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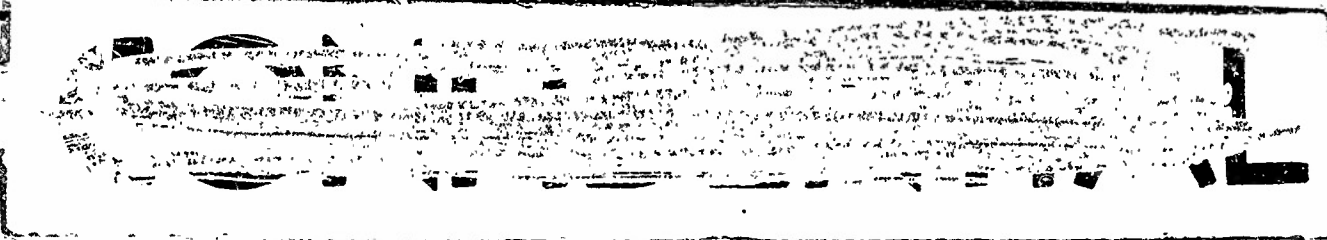
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Design of the Lark and Loon Guidance Computer - and Appendixes A-E (Technical Memorandum)

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U. S. Naval Air Missile Test Center, Point Mugu, Calif.

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Two automatic guidance computers were designed and constructed for the surface-to-air missile Lark and the surface-to-surface missile Loon. The guidance task and the coordination of missile and computer are discussed in relation to design features of the computer. Since on-off radio transmission is used in this guidance system, the computer is also of the on-off type, and the peculiarities of such a system are discussed. In both computers, the components used in the guidance loop have similar properties, therefore, the two computers differ only as stipulated by the two guidance tasks. A description is given of the computer system and those properties of the computer that enter into the guidance loop are explained. The design data of the two computers are presented and a schematic is included.

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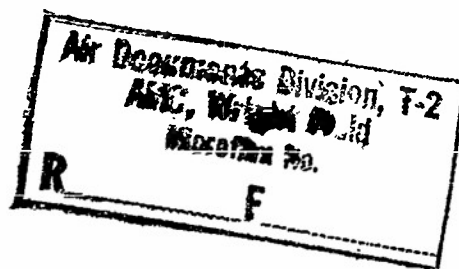
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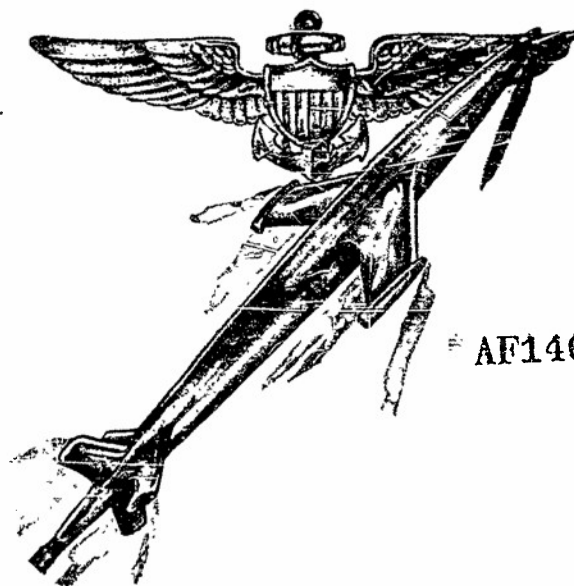
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DESIGN OF THE LARK AND LOON GUIDANCE COMPUTER

TECHNICAL MEMORANDUM REPORT NO. 35

1 FEBRUARY 1950

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TECHNICAL MEMORANDUM NO. 35

DESIGN OF THE LARK AND LOON GUIDANCE COMPUTER

1 FEBRUARY 1950

BUREAU OF AERONAUTICS

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Foreword

The U. S. Naval Air Missile Test Center was established at Point Mugu, California, by the Secretary of the Navy (SecNav ltr Op-24/mad Serial 1873P24 dtd 17 September 1946) effective 1 October 1946. It is an activity of the ELEVENTH Naval District. The Bureau of Aeronautics exercises management and technical control over this activity.

The primary mission of the Naval Air Missile Test Center is the testing and evaluation of guided missiles and their components. NAMTC is assigned cognizance over all facilities at Point Mugu, California, and outlying facilities on San Nicolas Island and the Santa Barbara Channel Islands, collectively referred to as the Sea Test Range.

Commander, Naval Air Missile Test Center....Captain D.S. Fahrney, USN
Commanding Officer, Naval Air Station....Captain E.W. Parish, Jr., USN
Director of Tests, Naval Air Missile Test Center....Captain W.T. Hines, USN
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This report is distributed to the Chief of Naval Operations, Attention Op-322F2 (30 copies) and to the Bureau of Aeronautics, Attention TD-4 (10 copies).

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design of the LARK and LOON guidance computer

summary

Two automatic guidance computers were designed and constructed at NAMTC for the surface-to-air missile LARK and the surface-to-surface missile LOON. They were of similar design. The computer receives its input from two ground radars that track the missile and the target, thereby controlling the radio commands transmitted to the missile for guiding it on the line of sight, or any other line, to a target.

Extensive studies of the desirable properties in the guidance system were conducted with a view to utilizing best the inherent ability of the missile to follow commands. These studies are not included in this report but the guidance task and the co-ordination of missile and computer are discussed in relation to design features of the computer.

Since an on-off radio transmission is used in this guidance system, the computer output is also of the on-off type. In this report, the peculiarities of such a system are discussed.

The LARK computer has been in satisfactory operation since April, 1948, and the LOON computer, since May, 1949.

introduction

In test programs at NAMTC, manual control has been used for guiding the surface-to-air LARK missile and the surface-to-surface LOON. To avoid the effect of unpredictable human judgment on test results, an automatic guidance computer for each of these missiles was designed and built at NAMTC. In both computers, the components used in the guidance loop (i.e.,

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missile, radar set, and radio transmission system) have similar properties; therefore, the two computers differ only as stipulated by the two guidance tasks.

This report contains a description of the computer system and explains those properties of the computer that enter into the guidance loop. An account of the investigation of the guidance loop, which includes stability, and causes and magnitudes of errors, is not contained in this report.

The computer output and the radio transmission system are of an on-off type. Proportional control is obtained by a variation of the interval between, and the duration of single commands. This results in a computer operation that combines the properties of proportional and on-off control. The adaptation of such a system to the limiting effect of the maximum acceleration of a missile is investigated in this report. This problem also exists in proportional control.

The introduction of the error rate into a guidance loop is necessary for damping the motion of the missile. An interesting problem in any guidance system is to utilize this error rate despite the presence of noise. The properties of the nonlinear computer system, which are relevant to this problem, are discussed in detail.

An on-off radio system transmits only a few different types of commands. This suggests not only a decrease in the radio transmission time for each command, but also a reduction of the number of commands. An investigation of this problem was made in regard to the LOON computer.

The theoretical results of the investigation are summarized in Section 17. They reveal that the computer, although simple, has some desirable attributes. To obtain these properties in a proportional system would require the use of a number of "tricky" components.

The design data of the two computers are presented in Section 15 and 16, and Figures 15a and 16a show a schematic. The detail designs will be published in separate reports.

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symbols

- C capacity of the computing circuit condenser (Fig. 6b)
- C_{sm} capacity of the smoothing circuit condenser (LOON computer input circuit, Fig. 16a)
- D distance between missile and guidance station (Fig. 1a)
- $E \approx \frac{1}{\rho} \frac{R_0}{R_B} V_B$: (nominal) minimum value of the error voltage V , which causes continuous full command (Sect. 5)
- E_0 voltage per unit of error angle applied to the radar potentiometer
- G value of the voltage V_r , which causes closing of one of the computing relay contacts (Sect. 7)
- H half amplitude of V_i , caused by a radar hunting oscillation (Appendix C)
- K command value (Sect. 4)
- K_h command value during radar hunting (Sect. 10B)
- R with subscript: resistance of the resistors shown in Figure 6a, 6b, 15a, 16a
- T damping time constant of the guidance equation: $T \approx R_0 C$ (Sect. 5 and 6)
- T_h time lag caused by hunting (Sect. 10b)
- T_M time constant (time lag) with which the missile follows a command (Sect. 3)
- T_i time lag between a missile motion and V_i (T_i approximately equals the time lag of the input circuit)
- V error voltage, i.e., voltage proportional to the distance of the missile from the line of sight (Sect. 2)
- V with subscript: voltages at the points of the computer shown in Figure 6a, 6b, 15a, 16a
- $\bar{V} = \left| \frac{V+TV}{E} \right| - \frac{g}{E}$ (Sect. 7)
- V_{av} average value of V during t_t (Sect. 12)

