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An Application of Kinematic Analysis to Tropical Weather

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(None)

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(None)

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An attempt was made to apply wind and pressure kinematics for the accurate weather forecasting in tropical areas. Fundamental theories are re-evaluated, a basis is determined on which a physical explanation for the existing weather conditions may be made, and a search is made for empirical rules based on statistical correlations. The basic reasoning is started out from Newton's laws of motion, to derive the vorticity equations. From this, an expression for divergence is obtained, which is one of the fundamental factors for the determination of the cause of large scale weather phenomena. The theory of divergenesis is then applied to an easterly wave to determine the north-south distribution of the weather, and westerly droughts are analyzed to obtain further conclusions helpful in forecasting.

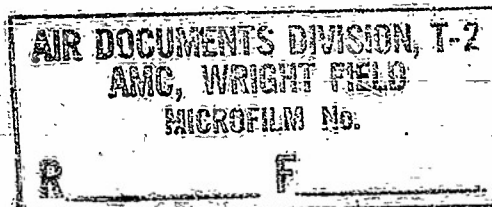
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TECHNICAL REPORT 105-51

AN APPLICATION OF KINEMATIC ANALYSIS
TO
TROPICAL WEATHER



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AN APPLICATION OF KINEMATIC ANALYSIS

TO

TROPICAL WEATHER

EARL C. KINDLE

CWO USA

2 MAY 1945

AAF WEATHER WING
REGIONAL CONTROL OFFICE, 9TH WEATHER REGION
MORRISON FIELD, FLORIDA

FOREWARD

This report consists of some notes on the discussions given by the Ninth Weather Region tropical technical consultant team, 1944.

Since the trend of research is toward the kinematics of wind and pressure, a large portion of this paper will deal with a discussion of certain concepts of dynamic meteorology from a physical viewpoint.

In a few cases, the physical interpretation of some mathematical representations will sacrifice rigorism for purposes of clarity and simplicity.

ERRATA SHEET

AN APPLICATION OF KINEMATIC ANALYSIS TO TROPICAL WEATHER
Earl C. Kindle, CWO, USA.

2 May 1945
AAF WEATHER WING
REGIONAL CONTROL OFFICE
9th WEATHER REGION
MORRISON FIELD, FLORIDA

1. Page 6, eq. (2a) " v " in the term " $\frac{v}{r}$ " should be a capital " V ".
2. Page 23, third paragraph, 6th line, " > 0 " should be inserted before "divergence."
3. Page 26, in Fig. 12, " $r_t = \infty$ ".
4. Page 27, last 2 lines, and 1st line of page 28, statement that there is "greater radius of curvature" is correct, but reason given is incorrect. Correct reason: equation #(24) applied to Fig. 13.
5. Page 41, Figure 24, change to: " c never $\geq \cos \psi$ ".

AN APPLICATION OF KINEMATIC ANALYSIS TO TROPICAL WEATHER

INTRODUCTION

Prior to the outset of the present conflict, the requirement for accurate forecasting in tropical areas was comparatively small. Commensurate with this requirement, the development of forecasting techniques was also limited. However, after the beginning of hostilities, there was created immediately an urgent demand for forecasts in tropical areas, both for actual operations in those regions and the movement of aircraft through tropical areas to the active theaters. At first, we, as forecasters trained and experienced in middle latitude weather, discovered very rapidly that much of our theoretical meteorology and forecasting techniques as applied to middle latitudes were ineffectual in meeting the requirements of operational forecasting in latitudes less than 25 degrees. Then quite naturally the most obvious and expedient development was the rise of empirical forecasting rules based on our own limited experience and the experience of older civilian forecasting agencies that were already active in the area. These rules enabled the forecasters to meet the problems of forecasting for the aircraft operations that faced them at that time. In this process there was a tendency for us to disregard practically all of the knowledge of theoretical meteorology that we then possessed, as very little of the theory seemed applicable to the problem of making route and terminal forecasts for aircraft operations in this area.

These methods, however, still leave very large gaps in forecasting percentages. To close these gaps, there are two approaches that might be

considered: (1) Further search for empirical rules based on statistical correlations which are becoming less and less obvious with time. (2) A search for newer instruments and charts that might reveal the intricate nature of some of these concealed correlations. (3) Re-evaluation of our fundamental theories and determining a basis on a physical explanation for the existing weather conditions might be made. By understanding the reason for the empirical correlations, it becomes possible to determine the conditions under which the empirical rules do not apply, and therefore logical corrections for the resulting discrepancies can be made. For example, if an empirical rule verified 70% of the time, from that viewpoint alone, we concede a 30% "bust." However, if we can physically explain a specific weather distribution and the previously established empirical correlation, we should be able to determine when the correlation is invalid and make logical compensations therefrom.

So that no significant weather producing force will be neglected, it seems necessary that we start with the basic concept of atmospheric motion that applies universally at the Equator, at the North Pole, in middle latitudes, in a wash bowl or in a pitot-tube. From there we can, by calculation and evaluation, eliminate the factors that are not significantly pertinent.

From Newton's laws of motion, which we can consider sufficiently reliable to start out basic reasoning, the derivation of the vorticity equation may be obtained. From this we can obtain an expression for divergence which is one of the fundamental factors we wish to investigate in order to determine the cause of large scale weather phenomena. Assuming

the solenoid factor, the vertical velocities and friction to be negligible. Rossby gives the following expression:

$$(1) \quad \frac{d(f + \zeta)}{dt} = - (f + \zeta) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

The left-hand side of the above expression gives the time rate of change of the absolute vorticity, where f equals the component of vorticity given by the rotation of the earth and is equal to $2 \Omega \sin \phi$. ζ is the vorticity relative to the earth and is equal to $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ using normal cartesian coordinate system.

On the right-hand side above, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ equals horizontal divergence. From this equation, it is readily seen that the rate of change of the relative vorticity is a pertinent factor in the distribution of convergence. If there is no convergence or divergence, the rate of change of the relative vorticity depends on the rate of change of latitude, as the latitude is the only variable in f . By analyzing the degree of deviation of the actual changes of vorticity from the changes that would be produced by changes in f alone, we can draw some conclusions about the distribution of convergence. In order to analyze convergence employing these concepts, a thorough understanding of the values and terms used is necessary.

VORTICITY

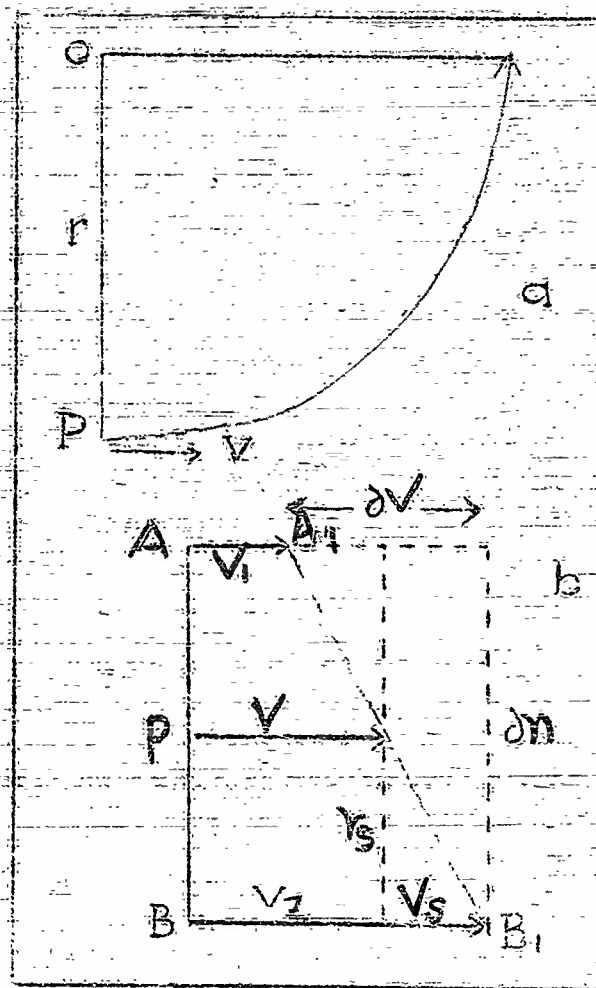
The first of these concepts to be reviewed will be vorticity. We will be concerned with a definition and a physical description, and then an attempt will be made to show how this physical concept may be expressed mathematically.

Vorticity is defined as a tendency of a body to spin. Consider any mass at point P rotating about a point O with linear velocity V, it is apparent that one of the components of spin is the rate of rotation of the mass about point O. Consider direction of V positive; r positive for cyclonic motion and negative for anticyclonic motion. The rotational component, ζ_r , is directly proportional to the linear velocity and is inversely proportional to the radius of curvature r. Using appropriate units, it may then be stated that the component of vorticity due to rotation is equal to the velocity divided by the radius of curvature of the stream flow.

FIG. 1

$$\begin{aligned} \zeta_r &= KV \\ \text{eq. 2} \quad \zeta_r &= K \frac{1}{r} \\ \zeta_r &= \frac{V}{r} \end{aligned}$$

$$\begin{aligned} \zeta_s &= \frac{V_s}{r_s} = \frac{\frac{1}{2} \partial V}{\frac{1}{2} \partial n} = \frac{\partial V}{\partial n} \\ \text{eq. 3} \quad \zeta &= \zeta_r + \zeta_s \\ \zeta &= \frac{V}{r} + \frac{\partial V}{\partial n} \end{aligned}$$



Another factor that tends to produce a spin is the shearing stress acting on the mass. If we have the velocity distribution of V_1 and V_2 to the left and right of point P, respectively, where V_2 is greater than V_1 , ($V_2 - V_1$ equals $\partial V > 0$). There will be a tendency for this shear to produce a spin in the mass M. This will be reflected by a rotation, in a specified unit of time, of axis AB to position indicated by $A_1 B_1$. (See Figure 1b.)

Using n as a coordinate normal to the velocity V and measured positively to the right of flow, $\frac{\partial V}{\partial n}$ will give an expression for this shear.

Using the same system in measuring this rotation as was used in the previous case, to an observer at point P moving with velocity V , this component of rotation is equal to $\frac{V_s}{r_s}$. And by construction, V_s equals $\frac{1}{2} \partial V$ and r_s equals $\frac{1}{2} \partial n$, thus $\frac{V_s}{r_s} = \frac{\partial V}{\partial n}$. This will be the spin produced by the shear factor, and will be referred to as $\mathcal{S}_s$.

This $\frac{\partial V}{\partial n}$ is the shear of the wind flow across P normal to the direction of motion divided by the distance between the two values of wind measured. By inductive analysis it can be seen that there are no other factors that tend to produce a sense of rotation in a horizontal plane other than those just mentioned. The vorticity in this plane is equal to $\mathcal{S}_r + \mathcal{S}_s$ which is the sum of the shearing stress acting on the mass plus the rotational motion of the mass itself, then substituting.

$$(3) \quad \mathcal{S} = \mathcal{S}_r + \mathcal{S}_s = \frac{V}{r} + \frac{\partial V}{\partial n}$$

This is an expression in a coordinate system that is not identical with the cartesian coordinate system, which is normally used. It seems advisable here to show how this expression may be given in the cartesian

coordinate system and still be a valid evaluation of the same physical concept. If the previous distribution of velocity is superimposed along the x axis such that V is numerically equal to u, V_1 is equal to u_1 , and V_2 is equal to u_2 ; we should be able to transpose $\frac{V}{r} + \frac{\partial V}{\partial n}$ into an expression in the xy system.

