REEL-C 6633

A.T.I 165981

UNCLASSIFIED

THE CALIBRATION OF THE SLOTTED SECTION FOR PRECISION MICROWAVE MEASUREMENTS

ARTHUR A. OLINER

MICROWAVE RESEARCH INSTITUTE
POLYTECHNIC INSTITUTE OF BROOKLYN
55 Johnson Street
Brooklyn 1, New York

(Report R-267-52, PIB-206. This work was performed for AFCRC under Contract AF 19(122)-3).

MARCH 1952

AIR FORCE CAMBRIDGE RESEARCH CENTER

CAMBRIDGE, MASSACHUSETTS

TABLE OF CONTENTS

I. INTRODUCTION

The function of the slotted section at microwave frequencies is to yield information concerning the properties of the standing wave existing in the waveguide; of particular interest are the locations of the voltage minima, and the VSWR (voltage standing wave ratio). Various small errors can arise in connection with such measurements which are of little practical consequence when ordinary (rough) measurements are taken, but which must be considered and compensated for when precise results are desired. The three principal error sources are: (a) the deviation from square law behavior of the detector attached to the probe, (b) the admittance associated with the probe, and (c) the presence of the slot in the slotted section, and other discontinuity effects inherent in the equipment. Error source (a), which affects VSWR measurements, may become quite serious depending upon the equipment used and consequently methods for its elimination are well known.[1] The distortion of the standing wave pattern due to the probe admittance may generally be minimized sufficiently by permitting only small probe insertion depths and by tuning the probe for maximum pickup; a discussion of probe effects[2] and a method of correction[3] for large insertion depths are given in the literature. This report is concerned solely with error source (c).

The presence of the slot in the slotted section introduces two effects: (a) the slot loads the waveguide and produces a slight shift in its guide wavelength[4] and characteristic impedance, and (b) the end of the slot introduces a discontinuity effect. Other discontinuity effects may be

---

1 For example, H. M. Barlow and A. L. Cullen, "Microwave Measurements", Constable and Company Ltd., London, 1950, pp. 128-130.

2 ibid., p. 139 ff.
C. G. Montgomery, "Technique of Microwave Measurement", McGraw-Hill Book Co., 1947, Sec. 8.4

3 W. Altar, F. B. Marshall, and L. P. Hunter, "Probe Error in Standing Wave Detectors", Proc. I.R.E., 34, pp. 33P - 44P, Jan. 1946.

4 C. G. Montgomery, op. cit., Sec. 8.3. Expressions for the shift are given.

introduced by couplings, the use of an adapter, stub or bead-supports if coaxial line is used, inaccuracies of manufacture, etc. The shift in guide wavelength (except in coaxial line), of the order of 1/2 percent, is not disturbing since the guide wavelength is accurately measured in the slotted section. The shift in characteristic impedance, also small, introduces a slight error, however, in that measured impedances are normalized to the shifted rather than to the desired unshifted value of characteristic impedance. In addition, the shift in characteristic impedance produces a reflection at the end of the slot, although this effect is less significant than that due to the geometrical discontinuity produced by the end of the slot. For common slotted sections at X-band, the end of the slot results in an insertion VSWR of between 1.01 and 1.02. Other discontinuities which may exist in the associated equipment can easily produce even larger insertion VSWR's. These discontinuity effects introduce errors in the location of the voltage minimum, and in the measurement of the VSWR. This report presents a simple and direct calibration scheme that yields the corrections to the location of the voltage minimum and to the VSWR so that the resulting values correspond to what would be obtained in a waveguide without a slot and with no discontinuity effects present.

Methods of compensating for the discontinuity effects have been considered in the literature. For example, M. H. Oliver[5] treats the discontinuity or discontinuities present as a lumped two terminal-pair network, gives methods for its measurement, and presents schemes for subtracting out the parameters of this network in order to correct a measured impedance. He does not consider the shift in characteristic impedance of the slotted section. While his method is suitable for the measurement of simple terminations where a value of impedance is directly obtained, it is too cumbersome for precision methods for two terminal-pair structures which employ an averaging scheme for the input data and do not present the impedance values until the very end. The calibration scheme presented herein corrects the input data directly. F. Tischer[6] describes a measurement method for determining the maximum possible error in the measurement of VSWR due to the presence of discontinuities; his result is in agreement with Eq. (2.9) of this report. While the considerations are directed toward the input data, they are evidently incomplete; the remainder of his paper is not of concern here. The method employed in this report is an

---

5 M. H. Oliver, "Discontinuites in Concentric-Line Impedance-Measuring Apparatus", Proc. I.E.E., London, Part III, 97 , p. 29, Jan. 1950. Also, discussion, p. 242.

6 F. Tischer, "Die Genauigkeit der Impedanzmessung bei Mikrowellen", Transactions of the Royal Institute of Technology, Stockholm, Sweden, No. 36, 1950.

extension of the one used by N. Marcuvitz[7] and co-workers at the M.I.T. Radiation Laboratory. Their considerations were limited to the measurement of lossless structures, and only the discontinuity effects were taken into account. By a slight extension, discussed herein, the guide wavelength shift is readily compensated for. It is further shown that it is possible by means of appropriate simple factors to correct the values of the location of the voltage minimum and the VSWR in the measurement of dissipative structures from the calibration curve valid directly for lossless measurements only.

Part II discusses the calibration procedure in principle and in application, while Part III presents the theoretical basis for the procedure; the Appendix contains a detailed instruction procedure for technicians. Part II begins with a description of the calibration procedure and the principle underlying it, and includes a sample calibration curve for an X-band slotted section to illustrate the quantitative aspects. The corrections to the VSWR and the location of the voltage minimum for both lossless and dissipative structures are discussed and expressions for the maximum possible corrections for these quantities are given in terms of the insertion VSWR of the slotted section being calibrated, the latter quantity being obtainable from the calibration curve itself. Section B of Part II considers a number of applications for the calibration procedure, and first presents remarks concerning reference plane shifts and electrical length measurements appropriate to discontinuities located in a length of waveguide. The modifications in the calibration procedure pertinent to the following special situations are then discussed: (a) when coupling structures are associated with the variable short in addition to the slotted section, (b) when the slotted section and the waveguide containing the discontinuity are of different cross-sectional shape or dimensions, and (c) when a so-called "relative" equivalent network is desired for a junction between two dissimilar waveguides.

The derivations of the correction relations and associated expressions are given in Part III. A theoretical expression is derived first for the calibration curve obtained for purely reactive terminations. The modifications necessary when the terminations are dissipative are then considered, and the appropriate correction relations are derived. Finally, the effect on the calibration curve of an error in the variable short is examined, and a method of testing for such an error is described. The Appendix, in presenting detailed instructions on taking the measurements, treating the data, and obtaining the pertinent corrections from the calibration curve, is essentially self-contained.

---

7 N. Marcuvitz et al., "The Representation, Measurement, and Calculation of Equivalent Circuits for Waveguide Discontinuities with Application to Rectangular Slots", Polytechnic Institute of Brooklyn, 1949, Chap. IV, pp. 6, 7.

II. THE CALIBRATION PROCEDURE

A. The Calibration Curve: Principle and Procedure

The basic slotted section measurements of VSWR and the location of the voltage minimum are affected by the variety of errors inherent in the slotted section. We treat here only those effects introduced by discontinuities due to the end of the slot and possible adaptors, bead supports, etc., and by the presence of the slot in creating a guide wavelength and characteristic impedance slightly different from that of a guide without the slot. The calibration procedure to be described compensates rigorously for these effects in a simple fashion, and, in additi , corrects the input data directly.

The arrangement of the equipment for a slotted section calibration is shown in Fig. 1; distances D and S are considered positive in the directions indicated. Although in Fig. 1, D and S are shown as actual distances from the reference plane indicated, it is sufficient in the discussion below to regard the symbols D and S as arbitrary indicator readings, where S represents the location of a short circuit in the variable short and D the location of the corresponding voltage minimum in the slotted section. The actual distances are then given by subtracting D_R and S_R, respectively, from the D and S readings, where the former quantities (subscript R signifying "reference") are the indicator readings corresponding to a short circuit at the interface between the end of the slotted section and the variable short, or, alternatively, from some other convenient reference plane or planes. The "slotted section" to be calibrated includes, in addition to the slotted section proper, any adaptor, couplings, bead supports, etc., if used. The calibration procedure assumes that the variable short is perfect; the effect of an imperfect variable short is considered in Part III, Sec. C.