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- $g = \frac{R_c + R_i}{R_c} G$: value of the error voltage V which, for $V_c = 0$, causes one of the computing relay contacts to close (Sect. 7)
- $h = \frac{\pi H}{2E}$: dimensionless value for the radar hunting amplitude
- k total change of course caused by commands during guidance of the missile. (Sect. 13)
- k_1 change of missile course caused by one command (Sect. 11)
- k_L course changes } the launching dispersion (Sect. 13a)
- k_f which are commanded } the finite value of k_1 (Sect. 13b)
- k_e because of } gyro drift (Sect. 13c)
- n number of commands required to achieve k (Sect. 13)
- n with subscript: corresponds to k with subscript.
- $s = \frac{v\dot{k}_0}{y_E}$ stiffness constant of the guidance equation (Sect. 5)
- t time
- Interval during which:
- t_+ the plus contact } of the computing relay closes (Sect. 4)
- t_- the minus contact }
- t_c any one of the contacts }
- t_d interval during which both contacts of the computing relay are open (Fig. 7c)
- $t_t = t_+ + t_- + t_d$: period of time of one command cycle (Sect. 4)
- t_h period of a full hunting oscillation (Fig. 10b and App. C)
- v missile speed
- y missile error, i.e., distance of missile from the line of sight (Fig. 1a)

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- $y_G = \frac{y_E}{E}$ g: error gap, i.e., minimum value of y which, for $V_c = 0$, causes a command (Eq. 7b)
- y_E nominal full command error corresponding to E . The actual full command error is approximately $y_G + y_E$ (Sect. 5)
- y_L missile error after launching at the beginning of guidance (Sect. 13A)
- ϵ gyro drift caused by an unbalanced gyro (Sect. 13C)
- γ relay hysteresis; i.e., minimum change of the voltage V_r , which is necessary to switch one contact of the computing relay in and out (Fig. 7a)
- κ course angle of the missile with reference to the line of sight (Fig. 1)
- $\dot{\kappa}_0$ maximum average rate of change of the missile course (Sect. 4)
- κ_L course angle κ after launching at the beginning of guidance. (Fig. 13a)
- $\rho = R_c/R_i$
- τ time lead of the computer (Sect. 12)
- $\omega = \frac{2\pi}{t_h}$: radians per second of a hunting oscillation

task, equipment and method

1. THE PURPOSE OF GUIDANCE

To avoid failures incident to human operation, it was found necessary that, by means of an automatic guidance computer, both the LARK and the LOON missiles be brought as close to the target as possible, with consideration taken of the inherent properties of the components already in use for manual control, that is, radar set, missile, and radio system. Another requirement was that only small course changes occur near the end of the controlled flight.

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The LARK is a surface-to-air G/M, and the target and missile are each tracked by separate ground radars, specifically, "target radar" and "missile radar" (see Figure 1a), which are located close to each other in the vicinity of the launching site. The computer transforms information obtained from the radars into suitable radio commands so that the LARK is guided in both azimuth and elevation. For tests that require the LARK to be guided on a straight line, the missile radar alone is used. The surface-to-surface missile LOON is to be guided automatically close to a surface target, in azimuth only. As long as the target is stationary, only the missile radar is needed.

It may be assumed that the LOON target is stationary or moves relatively slow. Guidance errors, therefore, are caused mainly by drift of the missile control gyro and a changing wind. The target of the LARK is expected to have only moderate motions; however, being an aircraft, its motions and the effect of wind are still the principal causes of errors.

A system for obtaining the shortest possible radio transmission time already used in manual control was introduced into the LOON computer.

Line-of-sight control was chosen after careful comparison of this method with proportional navigation. In a radar homing missile, the accuracy of the radar output increases as the missile approaches the target. The proportional navigation method takes advantage of this feature. In a missile guided by ground radar, the accuracy decreases as the missile approaches the target. Conventional line-of-sight navigation makes better use of the radar information that is received during the earlier part of the guidance period. Furthermore, line-of-sight guidance, which requires only azimuth angle information from the radar, permitted a solution of the task at short notice.

For one important part of the test program, the time of flight of the LARK until it meets the target is required to be only about one-half minute; this is not much longer than the time required for correction of the launching dispersion. The time of flight of the LOON, however, averages about one-half hour.

Identical systems are used for both azimuth and elevation guidance of the LARK. The description that follows refers to the azimuth axis.

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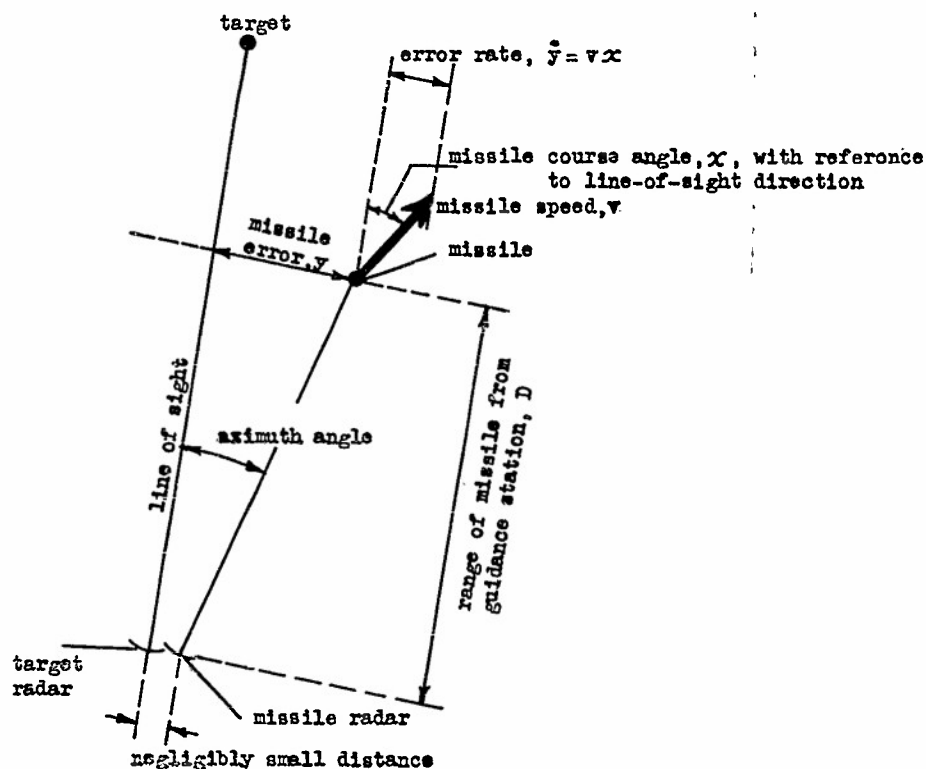


Fig. 1a. Designations and symbols. Azimuth angle and course angle are small.

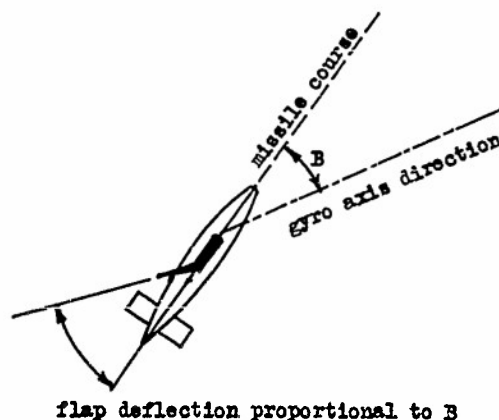


Fig. 3a. A commanded gyro precession causes a deviation B of the gyro axis direction from the missile axis direction, the latter being equal to the flight path direction of the missile. A flap deflection proportional to B causes a rate of change of the missile course proportional to B and, thereby, a decay of B with the missile time constant T_M .

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2. RADAR PROPERTIES

At present, two SCR-584 radar sets are used for guiding the LARK. For LOON guidance, one SP-1M radar is used. A beacon has been installed in the LOON. A beacon is not being used in the LARK but a corner reflector is mounted on the target drone.

The voltage indicating the azimuth angle (azimuth voltage) is derived from the radar. When the missile is guided on a fixed line, the azimuth voltage of the "missile radar" is multiplied by the range D (distance of the missile from the guidance station) in order to form the error voltage V (voltage proportional to the deviation of the missile from the line, which is the error Y). If the line of sight is established by a moving target, the difference of the azimuth voltages of missile and target radar is used instead.

The noise in the azimuth voltage is caused mainly by radar hunting. As long as the missile is at close range, the radar hunting remains usually within small amplitudes, for example, 2 or 3 mils, and at a rather high frequency, i.e., 1 cps. At greater range, when the missile return becomes weak, larger hunting amplitudes occur, (10 mils with a frequency of 1/3 cps or less). Occasionally, such large oscillations may occur even at close range; however, because the hunting angle is multiplied by the distance, these conditions finally become increasingly worse and determine the attainable guidance accuracy at great range.

In LOON flights, it appeared probable that the radar might lose the missile temporarily. If that occurs, command transmission is interrupted; means must be provided for a rapid self-adjustment of the computer to the new conditions after the missile has been recaptured.

