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NO. 105-69)

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27 April 1951

THEORY AND DESIGN
of a
GRADIENT WIND SCALE

1. Air Weather Service Technical Report No. 105-69, subject as above, will be changed as indicated in paragraph 2, effective upon receipt.

2. The following pen and ink changes will be made in Table A:

a. Page 12 -

Change the value "120" for $V_k = 20$, $V_g = 200$ to read "128."
Change the value "92" for $V_k = 80$, $V_g = 280$ to read "100."

b. Page 13 -

Delete the last line ($V_g = 160$, $G = 261$).

BY COMMAND OF BRIGADIER GENERAL SENTER:

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AIR WEATHER SERVICE
TECHNICAL REPORT 105-69

THEORY AND DESIGN
of a
GRADIENT WIND SCALE

November 1950

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AIR WEATHER SERVICE
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Andrews Air Force Base
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Air Weather Service Technical Report 105-69, "Theory and Design of a Gradient Wind Scale," is published for the information and guidance of all concerned.

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PREFACE

Numerous gradient-wind scales have been developed and reported upon in the literature. Although the principle of these scales may be applied to any map projection or scale, a different working model must be made in each case, and most of them are cumbersome to use. The increased emphasis being placed on upper-wind analysis and forecasting necessitated the development of a simple yet accurate scale with universal applicability to all maps used by Air Weather Service. Major Arthur F. Gustafson, of Headquarters, AWS, has developed such a scale and reports its design and use herein. The Weather Central at Hq. AWS, after trying various scales and tables for computing gradient winds, found the one described here to be the most convenient.

THEORY AND DESIGN
OF A
GRADIENT WIND SCALE

Because of its wide scope of operations, the AWS must use various base maps with a wide diversity of scales and projections. For each different scale or projection, the appropriate geostrophic wind scale is different. Geostrophic wind scales for three or five mb isobar intervals are currently printed on most AWS base plotting charts. At many stations geostrophic wind scales for a 200-foot constant-pressure contour interval are already in use. It is anticipated that appropriate wind scales for use on constant pressure analysis will also be printed on the base charts. In any event, the theory underlying their design can be found in a paper by L. P. Harrison.[1]

By utilizing available (or easily constructed) geostrophic wind scales, a new gradient wind scale has been developed which is practically <u>independent</u> of the map scale and projection and therefore applies <u>universally</u> to all maps. The scale consists simply of a series of properly spaced arcs and is easily constructed. Used together with an appropriate geostrophic wind scale and a set of tables, this scale will give the gradient wind speed conveniently and accurately for all of the map projections used in the AWS. The simple theory underlying the design and use of such a scale is described below. The necessary tables are included at the end of this report.

1/ Harrison, L. P., 1946: Fundamental Relationships Involving Field of Pressure and Geopotential. (April) pp. 22-31.

The gradient wind equation can be written:

$$(1) \quad \frac{1}{R'} G^2 + fG = fV_g = g\frac{\Delta z_p}{\Delta n}$$

where: R' is the radius of curvature of the "horizontal"[2/] trajectory (plus for cyclonic and minus for anticyclonic curvature).

$f = 2\Omega \sin\phi$ is the Coriolis parameter.

G is the gradient wind.

g is the acceleration of gravity.

V_g is the geostrophic wind.

Δz_p is the contour interval.

Δn is the spacing between adjacent contours.

