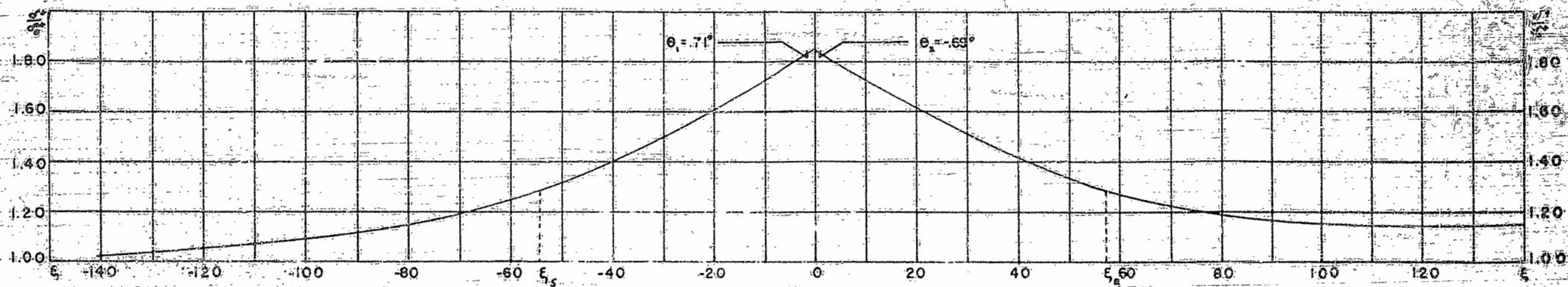


FIGURE 5. SURFACE PRESSURE DISTRIBUTION AND STREAMLINE PATTERN

(a) $Re_{fx} = 5972$; $M = 1.235$; $\frac{\bar{P}}{P_1} = 1.038$

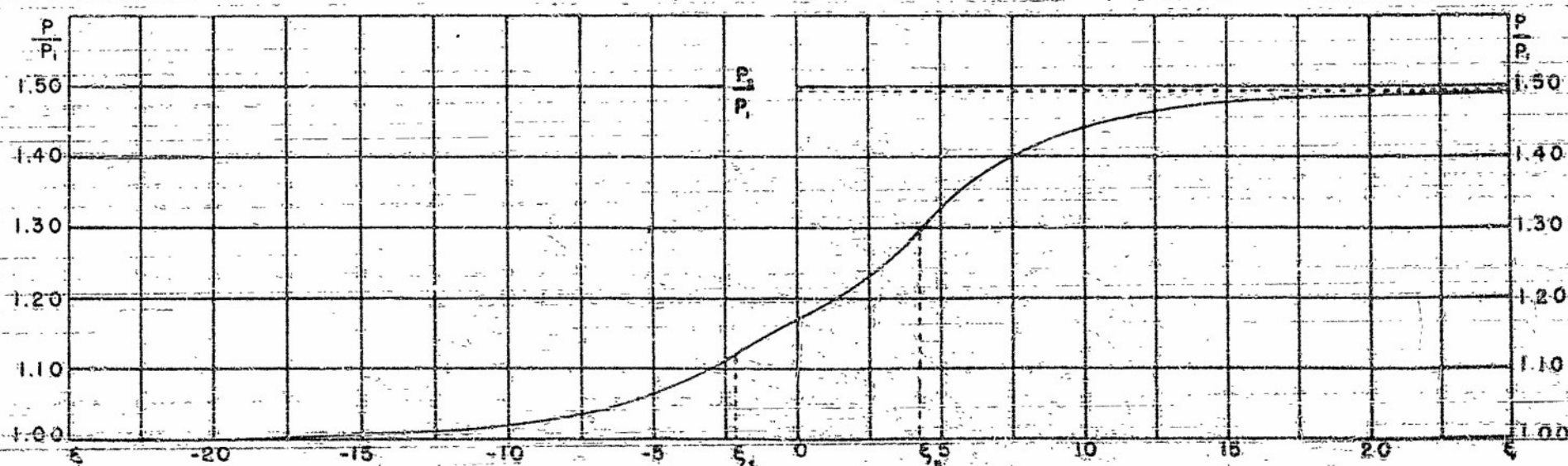
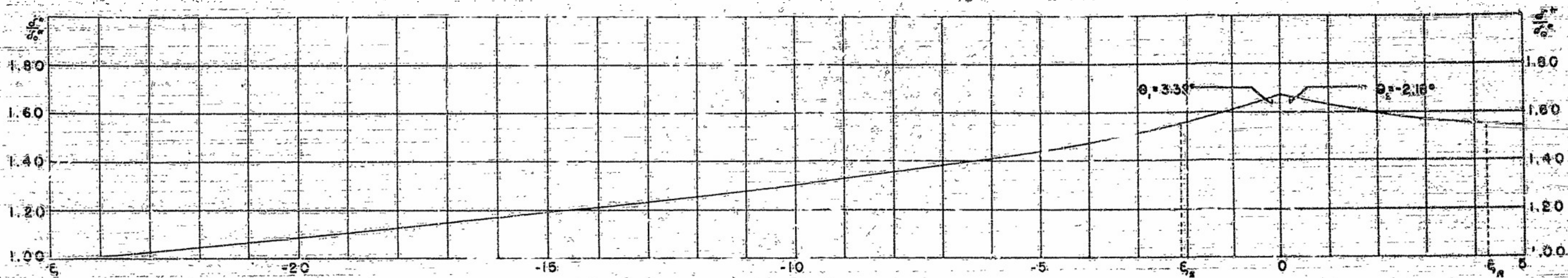