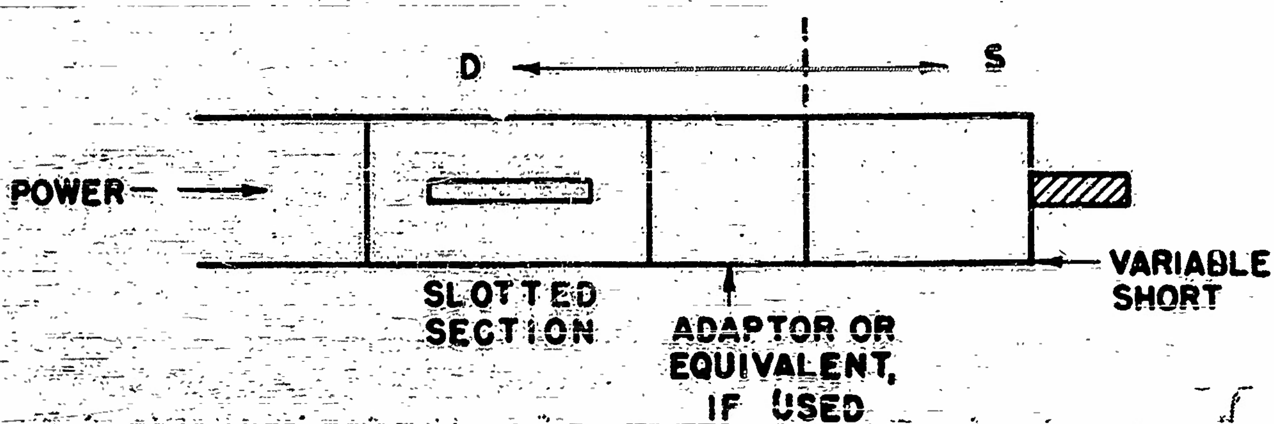

Fig. 1 Arrangement for Slotted Section Calibration.

The data for the calibration curve are obtained by taking a series of values of D and S over the range of travel of the probe carriage; in addition, measurements are made of the guide wavelengths λ_{gd} and λ_{gs} in the slotted section and variable short, respectively.

In order for the calibrated slotted section to correspond to an ideal or perfect slotted section, we require

$$D_{corrected} = - S \quad , \qquad (2.1)$$

the negative sign appearing because of the opposite directions assigned to D and S, as seen in Fig. 1. This specification implies that all discontinuity effects have been eliminated and that the guide wavelength in the slotted section is identical to that in the variable short. It is preferable to satisfy requirement (2.1) in two steps: (a) constructing a calibration curve according to the specification

$$\frac{D_{corrected}}{\lambda_{gd}} = - \frac{S}{\lambda_{gs}} \qquad (2.2a)$$

or, equivalently,

$$D_{corrected} = - S \ \lambda_{gd}/\lambda_{gs} \quad , \qquad (2.2b)$$

and (b) multiplying the corrected D value by the wavelength ratio $\lambda_{gs}/\lambda_{gd}$. The $D_{corrected}$ in specification (2.1) is, of course, different from that in (2.2); the latter is employed in the discussion below. Defining a correction Δ D, to be added to the directly obtained voltage null D to yield the corrected value, as

$$\Delta D = D_{corrected} - D \quad ,$$

we have from (2.2b)

$$\Delta D = - (D + S \ \lambda_{gd}/\lambda_{gs}) \quad , \qquad (2.3)$$

so that the corrections Δ D may be obtained from a curve of

$$- (D + S \ \lambda_{gd}/\lambda_{gs}) \ \text{vs.} \ D.$$

Such a curve is sketched in Fig. 2; the shape of the curve itself is discussed in Part III, Sec. A.

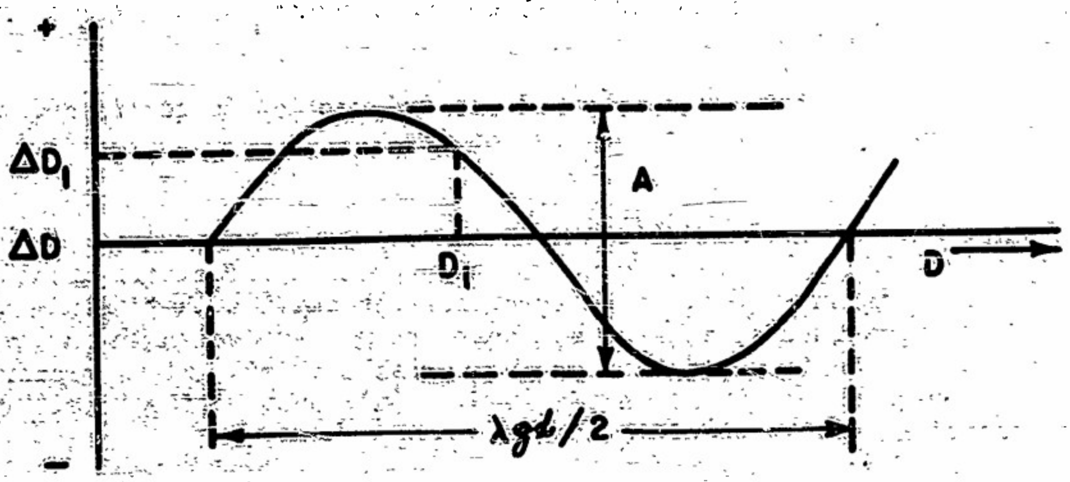

Fig. 2 Sample Calibration Curve.

A plot of $-(D + S)$ vs. D, yielding corrections in accordance with specification (2.1) rather than (2.2b), is not recommended because of the slope introduced by the unequal guide wavelengths in the slotted section and variable short. The slope creates a number of undesirable features, the principal one being the involved modifications necessary when dissipative structures are measured. Impedance values obtained from corrections made according to specification (2.2b) are correctly normalized[8] to the characteristic impedance of a guide without the slot, but the resulting transmission line representation for the slotted section is peculiar[8] in that its characteristic impedance is identical with that of the variable short while its guide wavelength is not. The peculiarity is eliminated, and the equivalent of specification (2.1) obtained, if distances $D_{corrected}$ are made relative to λ_{gs} by multiplying by the ratio $\lambda_{gs}/\lambda_{gd}$. Since all D readings must eventually be subtracted from D_R, and since both D and D_R must be corrected by the same calibration curve, the abscissa level of the calibration curve is unimportant and for convenience we draw the abscissa axis through the average of the curve as indicated in Fig. 2.

---

8 This was pointed out to the writer by H. M. Altschuler of the Microwave Research Institute of the Polytechnic Institute of Brooklyn.