Dividing equation (1) through by f we obtain

$$(1') \quad \frac{G^2}{fR'} + G = V_g$$

If a curvature parameter V_K is defined as

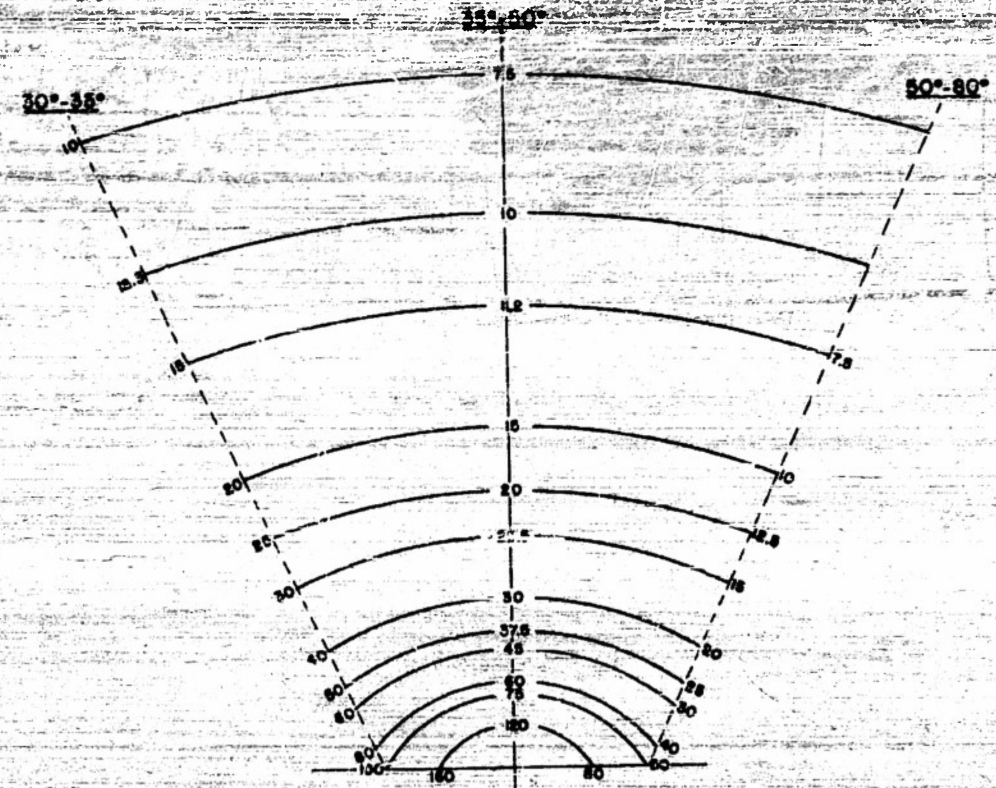
$$(2) \quad V_K = \frac{g\Delta z_p}{fR'}$$

equation (1) can be written:

$$(3) \quad \frac{V_K}{g\Delta z_p} G^2 + G = V_g$$

2/ "Horizontal" curvature is defined in Holmboe, Forsythe & Gustin, 1945, pp. 178-80.

FIG 1



CURVATURE SCALE FOR
POLAR STEOGRAPHIC (1:20X10⁶ AT 60°)
N=2

From equations (1) and (2) it can be seen that V'_K is defined in the same way as V_g except that Δn is replaced by R'. The curvature parameter V'_K can therefore be measured, using a geostrophic wind scale, by scaling off, <u>along the proper latitude</u>, a distance corresponding to <u>R'</u> in place of Δn. A "curvature scale" consisting of a series of circular arcs placed at distances from a reference point which are proportional to their radii, is used as an aid in making these measurements. An example of such a scale in which the arcs are placed at distances equivalent to one-half the radii of their corresponding circles is shown in Figure 1.

The ratio (1:2) is most convenient for maps whose scale is 1:20 x 10^6 or thereabouts. For larger scale maps (e.g. 1:10^7 or 1:12.5 x 10^6) a ratio (1:3) is preferable in order to avoid having too large distances representing significant radii of curvature. The speed V_K measured using these fractional parts of R' will of course be two or three times as great as the V'_K defined in equation (2). In fact, if $R' = NR$ then $V_K = NV'_K$. Substituting this in equation (3), gives:

$$(4) \quad \frac{V_K}{N g \Delta z_p} G^2 + G = V_g \qquad (N = 2, 3)$$

If V_K, G and V_g are in knots and Δz_p is in feet, equation (4) becomes:

$$(5) \quad \frac{V_K}{11.25\, N\, \Delta z_p} G^2 + G = V_g$$

Tables giving G as a function of V_K and V_g can be constructed from equation (5). Two sets of such tables, one (Table A) for $N\Delta z_p = 400$ (e.g. $N = 2$, $\Delta z_p = 200'$) and another (Table B) for $N\Delta z_p = 600$, are included at the end of this report.