FIGURE 5. (b) $R_{\text{max}} = 6.88; M = 3.00; \frac{P}{P_1} = 1.233$

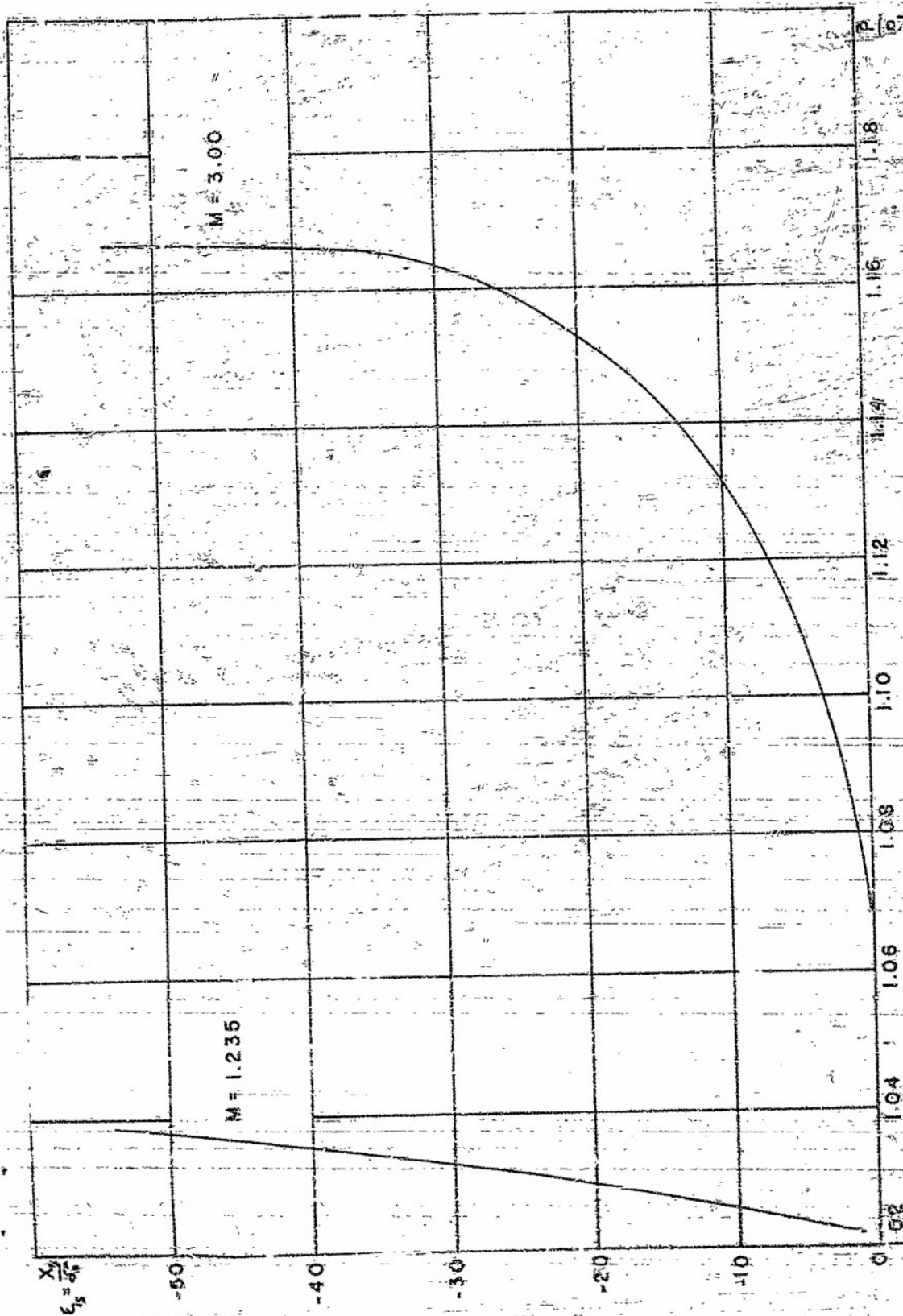


FIGURE 6 LOCATION OF SEPARATION POINT UPSTREAM OF INCIDENT SHOCK WAVE X_3/s_1
EFFECT OF MACH NUMBER $R_{sc} = 3972$

(a)