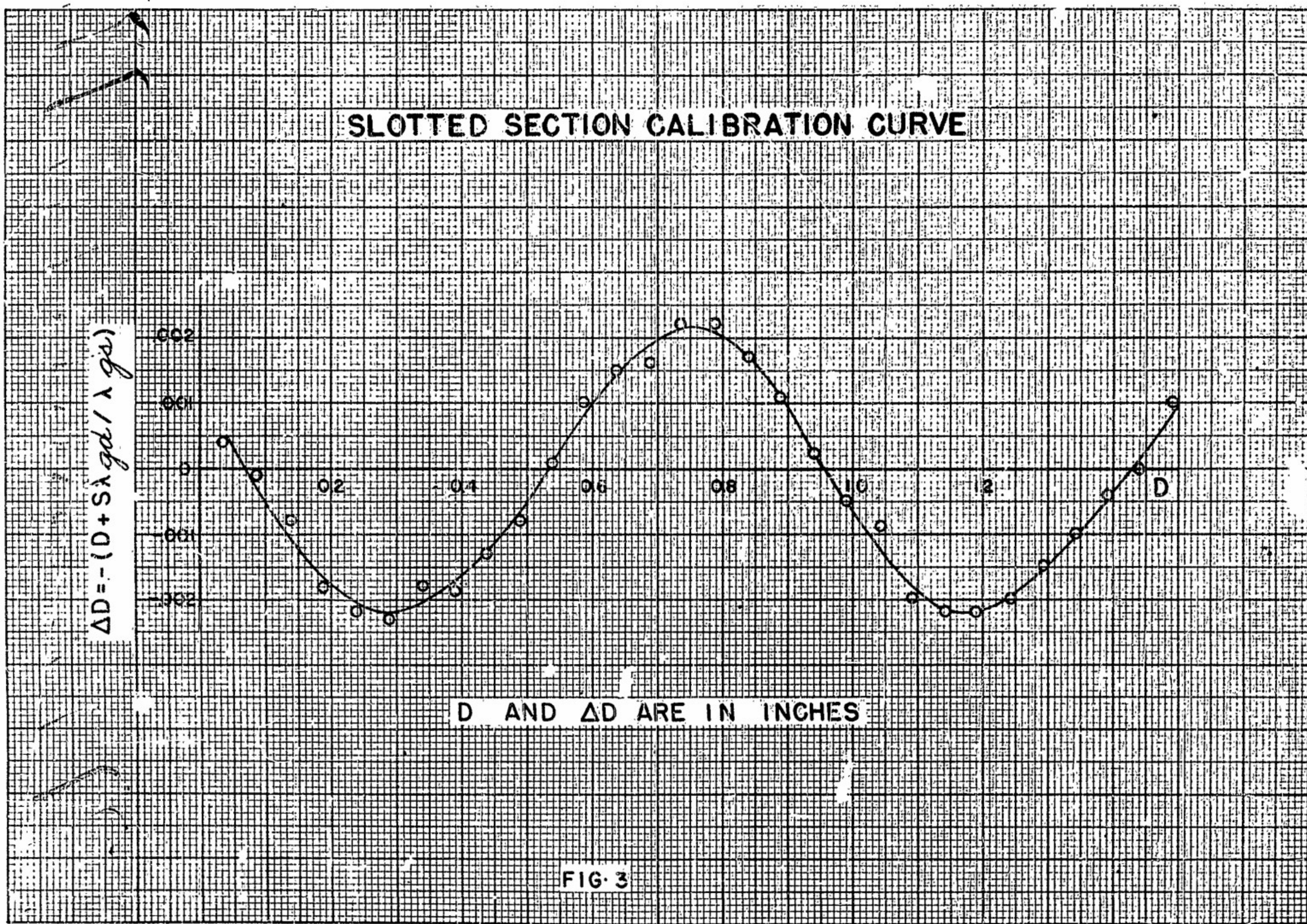
SLOTTED SECTION CALIBRATION CURVE
ΔD = -(D + Sλgd/λgs)
.002
.001
0
-.001
-.002
.02
.04
.06
.08
.10
.12
D
D AND ΔD ARE IN INCHES

FIG. 3

If an error is made in the measurement of $\lambda_{gd}/\lambda_{gs}$, a slight slope will appear in the calibration curve; the slope should be eliminated by altering this ratio accordingly. Since this is a sensitive test of the accuracy of the ratio, the resulting value may be used as a more accurate means of determining either λ_{gs} or λ_{gd}, whichever is felt to be less reliable. At X-band, the ratio for a particular slotted section and a particular variable short after a number of calibrations has been found to be $1.0025 \pm .0004$. For different variable shorts the value varies slightly, but the precision remains the same.

A sample calibration curve of $-(D + S\,\lambda_{gd}/\lambda_{gs})$ vs. D is shown in Fig. 3, where the original data points are included. The calibration curve, for a Yale and Towne slotted section at a free space wavelength of 3.200 cms., is seen to indicate an insertion VSWR of 1.016 (see Eq. (2.5a)), and has a ratio $\lambda_{gd}/\lambda_{gs}$ equal to 1.0027. The precision of the measurements is seen from the scatter of the points to be $\pm$ 0.0003 inches or better. It is understood that the calibration is valid only at the frequency at which it is made.

To summarize the calibration procedure: A series of values of D and S are taken over the range of travel of the probe carriage. A plot of $-(D + S\,\lambda_{gd}/\lambda_{gs})$ vs. D is made, the abscissa axis is drawn through the average of the curve, and the resulting ordinate is labelled $\triangle$ D. The value of $D_{corrected}$ is then given by

$$D_{corrected} = (D + \triangle D)\,\lambda_{gs}/\lambda_{gd} \quad . \tag{2.4}$$

An accurate value for the insertion VSWR of the slotted section and associated discontinuities is obtainable directly from the calibration curve. If these discontinuities are small, the calibration curve is sinusoidal in shape, and the insertion VSWR is related to the double amplitude A of the curve (see Fig. 2) by

$$\text{Insertion VSWR} = 1 + 2\pi\,A/\lambda_{gd} \quad . \tag{2.5a}$$

If the difference between the slot insertion VSWR and unity be expressed as ϵ, then (2.5a) becomes

$$\epsilon = \kappa_d\,A \quad , \tag{2.5b}$$

where $\kappa_d = 2\pi/\lambda_{gd}$, and the maximum possible correction $\triangle$ D is related to ϵ as

$$|\triangle D|_{max.} = A/2 = \epsilon/2\,\kappa_d \tag{2.6}$$

If the discontinuities are large, the calibration curve becomes more saw-tooth-shaped, and the appropriate expressions corresponding to relations (2.5b) and (2.6), respectively, valid for a lossles discontinuity of any size, are

$$\epsilon = 2 \tan \phi \, (\tan \phi + \sec \phi) \tag{2.7}$$

where $\phi = \kappa_d \, A/2$, and

$$\left| \Delta D \right|_{max.} = \frac{1}{\kappa_d} \tan^{-1} \left[\frac{\epsilon}{2\sqrt{1+\epsilon}} \right] \tag{2.8}$$

The calibration curve discussed above was obtained using purely reactive terminations and is therefore directly applicable for lossless structures only. Since the termination for the slotted section discontinuities is no longer purely reactive when dissipative structures are measured, one must investigate the effect on the D values of a dissipative termination; in particular, one desires the relation between the curve of $-(D+S\ \lambda_{gs}/\lambda_{gd})$ vs. D for lossless terminations and that for dissipative terminations. Such an investigation is carried out in Part III, Sec. B, where it is shown that under the restriction $\epsilon/(r_m - 1) \ll 1$, where r_m is the measured VSWR value and ϵ has been defined above, the calibration curve may be used together with appropriate simple correction factors to obtain the corrections to both the D and VSWR values in the measurement of dissipative structures. Since ϵ is generally small, the vast majority of measurements are included under this restriction; if $\epsilon = 1.016$, say, as in the calibration curve of Fig. 3, the correction relations may be used for measured VSWR's above 1.10.

The correction relations for dissipative structures are given in the Appendix as Eq. (A3) for the corrected value of D and as Eq. (A4) for the correction to the VSWR value. By reference to relations (A4) and (2.5b), the maximum possible correction Δr is related to the slotted section insertion VSWR by

$$\left| \Delta r \right|_{max.} = \epsilon \, r_m \ , \tag{2.9}$$

in agreement, since ϵ is small, with the well known relation concerning VSWR's of structures in tandem. From relation (A3) it is seen that for high r_m, the correction to D reduces to that for no loss, while for r_m approaching unity the correction diverges. The latter behavior is consistent with the fact that the effect of the location of the voltage minimum on impedance values becomes inappreciable as the VSWR approaches unity. For values of r_m for which ϵ is no longer much less than $(r_m - 1)$, the expressions become rather cumbersome; for D, the correction is rather unimportant, however, as mentioned above. Furthermore, in our experience in the measurement of two ~~terminal-pair networks r_m has rarely been less than 1.10.~~

B. Other Applications of the Calibration Procedure

1. Additional Remarks on Two Terminal-Pair Structures

A few further remarks are appropriate if the structure to be measured consists of a discontinuity located in a length of waveguide. Since the equivalent circuit measurements are intended for the discontinuity alone, the waveguide housing the discontinuity must be of the same cross-section dimensions as the slotted section and variable short. The procedure described in Sec. A of the Appendix specifies the reference planes at the ends of the slotted section and the variable short, i.e., at D_R and S_R; if reference plane shifts are required, the shifts in both input and output guides should be made relative to the variable short guide wavelength, λ_{gs}, by virtue of the calibration scheme.