For use on surface charts with an isobar interval of Δp, the gradient wind equation corresponding to equation (4) is:

$$(4') \quad \frac{V_K}{N \alpha_0 \Delta p} G^2 + G = V_g$$

(where α_0 is the specific volume at sea level).

For V_K, G and V_g in knots, Δp in millibars and $\alpha_0 = 800$ m^3/ton, the equation corresponding to (5) is:

$$(5') \quad \frac{V_K}{301.1\, N\Delta p} G^2 + G = V_g$$

For $N = 3$ and $\Delta p = 5$ mb, $301\, N\Delta p \doteq 4500$

but for $N\Delta z_p = 400$, $11.25\, N\Delta z_p = 4500$

Table A may therefore be used for computing the gradient wind on surface charts provided the geostrophic scale is for a 5 mb interval and provided a $N = 3$ curvature scale is used. If the geostrophic wind scale is for 3 mb the same Table A can still be used provided the arguments V_K are relabeled as 3/5 of their former values. For example, the column labeled $V_K = 25$ knots would be relabeled $V_K = 15$ knots, etc.

As mentioned previously, the exact value of V_g is found by scaling off the distance corresponding to R on the geostrophic wind scale at the proper latitude. Fortunately, however, the accuracy with which V_K must be measured is not so critical as it is for V_g. The various arcs can therefore be labeled corresponding to their V_K values at two or three fixed latitudes, and these values can be used to a good approximation over a range of latitudes around them. The range of latitudes for which the values of V_K at a fixed latitude can be used depends on the type of map projection being used. If, for example,[3] the map scale varied according to the sine of the latitude, the same contour spacing would everywhere represent the same geostrophic speed. For such a map, the range would be from 0° to 90° latitude. The polar stereographic projection, on the other hand, is an example in which the scale decreases with increasing latitude (and $\sin \phi$) so that the same map spacing of contours represents even larger variations of the geostrophic wind with latitude than that due to the variations of the Coriolis parameter. On a 30°-60° Lambert conformal projection, the scale increases with latitude above 45° N; and the variation of the geostrophic wind or V_K is therefore much smaller.

The curvature scale shown in Figure 1 is for a polar Stereographic map ($1:20 \times 10^6$ at 60° N) and has been labeled corresponding to its V_K values at approximately 32°, 41°, and at 60°. This choice gives a convenient (for labeling) 4:3:2 ratio of the respective V_K's. The latitude ranges in which each set of these fixed latitude V_K's can be used are 20°-35°, 35°-50°, and 50°-[illegible]0° respectively. For a Lambert conformal map only two fixed latitudes are necessary. Taking these as approximately

3/ See Williams, A. F., A Geostrophic Map, Bull. Amer. Met. Soc., November 1950.

35° and 40° gives a convenient 4:3 ratio of the V_K's and covers the latitude ranges 30°-40° and 40°-60° respectively.

