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The Secondary Flow in a Cascade of Aerofoils in a Non-Uniform Stream

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FARNBOROUGH, HANTS.

REPORT No: AERO. 2317

THE SECONDARY FLOW
IN CASCADE OF AEROFOILS
IN NON-UNIFORM STREAM

by

H. B. SQUIRE, M.A.

and

K. G. WINTER, B.Sc.

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ROYAL AIRCRAFT ESTABLISHMENT, FARNBOROUGH

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Aerofoils in a Non-uniform Stream

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SUMMARY

A method is given of calculating the secondary flow in a cascade of aerofoils in a stream with a variation of approach velocity along the span of the aerofoils. The results are compared with measurements of the secondary flow at a cascade at a corner in a return circuit wind tunnel, where the velocity gradient is obtained as a normal consequence of flow through a diffuser. The agreement between theory and experiment is satisfactory and indicates that the theory is sound.

The pressure drop coefficient associated with the secondary flow as calculated from the measurements is 0.032, compared with a value of 0.019 given by the theory.

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1 Introduction

The existence of secondary flow downstream of a cascade of aerofoils, in a stream of non-uniform velocity, is well known. It consists usually of a pair of vortices in each passage through the cascade with the character shown in Fig.1. The extent of the secondary flow depends on the degree of non-uniformity of the approaching stream and two types of secondary flow can be distinguished. In the first type the stream is uniform except for a thin boundary layer at the walls. The secondary flow is then limited to layers near the end walls and in the wakes of the cascades. This type of motion has been investigated by Carter and Cohen¹. In the second type of flow the stream is non-uniform over a large part of the span of the vanes as will occur if the cascade is placed at the end of a diffuser in a wind tunnel. The vortices shown in Fig.1 thus extend over a large part of the cross section and eventually combine to form a pair of large vortices.

This note deals only with the second type of flow in which the region of non-uniformity is large compared with the gap between the vanes. In this case, although the non-uniformity of the stream may have been created by frictional effects, the development of the secondary flow system can be considered as being independent of friction and can be calculated from the equations of motion of an ideal fluid.

The preliminary theory given in Ref.2 is strictly only valid for small angles of deflection.

There is an analogy between the secondary flow and the motion of a gyroscope. If a disc, which is rotating about its axis of symmetry, is turned about an axis in its plane it develops an angular velocity about a third axis perpendicular to the first two. The analogy with the present problem is obtained by supposing the disc to lie originally in the plane containing the stream direction and the spanwise direction of the cascades. Its original rotation is associated with the velocity gradient along the spanwise direction of the cascades. The disc and the stream are then turned, on passage through the cascade, about an axis along the cascade span. The rotation of the disc about its third axis corresponds to the secondary flow of the character indicated in Fig.1.

2 Theory

2.1 General

We start with the two-dimensional flow which would be present if the approaching stream were uniform. For any cascade this flow can be defined by a velocity potential $\alpha(x,y)$ and a stream function $\beta(x,y)$ as shown diagrammatically in Fig.2; x and y are Cartesian co-ordinates in the plane normal to the span of the cascades and z is measured along the span.

The flow in one of the passages between the aerofoils and its extensions ahead of and behind the aerofoils lies between two stream lines $\beta = \text{constant}$, and we shall restrict consideration to a single passage bounded in this way. The lines $\alpha = \text{const.}$, $\beta = \text{const.}$ form an orthogonal curvilinear network made up of small squares and the elements of arc measured along these lines are $h_{\alpha\alpha}$ and $h_{\beta\beta}$ where

$$\frac{1}{h^2} = \left(\frac{\partial \alpha}{\partial x} \right)^2 + \left(\frac{\partial \alpha}{\partial y} \right)^2 = \left(\frac{\partial \beta}{\partial x} \right)^2 + \left(\frac{\partial \beta}{\partial y} \right)^2$$

Alternatively the quantity h can be regarded as proportional to the reciprocal of the velocity of the two-dimensional flow at any point in the field. We choose the scale of α and β so that h is equal to unity far ahead of the cascade; h will also be equal to unity far behind a cascade which gives a deflection without expansion or contraction.

We now consider the flow past a cascade for which the approach velocity distribution is non-uniform. Then, if the degree of non-uniformity is not large, the two-dimensional flow referred to above can be regarded as a first approximation to the actual flow. We assume that this is so and shall calculate the actual flow by treating it as a linear perturbation from the two-dimensional flow. The motion is studied by referring it to the orthogonal curvilinear co-ordinates α , β and z , the distance measured along the span of the cascade. Following the notation of Ref.3 the elements of length measured along the directions of α , β , z increasing are $h_1 d\alpha$, $h_2 d\beta$ and $h_3 dz$ respectively and for the present system $h_1 = h_2 = h$ and $h_3 = 1$ so that these elements of length are $h d\alpha$, $h d\beta$ and dz respectively and where h is independent of z . The velocity is $\underline{v}$ which has components (u, v, w) in the directions of (α, β, z) increasing. The vorticity is $\underline{\omega}$ with components (ξ, η, ζ) and from equation (35) of Chapter III of Ref.3 these are

$$\left. \begin{aligned} \xi &= \frac{1}{h_2 h_3} \left\{ \frac{\partial}{\partial \beta} (h_3 w) - \frac{\partial}{\partial z} (h_2 v) \right\} = \frac{1}{h} \frac{\partial w}{\partial \beta} - \frac{\partial v}{\partial z} \\ \eta &= \frac{1}{h_3 h_1} \left\{ \frac{\partial}{\partial z} (h_1 u) - \frac{\partial}{\partial \alpha} (h_3 w) \right\} = \frac{\partial u}{\partial z} - \frac{1}{h} \frac{\partial w}{\partial \alpha} \\ \zeta &= \frac{1}{h_1 h_2} \left\{ \frac{\partial}{\partial \alpha} (h_2 v) - \frac{\partial}{\partial \beta} (h_1 u) \right\} = \frac{1}{h^2} \left\{ \frac{\partial}{\partial \alpha} (h v) - \frac{\partial}{\partial \beta} (h u) \right\} \end{aligned} \right\} \quad (1)$$

The equation of steady motion of an ideal incompressible fluid in vector form is³

$$\underline{v} \times \underline{\omega} = \text{grad} \left(\frac{p}{\rho} + \frac{1}{2} \underline{v}^2 \right)$$

where p is the static pressure and ρ is the density. The components of this equation corresponding to α , β and z are

$$\frac{1}{h} \frac{\partial}{\partial \alpha} \left(\frac{p}{\rho} + \frac{1}{2} \underline{v}^2 \right) = v\zeta - w\eta \quad (2)$$

$$\frac{1}{h} \frac{\partial}{\partial \beta} \left(\frac{p}{\rho} + \frac{1}{2} \underline{v}^2 \right) = w\xi - u\zeta \quad (3)$$

$$\frac{\partial}{\partial z} \left(\frac{p}{\rho} + \frac{1}{2} \underline{v}^2 \right) = u\eta - v\xi \quad (4)$$

In addition we have the equation of continuity

$$\operatorname{div} \underline{v} = 0$$

or

$$\frac{\partial}{\partial \alpha} (hu) + \frac{\partial}{\partial \beta} (hv) + h^2 \frac{\partial w}{\partial z} = 0 \quad (5)$$