For symmetrical discontinuities, one can obtain the equivalent circuit parameters at reference planes symmetrically disposed with respect to the geometrical symmetry plane without knowing the location of the discontinuity within the housing waveguide; only the overall length of the housing structure need be known. The measurement method,[9] which will not be discussed here, yields the shifts ϕ_1 and ϕ_2, in electrical lengths, from D_R and S_R, respectively, to reference planes symmetrically disposed with respect to the geometrical symmetry plane of the discontinuity, as shown in Fig. 4. The location of the reference planes relative to the symmetry plane of the discontinuity, specified by $\phi_{2\ell}$, follows directly from the knowledge of the overall electrical length ϕ_L of the housing structure.

The electrical length ϕ_L is obtained by placing a short circuit first at the end of the slotted section, determining D_R, and then at the end of the empty housing waveguide when its other end is attached to the slotted section, obtaining D_L. ϕ_L is then given by

$$\phi_L = 2\pi\left[\frac{D_R - D_L}{\lambda_{gd}} - \frac{n}{2}\right], \quad n \text{ integer} \tag{2.10}$$

where n is so chosen as to make ϕ_L a minimum.

[9] For lossless structures: Footnote reference 7, Chap. II, pp. 26-30.

For dissipative structures: "The Precision Measurement of the Equivalent Circuit Parameters of Dissipative Microwave Structures", by L. Felsen and A. A. Oliner. To be issued.

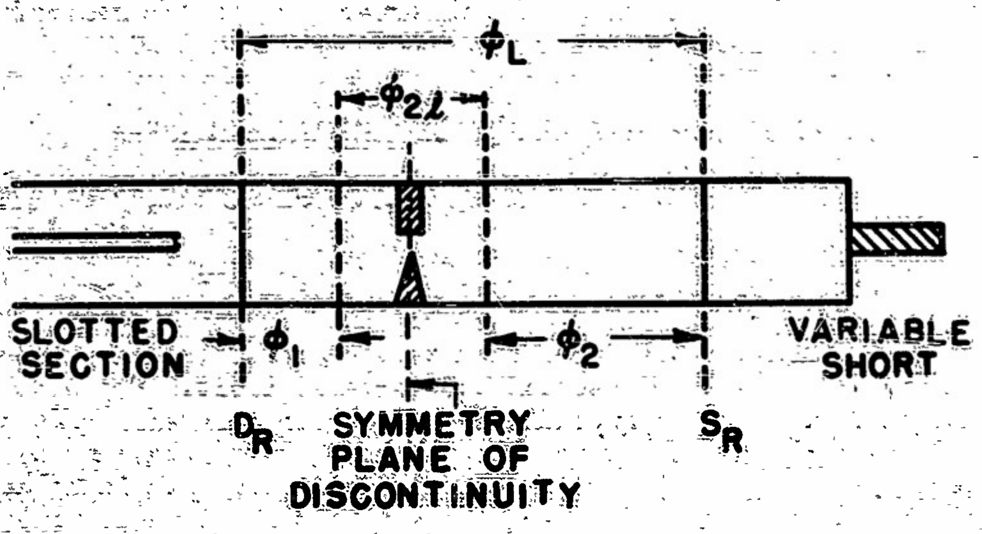

Fig. 4 Procedure for Symmetrical Discontinuities.

From Fig. 4 it is seen that

$$\phi_{2\ell} = \phi_L - (\phi_1 + \phi_2) \tag{2.11}$$

so that the distance 2ℓ between reference planes is $\lambda_{gs}\,\phi_{2\ell}/2\pi$. Length ϕ_L must be determined with the discontinuity absent; if this is impossible, a mechanical rather than electrical measurement of the overall length is necessary, thereby enhancing the possible error. With a mechanical length measurement, a value for the guide wavelength in the housing structure must be assumed, and a slight error in it may be serious as the housing waveguide is generally several wavelengths long; with the electrical measurement, the guide wavelength does not enter into the calculations until the very end and then in conjunction with a small distance, 2ℓ, so that the value chosen for it is not critical.

2. Coupling Structures Associated with Both the Variable Short and the Slotted Section

Suppose that the structure to be measured cannot be inserted directly between the slotted section and the variable short, but that coupling elements such as connectors or bends must be placed in between; Fig. 5 illustrates the case for right angle bends. Due to the presence of the bends, both the D and S values must be corrected.

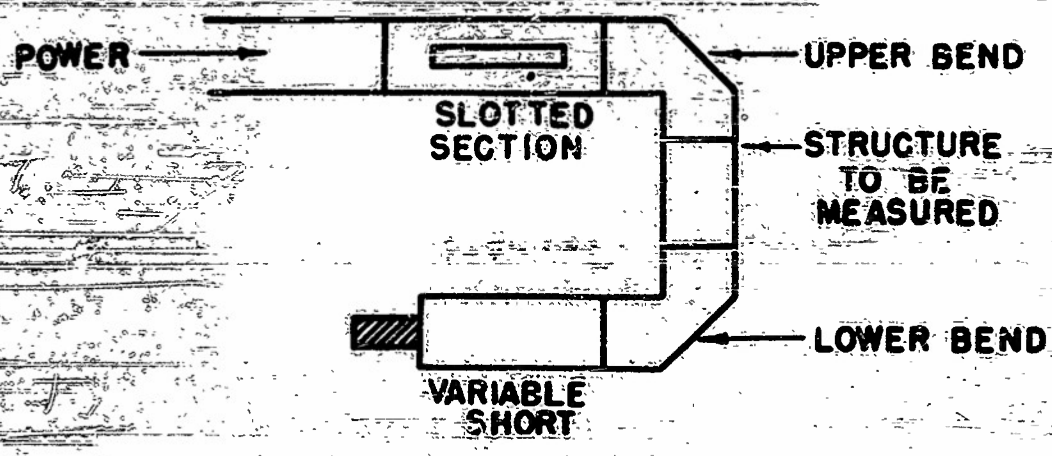

Fig. 5 Measurement Using Right Angle Bends.

The calibration curve for the D values is obtained by considering the upper bend as a part of the slotted section and arranging the equipment as shown in Fig. 6(a), taking a run of D vs. S values, and plotting the calibration curve of $-(D + S\,\lambda_{gd}/\lambda_{gs})$ vs. D as discussed in Sec. A. The ordinate ΔD of this curve then gives the correction to be added to any subsequent D reading obtained when some structure is inserted between the variable short and the upper bend.

The calibration curve for the values of S requires the apparatus to be arranged as indicated in Fig. 6(b); the calibration procedure now corrects for the presence of discontinuity effects in the lower bend. Another run of D vs. S values is taken, and the D values are corrected by using the calibration curve obtained from the arrangement of Fig. 6(a), including multiplication by $\lambda_{gs}/\lambda_{gd}$ according to Eq. (2.4). The ordinate of the calibration curve of $-(D_{corr.} + S)$ vs. S then gives the correction to be added to any subsequent S reading (including S_R) to yield the correct S value. The corrected D and S values are both relative to λ_{gs}.

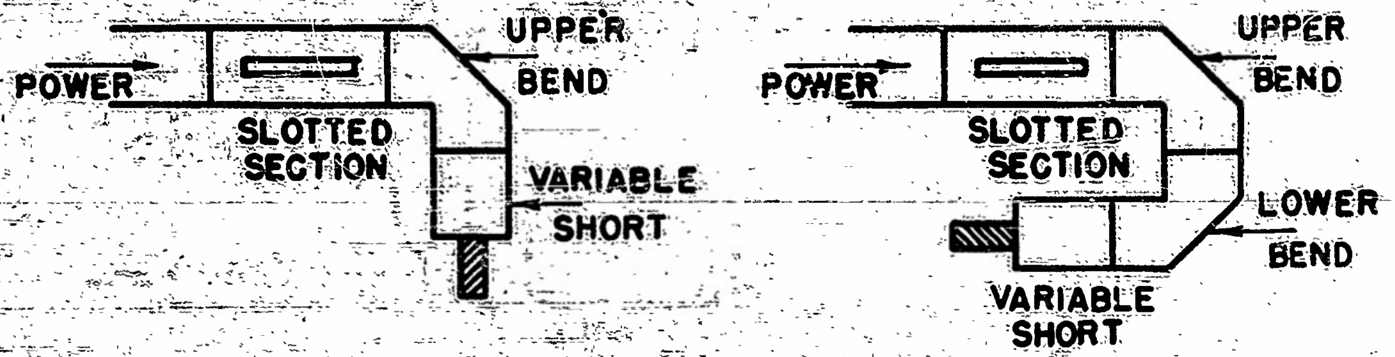

Fig. 6 Arrangement of Equipment for Calibration.