FIG. 2

In figure 2a

$$r \perp V; r' \perp V'$$

$$\frac{V}{r} = \tan \angle \theta$$

$$\frac{\partial V}{\partial x} = \tan \angle \alpha$$

$$\angle \alpha = \angle \theta$$

$$\therefore \frac{V}{r} = \frac{\partial V}{\partial x}$$

In figure 2b

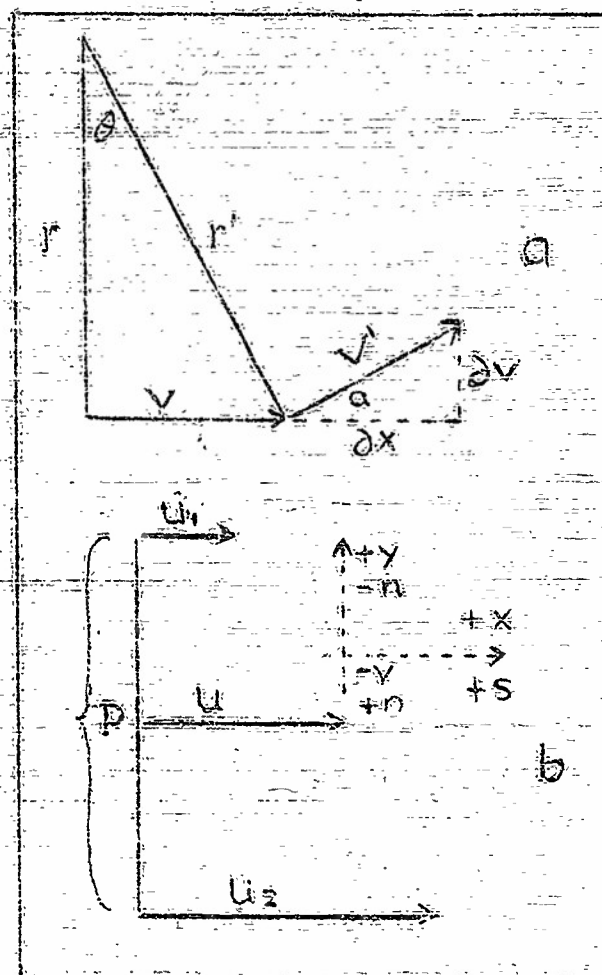
$$\partial V = \partial u$$

$$\partial n = -\partial y$$

$$\frac{\partial V}{\partial n} = -\frac{\partial u}{\partial y}$$

Substituting in eq. 3

$$\text{eq. 4 } \oint = \frac{\partial V}{\partial x} - \frac{\partial u}{\partial y}$$



DIVERGENCE

In this analysis, there has been assumed a direct correlation between divergence and convergence and the occurrence of weather. This, of course,

is based upon the relationship between divergence and convergence and vertical motion. An understanding of the mechanics of this relationship may be obtained by an investigation of the continuity equation.

The Continuity Equation. (eq. 5)

This equation is merely a mathematical accounting system for the mass and energy in any fluid motion; that is, it is an expression based upon the laws of conservation of energy and mass in relation to compressible fluids.

$$(5) \quad \text{div } \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

The left-hand member of this equation is a notation for total divergence of a given fluid mass; sometimes the notation $\text{div}_3 \vec{V}$ is used.

The right-hand member gives the sum of the divergence along all three axes of the x, y, and z coordinate system; hence a time rate of change of specific volume. From this, the continuity equation may also be expressed as

$$(6) \quad \text{div } \vec{V} = -\frac{1}{\alpha} \frac{\partial \alpha}{\partial t}$$

α represents the specific volume and ρ represents the density. As the specific volume is equal to the reciprocal of the density a third expression for the continuity equation is

$$(7) \quad \text{div } \vec{V} = -\frac{1}{\rho} \frac{\partial \rho}{\partial t}$$

It is obvious that any significant absolute value of the total divergence will be accompanied by a change in density and specific volume.

Consider the cube A in Fig (3a), with positive divergence occurring along all three axes. The net increase in velocity per unit distance along each axis is represented by the vectors extended along each respective axis. The right-hand portion of Fig. (3a) represents the expanded cube a unit of time later. This physical portrayal is in agreement with the previous mathematical expression showing that for positive values of divergence, there will be decreasing density values and increasing values of specific volume.

This consideration is made using a varying density factor. Considering the density constant makes the analysis of the relationship between horizontal divergence and vertical motion more apparent.

$$(8) \quad \text{div } \mathbf{V} = -\frac{1}{\rho} \frac{\partial \rho}{\partial t}$$

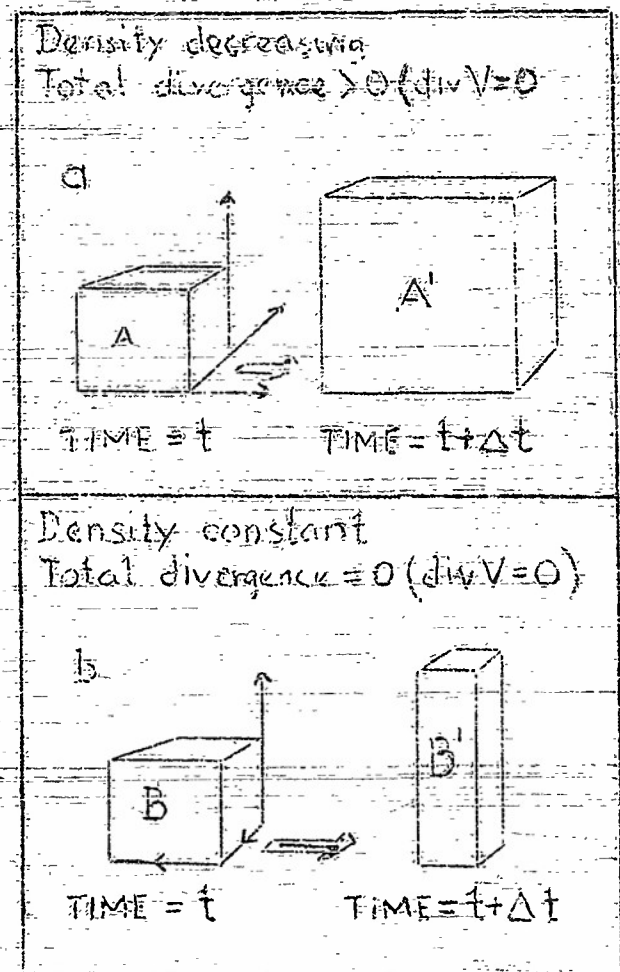
$$(9) \quad \text{then } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$(10) \quad \text{and } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial z}$$

With the density constant, the value of the total divergence must equal zero. From this, it is apparent that with horizontal shrinking (negative divergence), and with total divergence equal to nil, there must be vertical stretching to maintain constant specific volume and density.

For example, consider the fluid mass A in Fig. (3b) in which there is horizontal shrinking (negative divergence), the density is constant, and the total divergence is equal to zero

FIG. 3



($\text{div } \vec{V} = 0$): then the horizontal shrinking must be balanced by vertical stretching.

In summary, vertical convergence will accompany horizontal divergence and vertical divergence will accompany horizontal convergence.

THE UNITS OF DIVERGENCE

By breaking the divergence expression into units, the expression is obtained which gives t^{-1} as a final unit for divergence (this t^{-1} should not be confused with the t^{-1} unit used for measurement of angular velocity). Some authorities retain the units of distance in the expression for purpose of clarification.

For example, if the value of $-.5t^{-1}$ is a value given to determine the rate of divergence, it means that particles one (1) meter apart are converging (note the sign is negative) at a rate of five-tenths meters per second. It is readily seen that the same value is valid, if a different unit of distance is used, as long as the unit used to measure the distance in the denominator is the same as the distance unit in the velocity measurement. That is, the value of divergence given above implies that particles that are one (1) mile apart, are converging at a rate of five-tenths miles per second. It is obvious that the convergence value given here is much larger than is ever observed in atmospheric motion.

Fig. (4) gives some of the ranges of convergence and divergence commonly observed in the lower levels of the atmosphere with the approximate degree of weather that should be associated with that value.

SOME GENERAL RANGES FOR DIVERGENCE AND ASSOCIATED WEATHER

Convergence

<u>Value of</u>	<u>Approximate Weather Intensity</u>
$-0.00000001 \text{ sec}^{-1}$ or $-10^{-8} \text{ sec}^{-1}$	Very little weather
$-0.000001 \text{ to } -0.00001 \text{ sec}^{-1}$ or $-10^{-6} \text{ to } -10^{-5} \text{ sec}^{-1}$	Light precipitation
-0.0001 sec^{-1} or $-10^{-4} \text{ sec}^{-1}$	Heavy precipitation

Divergence

$0.00000001 \text{ sec}^{-1}$ or 10^{-8} sec^{-1}	Very light subsidence
$0.000001 \text{ to } 0.00001 \text{ sec}^{-1}$ or $10^{-6} \text{ to } 10^{-5} \text{ sec}^{-1}$	Moderate subsidence

Note that the order of magnitude of 10^{-6} , whether positive or negative, is so small, that it has very little influence in producing either subsidence or precipitation, and it requires divergence of an order of magnitude of -10^{-5} to -10^{-6} to produce vertical velocities great enough to produce even light precipitation. For what has been defined as heavy precipitation to occur, the magnitude of the divergence must be at least $-10^{-4} \text{ sec}^{-1}$.

If this value seems very small, it must be borne in mind that the unit of time used is seconds; and that in considering hours as the unit, the numerical value given for the divergence will be increased by a multiplication factor of 3600.

These approximations were obtained by computing the amount of moisture that would precipitate out of an average column of "maritime tropic air" with various amounts of stretching of that column, and comparing that result with the amount of convergence required to produce that stretching within a specific length of time. This, of course, is made on the assumption that the vertical velocity at the surface is zero, and that the layer in which the convergence is taking place is next to the surface. It is also true that the greater depth of the layer in which the convergence is occurring, the greater will be the vertical velocity at the top of the layer.

From this consideration, the greatest vertical velocity in a column of air, that is having convergence taking place in the lower layers and a spilling out (divergence) occurring in the upper levels, would be in the layer where the convergence changes to divergence. This is true, because in a positive direction, the vertical velocities increase below this level and decrease above it. This layer is devoid of any convergence or divergence and is referred to as the "level of non-divergence."

Consider the hypothetical air column in figure (5) with horizontal convergence taking place between z_1 and z_3 , and horizontal divergence occurring above z_4 . Assuming negligible changes of density, vertical divergence must exist between z_1 and z_3 and vertical convergence above z_4 . The vertical velocities at z_1 are equal to zero and with vertical divergence $\frac{\partial w}{\partial z} > 0$ between z_0 and z_3 , the vertical velocities increase with height. Then $w_3 > w_2 > w_1 = 0$.

Between z_3 and z_4 lies the indefinite layer in which there is no horizontal convergence or divergence ($\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$) and the vertical divergence is also equal to zero ($\frac{\partial w}{\partial z} = 0$), then the vertical velocity at

z_3 is equal to the vertical velocity at z_4 (w_3 equals w_4).

With the vertical convergence above z_4 ($\frac{\partial w}{\partial z} < 0$), the vertical velocities decrease with height, hence: $w_4 > w_5 > w_6$ Further $w_0 < w_1 < w_2 < w_3 = w_4 > w_5 > w_6$ Hence the maximum vertical velocities in such a column are between z_3 and z_4 (the level of non-divergence).

DIVERGENESIS

Bellamy of the University of Chicago, has derived the following expression for the time rate of change of divergence:

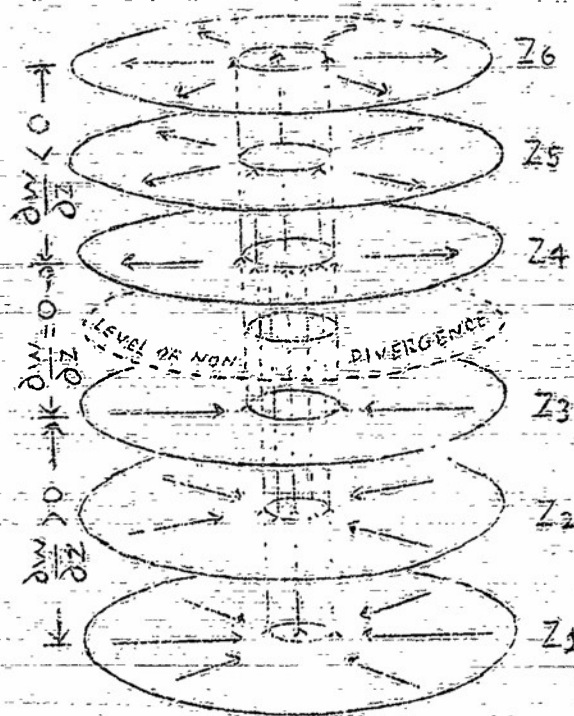
$$(11) \quad \frac{d(\text{div}_2 V)}{dt} = \text{div}_2 \frac{dV}{dt} + 2 \frac{\partial v}{\partial n} V \frac{\partial \alpha}{\partial s} - \left[\left(\frac{\partial v}{\partial s} \right)^2 + \left(V \frac{\partial \alpha}{\partial s} \right)^2 \right] - \frac{\partial w}{\partial s} \frac{\partial v}{\partial z} + \frac{\partial w}{\partial n} V \frac{\partial \alpha}{\partial z}$$

This equation was derived in Bellamy's s, n and z coordinate system, where the s axis is parallel to the horizontal wind component and positive in the direction of motion. The z axis has the same direction as the acceleration of gravity with negative values toward the center of the earth. The n axis is perpendicular to both s and z and is measured positively to the left of flow (notice that this s, n representation is somewhat different from that mentioned earlier, in that here, $dn > 0$ to left of flow instead of to the right).

$$(12) \quad \frac{d(\text{div}_2 V)}{dt} = \text{rate of change of the horizontal divergence with respect to time. When the value is positive, it indicates increasing divergence or decreasing convergence which Bellamy calls "divergensis;" when the value is negative,}$$

FIG. 5

$\frac{\partial \phi}{\partial z}$



decreasing divergence or increasing convergence is indicated; hence "convergence."

$$(13) \frac{div_2 dV}{dt}$$

= divergence of the accelerations. This states, that with a positive divergence of the accelerations, any current divergence of the horizontal wind field will increase, and any convergence will decrease. With negative divergence (convergence of the accelerations, any divergence of the horizontal wind field will decrease and any convergence will increase.

$$(14) 2 \frac{\partial v}{\partial n} V \frac{\partial \alpha}{\partial s}$$

= twice the product of the shear and curvature terms of the vorticity expression.