Also

$$\operatorname{div} \underline{\omega} = 0$$

so that

$$\frac{\partial}{\partial \alpha} (h\xi) + \frac{\partial}{\partial \beta} (h\eta) + h^2 \frac{\partial \zeta}{\partial z} = 0 \quad (6)$$

Elimination of $\left(\frac{p}{\rho} + \frac{1}{2} \underline{v}^2\right)$ from (3) and (4) gives

$$\frac{\partial}{\partial z} (w\xi - u\zeta) = \frac{\partial}{h\partial \beta} (u\eta - v\xi),$$

which can be written

$$\begin{aligned} & -u \left(\frac{1}{h} \frac{\partial \eta}{\partial \beta} + \frac{\partial \zeta}{\partial z} \right) + \frac{v}{h} \frac{\partial \xi}{\partial \beta} + w \frac{\partial \xi}{\partial z} \\ & = -\xi \left(\frac{1}{h} \frac{\partial v}{\partial \beta} + \frac{\partial w}{\partial z} \right) + \frac{\eta}{h} \frac{\partial u}{\partial \beta} + \zeta \frac{\partial u}{\partial z} \end{aligned} \quad (7)$$

From (5)

$$\frac{1}{h} \frac{\partial v}{\partial \beta} + \frac{\partial w}{\partial z} = -\frac{1}{h^2} \frac{\partial}{\partial \alpha} (hu) - \frac{v}{h^2} \frac{\partial h}{\partial \beta},$$

and from (6)

$$\frac{1}{h} \frac{\partial \eta}{\partial \beta} + \frac{\partial \zeta}{\partial z} = -\frac{1}{h^2} \frac{\partial}{\partial \alpha} (h\xi) - \frac{\eta}{h^2} \frac{\partial h}{\partial \beta}$$

Substituting in (7) we obtain

$$\begin{aligned} & \frac{u}{h^2} \frac{\partial}{\partial \alpha} (h\xi) + \frac{v}{h} \frac{\partial \xi}{\partial \beta} + w \frac{\partial \xi}{\partial z} + \frac{u\eta}{h^2} \frac{\partial h}{\partial \beta} \\ & = \xi \left[\frac{1}{h^2} \frac{\partial}{\partial \alpha} (hu) + \frac{v}{h^2} \frac{\partial h}{\partial \beta} \right] + \frac{\eta}{h} \frac{\partial u}{\partial \beta} + \zeta \frac{\partial u}{\partial z} \end{aligned} \quad (8)$$

If the approaching stream has velocity U , where U is in general a function of β and z , such that the velocity variation is not large, then the vorticity components η and ζ are small quantities of the 1st order and ξ is zero in the approaching stream (see equation (1)).

It will be assumed that on passing through the cascade the vorticity component ξ becomes a small quantity of the first order.

For the two-dimensional basic motion we should have

$$hu = U, v = w = 0, \quad (9)$$

and, since we are only considering small perturbations on this basic motion, we can regard v and w as small quantities of the first

order; also $\frac{\partial}{\partial \alpha} (hu)$ is a small quantity of the first order since it is zero for the basic motion (equation (5)). We shall neglect all quantities of the second order in equation (8) which becomes

$$\frac{u}{h^2} \frac{\partial}{\partial \alpha} (h\xi) = - \frac{u\eta}{h^2} \frac{\partial h}{\partial \beta} + \frac{\eta}{h} \frac{\partial u}{\partial \beta},$$

or

$$\frac{\partial}{\partial \alpha} (h\xi) = \frac{h^2 \eta}{u} \frac{\partial}{\partial \beta} \left(\frac{u}{h} \right). \quad (10)$$

From (2) and (4) we have

$$\frac{1}{h} \frac{\partial}{\partial \alpha} (u\eta - v\xi) = \frac{\partial}{\partial z} (v\xi - w\eta)$$

Now ξ , η , ζ and v , w are all quantities of the first order so that all terms in this equation except the first are quantities of the second order; hence to our order of approximation

$$\frac{\partial}{\partial \alpha} (u\eta) = 0,$$

so that $u\eta$ is independent of α and equal to its value for upstream. Hence

$$u\eta = U \frac{\partial U}{\partial z}, \quad (11)$$

since $u = U$ and $\eta = \partial U / \partial z$ in the approaching stream. Substituting for η from (11) and for u from (9), equation (10) becomes

$$\frac{\partial}{\partial \alpha} (h\xi) = \frac{h^4}{U} \frac{\partial U}{\partial z} \frac{\partial}{\partial \beta} \left(\frac{U}{h^2} \right)$$

or

$$\frac{\partial}{\partial \alpha} (h\xi) = h^2 \frac{\partial U}{\partial z} \left[-2 \frac{\partial}{\partial \beta} (\log h) + \frac{1}{U} \frac{\partial U}{\partial \beta} \right] \quad (12)$$

For the cascade with approach velocity variation only in the spanwise direction $\partial U / \partial \beta$ is zero. In other cases where the approach velocity variations are not large the second term on the R.H.S. can be neglected in comparison with the first term so that

$$-\frac{1}{2} (h^2) = -2h^2 \frac{U}{\pi} \frac{1}{r} (\log h);$$

the neglect of the term depending on U/r is in accordance with the approximations introduced above.

We now integrate (12) with respect to s from far upstream to far downstream obtaining

$$(h^2)_1 = -2 \frac{U}{\pi} \int_{-\infty}^{\infty} h^2 \frac{1}{r} (\log h) ds \quad (13)$$

where the suffix 1 denotes the value far downstream. This formula enables us to calculate the component vorticity along the stream direction downstream of the cascade.

Since the integration in (13) is along the stream lines of the basic flow, we can use a crude representation of the basic flow for the calculation of the integral. For this purpose the motion is assumed to be uniform in front of and behind the cascade, with $h = 1$ there; within the passage we assume that the basic motion is the irrotational flow in a circle about a vortex at the centre of the circle. For this latter motion

$$\alpha = r_0 \theta + \text{const.}, \quad \beta = r_0 \log (r/r_0), \quad h = r/r_0$$

where r , r_0 and θ are defined as in Fig. 3. Hence

$$\frac{1}{r} (\log h) = \frac{1}{r_0},$$

and

$$ds = r_0 d\alpha$$

in the passage, and thus

$$\int h^2 \frac{1}{r} (\log h) d\alpha = \left(\frac{r}{r_0} \right)^2$$

As a further approximation we shall assume that the width of the passage is small compared with the radius so that r does not differ greatly from r_0 across the passage. We may then put $r/r_0 = 1$ in the above expression which becomes

$$\int h^2 \frac{1}{r_0} (\log h) d\alpha = 0 \quad (14)$$

For the complete basic motion represented approximately by uniform approach flow, curved flow in the passage bounded by two circular arcs and uniform flow downstream, we find that the value of ζ in the outflow is, from (13) and (14),

$$v_1 = \frac{\partial w_1}{\partial z} - \frac{\partial v_1}{\partial \beta} = -2\varepsilon \frac{\partial U}{\partial z}, \quad (15)$$

where ε (Fig.3) is the total angle of deflection of the stream; we have put $h = 1$ downstream of the bend, and v_1, w_1 are the β, z components of velocity far downstream.