3. Discontinuity in a Waveguide Shape for Which No Slotted Section is Available

The application is illustrated by Fig. 7. It is assumed that we wish to obtain the equivalent circuit parameters of a discontinuity structure in a waveguide of unusual cross-section, normalized to the characteristic impedance of that waveguide, but that no slotted section is available for that guide shape. Such a measurement is possible using a slotted section in rectangular waveguide, say, with the additional requirements of an adaptor from the guide of unusual cross-section to rectangular guide, and a variable short in the former guide. Of course, the dimensions must be chosen so that only the dominant mode can propagate in each guide; the adaptor, furthermore, is tuned at the measurement frequency so as to minimize the discontinuity introduced by the adaptor itself, or the precision will suffer.

The slotted section of rectangular waveguide must be calibrated so that it has the properties of an equivalent slotted section in the waveguide of unusual cross-section, and so that the effects of the discontinuities introduced by the end of the slot, the adaptor, and the difference in characteristic impedances are eliminated. For this calibration, the variable short is placed immediately behind the adaptor and a run of D vs. S values taken, together with measurements of λ_{gd} and λ_{gs}. The procedure of plotting $-(D + S\ \lambda_{gd}/\lambda_{gs})$ vs. D is then followed, and the corrections of Sec. B of the Appendix are applied as needed.

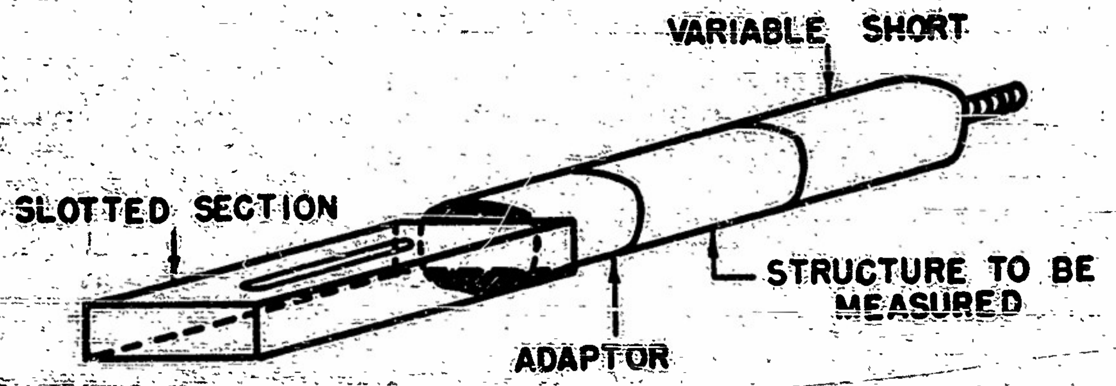

Fig. 7 Measurement of Discontinuity in Waveguide of Unusual Cross-section.

4. Junction Between Two Dissimilar Waveguides

In all of the preceding discussions it has been assumed that the equivalent circuit parameters were desired relative to some specific single waveguide, namely, the variable short guide. In the case of a junction between dissimilar waveguides, say a circular to rectangular waveguide junction with some slot at the junction plane, it is desirable to obtain a "relative" network representation,[10] i.e., with the parameters given as Z_{11}/Z_1, Z_{22}/Z_2, and Z_{12}^2/Z_1Z_2, and where Z_1 and Z_2 are the characteristic impedances of the two guides in question.

Such a measurement for the example indicated above would first require a calibration of the slotted section so that the D readings would be the same as in a guide with no slot. The measurement is then made by taking a run of D vs. S values with a variable short in circular waveguide placed behind the slotted section with the slot placed between them. The treatment of the resulting data is described in the literature. It is stressed here that this type of precision measurement requires two different variable shorts, one for the slotted section calibration and one for the actual measurement.

[10] Footnote reference 7, Chap. III, p. 1, 2.

III. THE THEORETICAL BASIS

A. The Calibration Curve for Lossless Structures

As discussed in Part II, Sec. A, the calibration curve is obtained by a plot of $-(D + S\,\lambda_{gd}/\lambda_{gs})$ vs. D. The same shape curve is evidently obtained if one plots $-(D/\lambda_{gd} + S/\lambda_{gs})$ vs. D/λ_{gd}, the difference being that the corrections will be relative to λ_{gd}. Although it is clearly preferable experimentally to deal with geometrical lengths, it is simpler in the derivations to utilize lengths normalized to the respective guide wavelengths. Therefore, for simplicity, in all of Part III the symbols D and S will signify D/λ_{gd} and S/λ_{gs}.

An expression for the shape of the calibration curve is most simply obtained from the tangent, or transformer, relation of A. Weissfloch[11]

$$\tan 2\pi (D - D_0) = \gamma \tan 2\pi (S - S_0) \tag{3.1}$$

where the notation follows that of N. Marcuvitz.[12] Parameters D_0, S_0 and γ characterize the discontinuity under consideration, and D and S are distances (relative to guide wavelength) from chosen reference planes T_1 and T_2 to a voltage null and a short circuit in the input and output guides, respectively; the pertinent lengths and their positive directions are shown in Fig. 8. The characteristic points D_0 and S_0 signify the reference plane locations corresponding to an equivalent network consisting only of a transformer of turns ratio $\sqrt{-\gamma}$; $(-\gamma)$ is furthermore the insertion VSWR of the two terminal-pair structure being measured.

If the insertion VSWR of the discontinuity is small, as it almost always would be for a slotted section calibration, one can write

$$-\gamma = 1 + \epsilon \quad , \quad \epsilon \ll 1 \quad . \tag{3.2}$$

Forming the $\tan 2\pi (D - D_0 + S - S_0)$, expanding and substituting Eq. (3.1), one obtains

$$\tan 2\pi (D - D_0 + S - S_0) = \frac{(1 + \gamma) \tan 2\pi (D - D_0)}{\gamma - \tan^2 2\pi (D - D_0)}$$

---

11 A. Weissfloch, "Eine Transformation über verlustlose Vierpole und seine Anwendung", Hochfrequenz und Elektroakustik, 60, pp. 67-73, 1942.

12 N. Marcuvitz, "On the Representation and Measurement of Waveguide Discontinuities", Proc. I.R.E., 36, pp. 728-735, June 1948.

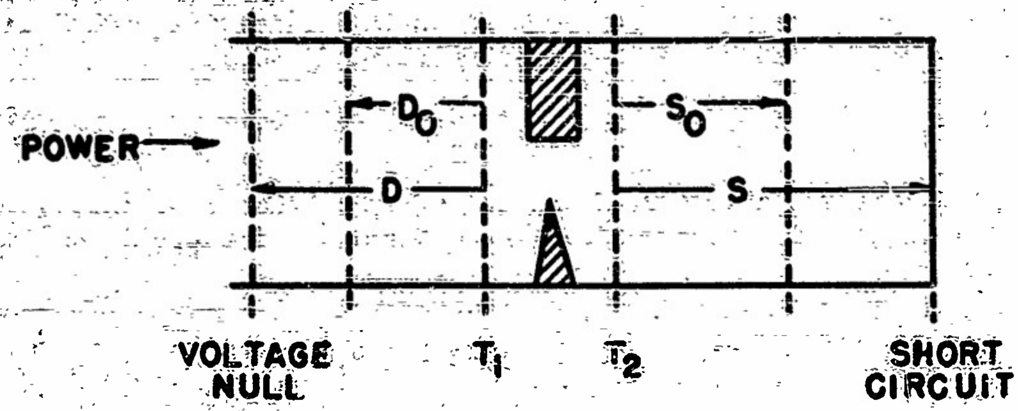

Fig. 8 Two Terminal-Pair Waveguide Structure.

Employing (3.2) for ϵ small, we have

$$D + S = D_0 + S_0 + \frac{\epsilon}{4\pi} \sin 4\pi (D - D_0) \quad , \tag{3.3}$$

the analytical expression for the negative of the calibration curve shown in Figs. 2 and 3. The additive constant $D_0 + S_0$, characteristic of the discontinuity being calibrated, is of no consequence since both D and D_R (see Part II) are corrected by the same calibration curve. Relations (2.5) are seen to follow directly from the amplitude $\epsilon/4\pi$ of (3.3) and the physical meaning of γ, recalling that distances in (3.3) are relative to guide wavelength. The sinusoidal shape of the calibration curve for small discontinuities, and its periodicity as shown in Fig. 2, also follow from Eq. (3.3).