$\frac{\partial v}{\partial n}$ = the shear and $\frac{\partial \alpha}{\partial s} = \frac{1}{r_s}$, where r_s = radius of curvature of the streamlines. If the

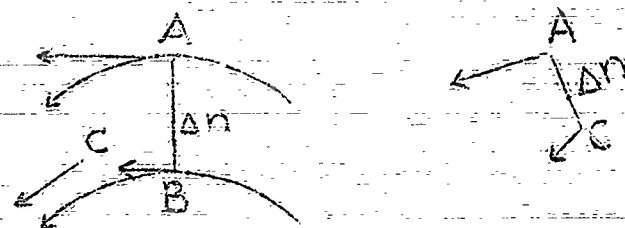
shear and the curvature have the same sign (i.e. both cyclonic or both anticyclonic), the sign of product is positive and implies that there will be increasing divergence with time or

"divergence." If the shear and curvature have opposite signs (i.e. shear

anticyclonic and radius of curvature cyclonic or shear cyclonic and curvature anticyclonic), the sign of the product will be negative, indicating decreasing divergence with time or convergenesis.

Bellamy gives the following physical interpretation of this term: (2)

"Consider a field of motion in which both $\frac{\partial v}{\partial n}$ and $\frac{\partial \alpha}{\partial s}$ are negative (see footnote) such as in the following:



At time $t=0$ At time $t=\Delta t$

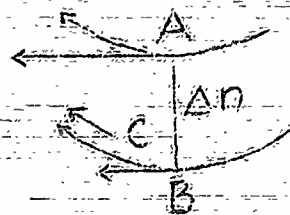
Since there is a shear present, parcel A will 'catch up' to some parcel, such as C, originally in front of it, so that, since there is also curvature of the streamlines, the term $V \frac{\partial \alpha}{\partial n}$ has become more divergent. To a first approximation the value of $\frac{\partial v}{\partial s}$ has

Footnote:

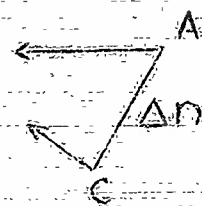
(Notice Bellamy considers a cyclonic rotation to be negative.)

not changed under these condition; so that there is a tendency for divergen-
sis. It is obvious that exactly the same condition results if both $\frac{\partial v}{\partial n}$ and $V \frac{\partial \alpha}{\partial s}$ are positive.

Consider a field of motion in which $\frac{\partial v}{\partial n}$ is negative and $\frac{\partial \alpha}{\partial s}$ is positive, such as in the following:



At time $t=0$



At time $t=\Delta t$

Since there is a shear present, parcel A will 'catch up' to some parcel, such as C, originally in front of it, so that, since there is also a curvature of the streamlines, the term $V \frac{\partial \alpha}{\partial n}$ will become more convergent. Since

$\frac{\partial v}{\partial s}$ will not change in a very short time under these conditions, there is

a tendency for convergenesis. The same result applies if $\frac{\partial v}{\partial n}$ is positive and $\frac{\partial \alpha}{\partial s}$ is negative."

$$(15) - \left[\left(\frac{\partial v}{\partial s} \right)^2 + \left(v \frac{\partial \alpha}{\partial n} \right)^2 \right]$$

the square of the rate of change of velocity in the direction of motion plus the square of degree of deviation of the streamlines from parallelism.

$\frac{\partial v}{\partial s}$ represents velocity divergence and $v \frac{\partial \alpha}{\partial n}$ represents the directional divergence, since the term involves the squares of these values, any absolute manifestation of either or both, regardless of sign, will give positive values within the brackets or negative value for the entire term: hence, convergence.

$$(16) - \frac{\partial w}{\partial s} \frac{\partial v}{\partial z} =$$

the product of the rate of change of the vertical velocities along the s axis and the rate of change of the s direction velocity along the vertical.

If these two values have the same sign, there will be a tendency for convergence and with opposite signs there will be a tendency for divergence.

$$(17) \frac{\partial w}{\partial n} v \frac{\partial \alpha}{\partial z} =$$

the product of the rate of change of the vertical velocities along the n axis, the velocity along the s axis, and the rate of change of the wind direction along the vertical. If the

rate of change of the vertical component along the n axis and the rate of change of the wind direction along the vertical have the same sign, the result is divergence; if they have the opposite sign, there will be a tendency for convergence. (Consider a veering wind to be positive.)

First approximations suggest that the shear curvature term (see equation 15) has generally a greater order of magnitude than the sum of the other terms when both the shear and curvature have significant values. At the present time, with the very limited amount of work that has been done with this concept, it seems that shear values greater than eight (8) miles per hour per two hundred (200) miles and radii of curvature less than eight hundred (800) miles, are sufficient to warrant a consideration of this term in evaluating the time rate of change of the horizontal divergence.

Any conclusion made from this type of analysis of the instantaneous horizontal wind field is pertinent only to the changes of divergence and convergence and is not necessarily relevant to the current weather distribution. It is obviously possible to have marked divergence and convergence occurring simultaneously. The proper deduction is that the divergence will decrease with time; and, if convergence continues, will become convergence. Similarly with convergence acting on a field of convergence, the convergence will increase; with divergence acting on a field of convergence, the convergence will decrease; and with divergence acting on

a field of divergence, the divergence will increase.

Bellamy also points out for systems that are moving in direction opposite to the wind component the effects of shear-curvature term will be reversed. As this discussion will be concerned mainly with Easterly Waves, which are systems that move with the wind, this factor will be neglected.

The consideration of the shear curvature factor should prove fairly helpful in making operational forecasts; however, it is not intended here to give the impression that this factor, used in an analysis, is a "cure-all" for forecasting tropical weather. In fact, the situations in which the shear or the curvature observed are too indefinite to form any conclusions of their effect on the changing weather intensities, will probably be more numerous than those in which this concept is applicable. However, it is believed that in many cases where there is a marked change in the degree of convergence within a forecast period, the presence of the aforementioned factor will make it possible to determine this intensification 12 to 24 hours previous to the change. More will be said concerning the practical application of divergenesis later in this report. It is not intended here to deliberately neglect the influence of the other terms in Bellamy's equation, but merely to attempt to evaluate the one that appears most pertinent and applicable. After more thorough investigations have been made by this and other interested agencies, there will probably be some reports more comprehensive in nature and broader in scope on the entire subject of divergenesis.

CONVERGENCE AND TRAJECTORIES

Earlier in this report the expression,

$$(1) \frac{d(f+\zeta)}{dt} = -(f+\zeta)(\text{div}_2 V)$$

was given and stated to be valid making the following assumptions:

- (a) negligible vertical motion
- (b) negligible friction
- (c) a zero solenoid factor.

This equation expresses a relationship between horizontal divergence and the time rate of change of the absolute vorticity $(f + \zeta)$. If there is no horizontal divergence, integration of the above equation yields:

$$(18) \quad (f + \zeta) = K$$

From equation (3) $\zeta = \frac{V}{r} + \frac{\partial V}{\partial n}$

In the middle of a broad stream

$$\frac{\partial V}{\partial n} = 0$$

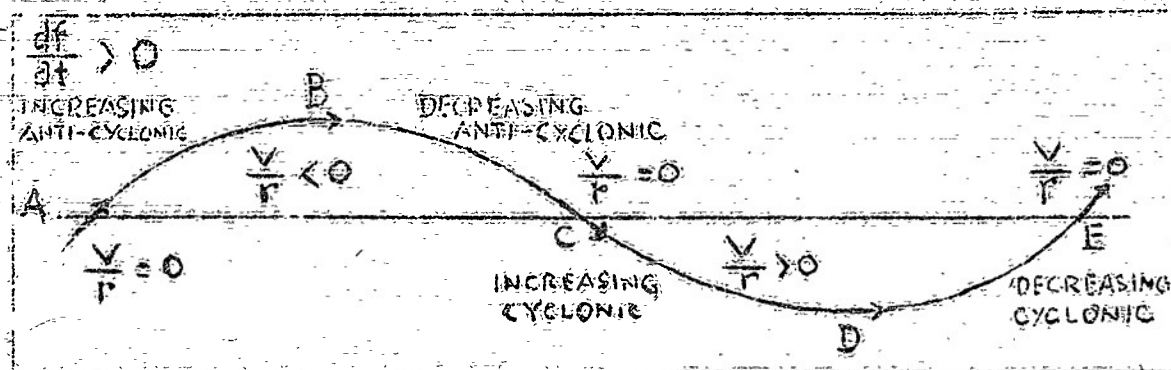
$$(19) \text{ Then } f + \frac{V}{r} = K$$

From this, assuming steady state conditions, the following analysis of trajectories can be made.

At point A in Fig. (6) the radius of curvature of the streamlines is equal to infinity (straight line flow). The flow is toward the north; the value of f is increasing and $f + \frac{V}{r} = K$, then $\frac{V}{r}$ must decrease or become anticyclonic. The curvature will continue to decrease (become more anticyclonic) as long as there is any crossing of the latitude lines from south to north. The increase of anticyclonic curvature will tend to deflect the path more and more into the east, decreasing the southerly component. Eventually at some point B, the flow will be from due west and will be curving toward the south due to the anticyclonic curl that has been acquired

after this point, with the resulting northerly components there will be decreasing latitude values with time; therefore increasing values of $\frac{V}{r}$. The absolute value of the negative vorticity will decrease, or the

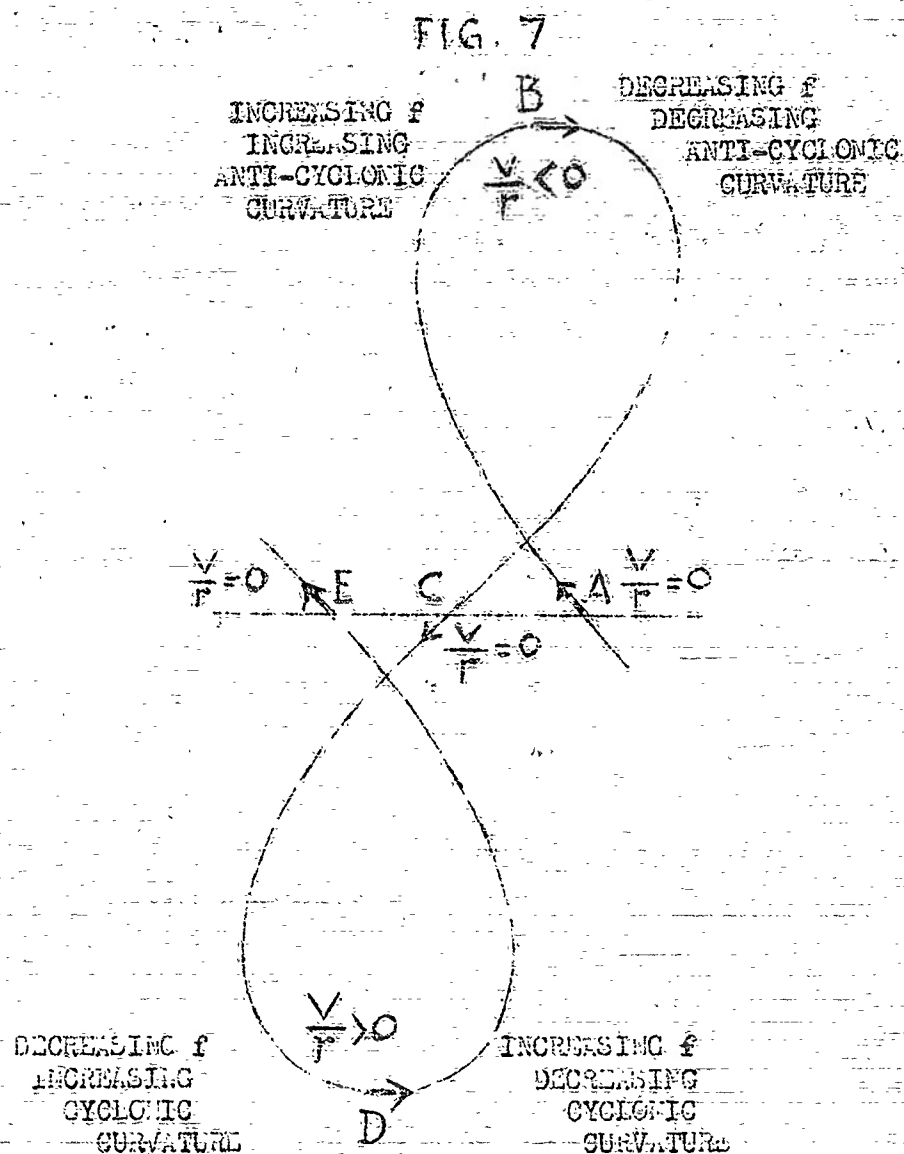
FIG. 6



curvature will become less anticyclonic. This process will continue as long as the flow is toward the south and the sign of the curvature will increase to zero at some point C, which is at the same latitude of point A. South of point C where the vorticity is still increasing the curvature will become positive (cyclonic), and the northerly component will then decrease as the cyclonic curvature increases. At some point D, the flow will again become westerly and here the cyclonic curl will tend to deflect the particle to the north and thereafter decrease the cyclonic curvature. At some point E (at the same latitude of A and C) the value of $\frac{V}{r}$ will again become equal to zero. The points of maximum anticyclonic and cyclonic curvature will be at points B and D respectively and the points of straight line flow are at A, C, and E. From this it is obvious that a perturbation in the westerlies will result in a series of stable sinusoidal waves.