If the motion settles down to a steady secondary flow far downstream superposed on the general stream we shall have $\partial u / \partial x = 0$ in (7), so that the equation of continuity becomes

$$\frac{\partial v_1}{\partial \beta} + \frac{\partial w_1}{\partial z} = 0. \quad (16)$$

Introducing a stream function of the secondary motion defined by

$$v_1 = \frac{\partial \psi}{\partial z}, \quad w_1 = -\frac{\partial \psi}{\partial \beta},$$

(16) is satisfied and (15) becomes

$$\frac{\partial^2 \psi}{\partial \beta^2} + \frac{\partial^2 \psi}{\partial z^2} = -2\varepsilon \frac{\partial U}{\partial z}, \quad (17)$$

It is assumed that there is no flow across the extensions of the cascades denoted by $\beta = \pm b$ (Fig.3) or at the end walls $z = 0, z = 2\ell$, so that the boundary conditions may be taken to be $\psi = 0$ for $\beta = \pm b$ and for $z = 0, z = 2\ell$.

The theory given above is a development of Karman and Tsien's theory for small deflections⁴; there are differences between the two theories with regard to the retention of some second order terms.

2.2 Solution for a bend of high aspect ratio

If the width of the passage is small compared with the height, the secondary flow will be mainly parallel to the span of the aerofoils except near the end walls. The solution of (17) corresponding to this type of flow is

$$\psi = (\beta^2 - b^2) \varepsilon \frac{\partial U}{\partial z}, \quad (18)$$

$$v_1 = (\beta^2 - b^2) \varepsilon \frac{\partial^2 U}{\partial z^2}, \quad w_1 = -2\varepsilon \beta \frac{\partial U}{\partial z}$$

provided that $b^2 \frac{\partial^3 U}{\partial z^3}$ is negligible in comparison with $\frac{\partial U}{\partial z}$; for

the velocity distribution considered below, this condition is satisfied except near the stations marked A in Fig.4.

The expression (18) may be regarded as a particular integral of (17) and is the required solution away from the end walls and from A (Fig.4).

Near the end wall $z = 0$ we have to add a complementary function to this particular integral, to satisfy the boundary condition there. This complementary function is the well-known 'surface wave' type of solution of Laplace's equation and the complete integral is

$$\psi = (\beta^2 - b^2) \frac{\partial U}{\partial z} + \sum_{n=0}^{\infty} B_n \exp \left[-\frac{(2n+1)\pi z}{2b} \right] \cos \left[\frac{(2n+1)\pi \beta}{2b} \right] \quad (19)$$

This form satisfies the condition $\psi = 0$ for $\beta = \pm b$, and the coefficients B_n are determined by the condition $\psi = 0$ for $z = 0$. Putting $z = 0$, $\psi = 0$ in (19) we obtain

$$(\beta^2 - b^2) \left(\frac{\partial U}{\partial z} \right)_{z=0} + \sum_{n=0}^{\infty} B_n \cos \left[\frac{(2n+1)\pi \beta}{2b} \right] = 0.$$

Hence

$$B_n = \frac{32(-)^n b^2}{(2n+1)3\pi^3} \left(\frac{\partial U}{\partial z} \right)_{z=0},$$

so that the coefficients decrease rapidly with increase of n .

We may thus to the order of accuracy with which we are concerned neglect all except the first term in the series in which case we may replace the form (19) by the expression

$$\psi = (\beta^2 - b^2) \frac{\partial U}{\partial z} \left[1 - \exp \left(-\frac{\pi z}{2b} \right) \right] \quad (20)$$

The values of the two component velocities are then

$$v_1 = \frac{\pi}{2b} (\beta^2 - b^2) \frac{\partial U}{\partial z} \exp \left(-\frac{\pi z}{2b} \right) + (\beta^2 - b^2) \frac{\partial^2 U}{\partial z^2} \left[1 - \exp \left(-\frac{\pi z}{2b} \right) \right], \quad (21)$$

$$v_2 = -2\beta \frac{\partial U}{\partial z} \left[1 - \exp \left(-\frac{\pi z}{2b} \right) \right] \quad (22)$$

The equations (20), (21), (22) are valid for the upper half of the channel $0 \leq z \leq l$.

3 Experiments

An experimental check of the theory was made at the first bend of the R.A.E. $4' \times 3'$ Wind Tunnel. The velocity distribution approaching the corner (Fig.4) is such as to give an almost linear variation of velocity for the first 15 inches away from the wall. The centre passage of the cascade which has a span of 6 ft. and a width $2b = 4.8$ ins., was used. The experiments consisted of measurements with a yawmeter, of the angles of deflection of the stream in the β and z directions. The traverses covered the upper part of the span of the turning vanes over which the approach velocity variation was appreciable. Because of the uncertainty of the main stream velocity the angles were obtained

by reading the pressure difference between the two tubes of the yawmeter for several angular settings and interpolating for zero difference. The slopes of the curves then also give the local velocity.

To avoid interference from neighbouring passages, the trailing edge of the outer of the two turning vanes enclosing the centre passage was extended as shown in Fig.5. The heads of the yawmeter were inserted about $\frac{1}{4}$ in. into this passage.

4 Comparison of experiments with theory

A direct comparison of the measured secondary velocities with the theoretically predicted values is given in Figs.6 and 7. Fig.6 shows the velocities in the spanwise direction and Fig.7 those normal to the span. In both cases the velocities are made non-dimensional by dividing by the mean speed U_M across the tunnel section approaching the corner.

The theoretical values of v_1 and w_1 are given by equations (21) and (22) using the velocity distribution given in Fig.4.

It will be seen in Fig.6 that agreement between theory and experiment is quite good. In Fig.7 the agreement in the non-linear region is not so good though one might say that the experimental curve is a smoothed out version of the theoretical curve. This really means that second order effects are coming into play. The exponential decay law predicted near the end wall is followed closely.

In Fig.8 a complete comparison of experiment with theory over the whole region considered is given by plotting the streamlines of the lateral motion, i.e. the lines $\psi = \text{constant}$. The values of appropriate to each streamline have been made non-dimensional by dividing by $U_M b$. The theoretical streamlines are given by equation (20) using the velocity distribution of Fig.4.

The experimental streamlines have been obtained by integration of the secondary flow velocities. Except in the region near the end wall, the steps in the process are as follows. We choose the streamline along the boundaries of each passage as $\psi = 0$. Then the value of ψ at any point (β, z) is given by

$$\psi = - \int_{-b}^{\beta} w_1 d\beta .$$

where the integration is performed at $z = \text{constant}$. In order that both vanes bounding a passage shall be streamlines it is necessary that

$$\int_{-b}^b w_1 d\beta = 0 \quad \text{for } z = \text{constant}$$

and this condition was satisfied by adjusting the zero of the measurements. The maximum adjustment required was a change of the zero of w_1/U_M of 0.04.

Near the end wall the streamlines were obtained from the integral $\int_0^z v_1 dz$. To join the streamlines the values of ψ given

by the v_1 measurements were plotted against z and the value of z taken, for which v_1 is a maximum. Call these values z_0, z_0' .

At z_0 since $\frac{dv_1}{dz} = 0$, the value of v_1 must be adjusted to zero and

v_1 will then be given by

$$v_1 = v_{10} + \int_{z_0}^z v_1' dz, \quad v_{10} = \text{constant.}$$

The maximum adjustment required was a change of zero of v_1/U of 0.04.