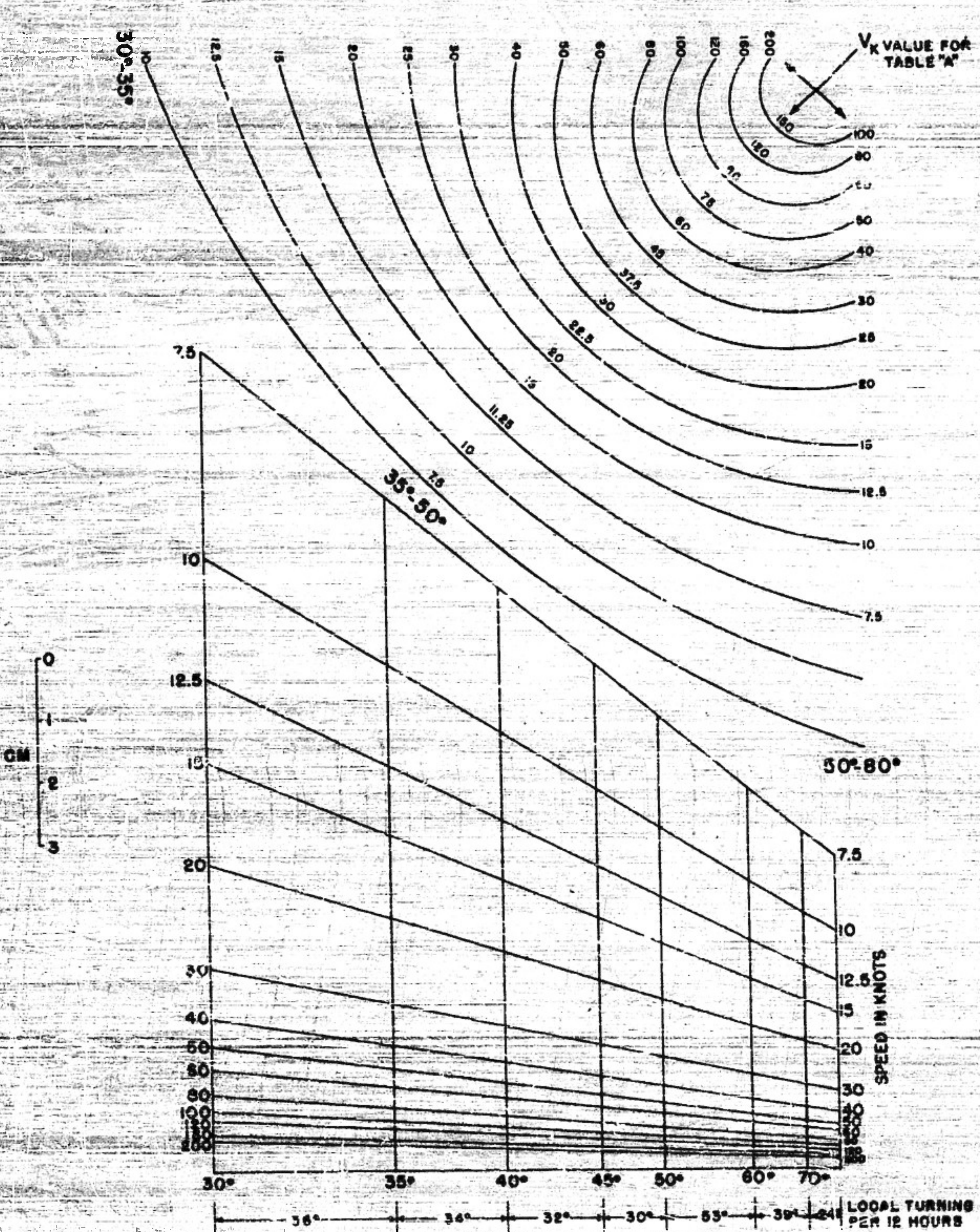
Now if the arcs are spaced according to their radii, the <u>exact</u> value of V_K can always be scaled off by placing the curvature scale on the geostrophic scale at the exact proper latitude. If this added accuracy is not desired, however, the arcs need not be spaced at distances proportional to their radii of curvature. The spacing is then determined by convenience in labeling and by economy of space used. In this form the curvature scale may as well be put on the same piece of plexiglass as the geostrophic wind scale since it can no longer be used for exact scaling of V_K. An example of this form of the scale together with the corresponding geostrophic wind scale is shown in Figure 2.

In order to make the basic principles of the scale more easily understood, the discussion thus far has purposely avoided distinguishing between the <u>contour</u> curvature and the curvature of the <u>trajectory</u>. Obviously when an arc on the curvature scale is fitted to the contours, the contour curvature, not the trajectory curvature, is being measured. In the following, however, it will be shown that this discrepancy can be taken care of by an adjustment in the latitude along which Δn and φ are scaled.