B. The Correction Relations for Dissipative Structures

The calibration curve obtained using purely reactive terminations is not directly applicable to measurements on dissipative structures; to determine its applicability to such measurements one must investigate the effect of lossy terminations. In this connection, a derivation is given below of the analytic expression for the calibration curve of - (D + S) vs. D, for the case of small discontinuity effects and for lossy terminations. The relation between this curve and that for purely reactive terminations is brought out, and a method of using the calibration curve to determine the corrections to the measured VSWR's is included.

The parameters D_o, S_o, and Υ of the tangent relation (3.1) are purely real for a lossless structure, as for example the slotted section considered now. However, when the termination is dissipative both D and S are complex. Their relation to the measured quantities are:

$$D = D_r + j D_i \quad ; \quad S = S_r + j S_i \tag{3.4}$$

where D_r and S_r are the distances from the reference planes to the voltage minima in the input and output guides, respectively, and where D_i and S_i are given by

$$\tanh 2\pi D_i = \frac{1}{r_m} \quad ; \quad \tanh 2\pi S_i = -\frac{1}{r_a} , \tag{3.5}$$

r_m (measured) and r_a (actual) being the VSWR's in the slotted section and output guide, respectively. Let

$$y = 2\pi (D - D_o) \quad , \quad x = 2\pi (S - S_o) \tag{3.6}$$

so that Eq. (3.1) becomes

$$\tan y = \Upsilon \tan x \tag{3.7}$$

Forming the tan (y + x), expanding, substituting (3.7), and employing (3.2) since the discontinuity is small, one obtains

$$\tan (y + x) = \frac{\epsilon \tan y}{(1 + \epsilon) + \tan^2 y} \tag{3.8}$$

Since y is complex, it is not permissible to drop the ϵ in the denominator unless $(r_m - 1) \ll \epsilon$; for $r_m \cong 1$, $\tan^2 y \cong -1$. The correction relations to be obtained below are thus valid under this restriction. Also, since ϵ is small, tan (y + x) can be replaced by (y + x), and (3.8) becomes

$$y + x = \frac{\epsilon}{2} \sin 2y \tag{3.9}$$

Combining definitions (3.4) and (3.6) one sees that

$$y + x = 2\pi \left[(D_r + S_r) - (D_o + S_o) \right] + j\, 2\pi (D_i + S_i)$$

and

$$\sin 2y = \sin 4\pi (D_r - D_o) \cosh 4\pi D_i + j \cos 4\pi (D_r - D_o) \sinh 4\pi D_i .$$

so that (3.9) yields for its real and imaginary parts

$$(D_r + S_r) - (D_o + S_o) = \frac{\epsilon}{4\pi} \sin 4\pi (D_r - D_o) \cosh 4\pi D_i \qquad (3.10)$$

$$2\pi (D_i + S_i) = \frac{\epsilon}{2} \cos 4\pi (D_r - D_o) \sinh 4\pi D_i \qquad (3.11)$$

When r_m approaches infinity (short circuit termination), D_i approaches zero, D_r and S_r become simply D and S, and Eq. (3.10) becomes the analytic expression (3.3) for the calibration curve of $-(D + S)$ vs. D for the lossless case; Eq. (3.11) then vanishes. It is seen, therefore, that when dissipative structures are measured, the D readings may be corrected by using the calibration curve obtained for the lossless case provided that these corrections are multiplied by the factor $\cosh 4\pi D_i$. By reference to (3.5) this factor becomes

$$\frac{r_m^2 + 1}{r_m^2 - 1} \qquad (3.12)$$

in accord with relation (A3) of the Appendix, where r_m is the measured VSWR value corresponding to the D_r value involved.

The presence of a discontinuity in the slotted section will affect the VSWR value also, so that the measured value r_m is different from the value r_a without the discontinuity. It is well known that r_m will fall between the limits of $r_a(-Y)$ and $r_a/(-Y)$, since $(-Y)$ is the insertion VSWR of the slotted section discontinuity. For a slotted section alone, the value of $(-Y)$ is generally about 1.02, so that r_m is different from r_a by 2 percent or less. Usually this may be neglected, but if not, the difference between r_m and r_a may be determined from the calibration curve by the use of Eq. (3.11). Using expressions (3.5), consider:

$$\tanh 2\pi D_i + \tanh 2\pi S_i = \frac{1}{r_m} - \frac{1}{r_a} = \frac{\sinh 2\pi (D_i + S_i)}{\cosh 2\pi D_i \cosh 2\pi S_i} \qquad (3.13)$$

Since the difference between r_m and r_a is small, (3.13) may be written as

$$\frac{2\pi (D_i + S_i)}{\cosh^2 2\pi D_i} = \frac{r_a - r_m}{r_m^2} \quad . \qquad (3.14)$$

Defining Δr as

$$\Delta r = r_a - r_m \quad , \tag{3.15}$$

and substituting for the cosh term of (3.14) (by expressing it in terms of tanh and using (3.5)), one obtains

$$2\pi (D_i + S_i) = \frac{\Delta r}{r_m^2 - 1} \tag{3.16}$$

which is seen to be the left hand side of Eq. (3.11). The sinh term of the right hand side may be shown to be equal to $2r_m/(r_m^2 - 1)$, so that Eq. (3.12) becomes

$$\Delta r = 2 r_m \left[\frac{\epsilon}{2} \cos 4\pi (D_r - D_o) \right] \tag{3.17}$$

or

$$\Delta r = -(4\pi r_m) \left[\frac{\epsilon}{4\pi} \sin 4\pi (D_r - \frac{1}{8} - D_o) \right] \quad . \tag{3.18}$$

The term in brackets of Eq. (3.18) is to be compared to the right hand side of Eq. (3.3), the expression for the negative of the calibration curve; they differ by the additive constant $D_o + S_o$ and the 1/8 subtracted from D_r. As mentioned earlier, the constant $D_o + S_o$ is in effect set equal to zero since the abscissa axis is drawn through the average of the curve. Thus, Eq. (3.18) states that the correction Δr (which is to be <u>added</u> to r_m to give the corrected VSWR value) is obtained from the calibration curve by multiplying by $4\pi r_m$ the ΔD value corresponding to $D_r - 1/8$, not D_r. If the calibration curve is plotted in absolute lengths rather than guide wavelengths, Δr is given by the ΔD value at $D_r - \lambda_{gd}/8$, multiplied by $4\pi r_m/\lambda_{gd}$, as specified by relation (A4) of the Appendix.

Equation (3.17) may be written as

$$r_m = \frac{r_a}{1 + \epsilon \cos 4\pi (D_r - D_o)} \quad ,$$

so that the maximum and minimum values for r_m are:

$$r_m = \frac{r_a}{1 - \epsilon} \quad ; \quad r_m = \frac{r_a}{1 + \epsilon} \quad .$$

Employing (3.2), we obtain

$$r_m = r_a(-\gamma) \quad , \quad r_m = r_a\left(\frac{1}{-\gamma}\right)$$

for the limiting values, in agreement with Eq. (2.9).

C. The Effect of an Error in the Variable Short

The calibration procedure assumes that the variable short is perfect; if the calibration curve is in large part due to an error in the variable short and if corrections are applied only to the D values, then, except in special circumstances, an error will be made in the data, as shown below. Error in the variable short is introduced if its guide walls are not uniform, if the motion of the plunger is not perfectly parallel to the guide axis or includes a lateral motion periodic with a rotation of the micrometer spindle, etc. In what follows, the value of γ in the tangent relation procedure[12] for a lossless structure is obtained first assuming that the calibration curve is due solely to an error in D and then solely to an error in S, and the results examined. A simple experimental method is then given for determining the seat of the error.