The differences between theory and experiment are more apparent in Fig.8, the maximum value of $-\frac{v_1}{U_M b}$ being 0.255 from the measurements and 0.204 from the calculations. However the general characteristics of the flow are the same and the experiments confirm that the physical basis of the theory is sound. The discrepancies can be attributed to the second order effects which have been neglected in the theory. For example, consider an angular deflection of the flow of 25° at the end of a passage. Assuming this deflection to grow linearly round the curved part of the passage then the lateral deviation of a streamline through the passage is about 8 ins. This deviation is sufficient to modify the velocity gradient appreciably, and, in fact, the measurements show that the velocity gradient along the centre-line of the passage becomes reversed for the first 5 ins. away from the wall.

5 Induced drag

The power loss associated with the secondary flow is given by

$$\frac{1}{2} \int_0^{2\pi} \int_{-t}^b U (v_1^2 + w_1^2) dp dz$$

We may therefore define an induced drag pressure drop coefficient

$$C_{p_i} = \frac{1}{4b\epsilon} \int_0^{2\pi} \int_{-t}^b \frac{U}{U_M} \left\{ \left(\frac{v_1}{U_M} \right)^2 + \left(\frac{w_1}{U_M} \right)^2 \right\} dp dz.$$

C_{p_i} has been calculated both by integration of the measured secondary flow and from the theoretical distributions. The respective values are 0.032 and 0.019. The measured value is thus of the order of half the profile drag pressure drop coefficient² and should be taken into account in estimating corner loss.

It is possible that there is some recovery of the energy in the secondary flow when mixing between the flows from adjacent passages takes place, and in the case of a wind tunnel there may be some recovery at subsequent corners. However it is unlikely that this recovery would exceed 50%.

6 Conclusion

The experiments show that the theory gives a good approximation to the secondary flow in a cascade with a 90° corner and a large variation of approach velocity.

LIST OF SYMBOLS

x,y	Cartesian co-ordinates in plane normal to span of cascade.
α,β	Orthogonal curvilinear co-ordinates in (x,y) plane.
z	Distance measured along span from one end wall.
h	The elements of length in $(\alpha\beta)$ co-ordinates are $h\alpha$ and $h\beta$.
r	Radius at any point of passage between two vanes.
r_0	Mean radius of passage.
$2b$	Width of passage at exit.
2ℓ	Height of passage at exit.
θ	Angle of deflection at corner up to point α .
ϵ	Total angle of deflection at corner.
$\underline{v}$	Velocity vector with components u,v,w in directions α, β, z .
U	Velocity of approach to corner.
v_1, w_1	Values of v, w downstream of corner.
$\underline{\omega}$	Vorticity vector with components ξ, η, ζ in directions α, β, z .
ψ	Stream function such that
$v_1 = \frac{\partial \psi}{\partial z} \quad w_1 = - \frac{\partial \psi}{\partial \beta}$	
p	Static pressure.
ρ	Density.

LIST OF REFERENCES

<u>No.</u>	<u>Author</u>	<u>Title, etc.</u>
1	Carter and Cohen	Preliminary investigation into the three dimensional flow through a cascade of aerofoils. Power Jets Report No.R.1180. A.R.C.10065 Feb., 1946.
2	Winter	Comparative tests of thick and thin turning vanes in the 4 ft. x 3 ft. wind tunnel. R.A.E. Report No. Aero.2217. A.R.C.10976 Aug., 1947.
3	Ed. Goldstein	Modern Developments in Fluid Dynamics. The Clarendon Press. p.100.
4	Karman and Tsien	Lifting-line Theory for a Wing in non-uniform Flow. Quarterly of Applied Maths. Vol.III p.1 April, 1945.

Attached:

Figs.1 to 8 Drg.Nos.22475.S to 22483.S

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FIG. 1.

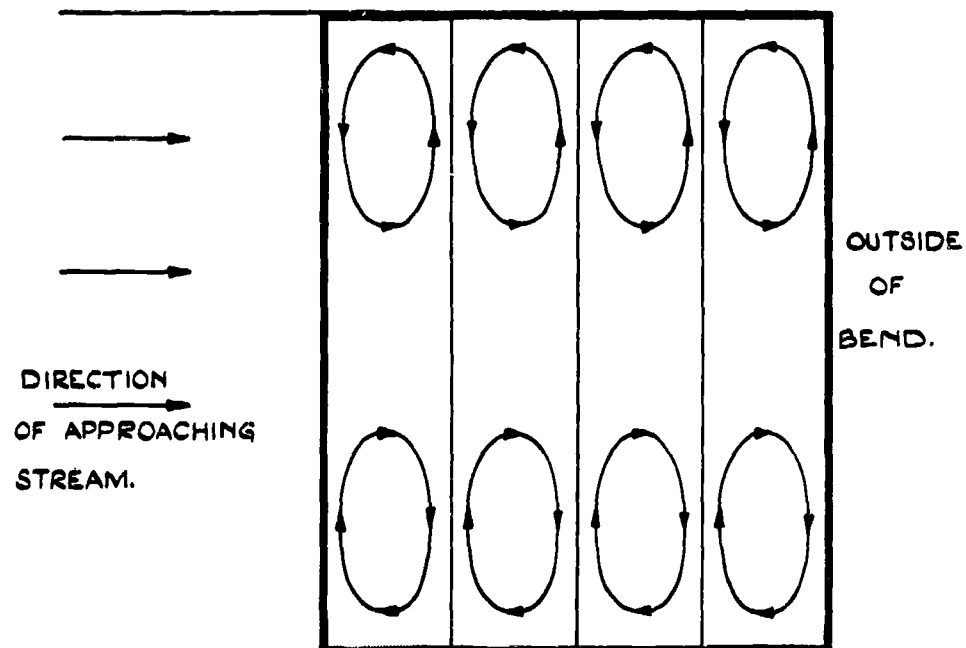


FIG.1. CHARACTER OF SECONDARY FLOW AT OUTLET OF A CASCADE IN A DUCT.

FIG.2.