If the wind direction is assumed to be always the same as the geostrophic wind direction, the "horizontal" curvature of the trajectory $K = 1/R'$ is related to the curvature of the contours K_g by the equation:

$$(6) \quad K = K_g + \frac{1}{G}\frac{\partial \psi}{\partial t} \quad \text{or} \quad \frac{1}{R'} = \frac{1}{R_g} + \frac{1}{G}\frac{\partial \psi}{\partial t}$$

FIG 2



GEOSTROPHIC AND GRADIENT WIND
SCALE FOR 1:20 X 10⁶ POLAR
STEREOGRAPHIC MAP (TRUE AT 60°)
AND 200FT CONSTANT PRESSURE CONTOUR INTERVAL

where $\frac{\partial\psi}{\partial t}$ is the local time rate of turning of the geostrophic wind.[4]

Substituting this in equation (1) we have:

$$\left(\frac{1}{R_g} + \frac{1}{G}\frac{\partial\psi}{\partial t}\right)G^2 + fG = fV_g$$

or

$$(7) \qquad \frac{1}{R_g}G^2 + \left(f + \frac{\partial\psi}{\partial t}\right)G = fV_g$$

Comparing equations (1) and (7) it is seen that R_g may be used in place of R' provided the latitude is adjusted in such a way as to account for the difference between f and $f + \frac{\partial\psi}{\partial t}$. For cyclonic (counterclockwise) local turning an adjustment should be made such that smaller values of V_g and V_K are obtained (usually toward higher latitudes) whereas for anticyclonic turning the adjustment should be made so as to give larger values of V_g and V_K. The local turning in degrees per 12 hours, necessary to require an adjustment from one reference latitude to another (e.g. from 35° N to 40° N), are given below. These values, though independent of the map scale, are unfortunately not independent of

---

4/ The treatment here, as implied from the beginning by the use of the "horizontal" curvature, assumes either that the flow is horizontal or that the effects of the vertical motion can be neglected. The main effect neglected is that on the curvature of the trajectory due to a combination of vertical motion w and turning of the wind with height $\partial\psi/\partial z$. The term $w/G\ (\partial\psi/\partial z)$, which should be added to the right hand side of equation (6) due to this effect, can be as large or larger than the term $1/G\ \partial\psi/\partial t$. Due to the difficulty in determining w, however, it is usually neglected.

the type of map projection.[5] Values are therefore given for the most common projections used in the AWS; i.e. the polar stereographic, the Lambert conformal 30°-60° N, and the Lambert conformal 30°-75°.

Degrees per 12 hrs of Local Turning Corresponding to Latitude Change $\phi_1 \longleftrightarrow \phi_2$

(Values given are average of change from ϕ_1 to ϕ_2 and from ϕ_2 to ϕ_1)