FIGURE 6. (b) EFFECT OF REYNOLDS NUMBER $M=3.00$

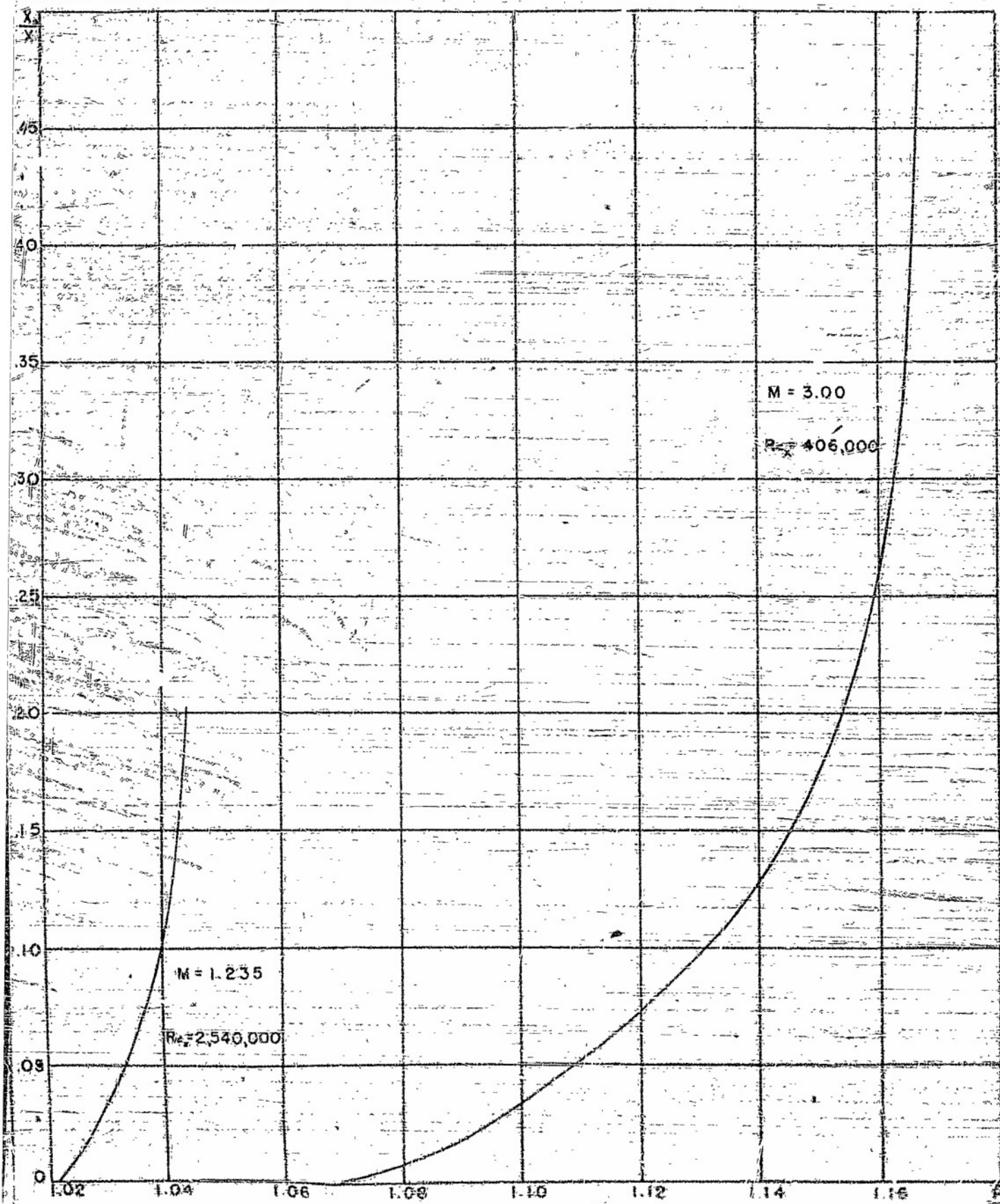


FIGURE 6. (c) LOCATION OF SEPARATION POINT UPSTREAM OF INCIDENT SHOCK WAVE, Y/X
EFFECT OF MACH NUMBER $Re_{T^*} = 3972$

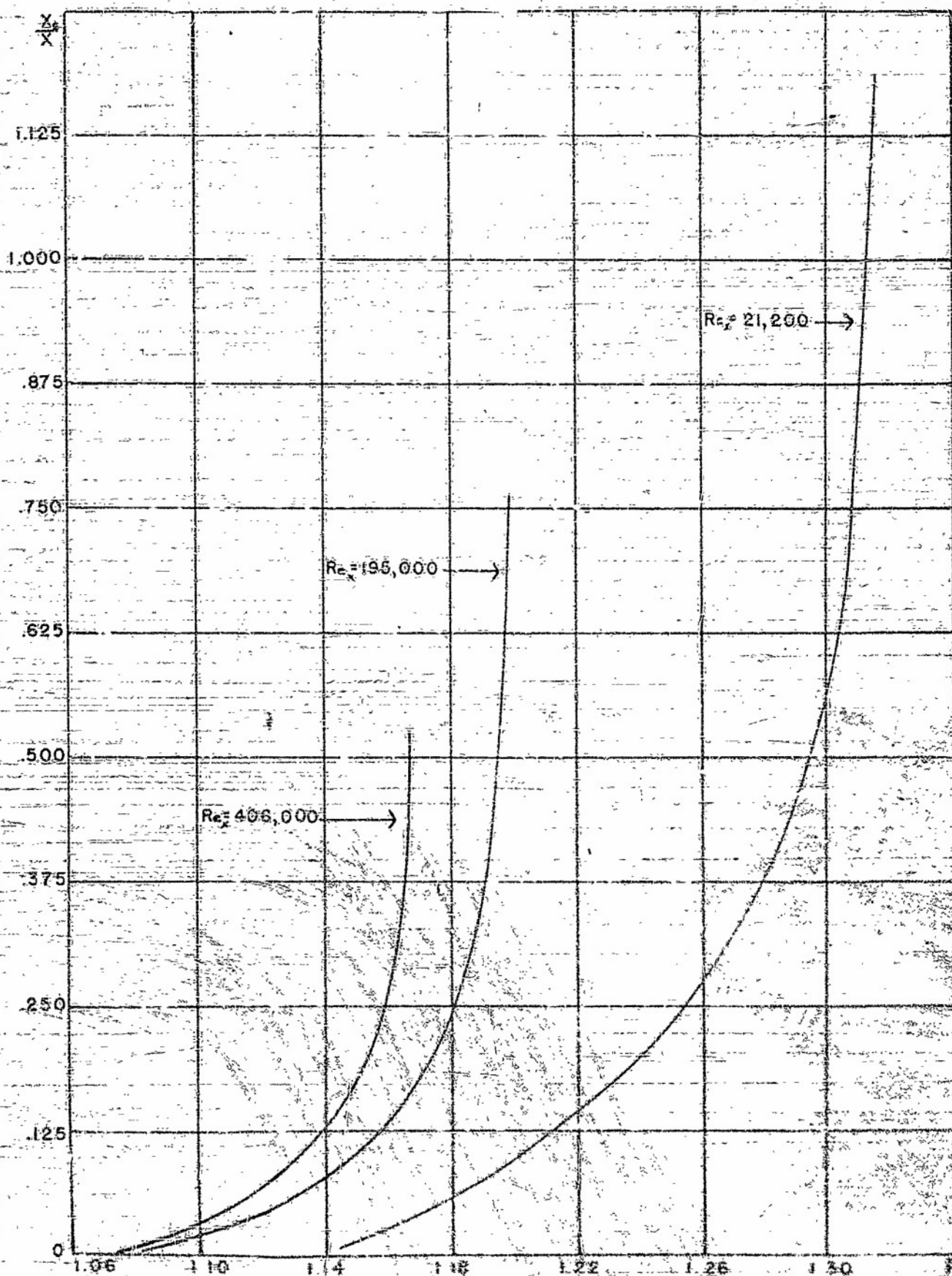


FIGURE 6 (a) EFFECT OF REYNOLDS NUMBER
M = 3.60

References

1. Liepmann, H. W.: Investigations of the Interaction of Boundary Layer and Shock Waves in Transonic Flow. Final Report to U.S.A.A.F. (1946). Contract #W33-038ac-1717(11522) C.I.T. Also, Journal of the Aeronautical Sciences, December, 1946.
2. Liepmann, H. W., Ashkenas, H., and Cole, J. D.: Experiments on Transonic Flow. Report to U.S.A.A.F., June, 1947. Supplement #6 to Contract W33-038ac-1717(11522), C.I.T.