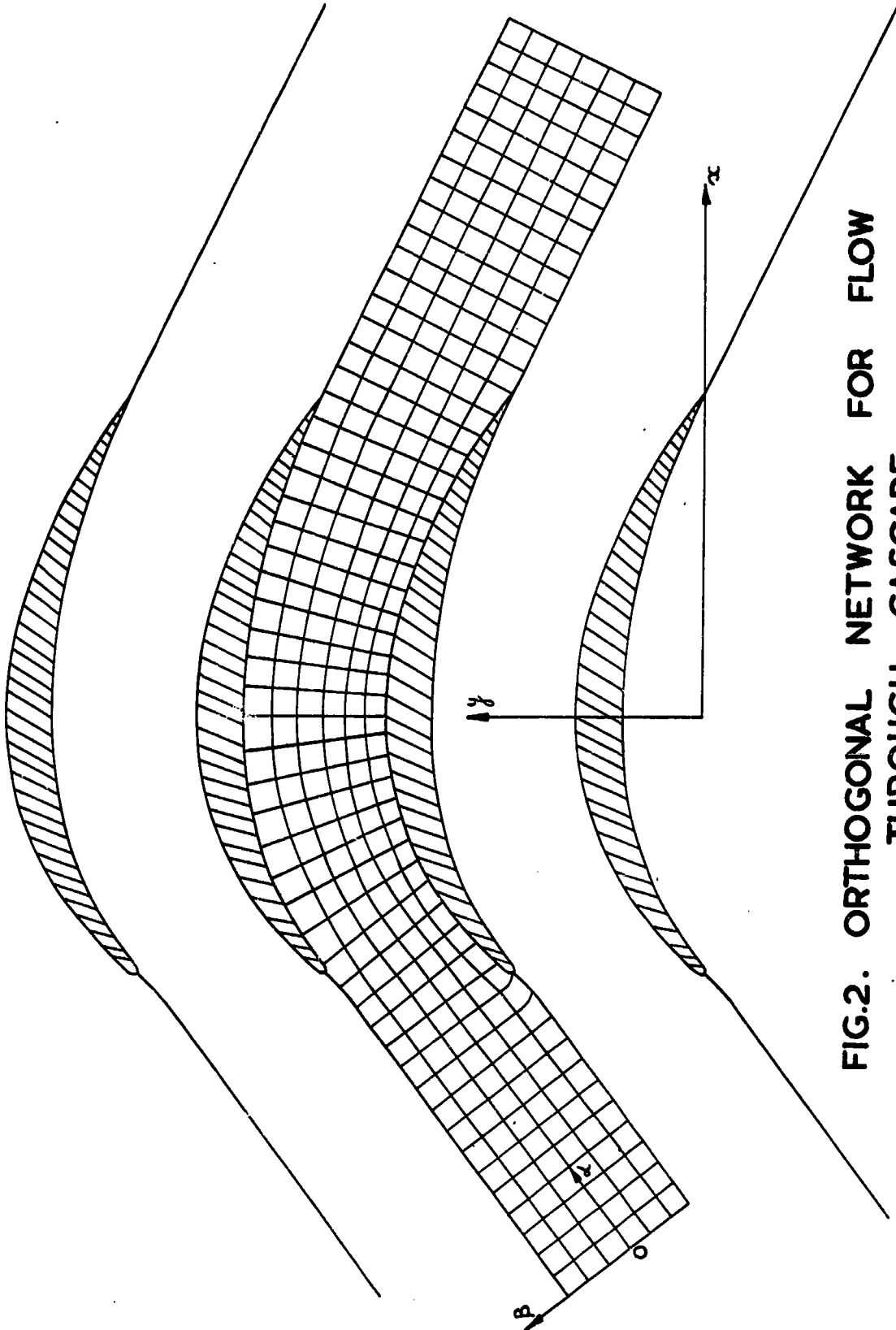
FIG.2. ORTHOGONAL NETWORK FOR FLOW
THROUGH CASCADE.

FIG. 3.

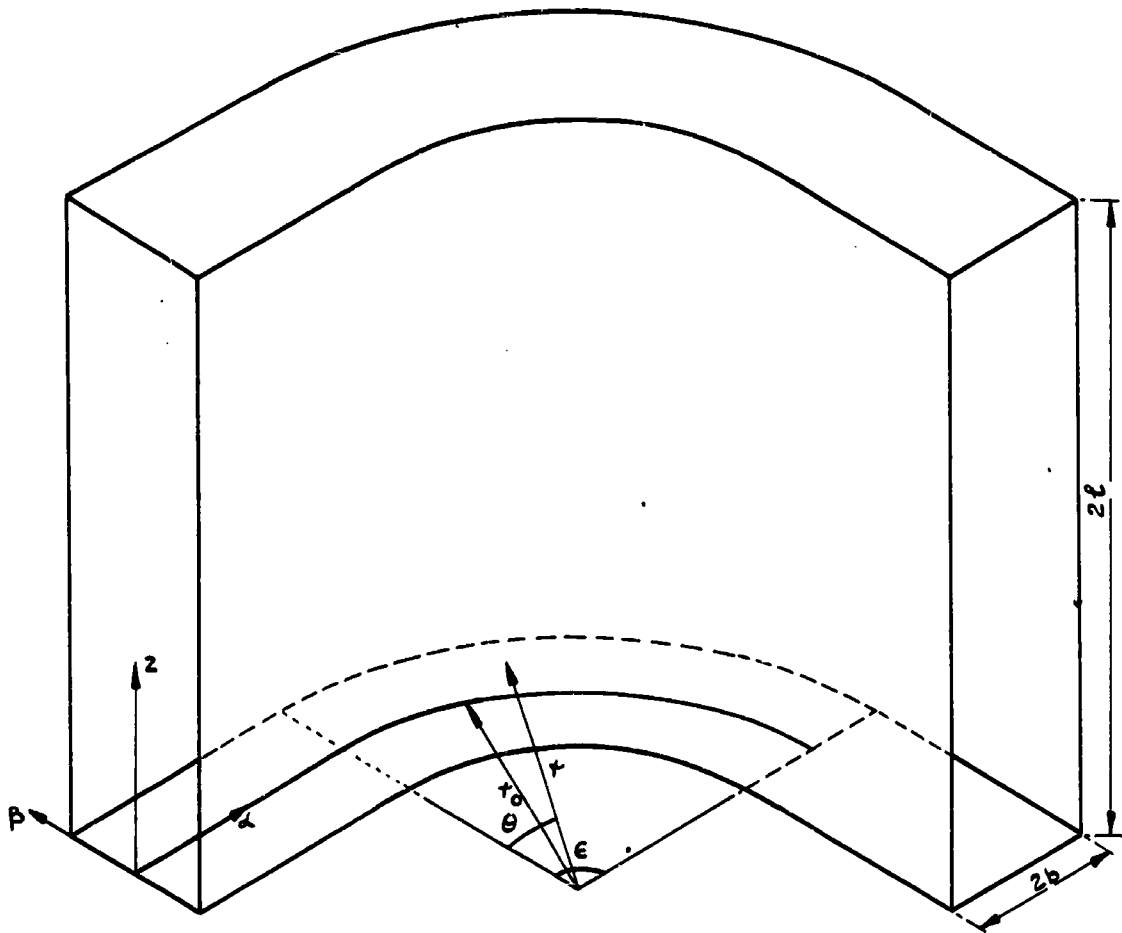


FIG.3. SKETCH ILLUSTRATING CHANNEL USED
TO REPRESENT ONE PASSAGE OF CASCADE.

FIG. 4.