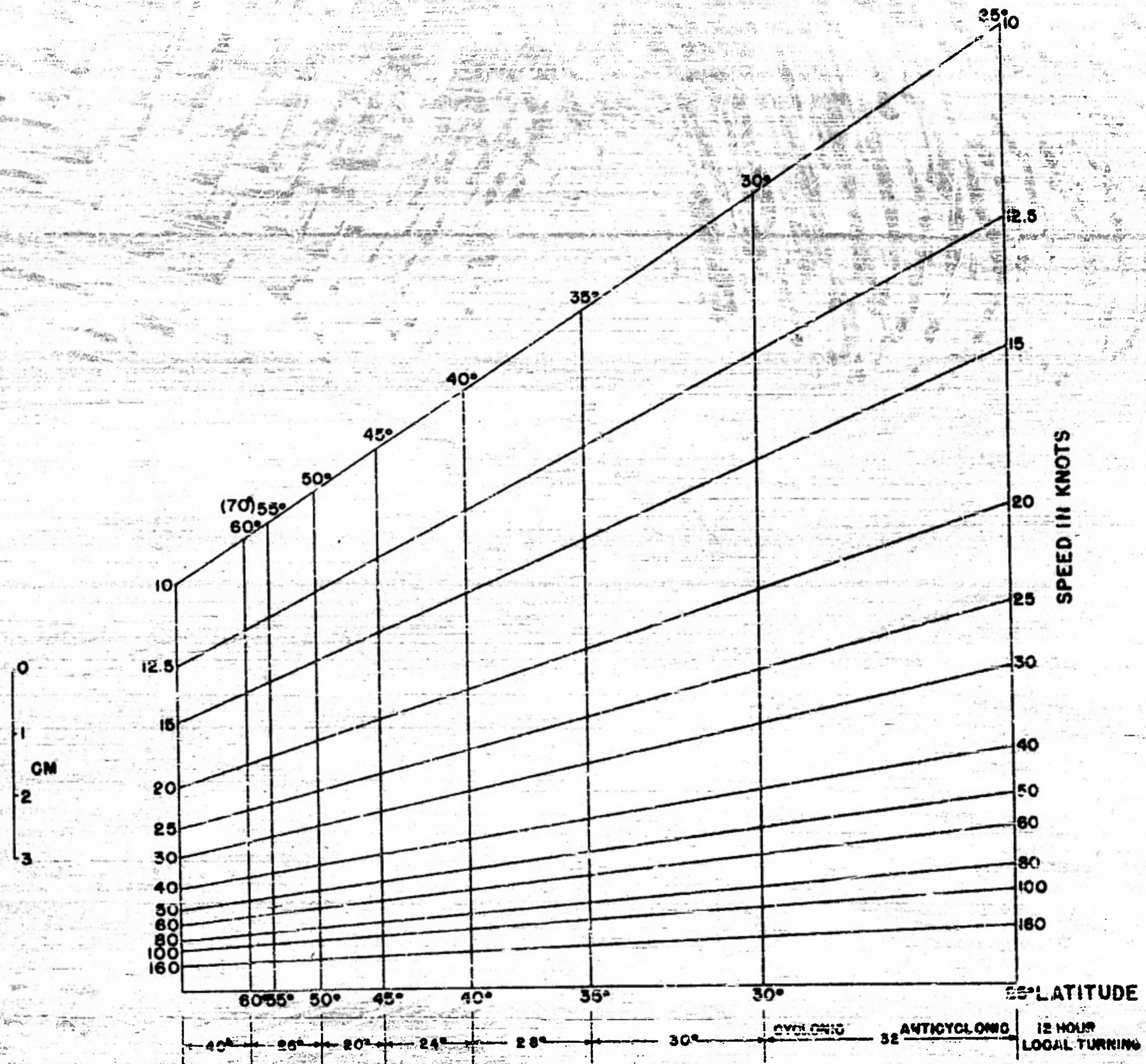
$\phi_1 \longleftrightarrow \phi_2$	Polar Stereographic	Lambert Conformal 30°-60°	Lambert Conformal 30°-75°
25 - 30	±27	±32	±34
30 - 35	36	30	32
35 - 40	34	28	30
40 - 45	32	24	27
45 - 50	30	20	24
50 - 55	28	16	20
55 - 60	25	±10	16
60 - 70	39	--	±14
70 - 80	±24	--	--

5/ It can be shown that the local turning $(\partial\Psi/\partial t)_{1,2}$ corresponding to a change in latitude from ϕ_1 to ϕ_2 is:

$$-(\partial\Psi/\partial t)_{1/2} = f_{\phi_1} - \frac{K_{\phi_1}}{K_{\phi_2}} f_{\phi_2} = 360 \left(\sin \phi_1 - \frac{K_1}{K_2} \sin \phi_2\right)$$

where K_ϕ is the scale variation as defined by L. P. Harrison (op cit).

FIG 3



GEOSTROPHIC WIND SCALE
FOR 200 FT CONTOUR INTERVAL ON 30°-60°
LAMBERT CONFORMAL 1:12.5 X 10^6

930951

In Figure 3 a geostrophic wind scale is shown for a Lambert conformal 30°-60° map on which the approximate amounts of turning corresponding to an adjustment from one reference latitude to another have been entered.

The manner in which these values are used is illustrated by the following example. Suppose the point in question is at 33° and the local rate of turning has been estimated as 30° per 12 hours. On the Lambert conformal 30°-60° map, 30 degrees of turning is seen to correspond to about a 5° adjustment of the latitude. The adjusted latitude is therefore taken as 38°. On the polar stereographic, on the other hand, the adjusted latitude would be $33° + \frac{30}{36} \times 5° = 37°$.

Around 60° or 70° on the Lambert conformal 30°-60° map, an adjustment may be required which cannot be identified with any latitude[6]. A point representing 40° of cyclonic turning from 60°, however, can be located on the type of geostrophic wind scale shown in Figure 2, as follows: — Extend the 15 knot line from 60° until its Δn value is equal to the Δn for the 20 knot line at 45°. From this point draw a line parallel to the 60° N latitude line and then extend the remaining $V_g =$ const. line as shown in Figure 2.