In the easterlies this inertia component produces an unstable pattern

of the trajectories. Consider a southerly straight-line flow as is represented at point A in Fig. 7. The air is moving toward the north



and will tend to develop anticyclonic curvature. This tendency will continue as long as the path is in the direction of increasing latitudes. The maximum anticyclonic curvature will occur at some point B where the curl has caused the path to be directed from due west. At this point the path

is deflected into the south by the acquired anticyclonic curl and the decreasing latitude values that result will be accompanied by a decrease in the anticyclonic curvature. The anticyclonic curvature will continue to decrease as long as a particle is moving from the north and at some point C which is at the same latitude as point A, the value of $\frac{v}{r}$ will again become equal to zero. South of point C as the curl is continuing to increase the curvature will become cyclonic as the flow is toward the south. This increase will be terminated at some point D where the flow again becomes westerly and has a tendency to be deflected into the north by the acquired cyclonic curl. With this northward movement, the cyclonic curvature will decrease and become straight line flow at some point E. Notice that with westerlies any increase in the absolute value of the curvature term will decrease the cross latitude component of the flow, thereby producing a stabilizing influence on the wave formation. In the easterlies, however, any increase in the absolute value of the curvature term will increase the latitude factor and thereby tend to produce unstable systems as in Fig. (7).

EFFECT OF DIVERGENCE ON TRAJECTORIES

Experience has shown that the actual trajectories observed in the tropics are of a very different nature from the theoretical trajectories suggested by Fig. (7). At first, there may be some tendency to doubt the validity of the vorticity equation. It may be more practical, however, to re-evaluate the previously made assumptions and determine if any of them are significant enough to explain the observed deviation.

Of all the factors, that were assumed negligible, the horizontal divergence appears most likely to be one that would have the greatest influence on the time rate of change of the relative vorticity.

If such a relationship between divergence and trajectories satisfactorily accounts for the deviation of actual trajectories from inertia trajectories, it seems obvious that an investigation of weather distribution in conjunction with the past and present streamline analysis, would be a tool of definite forecasting value.

CONVERGENCE AND DIVERGENCE IN THE EASTERLIES

Consider first an easterly flow with an infinite radius of curvature of the streamlines; this flow would of course continue to remain easterly (see AC Fig 8a), if the assumptions made on page 19 were valid. However, if a field of divergence is superimposed on the easterly flow, it would definitely affect the paths of the air particles. From equation (1), if $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} > 0$ (divergence), then $\frac{d\zeta}{dt}$ must be negative (flow becoming anticyclonic) and the trajectory would be deflected to the north as indicated by AB in Fig. (8a). If $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} < 0$ (convergence), $\frac{d\zeta}{dt} > 0$ and particles would be deflected to the south with cyclonic curvature being set up:

When a latitude factor is introduced as in the case of easterlies with a northerly or southerly component, the analysis becomes a little more complex.

In Fig. (8b) with a southeasterly straight-line flow and no convergence taking place, the latitude effect would tend to curve the particles more

into the north (see AB in Fig. (8b)). If, however, it is observed that the southeasterly straight-line flow does not become anticyclonic but remains straight-line flow, then substituting in the vorticity equation, $\frac{df}{dt} > 0$ and $\frac{d\beta}{dt} = 0$, then $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ must be less than zero; that is, convergence is taking place, preventing the flow from becoming anticyclonic.

(See AC in Fig. (8b).) In the trajectory suggested by AD in Fig. (8b), the convergence must be even greater as the acquired curl is opposite to that which would be produced by a latitude effect alone. Divergence occurring with straight-line southeasterly flow will tend to cause the flow to become anticyclonic much much more rapidly.

Fig. (8c) shows the effect of convergence on straight-line northeasterly flow.

RELATIONSHIP OF TRAJECTORIES TO STREAMLINES

Actual trajectories can be determined from the following three factors:

- (a) streamline patterns
- (b) velocity of the streamlines
- (c) velocity of the wind.

Let c = velocity of the streamlines

r_t = radius of curvature of trajectories

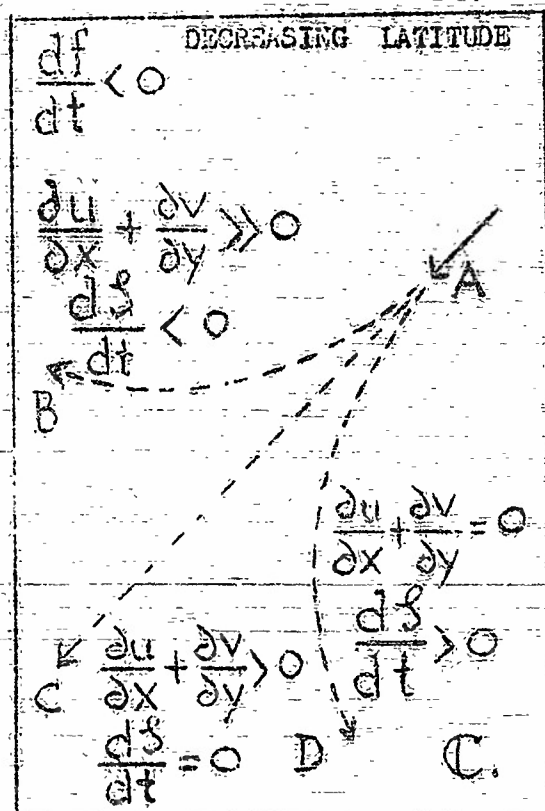
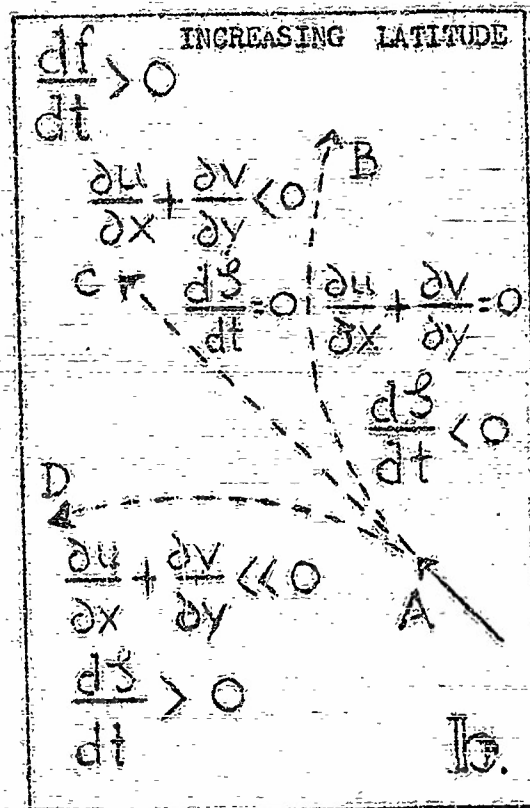
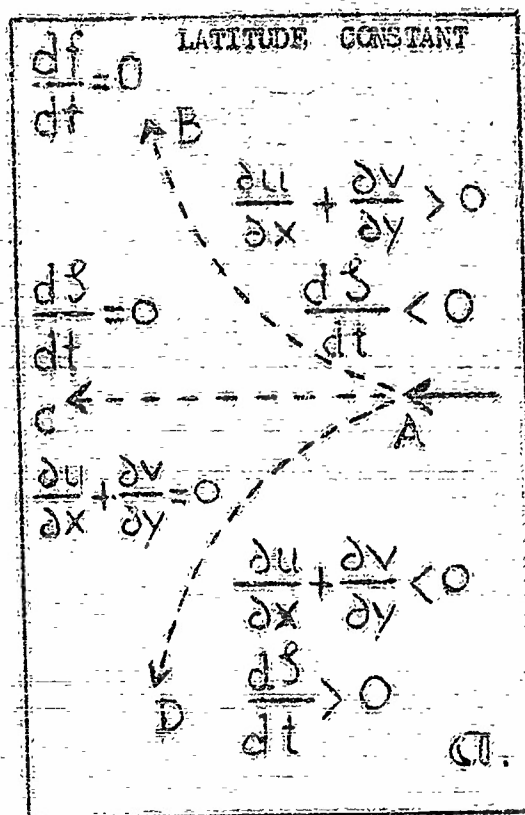
r_s = radius of curvature of streamlines.

ψ = angle formed by direction of wind with direction of motion of system

$\frac{\partial \psi}{\partial t}$ = change of wind direction with time at a given point fixed to the earth.

Consider a backing wind is positive.

FIG. 8



The following relation is given by Blaton: (20) $\frac{\partial \psi}{\partial t} = \frac{v}{r_t} - \frac{v}{r_s}$

assuming no structural changes of the system, the local change of wind direction is also given by $\frac{\partial \psi}{\partial t} = -C \frac{\partial \psi}{\partial x}$ where the x-axis is parallel to the direction of motion of the streamlines and for simplification here, x will be considered positive to the west.

For circular systems Petterssen gives $\frac{\partial \psi}{\partial x} = \frac{1}{r_s} \cos \psi$ (Eq. 21)

then $\frac{\partial \psi}{\partial t} = -\frac{C}{r_s} \cos \psi$ and substituting in (20)

$$(22) \quad \frac{r_s}{r_t} = 1 - \frac{C}{V} \cos \psi$$

This expression is for closed circular systems and does not hold for a symmetrical sinuoidal streamline pattern with which this discussion is largely concerned.

For the circular system in Fig. (9)

FIG. 9

$$\psi_1 = \psi_2 = \psi_3$$

$$\psi_3 - \psi = \psi_2 - \psi = \psi_1 - \psi$$

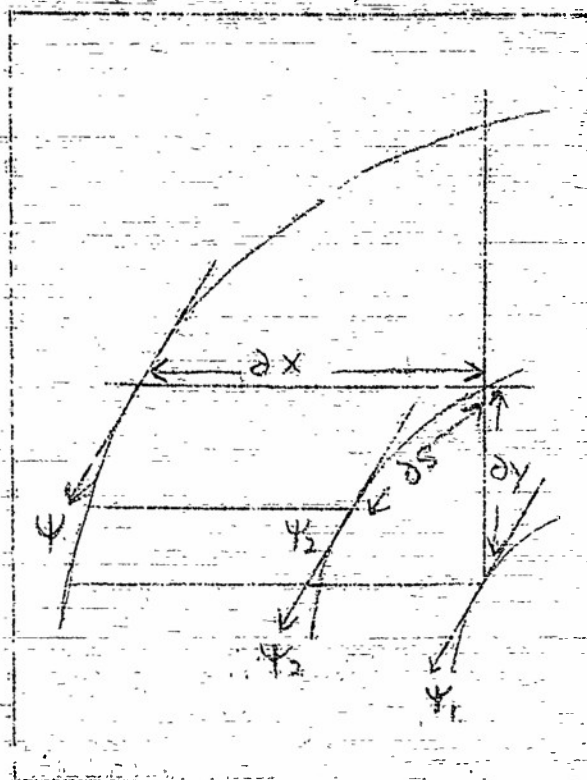
$$\partial x \cos \psi = \partial s$$

Then $\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial s} \cos \psi$

and $\partial y \sin \psi = \partial s$

Then

$$(23) \quad \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial s} \sin \psi$$



But this is not valid for sinusoidal patterns because in symmetrical sinusoidal streamline patterns

FIG. 10

$$\frac{\partial \Psi}{\partial y} = 0$$

$$\frac{\partial \Psi}{\partial x} = \frac{\partial \Psi}{\partial s} \frac{1}{\cos \psi} = \frac{1}{r_s} \frac{1}{\cos \psi}$$

Substituting in (20) we get:

$$(24) \quad \frac{r_s}{r_t} = 1 - \frac{c}{v \cos \psi}$$

This last expression will be used to investigate the trajectories of sinusoidal patterns in the easterlies.