3. Ackeret, J., Felbmann, F., and Rott, N.: Investigations of Compressor Shocks and Boundary Layers in Gases Moving at High Speed. Translated as N.A.C.A. Technical Memorandum No. 1113, (Jan., 1947)
4. Lees, L.: Remarks on the Interaction Between Shock Waves and Boundary Layer in Transonic and Supersonic Flow. Project Squid Technical Memorandum No. Pr-1, Nov. 1, 1947.
5. Lagerstrom, P., Cole, J. D., and Trilling, L. (C.I.T.): Viscous Effects in Compressible Flow. Paper presented at meeting of Institute for Fluid Mechanics and Heat Transfer, Pasadena, Cal. June 23, 1948.
6. Oswatitsch, K. and Wieghardt, K.: Theoretical Analysis of Stationary Potential Flow and Boundary Layers at High Speed. Translated as N.A.C.A. Technical Memorandum No. 1159 (April, 1948)
7. Lighthill, M.: The Propagation of Steady Disturbances in a Supersonic Stream Bounded on One Side by a Parallel Subsonic Stream. Proc. Cambridge Phil. Soc., Vol. 44 (part 2), pp. 264-80 (1948).

8. Parodniyev, A.: Laminar Boundary Layer in Compressible Flow. *Comptes Rendus Acad. Sci. U.S.S.R.*, Vol. 34, No. 8, 1942, pp. 213-219.
9. von Karman, Th.: On Laminar and Turbulent Friction. Translated as N.A.S.A. Technical Memorandum No. 1092 (1945)
10. Dryden, H. L., Murnaghan, F. D., and Bateman, H.: Report of the Committee on Hydrodynamics, National Research Council, Feb. 1932, p. 236.