The local turning $\frac{\partial \psi}{\partial t}$ can be estimated in various ways. The past 12 hour turning of the contours, for example, may often be taken as an indication of the instantaneous rate of turning, provided that approximately the same rate of turning can be anticipated judging from a 12-hour prognosis.

6/ The same is true for a 30°-75° Lambert conformal around 70° or 75°.

If the contour system is moving without appreciable change of shape, the local rate of turning can be estimated from the propagation speed of the system and the curvature of the contours. Rather than do this and then adjust the latitude as described above, however, it is simpler to "correct" V_x by multiplying it by the ratio:

$$\frac{G_I - c}{G_I}$$

where G_I is a first estimate of the gradient wind and c is the component of the propagation velocity in the direction of the geostrophic wind. When this "correction" to V_x is made, no adjustment is made in the latitude.

Although the gradient wind scale described above can be used anywhere on the map, it should be realized that the gradient wind speed is more representative of the true wind speed under certain circumstances than under others. Having been based on the assumption that the motion is frictionless, quasi horizontal and constant in speed, the gradient wind cannot be expected to give good results at low elevations over rough or mountainous terrain. For reasons mentioned above (see footnote 4), the gradient wind is also likely to be less reliable where the wind (or the geostrophic wind) veers or backs rapidly (e.g. 20° per 5,000 ft. or more) with height. It may also be stated both on theoretical and empirical grounds that the gradient wind speed is most applicable (as opposed to the geostrophic wind, for example) when the speed is high (e.g. > 50 knots) and when the curvature of the contours is cyclonic.

Fortunately when the geostrophic wind speed is weak, rawin, if not pibal or rabal observations, are more likely to be available since low elevation angles resulting in termination of SCR 658 rawin observations will not occur. Furthermore, in those situations where the curvature is anticyclonic (and the gradient wind is less reliable), the skies are more likely to be clear, permitting pibal or rabal observations. Under these conditions, weak winds or anticyclonic curvature (or both), observed wind speeds, if available, should be used to the fullest extent even though they differ from the computed gradient wind speed.

REFERENCES

Harrison, L. P., 1946: Fundamental Relationships Involving Field of Pressure and Geopotential. (April) pp. 28-31.

Williams, A. F., 1950: A Geostrophic Map. Bull. Am. Meteor. Society, November 1950.

Holmboe, J., Forsythe, G. E., and Gustin, W., 1945: Dynamic Meteorology. pp. 178-80.

TABLE A

$(F \Delta a_p = 400)$

Cyclonic

z \ K	7.5	10	12.5	15	20	25	30	40	50	60	80	100	120	160
10	10	10	10	10	10	10	9	9	9	9	9	8	8	8
15	15	14	14	14	14	14	14	12	13	13	12	13	11	11
20	19	19	19	19	18	18	18	17	17	16	16	15	14	14
25	24	24	23	23	22	22	22	21	20	20	19	18	17	16
30	29	28	28	28	27	26	26	25	24	23	22	21	20	18
35	33	33	32	32	31	30	29	28	27	26	24	23	22	20
40	38	37	36	36	35	34	33	31	30	29	27	26	24	22
45	42	41	40	40	38	37	36	34	33	32	29	28	26	24
50	46	45	44	44	42	41	40	37	35	34	32	30	28	26
60	56	54	52	51	49	48	46	43	41	39	36	34	32	29
70	65	62	60	59	56	54	52	49	46	44	41	38	36	33
80	72	69	67	66	[illegible]	[illegible]	58	54	51	49	45	42	39	36
90	80	77	75	72	[illegible]	65	63	59	56	53	48	45	42	38
100	87	84	82	79	75	72	69	64	60	57	52	48	45	41
110	95	91	88	86	81	77	74	68	64	61	56	51	48	43
120	104	98	95	92	87	82	79	73	68	64	58	54	51	45
140	117	112	108	104	98	93	88	81	76	72	65	60	53	48
160	131	125	120	116	108	102	97	89	83	79	71	65	61	55
180	145	138	132	127	118	111	106	97	90	85	76	70	62	56
200	158	150	143	137	129	120	114	104	97	91	82	75	70	63
240	184	173	165	157	146	136	129	117	109	102	91	84	78	69
280	206	195	185	176	163	152	143	130	120	112	92	92	86	76