We begin by introducing the notation to be employed below. Let D and S be the actual readings, and D_{oc}, S_{oc} and γ_c be the constants of the calibration curve. The tangent relation (3.1), for a subsequent measurement on some discontinuity, containing only unprimed quantities will refer to all uncorrected quantities; the relation for corrected values of D, and then S, will contain, respectively, D', S, D'_o, S'_o, and γ', and D, S'', D''_o, S''_o, and γ''. The correction ΔD is given by the negative of Eq. (3.3), with $D_o + S_o$ set equal to zero for the reason indicated there, and the correction ΔS is

$$\Delta S = \frac{\epsilon}{4\pi} \sin 4\pi (S - S_{oc}) \quad , \quad -\gamma_c = 1 + \epsilon \; ; \tag{3.19}$$

the derivation of ΔS parallels that of ΔD.

An expression for γ' is obtained by writing the tangent relation (3.1) for the uncorrected quantities as

$$\tan 2\pi (D' - \Delta D - D_o) = \gamma \tan 2\pi (S - S_o) \quad . \tag{3.20}$$

Since the value of γ' is given by the slope of the D' vs. S curve in the range $(D' - D'_o)$ and $(S - S'_o)$ small, expansion of (3.20) in this range yields (recognizing that D'_o and S'_o differ only slightly from D_o and S_o).

$$2\pi\,(D' - D_o) - 2\pi\,\Delta D \;=\; \gamma\, 2\pi\,(S - S_o) \qquad (3.21)$$

The expression (3.3) for ΔD may be rewritten as

$$\Delta D \;=\; -\,\frac{\epsilon}{4\pi}\,\sin 4\pi\,(D' - \Delta D - D_{oc} + D_o - D_o)\quad ,$$

which becomes, for $(D' - D_o)$ small,

$$\Delta D \;=\; -\,\epsilon\,(D' - D_o)\,\cos 4\pi\,(D_o - D_{oc} - \Delta D)$$

$$-\,\frac{\epsilon}{4\pi}\,\sin 4\pi\,(D - D_{oc} - \Delta D) \qquad (3.22)$$

Substitution of (3.22) into (3.21) yields

$$D'\,(1 + \epsilon\cos\phi') = \gamma S - \gamma S_o - (\epsilon/4\pi)\,\sin\phi' + D_o\,(1 + \epsilon\cos\phi') \qquad (3.23)$$

where $\phi' = 4\pi\,(D_o - D_{oc} - \Delta D)$. The variables in (3.23) are D' and S (ΔD is very slowly varying and may be considered constant), so that it may be rewritten as

$$D' \;\cong\; \frac{\gamma}{1 + \epsilon\cos\phi'}\; S + \text{Constant}' .$$

Since γ' is the slope of the curve of D' vs. S in the range considered above, we have

$$\gamma' \;\cong\; \frac{\gamma}{1 + \epsilon\cos 4\pi\,(D_o - D_{oc})} \qquad (3.24a)$$

or

$$\Delta\gamma' \;\cong\; -\,\epsilon\cos 4\pi\,(D_o - D_{oc}) \qquad (3.24b)$$

By similar considerations an approximate expression for γ'' may be obtained. Equation (3.1) is written as

$$\tan 2\pi\,(D - D_o) \;=\; \gamma\,\tan 2\pi\,(S'' - \Delta S - S_o) \qquad (3.25)$$

We consider the range $(D - D_o)$ and $(S'' - S_o)$ small, and obtain from (3.25)

$$2\pi\,(D - D_o) \;=\; \gamma\, 2\pi\,(S'' - S_o) - \gamma\, 2\pi\,\Delta S \qquad (3.26)$$

For $S'' - S_o$ small, Eq. (3.19) becomes

$$\Delta S = \epsilon (S'' - S_o) \cos \phi'' + \frac{\epsilon}{4\pi} \sin \phi'' \quad ;$$

upon substitution into (3.26), and separating out the constant terms (considering ΔS as constant), we have

$$D \cong Y (1 - \epsilon \cos \phi'') S'' + \text{Constant}'' \quad ,$$

where $\phi'' = 4\pi (S_o - S_{oc} - \Delta S)$. Since Y'' is the slope of the curve of D vs. S'' in the range considered above, one obtains

$$Y'' \cong Y (1 - \epsilon \cos 4\pi (S_o - S_{oc})) \qquad (3.27a)$$

or

$$\Delta Y'' \cong - \epsilon \cos 4\pi (S_o - S_{oc}) \qquad (3.27b)$$

Comparison of Eq. (3.24b) for the change in Y obtained by correcting the D values, with Eq. (3.27b) for the corresponding change resulting from a correction of the S values, indicates that the changes may be in the same or opposite directions depending upon the relation between the D_{oc} and S_{oc} of the calibration curve and the D_o and S_o of the structure being measured. For example, if $D_o = D_{oc}$ and $S_o = S_{oc}$, the corrections will be similar, while if $D_o = D_{oc}$ and $S_o = S_{oc} + 1/4$, the corrections will be opposite; it is therefore purely fortuitous whether or not the corrections will be the same. Although the above investigation was narrow in scope in that it considered only parameter Y for lossless structures, it indicates that it is necessary in general to know whether the calibration curve is correcting primarily for errors in the slotted section or for those in the variable short.

A simple experimental method for determining the primary source of the error is the following. Two calibration runs are taken; for the first, the variable short is placed directly after the slotted section in the usual fashion, for the second, a quarter wavelength length of line (carefully constructed, with negligible discontinuity effect) is inserted between them. Four calibration curves are then plotted: $-(D + S)$ vs. (a) D, (b) S for the first calibration run, (c) D, (d) S for the second calibration run; it is well to superimpose curves (a) and (c) and curves (b) and (d). Vertical shifts should be neglected, i.e., the abscissa axis should be drawn through the average of all curves. If all of the discontinuity effect is associated with the slotted section, then inserting the quarter wavelength length of line serves only to change the value of S by 1/4; the reverse is true if all of the discontinuity effect is inherent in the variable short. For the former case, the plotted curves would appear as in Fig. 9.

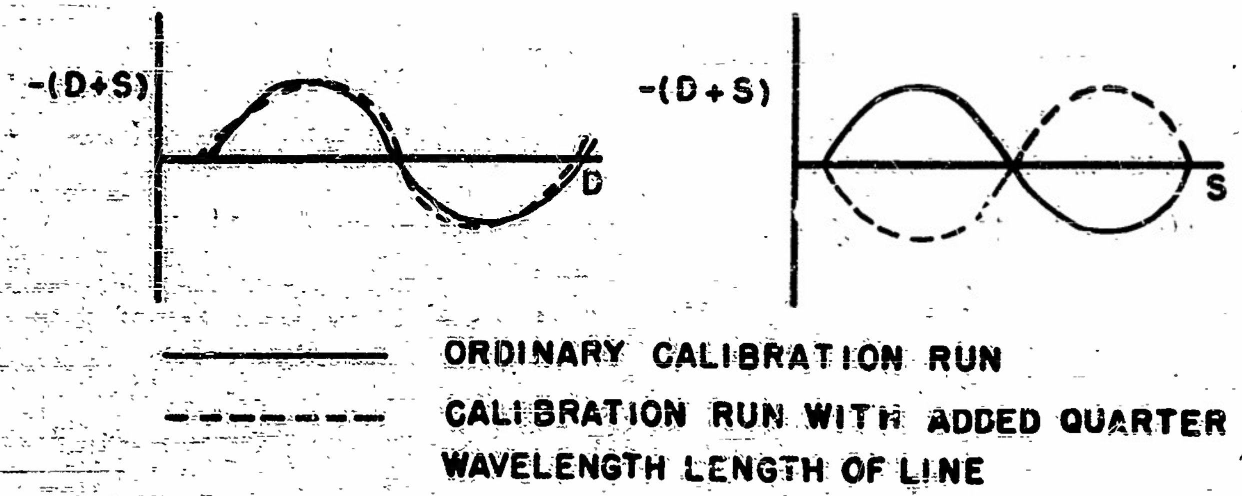

Fig. 9 Curves Obtained when the Discontinuity Effect is Solely in the Slotted Section.

If the inserted length of line is not discontinuity-free (the primary effect can occur at the junctions between the line and the other equipment) the procedure described above is unreliable; a partial check on the length of line is obtained by reversing it and taking a third calibration run. Since in practise it is desired to calibrate a discontinuity effect known to be associated with the slotted section, if the procedure described above does not result substantially in the curves of Fig. 9, the variable short should be rejected and a substitute one obtained.