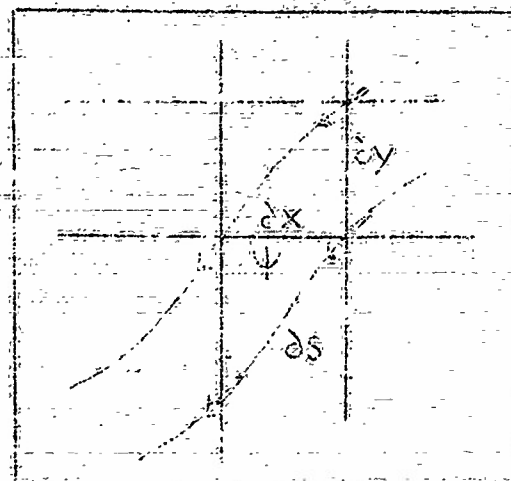
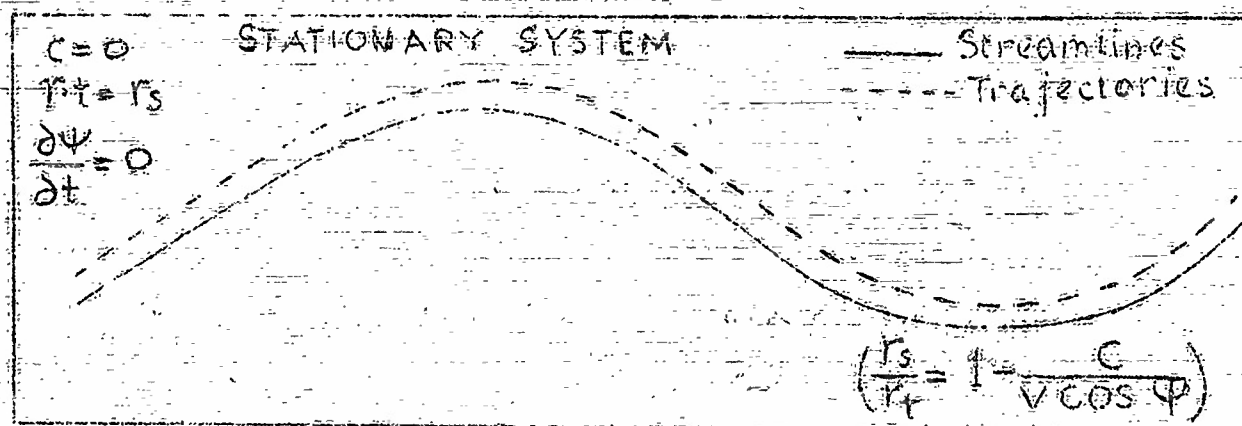
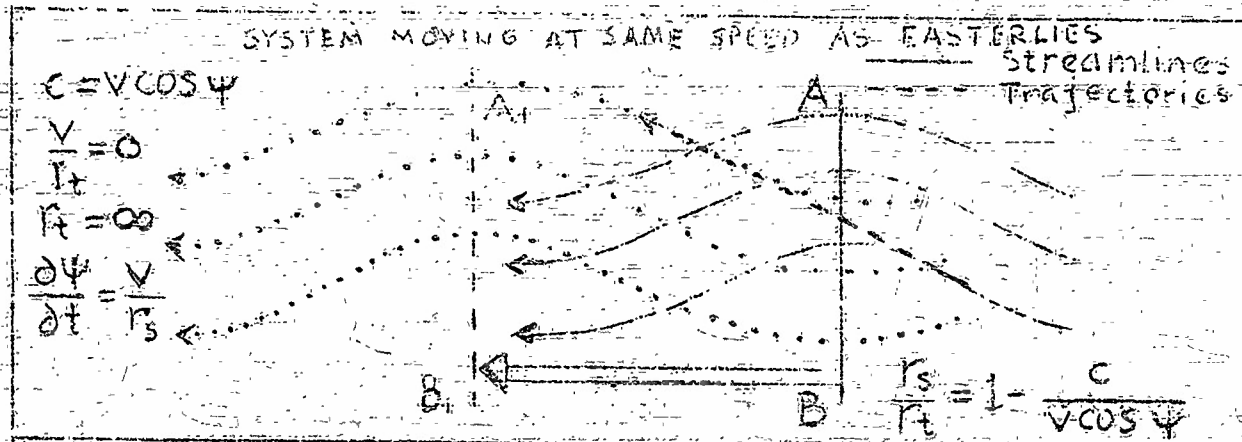


FIG. 11



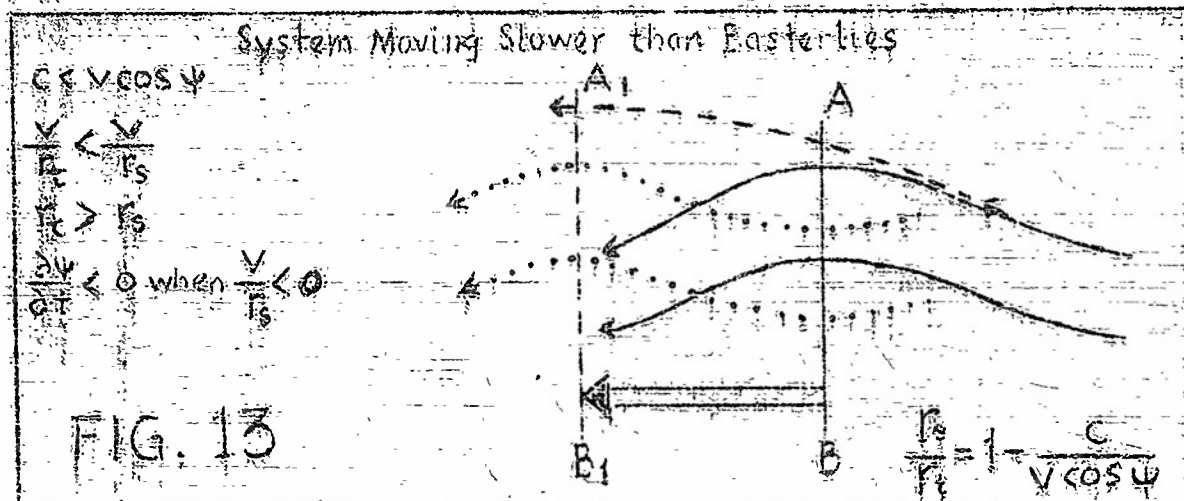
In Fig. (11) the perturbation is stationary, ($c = 0$) and therefore the radius of curvature of the streamlines is equal to the radius of curvature of the trajectories, the local wind direction is constant.

FIG. 12



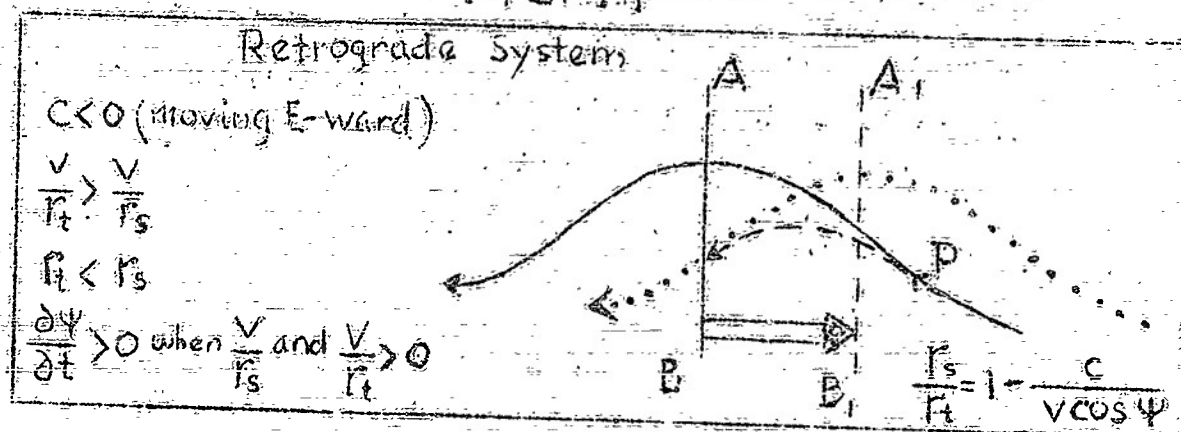
When the speed of the system equals the speed of the easterly component, the radius of curvature of the trajectories is infinite or the curvature of the trajectories is equal to zero. A particle at any position of the wave will not change its position with respect to the center of the wave, therefore it will not change its direction of motion but will move in a straight line. The only motion the particles will have with respect to an observer moving with the wave will be normal to the direction of motion (see Fig. 12).

When the winds are moving through a system, the easterly component is greater than the speed of the wave. As a result, the radius of curvature of the trajectories is greater than the radius of curvature of the streamline. In Fig. (13) a particle at any position P, which is east of the center of the wave, will have to move farther along the x axis than distance P Q before it starts turning southward; therefore, its path must have a greater wave length than the streamlines; also, since it will continue to be displaced northward until it reaches the crest at some point Q, the trajectories will have a greater amplitude than the streamlines. With a greater wave length and a greater amplitude there must also be a greater radius of



curvature. The sign of the local wind change is opposite the signs of the radii of curvature of the trajectories and streamlines, that is in the cyclonically curved positions of the streamline pattern, the local wind will veer, and in the anticyclonic portions, the local wind will back.

FIG 14

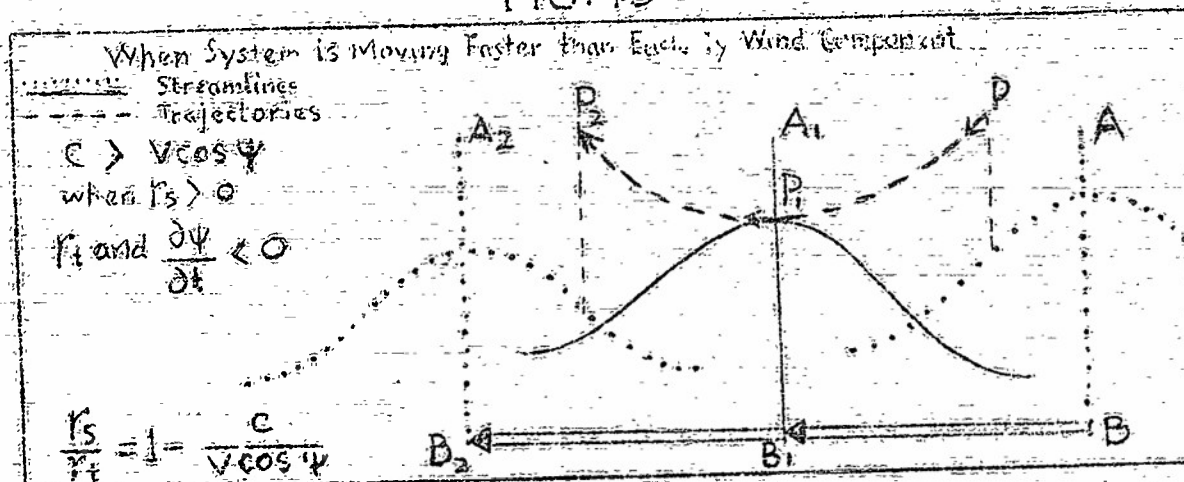


When the motion of a perturbation is retrograde, that is, moving toward the east, the radius of curvature of the trajectories is less than the radius of curvature of the streamlines. In Fig. (14) a particle at any point will be forced to the south before it reaches line AB; therefore amplitude and wave length of the path will be less than the amplitude and wave length of the streamlines. The local wind will veer with anticyclonic curvature of the streamline and back with cyclonic curvature.

When the speed of the wave exceeds the speed of the easterly component as indicated in Fig. (15), the sign of the curvature of the trajectories will be opposite to the curvature of the streamlines. Consider a particle at P when wave center is at AB. That particle will have moved to a point P1 when wave center is at A1B1 and will go to point P2 when wave

crest is at A_2B_2 . The trajectory of the parcel given by the dashed line in Fig. (15) is 180 degrees out of phase with the streamline curve.