3. MISSILE CHARACTERISTICS

The guidance properties of both missiles are similar. The direction of the fuselage axis differs from the course angle by an amount so small that it can be neglected in the considerations which follow. A gyro is used for maintaining the course; and for changing the course, a command is transmitted for precessing the gyro. The gyro precession rate is kept constant within narrow limits. The change of gyro direction, therefore, is proportional to the duration of the command transmission. This gyro precession creates a difference between the gyro and the direction of the fuselage (See Figure 3a). This angular

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difference B effects a control flap deflection (which is proportional to B), thereby causing a rate of change in the course of the missile. This causes a decay of B with a certain time constant T_M :

$$\frac{dB}{dt} + \frac{B}{T_M} = 0$$

The missile course finally attains the new gyro direction. It follows from this that:

- (a) After a time-lag T_M , the missile will follow a commanded course change.
- (b) This commanded change is obeyed by the missile with good accuracy.
- (c) The control system of the missile acts as a "smoothing circuit" for the command noise. Full commands transmitted during short periods are integrated by the gyro and this integral-decaying with T_M determines the flap deflection. In this respect, the gyro acts accurately like an RC smoothing circuit between the command and the rate of change in course of the missile.

Only the speed of the missile affects the time constant T_M of the LOON. During the climbing period, this speed is about 2/3 of the final speed and, therefore, T_M is 2/3 of its final value. During the later part of the flight, when accuracy is required from the guidance system, the missile, quite accurately, attains a certain speed; hence, T_M is well defined by:

$$T_M = 6.5 \text{ seconds } (\pm 10 \text{ per cent}).$$

The time constant T_M of the LARK is affected by the weight of the missile, the location of the center of gravity, the properties of the control pilot, the aerodynamic missile properties, and in addition by Mach number effects, missile speed, and air density; therefore, T_M varies within broader limits, for example within 1 and 2.5 seconds. In order to diminish the effect of this time lag, it was suggested that the command in the missile by-pass the gyro and effect directly a flap deflection. Such changes, however, did not appear practicable at that time.

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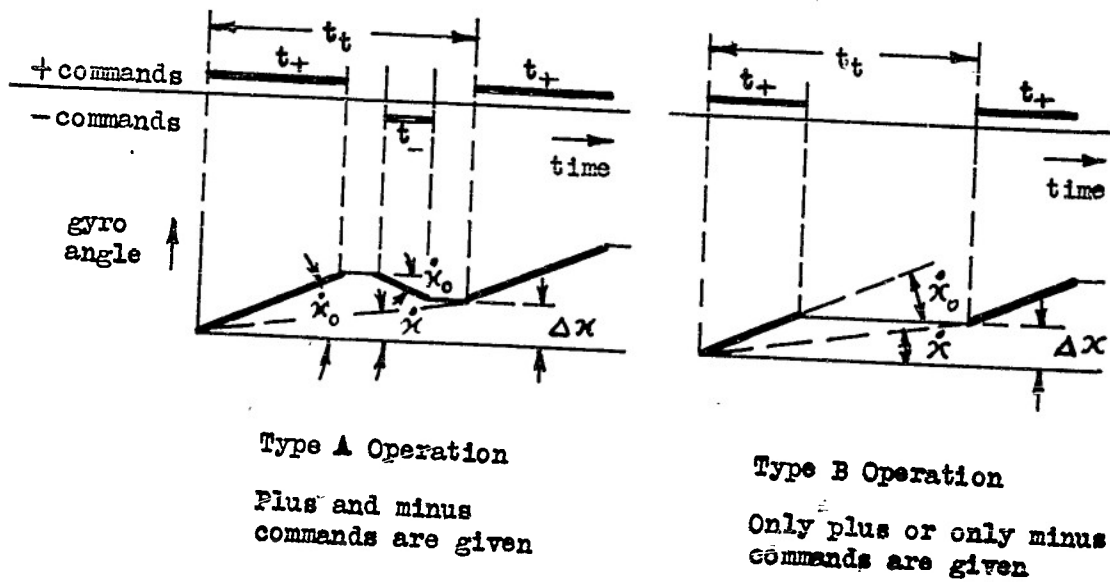


Fig. 4a. During command transmission, the gyro is precessing with a constant rate $\dot{\chi}_0$. An average gyro precession rate $\dot{\chi} = \dot{\chi}_0 \frac{t_+ - t_-}{t_t} = \dot{\chi}_0 K$ can be achieved either by plus and minus commands during a command cycle (Type A) or by commands of one sign only (Type B).

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For maintenance of the proper relationship between gyro and course of the missile, the maximum gyro precession rate should not exceed the maximum rate of change of the course as determined by the aerodynamic properties of the missile.

4. RADIO TRANSMISSION SYSTEM

The conventional radio transmitter and receiver combination for the two missiles was used with five channels available for the transmission of on-off commands. Transmission is effected for each channel by closing a contact. Ground tests showed that commands lasting longer than 30 ms will be transmitted reliably with an accuracy of about ± 15 ms. The transmission of commands shorter than about 50 ms is avoided.

Two radio channels are used for guiding the missile in azimuth. "Plus" commands, transmitted during an interval t_+ , change the course angle κ of the missile to the right and $\Delta\kappa$ becomes positive. "Minus" commands, transmitted during an interval t_- , cause a negative $\Delta\kappa$. If such commands are transmitted during a total period of time t_t , including intermissions, then the total change of course is proportional to $t_+ - t_-$, (See Figure 4a):

$$\Delta\kappa = \dot{\kappa}_0 (t_+ - t_-)$$

In this, $\dot{\kappa}_0$ is the gyro precession rate to the right that equals the maximum rate of course change resulting from a continuous right command. Let the average rate of change of the course during t_t be called $\dot{\kappa}$, then:

$$\dot{\kappa} = \dot{\kappa}_0 \frac{t_+ - t_-}{t_t} = \dot{\kappa}_0 K \quad 4a$$

The value

$$K = \frac{t_+ - t_-}{t_t} \quad 4b$$

is called the "command value" during the interval t_t . A command value $K = \pm 1$ means a continuous transmission resulting in $\pm \dot{\kappa}_0$.

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5. TASK OF THE COMPUTER

The task of the guidance computer is to transform the error voltage V into a proper command value for guiding the missile back to the line of sight. If the missile is to the right of the line of sight, that is, if its error y is positive, it will be necessary to change its course to the left by giving a negative command. Therefore,

$$K = -\frac{y}{y_E} \quad (y_E \text{ is a constant; } y_E > 0)$$

might be chosen. Since $\dot{y} = v\kappa$ (v = missile speed) and with Equation 4a, it follows that

$$\frac{v\dot{\kappa}_0}{y_E} y + \ddot{y} = 0$$

This equation indicates that the guidance motion would be self-exciting if the time lags of the system are considered, particularly T_M . In order to damp the motion, the computer should deliver the following command:

$$K = -\frac{1}{y_E} (y + T\dot{y}) = -\frac{1}{E} (V + T\dot{V}) \quad 5a$$

T is the "time constant" of the computer, which determines the damping influence of the error rate $\dot{y} = v\kappa$. This yields

$$s(y + T\dot{y}) + \ddot{y} = 0 \quad 5b$$

with

$$s = \frac{v\dot{\kappa}_0}{y_E} \quad 5c$$

$y + T\dot{y}$ is the error that the missile would attain after T seconds if it maintained its present course (See Figure 5a). For example should $\dot{y} = -\frac{y}{T}$, that is, should the missile be on a course that

would reach the line of sight after T seconds, then no command is given. y_E and E are the magnitudes of error and error voltage, respectively, to which the command value "1" is co-ordinated. The constant s signifies the "stiffness;" it is the acceleration of the missile per unit of error when $\dot{y} = 0$.

The guidance equation 5b, as written, shows stability for

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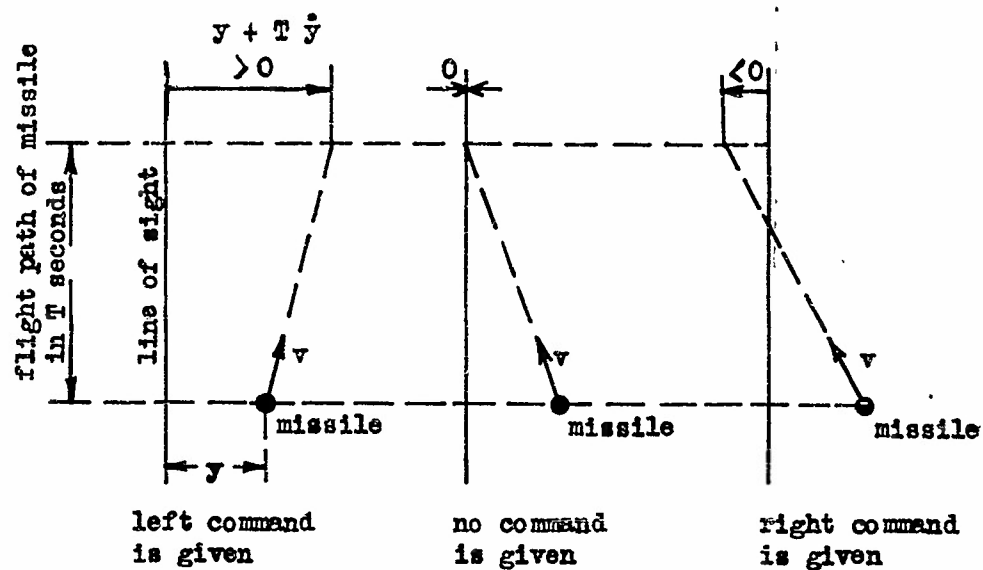


Fig. 5a. Relationship between the flight path of the missile and command.