APPENDIX: INSTRUCTIONS FOR TECHNICIANS

A. The Calibration Procedure

1. Measurements

The detector is assumed to be square law by appropriate calibration, and the probe insertion depth is taken to be small. Methods of calibrating the detector and accounting for greater probe insertion depth are given in the literature.[1,3] The probe should be tuned for maximum pickup in order to reduce the asymmetry effect introduced by the probe susceptance. In order to maintain constant guide wavelengths during the measurements, it is advisable to employ a frequency stabilizer or monitor. The standard precision procedure for locating a voltage minimum is to obtain the average position between equal power levels on each side of the minimum. As a check on the requirement of symmetry for the voltage minimum, one can obtain the minimum position from readings at two different power levels, and require that they agree to within 0.3 mil, say, at X-band.

The notation employed in the calibration procedure below is as follows:

- D is the indicator reading corresponding to the probe location at a voltage minimum in the slotted section.
- S is the reading corresponding to the location of the variable short.
- D_R (R signifying "reference") is the D reading corresponding to a short circuit located at the end of the "slotted section" nearest the load.
- S_R is the S reading corresponding to a short circuit located at the end of the variable short.
- λ_{gd} is the guide wavelength in the slotted section.
- λ_{gs} is the guide wavelength in the variable short.

The "slotted section" to be calibrated includes the slotted section proper together with any associated equipment such as adaptors, bends, etc. The calibration of any coupling element associated with the variable short is discussed in Part II, Sec. B, 2., and is not treated here. If the calibrated slotted section is to be used only for the measurement of single terminal-pair structures, it is not necessary to determine S_R. The equipment is arranged for the calibration procedure as shown in Fig. 1.

(1) Measurement of D_R: Since the D values generally appear as an arbitrary reading on an indicator attached to the slotted section, the following is employed to obtain this reading relative to some fixed reference plane, or what is equivalent, to a position $n\,\lambda_{gd}/2$ away from this

plane. Place a very smooth flat metal plate at the end of the "slotted section" (including any adaptors, etc.) nearest the load. The reading on the indicator corresponding to a voltage null produced by this termination is D_R. The geometrical distance of any other measurement relative to the reference plane chosen is then given by subtracting D_R from the indicator reading D (after correcting both D and D_R from the calibration curve). Disconnect the flat metal plate.

(2) Measurement of S_R: The plane of the end of the variable plunger is usually slightly different from the plane of its effective short circuit, and its location is generally given in terms of an arbitrary distance on a micrometer scale. In order to locate the plane of the effective short circuit relative to the end of the variable short, connect the variable short directly to the "slotted section" (plus adaptor, etc.). Set the probe carriage on the slotted section at D_R, and vary the position of the variable short plunger until the voltage null in the slotted section occurs at D_R. The corresponding variable short micrometer reading is S_R. The geometrical distance for any other location of the plunger relative to the above-mentioned reference plane is given by subtracting S_R from the reading S of the micrometer.

A more accurate determination of S_R is obtained by moving the variable short plunger and averaging the positions between equal power levels in the slotted section detector; a check should be made by setting the micrometer at S_R and determining in the usual averaging fashion whether or not a voltage null occurs exactly at D_R.

(3) Measure the guide wavelength λ_{gd} in the slotted section.

(4) Measurement of the guide wavelength λ_{gs} in the variable short: Locate D corresponding to a given setting S. Keeping the D value fixed, move the variable short plunger until a voltage null again occurs at D. The difference in the S settings is equal to $\lambda_{gs}/2$.

(5) Calibration run: Obtain a series of values of D and S over the range of travel of the probe carriage by varying S and tracking with D. Take about 20 approximately equally-spaced points to the half wavelength.

2. Treatment of Data

(1) Plot the graph of $-(D + S\,\lambda_{gd}/\lambda_{gs})$ vs. D. Draw a smooth curve through the distribution of points; the curve should be sinusoidal in shape and periodic in $\lambda_{gd}/2$. For increase precision the plot may be made on a large sheet of paper; the size should depend on the scatter of the data points.

(2) A slight slope will appear in the curve if any error is present in the ratio $\lambda_{gd}/\lambda_{gs}$; the slope should be eliminated by altering this ratio accordingly. The resulting value should be used to obtain a more accurate value for either λ_{gs} or λ_{gd}, whichever is felt to be less reliable.

(3) Draw the abscissa axis through the average of the curve. Label the ordinate axis as Δ D and the abscissa axis as D. The curve should look like Fig. 3.

B. The Correction Relations

1. For lossless structures

(1) Obtain the Δ D value corresponding to the D value in question from the calibration curve. The corrected value of D is given by

$$D_{corrected} = (D + \Delta D)\ \lambda_{gs}/\lambda_{gd} \qquad (A1)$$

(2) Repeat as in (1) to obtain the corrected value of D_R. The resultant $D_{corrected} - D_{R\ corrected}$ gives the location of the voltage minimum relative to the reference plane associated with D_R, in a waveguide of characteristic impedance and guide wavelength identical with that of the variable short used to calibrate the slotted section.

2. For dissipative structures

The corrections given below are valid under the restriction $\epsilon/(r_m - 1) << 1$, where r_m is the measured value of the VSWR and $(1 + \epsilon)$ is the insertion VSWR of the slotted section. The value of ϵ is obtained from the double amplitude A of the calibration curve as

$$\epsilon = 2\pi A/\lambda_{gd} \qquad (A2)$$

(1) Obtain the Δ D value corresponding to the D value from the calibration curve. The corrected value of D is given by

$$D_{corrected} = \left\{ D + \Delta D \left[\frac{r_m^2 + 1}{r_m^2 - 1} \right] \right\} \frac{\lambda_{gs}}{\lambda_{gd}} \qquad (A3)$$

(2) Repeat as in (1) for lossless structures to obtain the corrected value of D_R. The comments under (2) for lossless structures also apply here.

(3) The correction Δr, which is to be added to r_m to yield the corrected VSWR value, is

$$\Delta r = \frac{4\pi r_m}{\lambda_{gd}} \left[\Delta D \right]_{1/8} \qquad \text{(A4)}$$

where $(\Delta D)_{1/8}$ signifies that the value of ΔD is obtained from the calibration curve at $D - \lambda_{gd}/8$, rather than at D itself, where D is the indicator reading corresponding to the voltage minimum.

A modified procedure valid over a narrow frequency band:

The calibration procedure described above can be made valid over a narrow frequency band, say 1 percent or 2 percent, rather than at a single frequency, by a slight modification. Such a requirement is of interest when it is not easy or desirable to reset the frequency to the exact value at which the calibration was made. The modifications necessary are:

(1) The calibration curve should be plotted as $-(D/\lambda_{gd} + S/\lambda_{gs})$ vs. D/λ_{gd}; the corrections are then given as $\Delta D/\lambda_{gd}$. Any slope present should be eliminated as usual.

(2) The value used for the ratio $\lambda_{gs}/\lambda_{gd}$ in relations (A1) and (A3) should be that obtained on the day of measurement rather than on the day of calibration.

Two assumptions are implicit in the formulation above: (a) that the discontinuity effects associated with the slotted section are substantially constant with frequency, (b) that negligible slope would be introduced if the calibration curve were made at the slightly different frequency, since the ratio $\lambda_{gd}/\lambda_{gs}$ would be somewhat different. For assumption (a), a change in the discontinuity effect would alter the amplitude of the curve and produce a lateral shift in it; however, these effects may be neglected for small frequency shifts. The ratio $\lambda_{gd}/\lambda_{gs}$ may be written* as $\lambda_{gd}/\lambda_{gs} = 1 + \delta$, where $\delta = K\lambda_{gs}^2$, and is very small; it is easily shown that for $\lambda_{gs}/\lambda_o = 2$, where λ_o is the free space wavelength, the change in $\lambda_{gd}/\lambda_{gs}$ is given by

---

* See footnote reference 4, Eq. (11).

$$\Delta \left[\frac{\lambda_{gd}}{\lambda_{gs}} \right] = \delta\, 4 \left[\frac{\Delta \lambda_o}{\lambda_o} \right] .$$

For $\lambda_{gd}/\lambda_{gs} = 1.0030$, and for a frequency change of 2 percent, the change in the wavelength ratio is .0002, which is within experimental error. Assumption (b) is thus justified. The validity of assumptions (a) and (b) determine the extent of applicability of the modified procedure.