FIG. 15

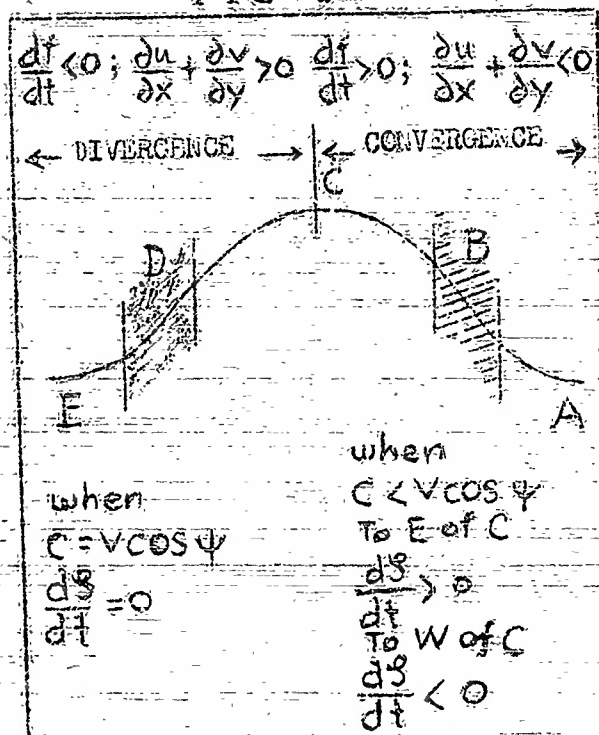


The local wind will have the same sign of the curvature of the trajectories and an opposite sign from the streamlines.

Examining a perturbation in the easterlies from this viewpoint the theoretical weather distribution shown in Fig. (16) is obtained. Consider theoretical sinusoidal wave in the easterlies as is indicated in Fig.

FIG. 16

(16), in which the speed of the system is equal to the component of the easterlies and the shear is equal to zero and unchanging. Everywhere between A and C, the flow is from the south but the trajectories are straight lines. From the discussion on page 22, convergence must be occurring between A and C to prevent these trajectories



from turning to the north. Between C and E the flow is toward the south. Here divergence must be occurring to prevent the trajectories from turning cyclonically.

At point B, the flow has the greatest southerly component and there $\frac{df}{dt}$ has its maximum value. With $\frac{dS}{dt}$ equal to zero at all points along this curve, the convergence must be greatest at B in order to maintain straight line trajectories. Similarly the divergence must be greatest at D.

Rearranging equation (1) and substituting plus convergence for minus divergence we get
$$\frac{d/dt (f+S)}{(f+S)} = \text{conv}_2 V$$

When the vorticity is anticyclonic, it is negative and decreases the value of $(f+S)$ thereby increasing the value of the entire left-hand member. Then it can be seen that with anticyclonic vorticity the amount of convergence necessary to balance a given time change of latitude and vorticity would be greater than for straight line flow. When the vorticity is cyclonic, that is, positive and additive to f , the denominator of the fraction is increased thereby decreasing the value of the entire left-hand member. Then with cyclonic vorticity the amount of convergence corresponding to given values of $\frac{df}{dt}$ and $\frac{dS}{dt}$ would be less than for straight line flow or anticyclonic vorticity.

The absolute value of $\frac{df}{dt}$ is greatest at the inflection points and in a simple system as shown in Fig. (16) there should be some tendency for the greater amount of convergence or divergence to be at these points. Also there should be more convergence between A and B than between B and C; therefore, with the ideal perturbation, the area of maximum convergence should be approximated by the shaded zone and the area of maximum divergence shown by dotted zone in Fig. (16).

The preceding discussion is based on the assumption that the radii of curvature of the trajectories are equal to infinity. This means that wind is geostrophic and $\frac{dS}{dt}$ is equal to zero.

If, however, the wind is blowing through a perturbation, the absolute value of $\frac{dS}{dt}$ will be displaced from zero, and is a pertinent factor in this analysis. The magnitude of $\frac{dS}{dt}$ depends on the curvature of the streamlines and difference between the speed of the easterly wind component and the speed of the system.

If the ideal perturbation in Fig. (16) were moving slower than the easterlies, everywhere between A and C, $\frac{dS}{dt}$ would be positive. This means the convergence is greater than when straight-line trajectories exist because the amount of convergence must be great enough not only to maintain straight line flow in opposition to the latitude influence, but must be even greater, for the curvature to be increasingly cyclonic.

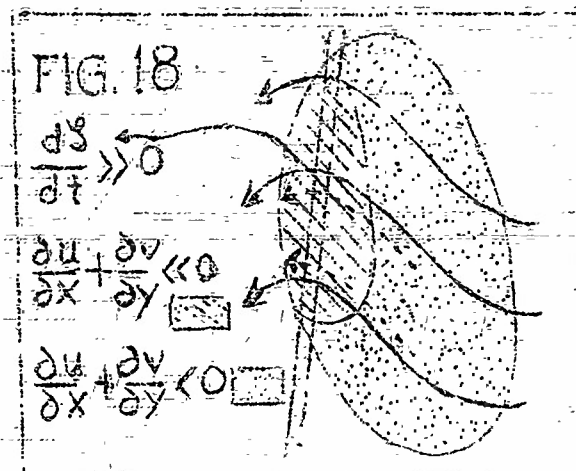
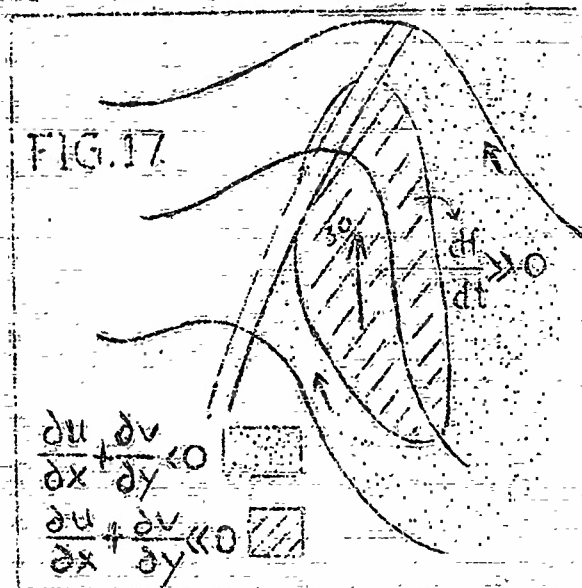
Between C and E the divergence must also be greater than in the case where the system is moving at a rate equal to the easterly wind component because the latitude factor would tend to cause increasing cyclonic curvature. The convergence then must be sufficiently large not only to prevent this tendency, but actually is large enough to cause the curvature to become increasingly cyclonic.

Summarizing, it may be said that, in general, perturbations, in which the wind is moving through the wave, will have greater subsidence to the west and more intense weather areas to the east of the wave crest than will an identical system which is moving at approximately the same speed as the easterly wind component.

The conclusions mentioned here are for simple ideal systems that are among the least common of those observed in the easterlies. The value to be obtained from such an analysis lies not in the application to a specific system that is identical with the ones suggested in Fig. (16). But it does seem reasonable that if an ideal or average synoptic situation can be explained thoroughly and logically from a physical viewpoint and the analyst has fixed in his mind the reasons for an ideal weather distribution, it is much less difficult for him to analyze the situations that appear synoptically and apply corrections for the deviation from a hypothetical situation.

For example, with a very strong straight line flow from near 180 degrees with a tendency to become cyclonic, there must be a great deal of convergence present to prevent such a flow from becoming anticyclonic, as the value of $\frac{df}{dt}$ is comparatively large. (See Fig. 17).

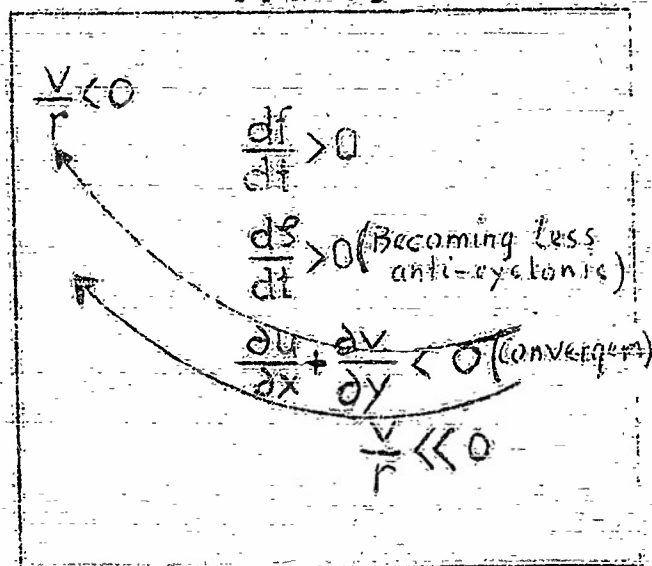
When the convergence is greater in the cyclonic portions of an easterly wave than in the anticyclonic portions, the value of $\frac{\partial s}{\partial t}$ must be greater in the cyclonic portions. This may be reflected by a more rapidly changing radius of curvature along the flow lines as is indicated in Fig. (18) in which the wind is blowing through the system.



This must be considered in conjunction with the difference with which the wind is moving through the various portions of the wave.

Occasionally on the back side of an anticyclone with no perturbation of any sort apparent, rather widespread areas of scattered showers occur. This situation might be explained by the fact that there is a tendency for decreasing anticyclonic vorticity with a flow toward the north. (See Fig. 19).

FIG. 19



WEATHER IN THE ADVANCE OF AN EASTERLY WAVE

When convergence exists to the west of an easterly wave, the increase of cyclonic vorticity must be greater than could be accounted for by the latitude factor alone ($\frac{df}{dt} > -\frac{dS}{dt}$): that is, there are two processes present, both of which tend to increase the cyclonic vorticity.

It was shown earlier that waves, in which the winds have easterly components equal to and greater than the speed of the systems, should have divergence to the west of the wave center. In one case the cyclonic curvature remains constant and, in the other the cyclonic curvature decreases. The cyclonic curvature of air parcels to the west of center line does, however, increase when the system is moving faster than the winds, (i.e., the center of the system is catching up with the parcels in the advance of the system, thereby forcing them into the more cyclonically

curved portions of the wave). (See Fig. 15).

But the observed speeds of the waves in the easterlies is between 10-20 miles per hour and, the speeds of the easterly winds must be considerably less than that, if the difference between the two is to produce any significant change of the radius of curvature of an air parcel. As the curvature term of the vorticity expression is directly proportional to the velocity, these small velocity values will result in small values of the term $\frac{v}{r_s}$. If the value of a positive $\frac{v}{r_s}$ remains small even after a positive increase, the order of magnitude of the change must also be small. From this it seems unlikely that the weather in advance of an easterly wave could be explained by the fact that the system is moving faster than the winds.

The most important effect of such a relationship would probably be to decrease the divergence in advance of the trough and decrease the convergence to the rear of the trough. This situation is often observed in portions of the weaker perturbations. The existence of weather in advance of an easterly wave cannot be explained by any of the above relationships between the speed of system and speed of the easterlies and these three conditions encompass all possible situations. Then again, it is suggested that the previously made assumptions be investigated to determine whether or not the assumptions are valid for all situations.

Up to this point a change of vorticity has been considered to be manifested entirely by a change in the cyclonic curvature, that is, the shear term remains constant. In most of the simple perturbations in the easterlies, this assumption appears to be a valid one. However, the order of magnitude of this term is too great to neglect entirely in the analysis

of all situations. It then remains for the analyst to determine whether or not it is necessary to consider a variable shear, by evaluating the curvature of the streamlines, the curvature of the trajectories, the weather distribution, etc.

It was previously stated that convergence occurring with a northerly flow would result in quite a large value for the time rate of change of the relative vorticity.

If, in an easterly wave, weather is occurring to the west of the center line and it is assumed that the increase of cyclonic vorticity is reflected entirely in the increase of cyclonic curvature, a closed center would be formed. However, as vorticity is a function of both shear and curvature, it is possible for the increase of vorticity to exist entirely in the increase of shear and satisfy completely the mathematical requirements of the vorticity equation, and not have any increase of cyclonic curvature accompanying convergence.

In Fig. (20), the easterly wave is such that the change of r_s between A and B is slight, so that a particle moving through such a system would receive a very small change of radius of curvature of streamlines.