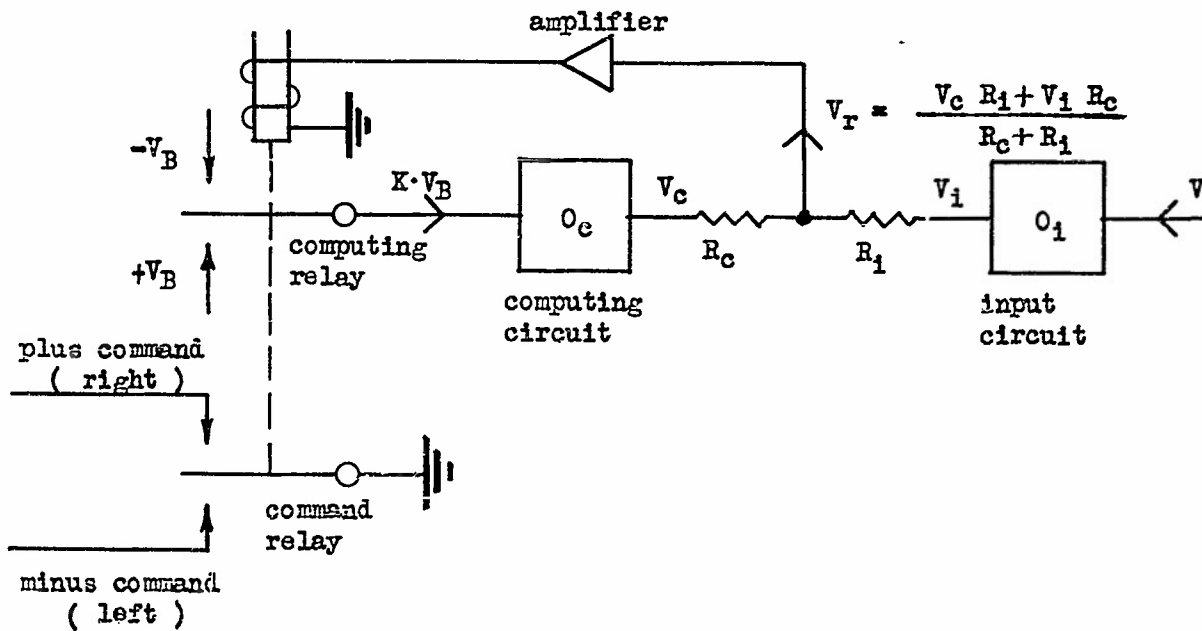


Fig. 6a. Schematic of the computer. The large arrows indicate the trend of operation.

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all possible positive values of s and of T and, therefore, gives no indication whatever for a suitable choice of these values. Only the introduction of the time lags and noise into the guidance equation determines the limits for these constants and thereby the miss that might be caused by a disturbance. For instance, should the target use a lateral acceleration $\ddot{Y}$, the miss will be approximately

$$y = A\tau_t^2 \ddot{Y}$$

In this equation, τ_t is the sum of all time lags in the guidance system and A is a constant that depends also upon other conditions of the guidance task, for example, time of flight, duration of the acceleration, radar noise, etc. The investigation of the conditions as they exist for the LARK showed, for line-of-sight control, a lower limit for A of approximately 2, although 10 is a more probable value.

Originally, it had been intended to insert other influences into Equation 5a. An integration influence, that is a term

$$\frac{1}{T_1} \int v dt$$

should counteract the effect of disturbances of long duration. The analysis for the LARK, however, showed that the time lag of the missile is not short enough in comparison with the total time of flight or compared with the period of time during which the target can possibly maintain a large acceleration, for permitting the successful use of such an influence. For the LOON, these conditions are more favorable and an integrating influence may be added possibly at a later date. Radar noise did not permit the use of a second-rate influence, namely,

$$T_2^2 \ddot{V}.$$

design and operation of the computer

6. COMPUTER SYSTEM

Fig. 6a shows a general schematic of the computer. The two contacts of a polarized relay (computing relay) are connected to the constant voltages $+V_B$ and $-V_B$, respectively. The arm of the relay, which is connected to the "computing circuit," may

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remain either in neutral position or at one of these contacts. Thus, the relay feeds a variable input, KV_B , to the computing circuit; KV_B is the average value of this voltage during a command cycle t_t . On the assumption that the operation, O_c , of the computing circuit is such that it evaluates this average voltage, it transforms KV_B into the "computed voltage" $V_c = O_c \cdot KV_B$. The error voltage V may be transformed by an "input circuit" into a voltage $V_i = O_i \cdot V$. By means of two resistors R_c and R_i , the influences of V_c and V_i are added, and the sum,

$$(V_c R_i + V_i R_c) / (R_c + R_i) = V_r$$

is used to operate the relay. If the sum is positive, the minus contact is closed, and vice versa. Since the relay is very sensitive, this sum is maintained at almost zero by the relay operation (For exceptions see Section 8). Consequently:

$$V_i = -V_c \frac{R_i}{R_c} = -V_c / \rho \quad 6a$$

and:

$$K = -\frac{\rho}{V_B} \frac{O_i}{O_c} \cdot V \quad 6b$$

By means of electrical or mechanical linkage, the command relay is operated simultaneously with the computing relay.

A proper command value may be derived by a suitable choice of the two circuits of the computer.

A smoothing circuit (integrating network) has been selected for both computers as the computing circuit (See Figure 6b). $R_o \ll R_B$. Hence, with $T = R_o C$:

$$V_c = O_c \cdot KV_B = \frac{R_o}{R_B} \frac{KV_B}{1 + T \frac{d}{dt}}$$

or, designating the saturation value of V_c as

$$\rho E = \frac{R_o}{R_B} V_B \quad 6c$$

$$K = \frac{1}{\rho E} (V_c + T \dot{V}_c) \quad 6d$$

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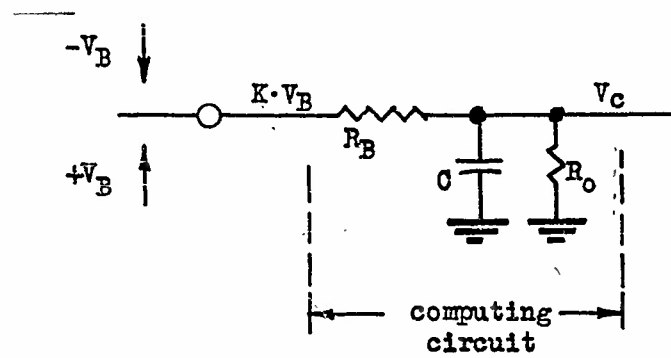


Fig. 6b. The computing circuit is an RC - smoothing circuit with a time constant $T = CR_O$; $R_O \ll R_B$.

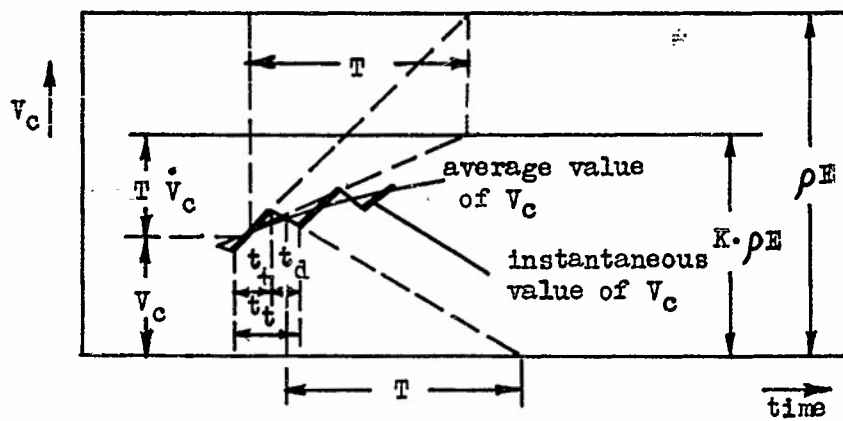


Fig. 6c. The instantaneous changes of the voltage V_C , as caused by the closing and opening of the relay contact, and the change of the average value of V_C .

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The derivation of this formula in greater detail is given in Appendix A (See Figure 6c).

The value $O_i=1$ was selected for the LARK computer; therefore, with 6a, the desired relationship of Equation 5a is achieved.

Figure 15a represents a schematic of the LARK computer, including numerical values for its elements. Figure 16a gives this information for the LOON computer. A smoothing circuit has been added at the input of the LOON computer:

$$K = - \frac{1}{\rho E} \frac{V + T \dot{V}}{1 + T_i \frac{d}{dt}}$$

Furthermore, a timing circuit maintains the computing time t_c at 3 seconds; this is the period during which the contact of the computing relay is closed.

7. THE RELAY GAP

The "relay gap" G is the amount of voltage V_r necessary to switch the relay arm to "plus" or "minus"; for $V_c=0$, the amount of error voltage necessary to close a relay contact is $G(R_c+R_i)/R_c=g$. The relay hysteresis γ is the minimum change of voltage V_r required to open and close a relay contact (See Figure 7a).