TABLE A (Continued)

Anticyclonic

$V_g \backslash V_K$	7.5	10	12.5	15	20	25	30	40	50	60	80	100	120	160
10	10	10	10	10	12	11	11	11	11	12	13	15		
15	15	16	16	16	16	17	17	18	19	21	28			
20	21	21	21	22	22	23	24	26	30					
25	26	27	27	28	29	30	32	37						
30	33	32	33	34	35	38	41							
35	37	38	39	40	43	46	56							
40	43	45	45	48	52	60								
45	[illegible]	51	53	55	62	104								
50	65	56	60	63	75									
60	68	71	76	93										
70	81	84	95	110										
80	95	104	120											
90	110	120												
100	127													
110	143													
120	158													
150	221													

GRADIENT WIND V IN KNOTS

Formula: $\dfrac{(V_K)\,V^2}{4{,}500} + V = V_g$

V_g = geostrophic speed (knots)

V_K = curvature parameter

for either:

$N = 3,\ \Delta z_p = 200$ ft

or:

$N = 3,\ \Delta p = 5$ mb

TABLE B

($N \Delta z_p = 600$)

Cyclonic

V_g \ V_K	10	12.5	15	20	25	30	40	50	60	80	100	120	160
10	10	10	10	10	10	10	9	9	9	9	9	9	8
15	15	15	14	14	14	14	14	14	13	13	13	12	12
20	19	19	19	19	19	18	18	18	17	17	16	16	15
25	24	24	24	23	23	23	22	22	21	20	19	19	18
30	29	28	28	28	27	27	26	25	25	23	23	22	20
35	33	33	33	32	31	31	30	29	28	27	25	24	23
40	38	37	37	36	35	35	33	32	31	29	28	27	25
45	42	42	41	40	39	38	37	36	34	33	31	29	27
50	47	46	45	44	43	42	40	39	37	35	33	32	29
55	51	50	49	48	47	46	44	42	40	38	36	34	31
60	55	54	54	52	50	49	47	45	43	41	38	36	33
70	64	63	62	59	58	56	53	51	49	45	43	41	37
80	72	71	69	67	65	63	59	57	54	50	47	45	41
90	80	79	77	74	72	69	65	62	59	55	51	48	44
100	88	86	84	81	78	75	70	67	64	59	55	52	47
110	96	94	91	87	84	81	76	72	68	63	59	56	50
120	104	101	98	94	90	87	81	77	73	67	62	58	53
140	118	115	112	106	102	98	91	86	81	74	69	65	59
160	134	129	125	118	113	108	100	94	89	81	76	71	64
180	148	142	138	130	124	118	109	102	97	88	82	76	69
200	161	155	150	141	134	128	118	110	104	94	87	82	73
240	185	180	173	162	153	146	124	125	117	106	98	91	82
280	213	203	195	182	171	163	149	138	130	117	108	100	90

TABLE B (Continued)

Anticyclonic

V_g \ V_K	10	12.5	15	20	25	30	40	50	60	80	100	120	160
10	10	10	10	10	10	10	11	11	11	12	12	13	16
15	15	16	16	16	16	16	17	17	18	19	22		
20	21	21	21	21	22	22	23	24	26	32			
25	26	26	27	27	28	29	30	33	37				
30	31	32	32	33	35	36	39	45					
35	37	38	38	40	41	43	50						
40	42	43	44	46	49	52	65						
45	48	50	51	53	57	62							
50	54	56	57	61	66	75							
55	60	62	64	69	77	95							
60	67	69	71	78	90								
70	78	83	87	99	130								
80	93	98	104	130									
90	107	114	124										
100	122	133	150										
110	138	154	191										
120	157	180											
140	198												
160	261												

GRADIENT WIND G IN KNOTS

Formula: $\frac{V_K}{6,750} G^2 + G = V_g$

V_g = geostrophic speed (knots)

$\bar{V}_K$ = curvature parameter