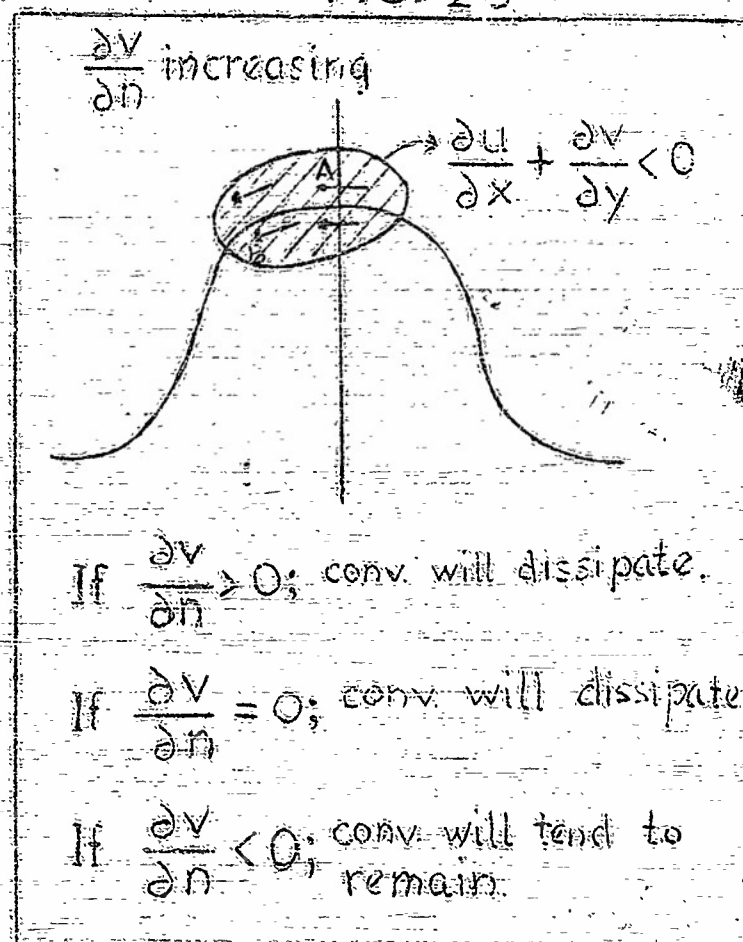
Also, let it be assumed that a particle moving between A and B will experience a noticeable increase of shear; this, plus an increase in the wind velocities could possibly provide a large enough increase in ζ to account for a significant amount of convergence to the west of the trough.

If the shear were initially cyclonic in the cyclonically curved portions of the wave, from Bellamy's divergenesis equation, any convergence would already be in the process of breaking down. (Cyclonic shear

plus cyclonic curvature equals divergenesis).

FIG. 20

If the shear were initially equal to zero, the presence of convergence would tend to produce cyclonic shear, and this in turn would decrease and dissipate the convergence.



If the shear were initially anticyclonic the association with cyclonic curvature would indicate increasing convergence, so that the convergence in advance of the wave would per-

sist much longer than in either of the other two cases. (Owing to the limit of error and representativeness of the pilot balloon observation, it seems that shear values of over eight (8) miles per hour are necessary in order for the above consideration to be significant).

Fig. (21) illustrates a stable wave in the easterlies in which convergence is occurring to the west of the trough. The following discussion will show how a hypothetical stable wave may be maintained with convergence occurring to the west of the trough line.

In Fig. (21) the wave is stable - not intensifying - with convergence in and to the west of the trough; the shear through the mass M is strongly anticyclonic. The presence of anticyclonic shear and cyclonic curvature

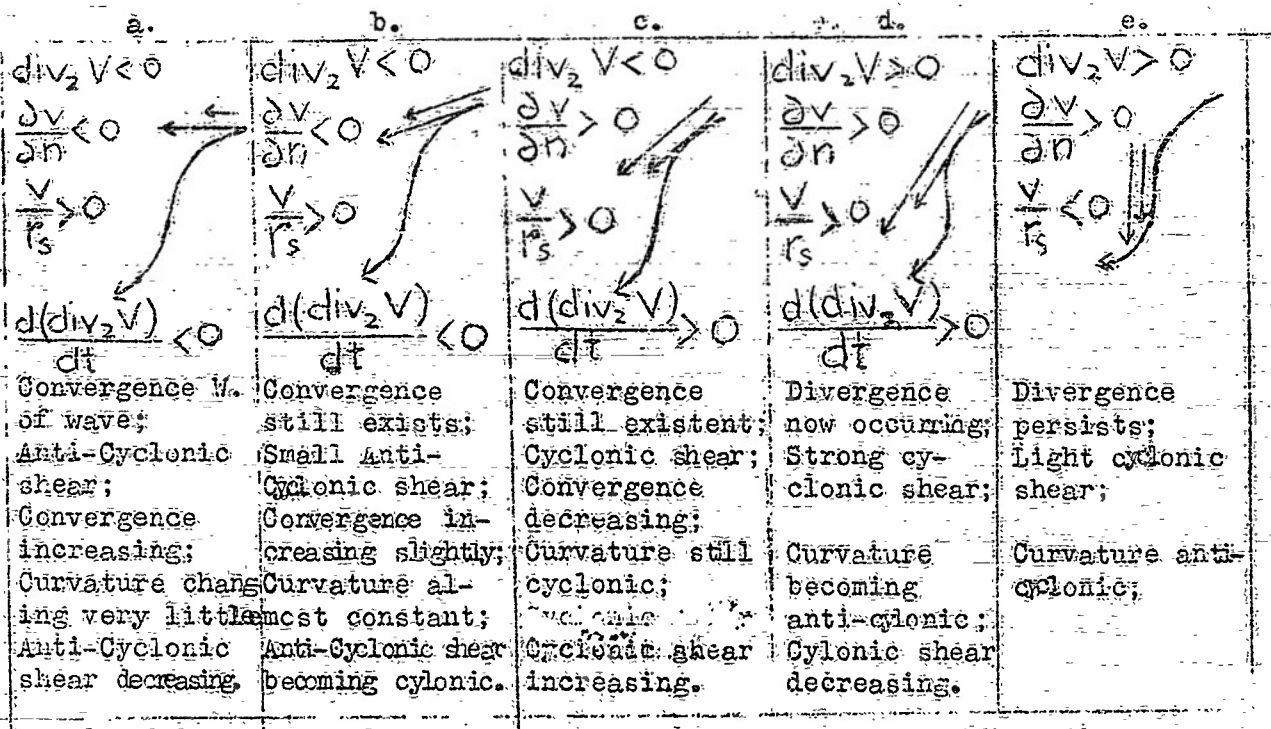


Figure 21

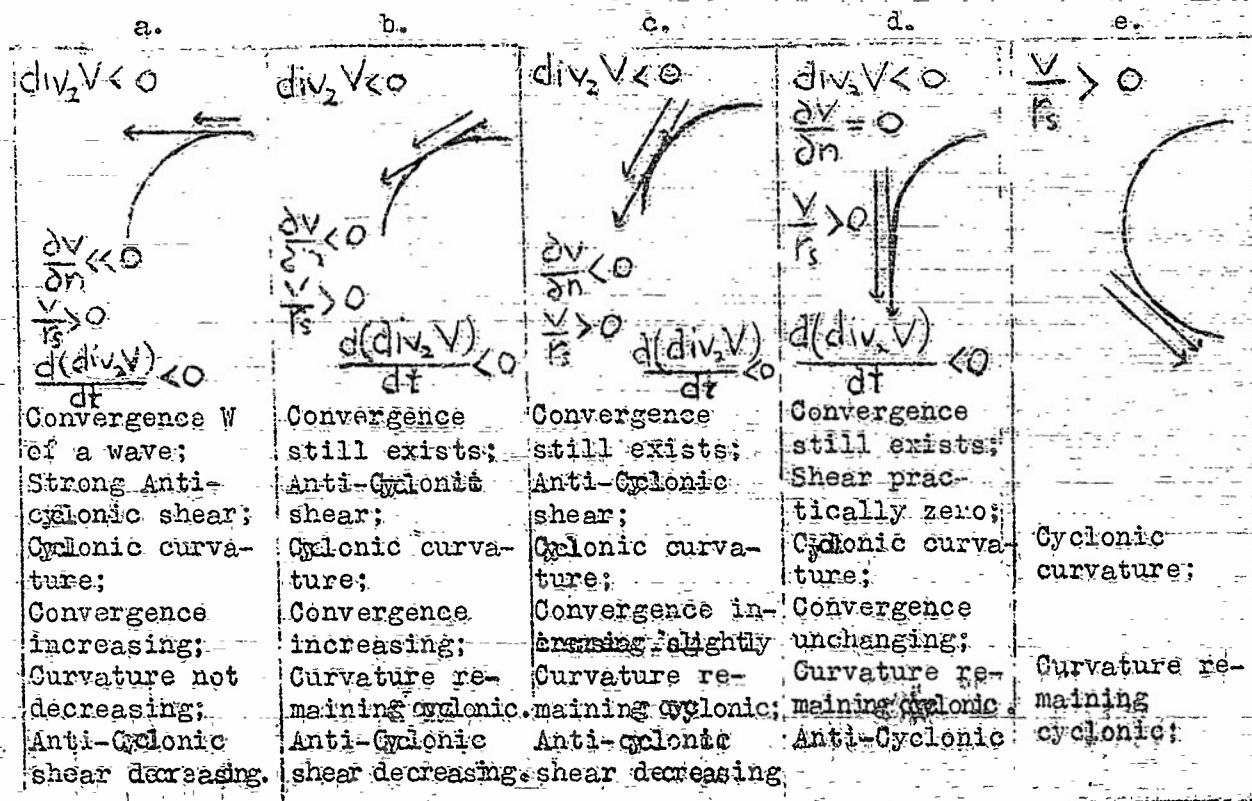


Figure 22

will tend to increase the current amount of convergence. This convergence must be balanced by a corresponding increase of the cyclonic vorticity. Let it be assumed that this increase will be almost entirely in the increase of the shear term (decreasing anticyclonic shear).

As the parcel is moving through the wave it will start acquiring a northerly component, as shown in Fig. (21b). This, plus the fact that the convergence has also increased, would indicate the rate of change of cyclonic vorticity is increasing. Again assuming this is manifested almost entirely in the change of shear, the curvature of the parcel will change very slightly. As the shear is only very slightly anticyclonic by this time the increase of convergence will be practically nil.

By the time the parcel has reached the position shown in Fig. (21c) the shear has become cyclonic. In view of the fact that the curvature is still cyclonic, "divergence" should start to act and decrease the convergence.

As long as there is convergence present, the cyclonic shear may increase and tend to further increase the rate at which the convergence changes to divergence as shown in Fig. (21d).

By this time the cyclonic shear is quite strong and will tend to increase the divergence until divergence is great enough to start decreasing the cyclonic curvature and at some point as is indicated by Fig. (21d), the curvature will become anticyclonic, thereby preventing the formation of a closed low.

In summary, it seems from this that with cyclonic shear present in the trough, any intensification of the trough or weather to the west of the center line, is temporary, and the trend would be toward dissipation

of the weather and a weakening of the trough. With anticyclonic shear present, the convergence should increase and the system will probably intensify but the formation of a closed system may depend on a number of other factors one of which may be the amount of anticyclonic shear originally present.

As shown by Fig. (7) any northerly flow in the easterlies, assuming no convergence, will tend to form a closed pattern of the streamlines. It was shown that divergence occurring in conjunction with this northerly component would maintain a stable perturbation in this flow. It was also shown that if convergence were present that there would be a tendency to increase the cyclonic shear as well as the curvature, which combination in turn would change the convergence to divergence thereby exerting a stabilizing influence and maintaining the wave pattern in the easterlies. In addition, Fig. (21) discusses the possibility of having the convergence remain active to the west of the center line and still prevent the formation of a closed cyclonic cell.

Fig. (22) is a parallel to Fig. (21) except in Fig. (22) an attempt will be made to show how a closed low may form on an easterly wave if the anticyclonic shear is initially great enough.

Sec. (a) of Fig. (22) shows the western portion of an easterly wave in which there is very strong anticyclonic shear present at the crest of a wave. The point to be made here is that even though there were marked increases of cyclonic vorticity which reflected large decreases of anticyclonic shear, the shear was initially large enough so that from Fig.

(22a) through Fig. (22d) the shear remained anticyclonic. This infers that

at no point along the path was there any tendency to decrease the convergence and if the cyclonic curvature merely maintained itself, the formation of a closed low would be set up as is indicated by Fig. (22e). In addition to the continuance of the convergence, the flow has become due northerly so that the latitude factor by this time is so strong it would require extremely strong divergence and increases of shear to cause the curvature to become anticyclonic.

In practically all cases when a west wind component was detected in advance of an easterly wave, a closed low center was found to exist or eventually developed.