The computer in Figure 6a may be operated in two different ways (See Figure 4a): for small and medium command values, either plus and minus commands, with short intervals, may be transmitted (Type A); or only one channel may operate at greater intervals (Type B). Whichever operation takes place is dependent upon the time lag of the relay and the relay gap. If one contact is closed, the voltage V_r changes and, if the time required for this voltage to cross the relay gap is less than the relay time lag, Type A will occur. If the time is greater Type B occurs.

In the LOON computer, the closing time lag of the relay is very short; the opening time lag is longer and approximately equal to t_c . The condition under which Type B takes place even for small command values may be derived from Figure 7b:

$$\frac{t_c}{2T} E < g \quad 7a$$

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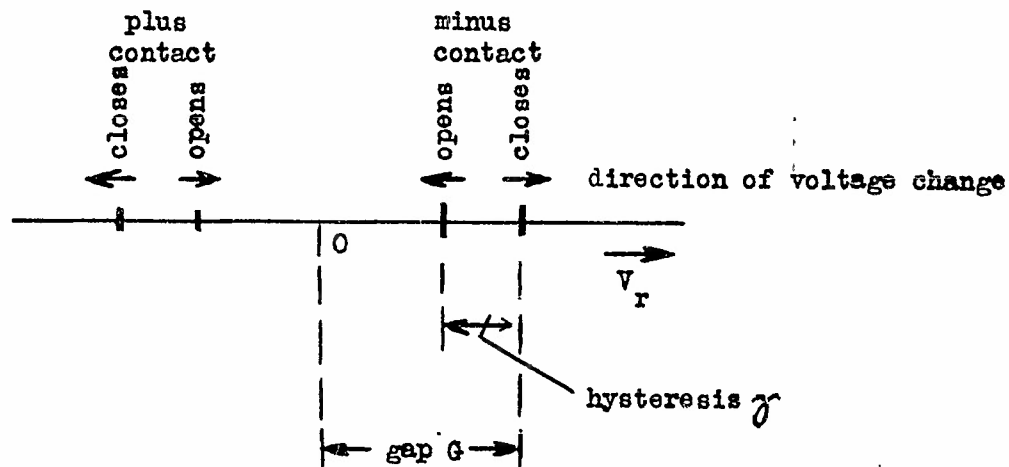


Fig. 7a. The gap G and the hysteresis γ of the computing relay.

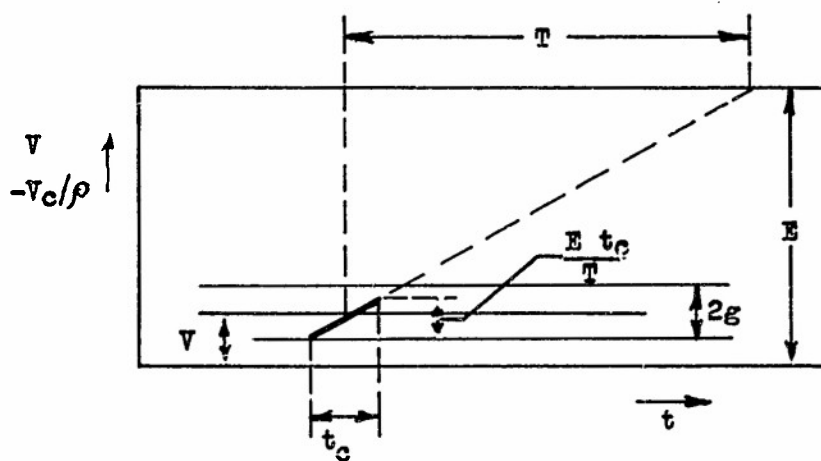


Fig. 7b. The condition where the relay contact opens before the condenser voltage has crossed the gap.

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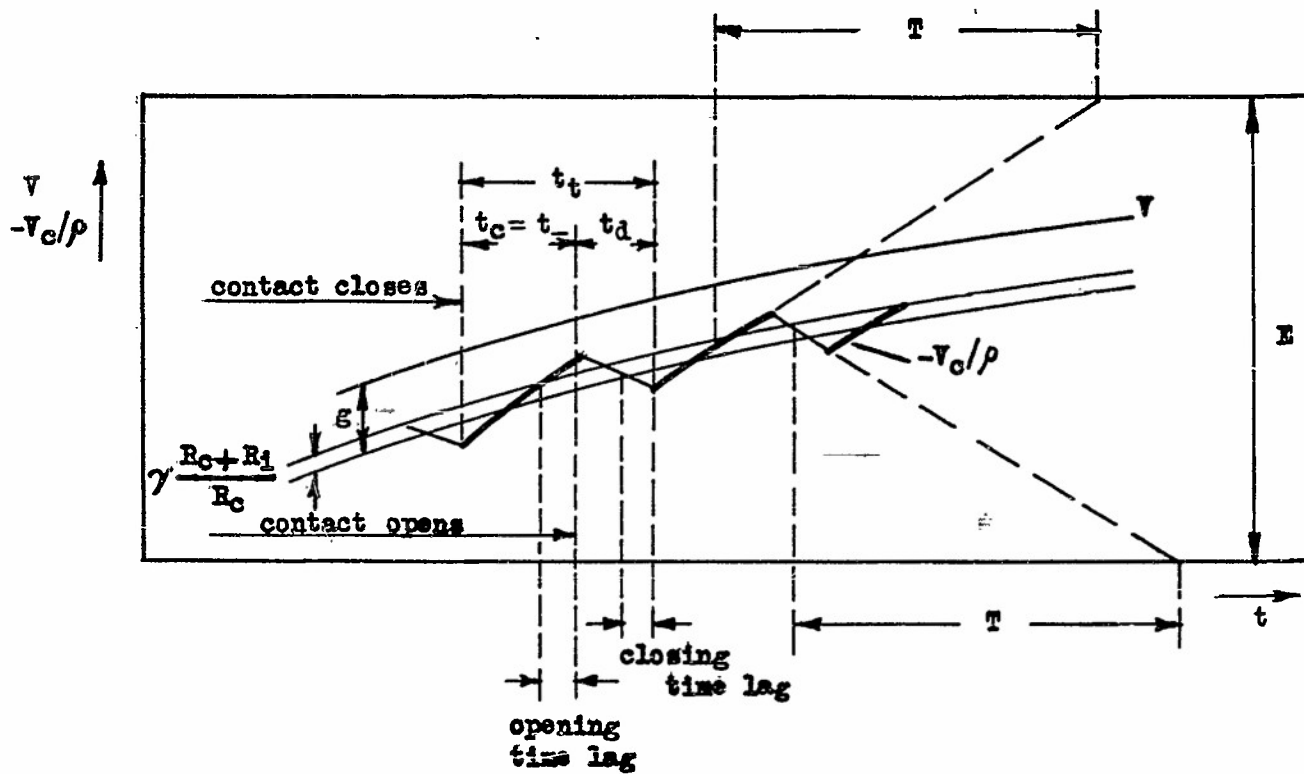


Fig. 7c. In the LARK computer, the duration of t_c and t_t is dependent upon the relay hysteresis and the relay time lags; however, the command value $K = t_c/t_t$ is independent of these characteristics and is dependent only upon $V \cdot g$ and its rate.

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Type B was chosen for both the LARK and the LOON computer. In both, the relay gap is rather large, that is, the relay remains in neutral position until the missile error reaches several per cent of y_E . This prevents reasonably small radar hunting amplitudes from causing commands. Furthermore, Type B gives a smaller number of commands.

In Figure 7c, V and $-V_c/\rho$ are plotted versus time. For example, if $V > 0$, the minus contact of the relay closes when $V - g > -V_c/\rho$. Figure 7c shows that the command is not dependent on V but on $V - g$. The formula:

$$K = -\bar{V} \text{ sign } (V + T\dot{V})$$
$$\bar{V} = \left| \frac{V + T\dot{V}}{E} \right| - \frac{g}{E} = \left| \frac{y + T\dot{y}}{y_E} \right| - \frac{y_G}{y_E} \quad 7b$$

represents the "normal" relay operation with inclusion of the gap influence (See Equation B1). This normal or proportional operation occurs only as long as the condenser voltage V_c can follow the error voltage V in such a manner that V_r remains continuously as close to $+G$ or to $-G$ as the relay deficiencies permit. This is so as long as the condition $1 > \bar{V} > 0$ is fulfilled continuously.

8. NONPROPORTIONAL OPERATION; TRAJECTORY SELECTOR

When the ability of the condenser voltage to follow the error voltage ceases, nonproportional operation begins. During nonproportional operation, the computer gives either zero or full command and disregards the error rate, that is, a command of that nature remains, regardless of the sign and the magnitude of $\dot{V}$, until V_r again becomes equal to $\pm G$; after which normal operation is resumed. Nonproportional operation begins or occurs under the conditions given in the following four examples (See Figure 8a).