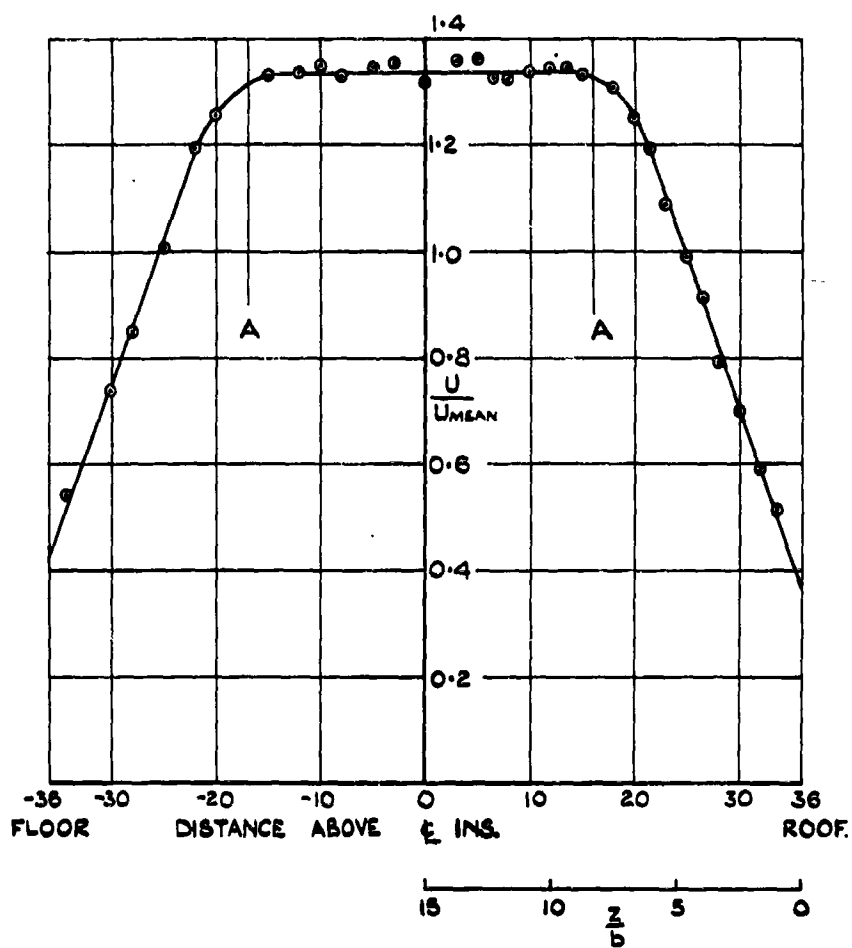


FIG. 4. VELOCITY DISTRIBUTION, APPROACHING FIRST CORNER OF 4'x 3' WINDTUNNEL.

FIG.5.

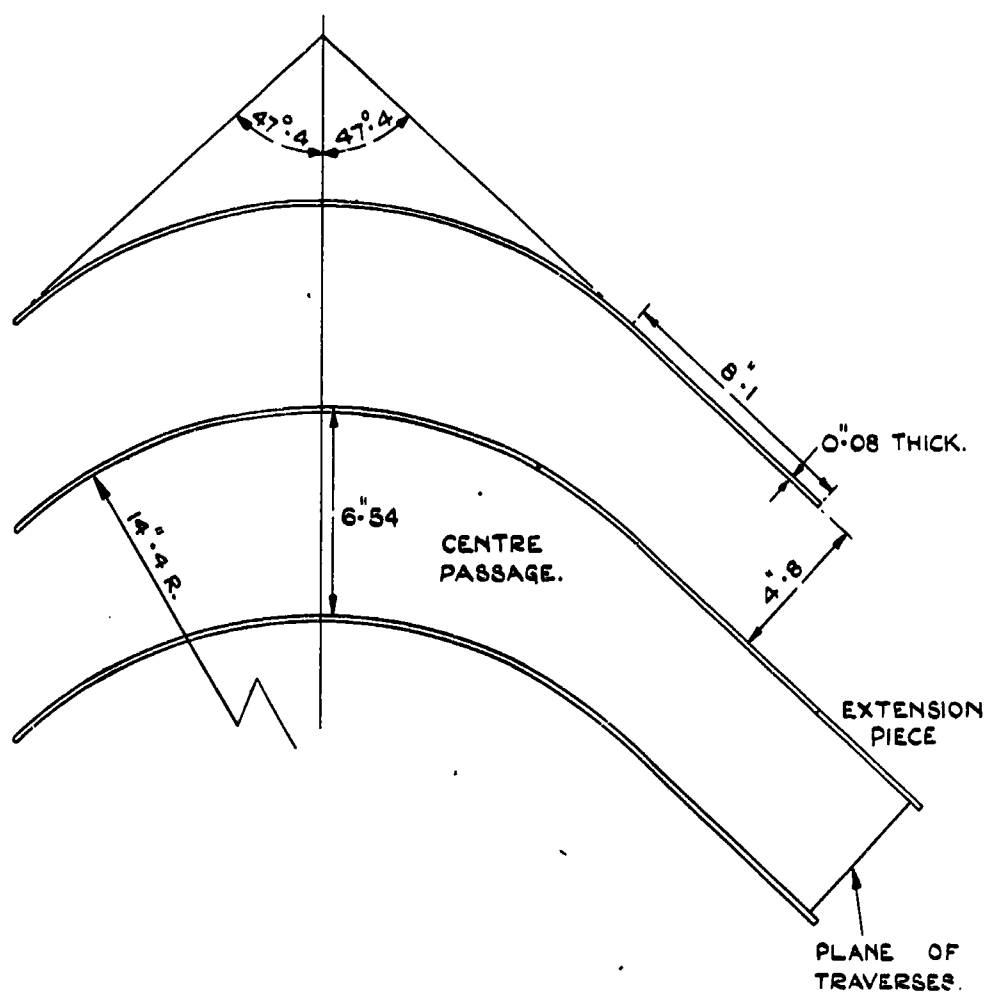
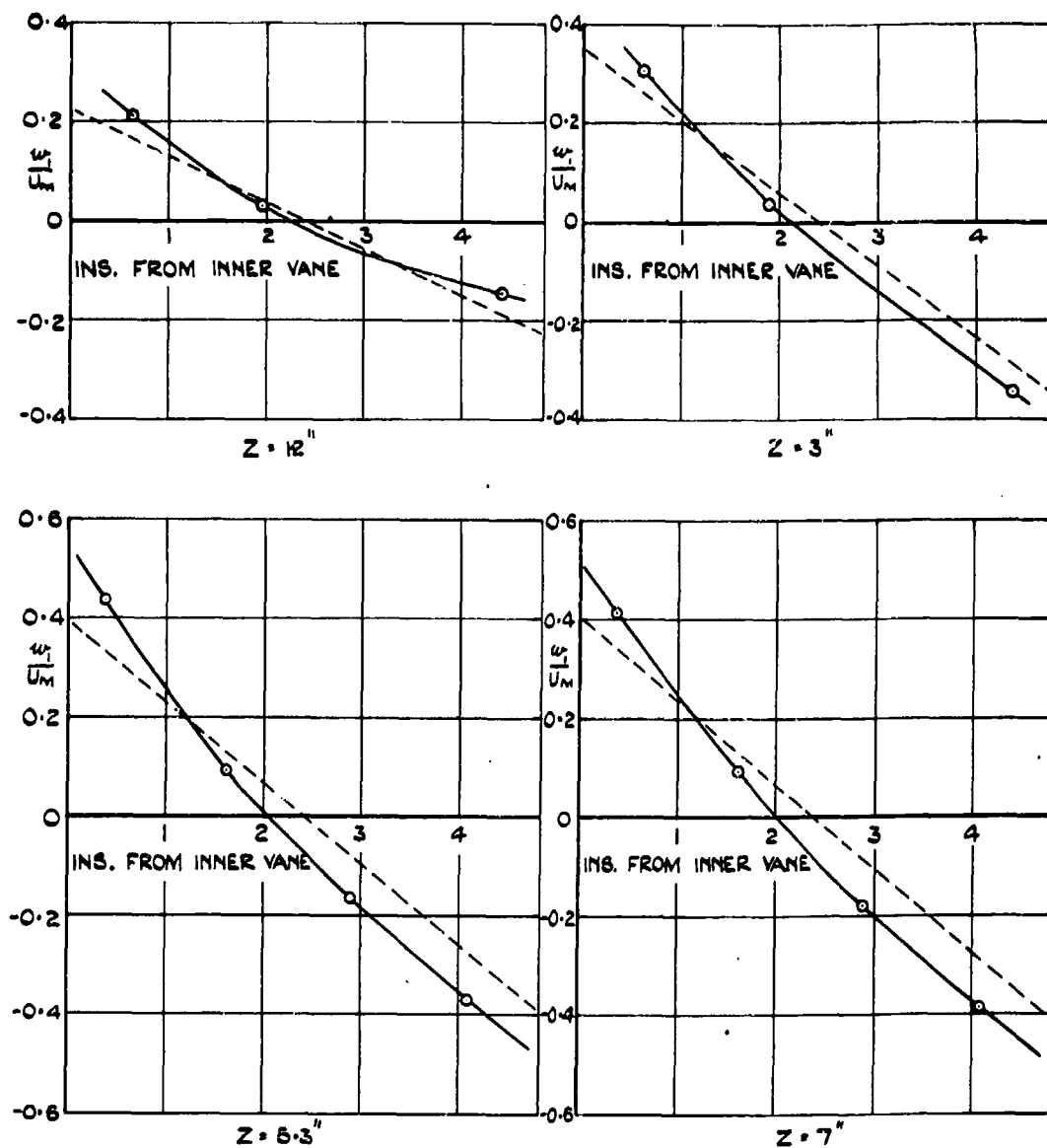