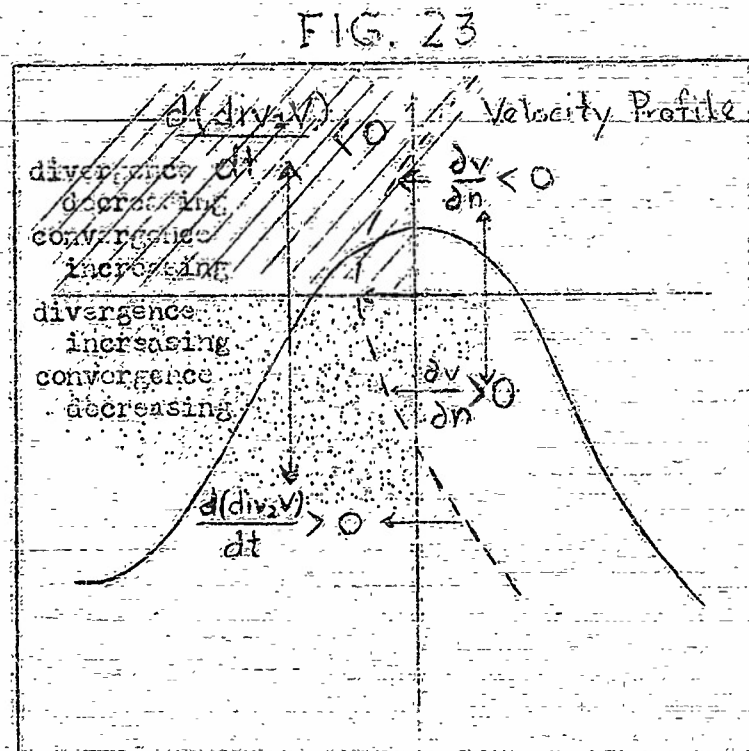
Another supporting observation is that practically all of the closed cells that form on easterly waves form in the northern portions of the wave. That is, the area of formation seems to lie to the north of the peak of the velocity profile of the easterlies which places the formation of the area of anticyclonic shear. An investigation of the intensity and position of this velocity profile in conjunction with the easterly indexing system proposed by Namias of the U. S. Weather Bureau, should lead to some very interesting results.

FURTHER APPLICATION OF DIVERGENESIS

Further use of the theory of divergenesis may be applied to an easterly wave to determine the north-south distribution of the weather. Consider perturbation in Fig. (23) in which the peak of the velocity profile of the easterlies is at B. Anticyclonic shear north of B results in a tendency for convergence to form to the west and in the trough if

none is already present, and if any is present there should be a tendency for that convergence to increase. To the south of B, the presence of cyclonic shear and curvature simultaneously should tend to dissipate any convergence present and intensify the divergence, if that factor is present.

As the peak of the velocity profile in the Caribbean is generally to the north of the continent of South America, the consideration of the shear curvature effect may help to explain the lack of weather that exists along the north coast of Venezuela and Columbia when easterly waves pass these regions.



WESTERLY TROUGHS

An analysis of westerly troughs employing a procedure similar to that used in the investigation of easterly waves may lead to some conclusions that will prove helpful in forecasting for this phenomenon.

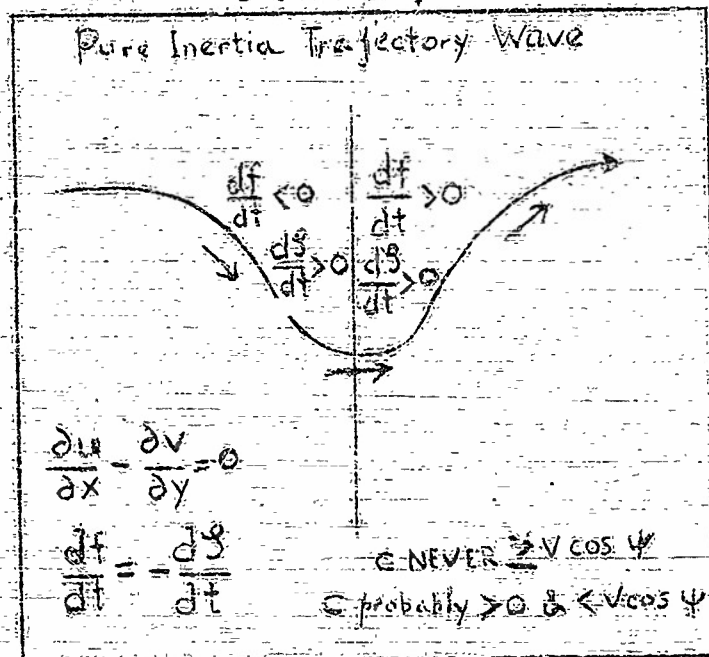
There are a few major differences between perturbations in the easterlies and those in the westerlies that should be considered in establishing the relationships between weather distribution, speed of movement,

intensification, etc. One of the major differences is that in the westerlies it is not necessary to have convergence and divergence present to maintain a stable wave. See Fig. (6).

An ideal perturbation in the westerlies would be devoid of any convergence or divergence whatsoever (i.e., a pure inertia trajectory wave).

Assuming the wave in Fig. (24) is a wave of this type, the following reasoning may be applied.

As the value of the divergence is equal to zero, the change of latitude must be balanced by the opposite change of vorticity. Then the wave could not move at a speed equal to or greater than the westerly wind component. If the system were stationary the curvature



ture of the streamlines would be equal to the curvature of the trajectories and as the trajectories are inertia trajectories, the curvature streamlines would be equal to the curvature of the inertia trajectories ($\frac{1}{R_s} = \frac{1}{R_t}$). If the system were retrograde, the curvature of the trajectories would be greater than the curvature of the streamlines. In nearly all of the actual observed situations the curvature of the streamlines exceeds the curvature of the inertia trajectories. This would imply that generally speaking if such an ideal wave did exist, it would be moving toward the east at a

speed somewhat less than the westerly component.

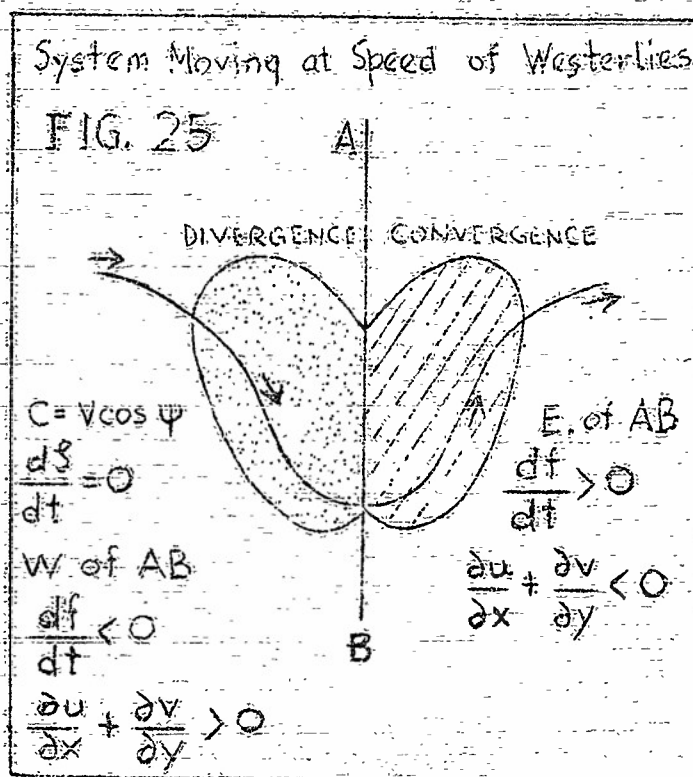
Next, consider the trough in Fig. (25) in which the speed of the trough is equal to the westerly component. Here the value of $\frac{d\zeta}{dt} = 0$ (i.e., the winds are geostrophic) and the convergence will be given by an examination of the changing

latitude value. Everywhere to the west of center line AB, divergence would be taking place and everywhere to the east of center line AB convergence would be occurring; the maximum amount of convergence and divergence will take place at the inflexion points. From this it can be said that with speed of the

trough approximately equal to the speed of the westerlies, convergence should exist in advance of the trough and divergence to the west of the trough.

When the speed of the trough exceeds the speed of the westerlies, the effect of curvature is added to the latitude factor in producing convergence in advance of the trough.

When weather exists to the west of the center line of the trough, again the increase of cyclonic vorticity must be greater than would be produced by the latitude factor alone. This, of course, could be explained if intensification of the trough were taking place, but



intensification of the trough is not necessary for convergence to occur west of the center line.

If the parcels of air are moving through the wave at fairly strong rate, the change of $\frac{dS}{dt}$ is of course fairly large. If a parcel moved from the portion of the trough marked P to the portion marked Q in Fig. (26)

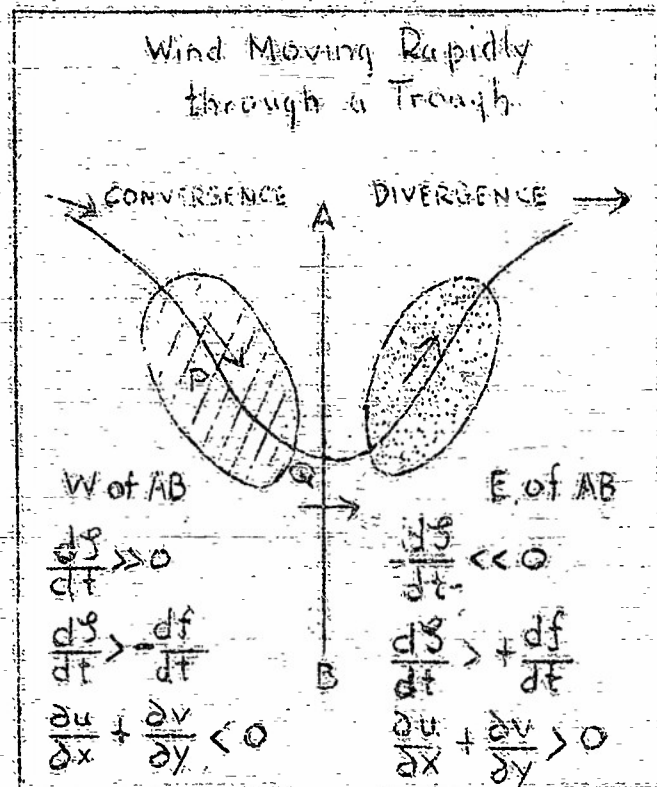
in a relatively short period of time, the value of $\frac{dS}{dt}$ becomes rather large. If the trough is sufficiently pronounced the resulting $\frac{dS}{dt}$ could be large enough to account for both the decreasing latitude factor and convergence. It is easily seen that the more rapidly a parcel of air is moving through a system, the greater will be the value of $\frac{dS}{dt}$.

Also this factor

should tend to offset the latitude factor to the east of the trough resulting in divergence in that area.

Then, a slow moving, stationary, or retrograde trough, with fairly strong westerly component, should have divergence to the east and convergence to the west of the center line. From the preceding statements, neglecting temporarily, intensifications of structure, the acceleration or deceleration of the trough would be reflected by a change in the

FIG. 26



distribution of weather within the trough. With the observance of either of these factors, information relative to the other may be concluded.

A verification of this hypothesis may occur in the westerly troughs that stagnate between Miami and Puerto Rico. There, quite often a trough will move down in a position to the northwest of Turk's Island with build-ups in advance of center line extending to Turk's Island. Nearly always, when the trough becomes stationary, the weather area will intensify and retrograde about to the position near Mayaguana Island.

The above discussion assumes the trough is manifested in the lower level circulation and again does not consider a variable shear. A similar application of a changing shear term may be considered when the consideration of the time changes of curvature and velocities warrant it.

When a westerly trough induces a trough in the easterlies, and the easterlies extend to any considerable depth, there should be convergence to the east and divergence to the west of the center line. If the displacement of the trough is to the east, there should result in a very rapid movement of the parcels in the easterly flow through the trough. This would affect a rather large positive value of $\frac{d^2}{dt}$ to be acting on the air to the east of the center line providing the induced trough is of even moderate intensity.

The northern part of an induced trough may be analyzed as any other deformation field.

In conclusion, the writer would like to emphasize that the material discussed in this report is in no way intended to supplant any of the existing forecasting techniques. It is suggested that these concepts be used to supplement the existing procedures by pointing out possible

applications of both new and old theories to the problem of explaining the occurrence of weather in the tropics. A large part of the popular methods of tropical forecasting have not been discussed. This omission has been prompted by either the nebular state of the current knowledge of some of these factors or the assumption that the subjects have been discussed in sufficient detail in earlier reports. A good deal of the material here is without doubt repetitions of other papers, but the inclusion of material of such a nature was felt necessary to maintain some manner of cohesion to this discussion.

2 May 1945

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