Case $\bar{V} < 0$ (See Figure 8a,A) $\bar{V}$ in Equation 7b, becomes negative. After this has taken place, the voltage V_r is within the gap and the computer gives no command. The loss of rate influence within the gap is desirable for limiting the effect of radar hunting. As shown in Figure 8a A, the case $\bar{V} < 0$ may occur for any value of the error voltage as long as $|V| < E$. In other words, during normal operation, the condenser voltage adjusts itself to the missile error; hence, the gap, so to speak, follows the motion of the missile.

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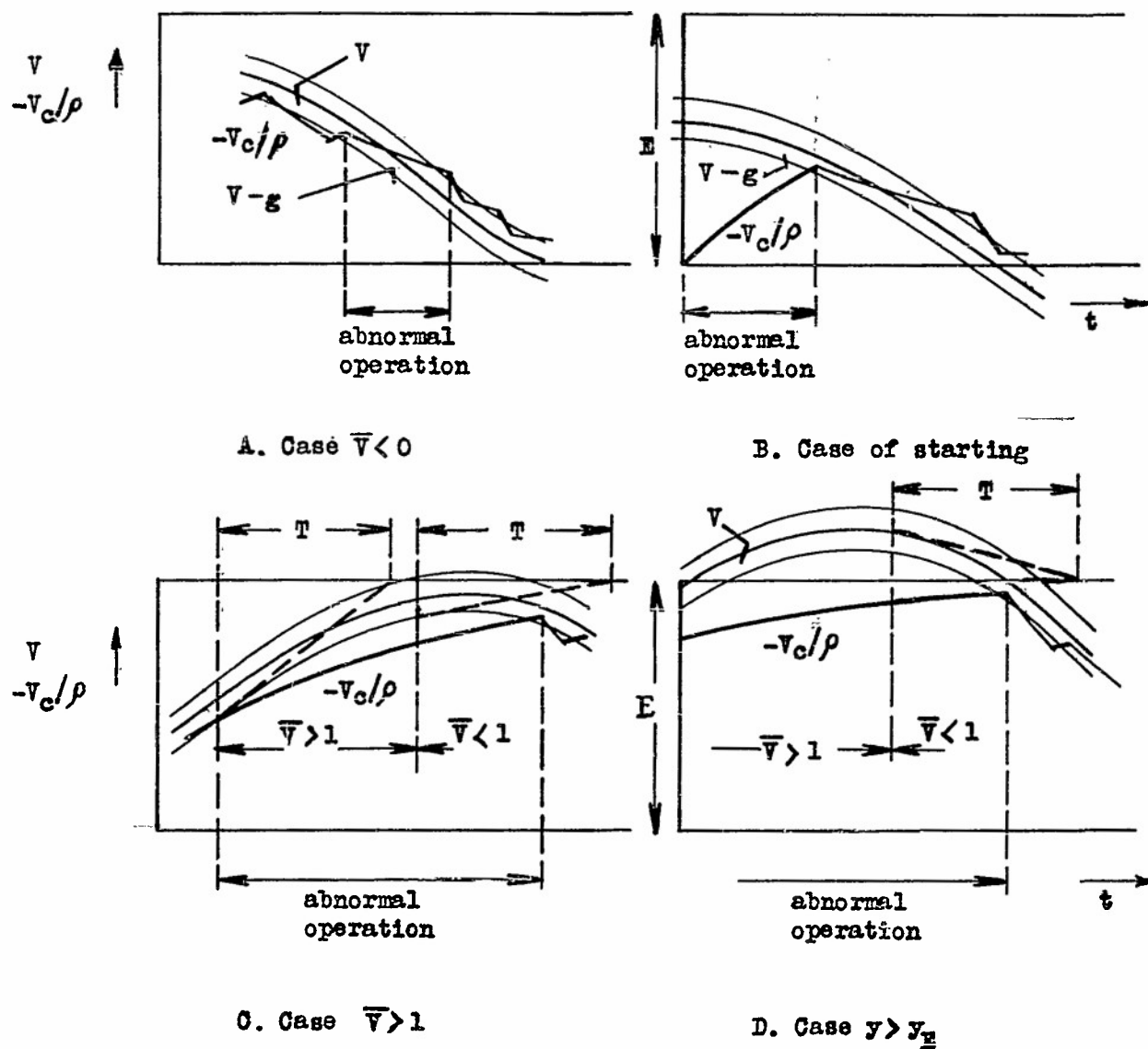


Fig. 8a. The four cases of nonproportional computer operation.

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Case of Starting (See Figure 8a, B) Should the computer be switched in at any instant when the condenser voltage does not correspond to the error voltage, full command is given until V_r reaches the gap. This is undesirable because the command thus created may be erroneous (See the example shown in Figure 8a, B); if the launcher is not located on the line of sight, the erroneous command may be avoided by directing the antenna of the missile radar toward the launcher and by switching the computing circuit in before launching. A "radar-off" switch is provided in the LOON computer for avoiding this necessity, viz., during the time that the switch is operated, the resistor R_B is by-passed and the condenser voltage adjusts itself almost immediately to the error voltage (See Figure 16a). This switch is also used should the radar, cease to follow the missile during the flight. After the radar is again on the missile the switch is cut out and the computer starts to give the correct command.

Case $\bar{V} > 1$ (See Figure 8a, C) $\bar{V}$ exceeds 1. After this has taken place, the relay contact remains closed and full command is given until the condenser voltage overtakes the error voltage. When the properties of computer and missile are properly co-ordinated, the motion of the missile, as a rule, will not call for full commands. If as a result of launching, the case $\bar{V} > 1$ occurs, it is not desirable to continue full command even after $\bar{V}$ has dropped below 1 (See Figure 8a, C). This can be avoided by cutting in the radar-off switch; however, the use of the radar-off switch would harm the guidance accuracy should the condition $\bar{V} > 1$ be caused by radar hunting.

Case $y > y_E$ (See Figure 8a, D) The error itself is so large that it results in full command. The condenser voltage cannot off-set the error voltage and full command is given. During a flight, this can happen only after $\bar{V}$ has become greater than one. If, as a result of an extremely large launching deviation, the conditions $\bar{V} > 1$ and $y > y_E$ occur, it would be more desirable to have the computer return to proportional command when $\bar{V}$ drops below 1 although y is still larger than y_E (See Figure 8a D).

In order to avoid these deficiencies, it is intended to add a "Trajectory Selector, TS" to the computer for achieving the following properties (See Appendix E):

- (a) In all conditions where the present computer is able to guide the missile without giving full command, the TS does not go into action.
- (b) If the flight path of the missile is such that the present

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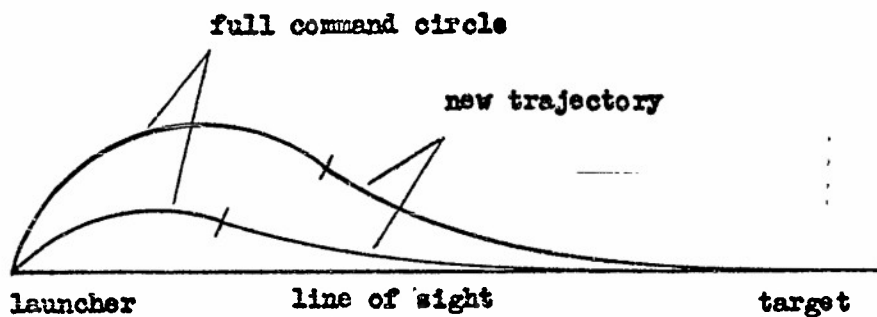


Fig. 8b. Missile flight path with trajectory selector (TS) for different launching angles. After launching, full command is given until the course of the missile has achieved a direction that coincides approximately with the tangent to the new trajectory chosen automatically by the TS out of a family of curves.

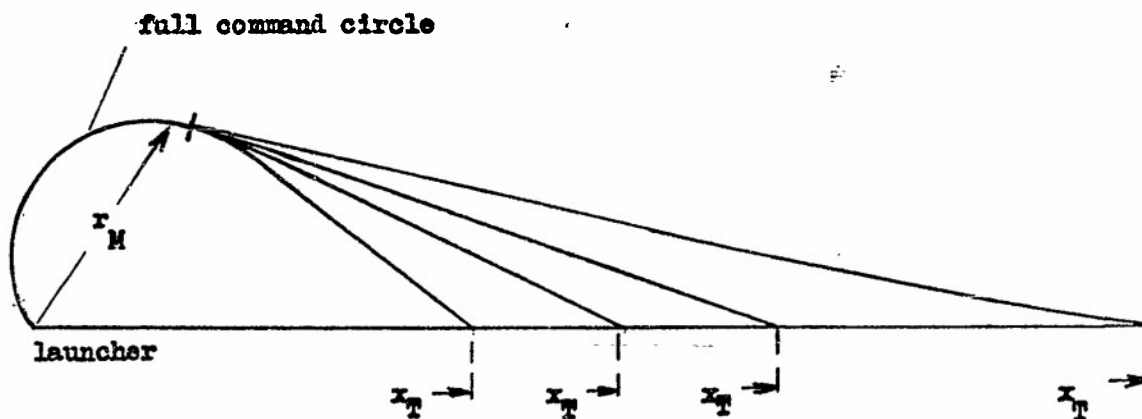


Fig. 8c. Same as Figure 8b. These approximately straight trajectories are attained by the TS design shown in Figure E3 of Appendix E. They are shown for the same launching dispersion, but different values, of target range x_T .