FIG.5. TURNING VANES AT FIRST CORNER OF 4' x 3' WIND TUNNEL.

FIG. 6.



Z = CO-ORDINATE ALONG SPAN OF TURNING VANES MEASURED FROM TUNNEL ROOF.

w_z = VELOCITY IN Z DIRECTION.

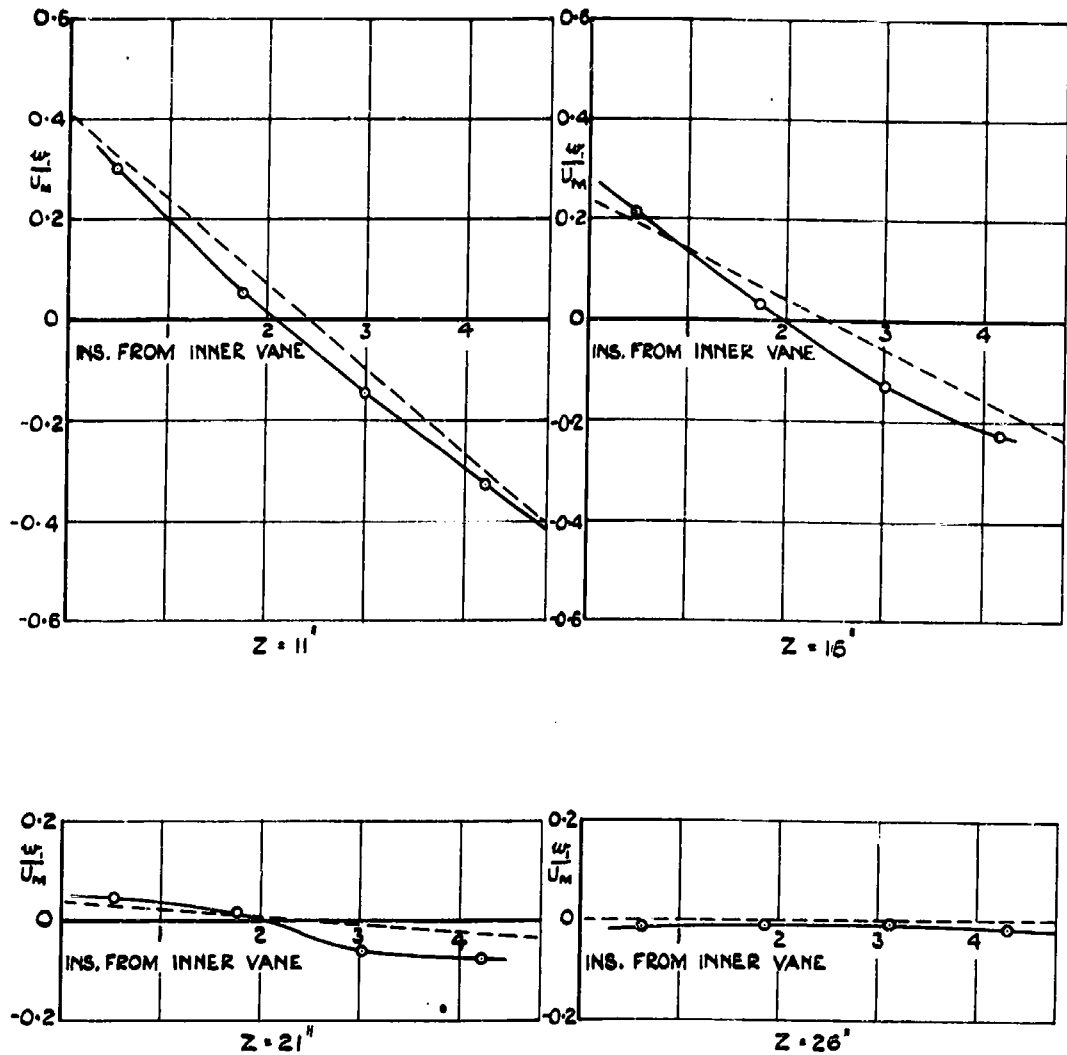
U_m = MEAN VELOCITY OF FLOW APPROACHING BEND.

—— MEASUREMENTS.

---- THEORY.

FIG.6. SECONDARY FLOW IN SPANWISE DIRECTION
COMPARISON OF THEORY & EXPERIMENTS.