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computer gives full command (for example, in event of a large launching deviation or if the missile is launched towards the guidance beam from any direction), the TS will cause the computer to continue the full command only until the missile is able to follow a new trajectory smoothly. Out of a family of possible trajectories, the TS selects automatically the trajectory that has the least deviation from the line of sight and which is compatible with the launching deviation; (See Figure 8b.)

- (c) The type of trajectory may be chosen so that the missile returns to the line of sight after correction of the launching deviation. (See Figure 8b). This type of trajectory avoids the necessity of introducing the target range into the TS, but its use is limited, for example, for the LOON, to launching angles smaller than 60 degrees and target ranges greater than 45 miles. On the other hand, the new trajectory may be approximately straight (See Figure 8c). If this type is chosen, the launching angle is unlimited, but the target range must be introduced accurately into the TS.

After the missile has reached a certain range, the TS is prevented from changing the chosen trajectory so that large hunting motions during the later part of the flight will not interfere with the trajectory.

9. RANGE INTRODUCTION

The multiplication of the azimuth error angle voltage by the range can be accomplished in two ways:

In the LARK computer, the voltage per unit of error angle E_0 applied to the radar potentiometer is increased proportionally to the range D^* (See Figure 15a); thus, the relationship between missile error y and command remains constant during flight. In the LOON computer, the voltage E is kept constant but the ratio R_c/R_i is increased during flight in such a way that y_E increases with increasing range (See Figure 16a and Figure 16b); thus, the gap y_G increases with the range D even faster than

*This method was copied from an error indicator for manual control of the LARK, developed by the Franklin Institute Laboratories for Research and Development.

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y_E increases (See Equation B2, lower). This is desirable because of the increasing influence of radar hunting with range.

If y_E changes during the guidance period, the guidance Equation 5a is no longer accurate (See Appendix D).

details of computer operation

10. RADAR HUNTING EFFECTS

Radar hunting introduces noise and inaccuracies into any guidance system and, consequently, limits the attainable guidance accuracy. At close missile range, the hunting amplitudes by themselves will not create full commands but the rate influence caused by hunting must be expected to exceed unity during almost the entire flight.

The following discussion treats mainly of the response of the LARK computer to this condition. Because of the predetermined long computing time of the LOON computer, $t_c = 3$ seconds, the results are directly applicable to this computer only when hunting amplitudes are small ($V_i < g$) and when hunting motions are slow when compared with 3 seconds. It is expected that the smoothing circuit in the LOON computer input will as a rule, maintain the amplitudes of V_i below g for faster hunting motions.

A. Small Hunting Amplitudes

As long as the hunting amplitude is smaller than the gap, no command is given. When a particularly large hunting motion of V exceeds the gap, it causes a command and, thereby, a rate of change of $-V_c/\rho$. The command ceases after t_c when $-V_c/\rho + g$ overtakes V (See Figure 10a, A). Since the change of V leading to this command might be caused partly by hunting and partly by a missile error, it is undesirable to have this command followed immediately by a countercommand. Figure 10a, B, shows that such a countercommand is avoided as long as the hunting amplitude of V_i is

$$H < g - \frac{t_c}{T} E \quad 10a$$

The right-hand side of this equation is the "effective" gap. For the LARK computer, t_c depends upon the frequency of the hunting oscillation (See Figure 10a A). For frequencies slower than 1 cps, that is, $t_c \sim 0.25$ seconds, the gap begins

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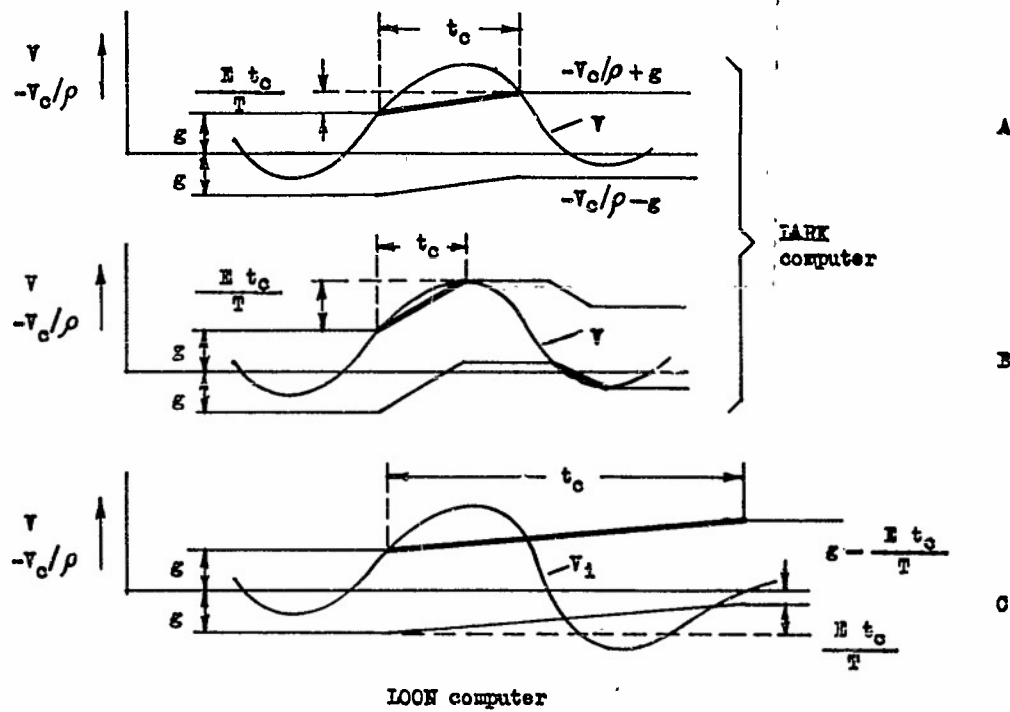


Fig. 10a. Response of the computer to a radar hunting motion having an amplitude smaller than the gap.

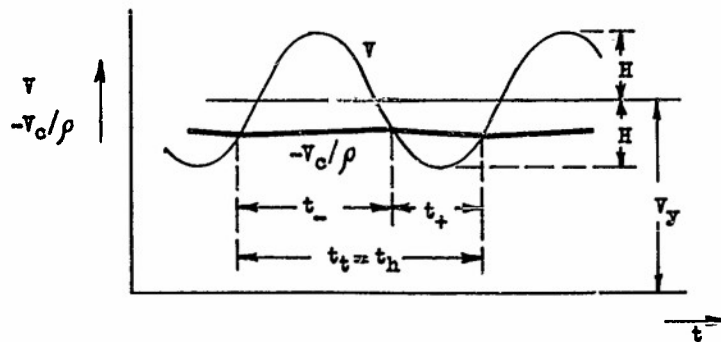


Fig. 10b. The input voltage V_i is composed of the part V_y corresponding to the missile error, and $H \sin \omega t$ caused by hunting. The condenser voltage V_c cannot follow a fast hunting oscillation. The zeros of $V_y + H \sin t + \frac{V_c}{\rho}$ determine the instants of time when the full command K changes the sign and, thereby, the command value K_h during hunting.

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to be less effective. Consider Equation 10a with $g/E=0.05$, $T=7$ seconds. For the LOON computer, see Figure 10a A, with $t_c=3$ seconds.

B. Sinusoidal Hunting

Sinusoidal hunting is investigated in Appendix C for hunting amplitudes H large compared with the gap and hunting oscillations which are fast compared with the computer time constant. This investigation yields the following results:

- (a) The hunting frequency does not affect the operation of the computer (See Equation C2 to C11). The subsequent results also hold for varying hunting frequency.
- (b) During hunting, only alternating full commands are given ($K=\pm 1$). The change of sign takes place every time the error voltage V , as caused by the missile error voltage V_y , and by hunting (i.e., $H \sin \omega t$), crosses the condenser voltage $-V_c/\rho$ (See Figure 10b); thus, the (average) command value K_h during hunting is dependent upon the missile error, the hunting amplitude, and the condenser voltage. As in any nonproportional operation, however, the missile error rate $\dot{V}_y$ does not influence directly the command value (See Equation C2 and C3).
- (c) For constant missile error and constant hunting amplitude, the command value K_h is smaller than K . Figures 10c and 10d show that this decrement, $K-K_h$, is considerable; however, it is only a part of the hunting amplitude and, therefore, is only of the magnitude of the inaccuracy in the radar output during hunting. When hunting commences, (See Figure 10e), the command value K_h drops to zero but then recovers; the smaller the hunting amplitude, the quicker the recovery. After the hunting amplitude decreases (See Figure 10f), and the error measurement is to be considered as more accurate, the command not only corresponds more closely to the error voltage, but an additional transient increase of command volume transmitted immediately after this change of hunting amplitude, makes up for previous loss.
- (d) During hunting with a constant amplitude, the computer does not create the command value $K=\pm \bar{V}$ according to Equation 7b but only the command value $K=\pm 1$, thus smoothing $|\bar{V}|$ to one. In doing this, the computer