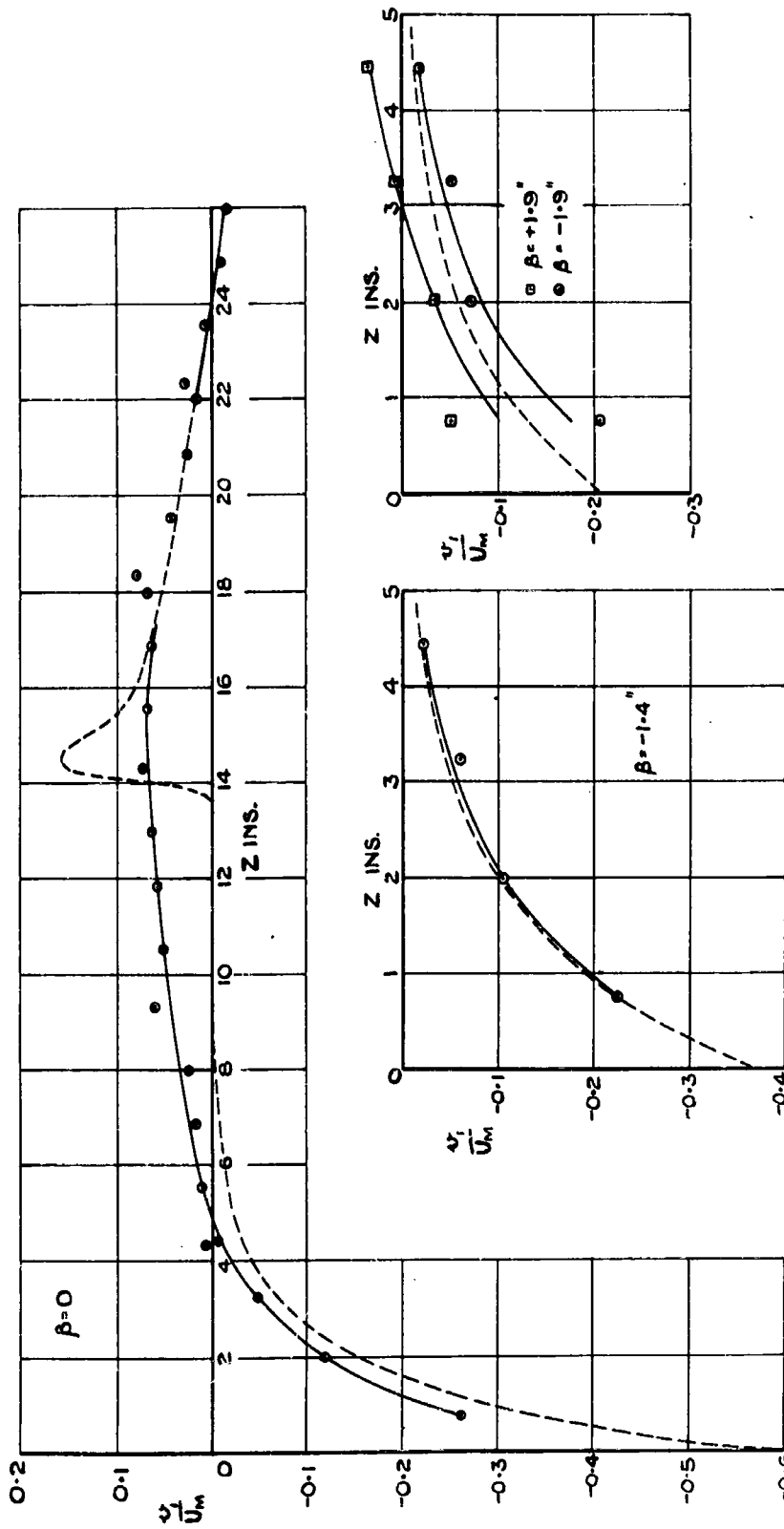
FIG. 6 (CONT.)



Z = CO-ORDINATE ALONG SPAN OF TURNING VANES MEASURED FROM TUNNEL ROOF.
 w_z = VELOCITY IN Z DIRECTION.
 U_m = MEAN VELOCITY OF FLOW APPROACHING BEND.
 — MEASUREMENTS.
 ---- THEORY.

FIG. 6 (CONT.) SECONDARY FLOW IN SPANWISE DIRECTION — COMPARISON OF THEORY & EXPERIMENTS.

FIG. 7.



Z = CO-ORDINATE ALONG SPAN OF TURNING VANES MEASURED FROM TUNNEL ROOF
 β = CO-ORDINATE NORMAL TO SPAN OF TURNING VANES. ZERO AT CENTRE OF GAP & POSITIVE TOWARDS OUTSIDE OF BEND.

v_1 = SECONDARY VELOCITY IN β DIRECTION.

U_m = MEAN VELOCITY OF FLOW APPROACHING BEND
 ——— MEASUREMENTS. - - - - - THEORY.

FIG. 7. SECONDARY FLOW NORMAL TO SPAN — COMPARISON OF THEORY & EXPERIMENTS.

FIG. 8.

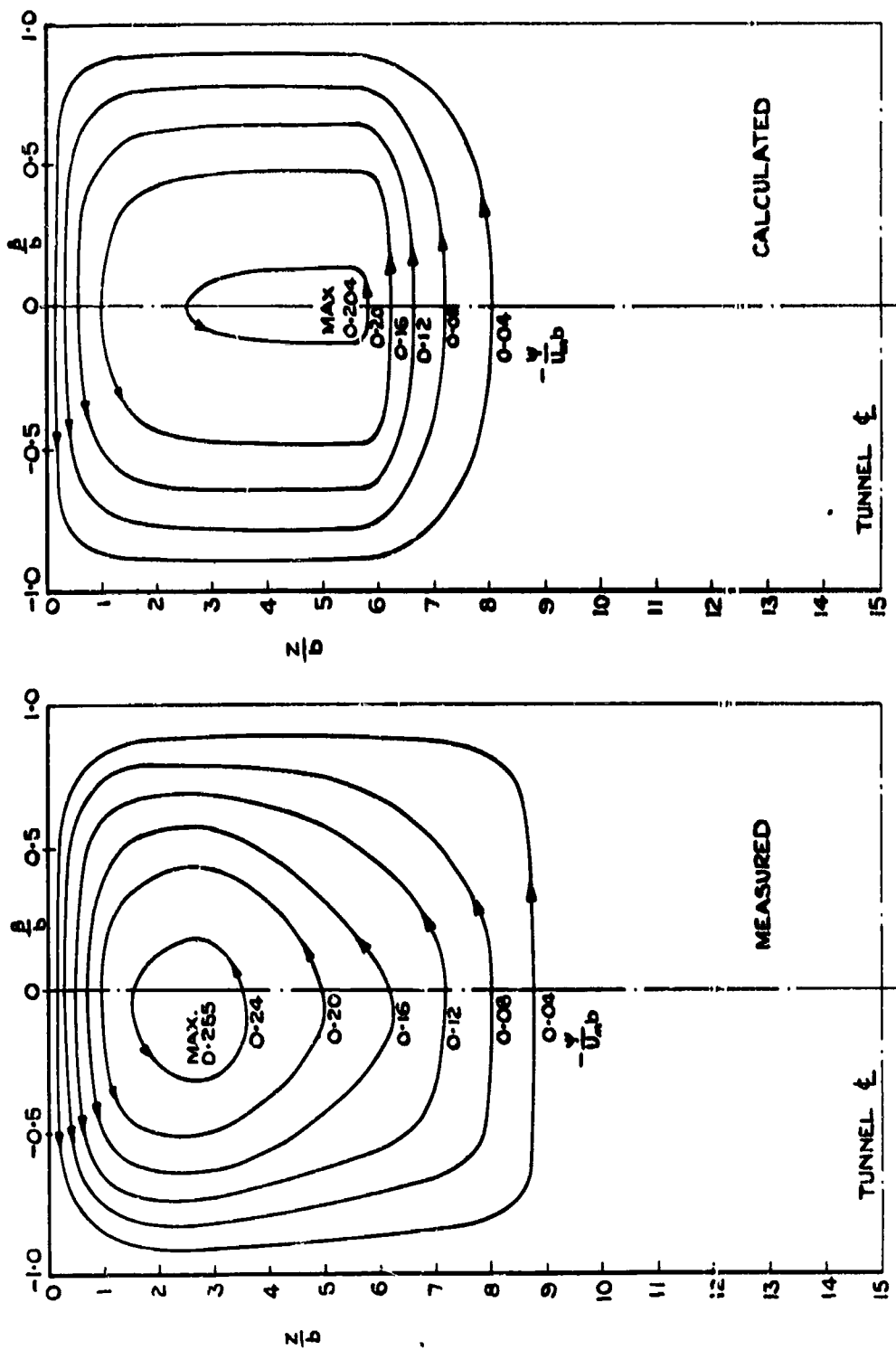


FIG. 8. COMPARISON OF MEASURED & CALCULATED SECONDARY FLOW STREAMLINES IN PASSAGE BETWEEN TWO TURNING VANES.