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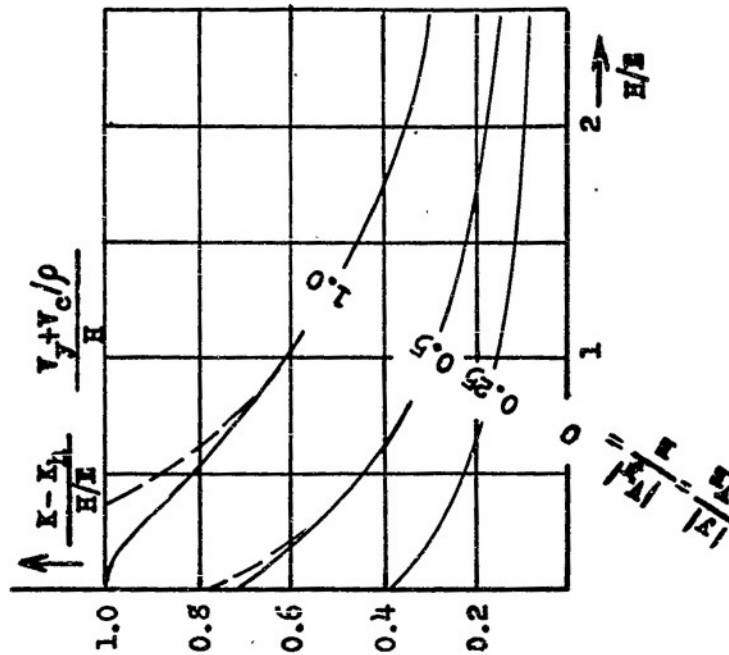


Fig. 10d. The ordinate is the difference of the command value K without hunting and the command value K_h in case of steady-state sinusoidal hunting compared with the hunting amplitude H/E . The dashed curves are derived from the approximation Equation C3; they coincide well enough with the correct curves as long as $y/y_E < 0.5$.

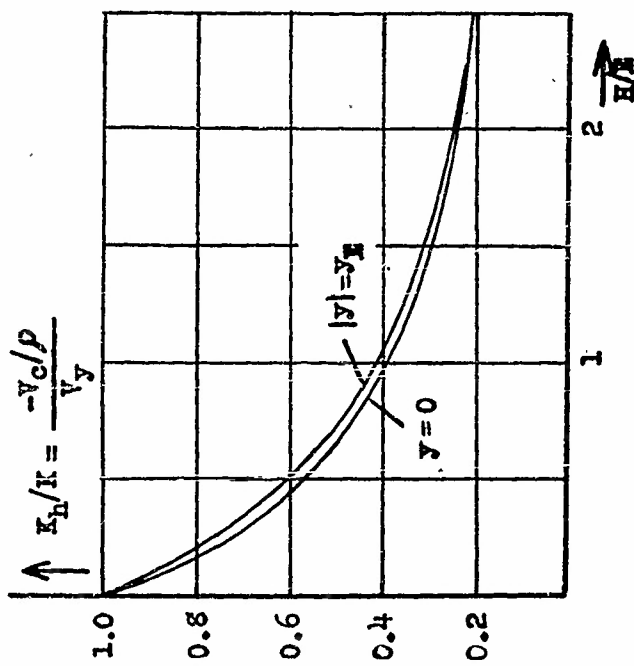


Fig. 10c. The ordinate is the command value K_h during steady-state sinusoidal hunting compared with the command value K without hunting. The abscissa is the hunting amplitude H in terms of the full command voltage E . Large hunting amplitudes diminish considerably the command value. K_h/K depends only slightly upon the missile error y in terms of y_E .

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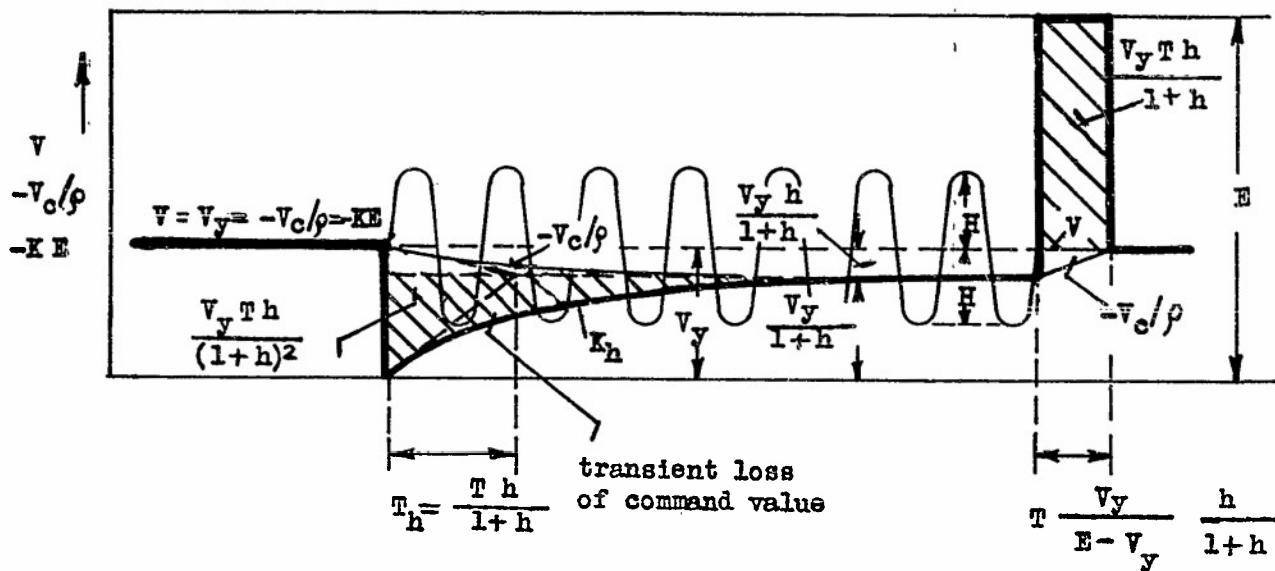


Fig. 10e. The variation of the command value K_h (indicated by the heavy line) and of the condenser voltage (of $-V_c/\rho$) for a constant missile error after the beginning ($t=0$) and after the end of a constant-amplitude hunting motion.

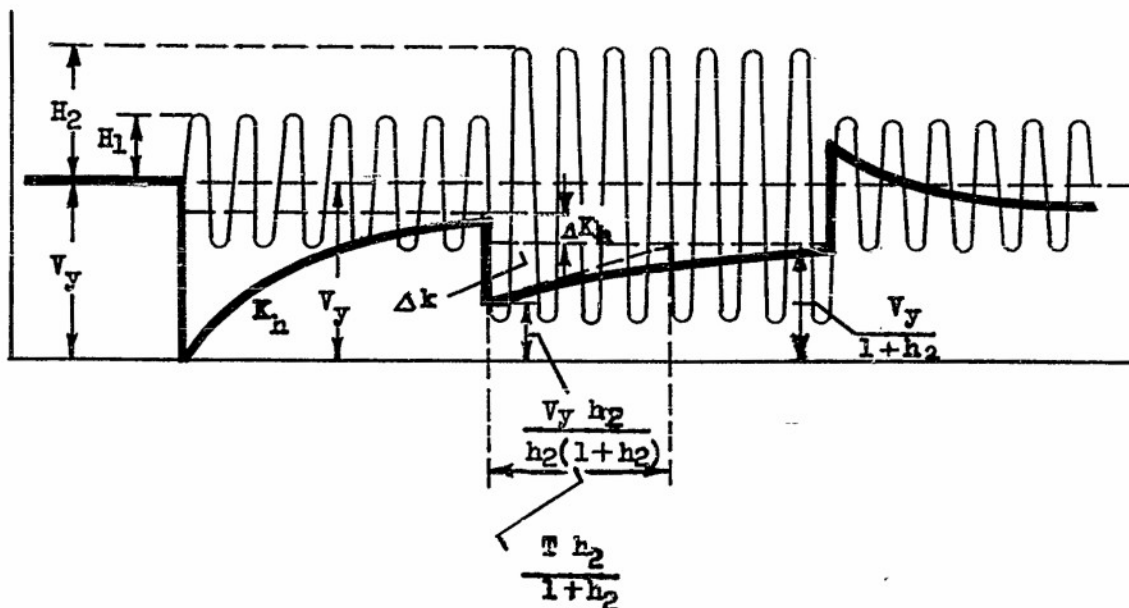


Fig. 10f. The variation of the command value K_h and of $-V_c/\rho$ for a constant missile error but a jumping hunting amplitude H . Every change of the hunting amplitude causes a transient change of K_h and a steady-state change ΔK_h .

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develops automatically a smoothing time lag T_h , that is, the condenser voltage $-V_c/\rho$ follows the missile error voltage with this time lag and, therefore, the command value K_h follows $V_y + T\dot{V}_y$ with the same time lag (See Equations C7 and Cg). Equation C8 shows that

$$T_h = T \frac{\frac{\pi}{2} \frac{H}{E}}{1 + \frac{\pi}{2} \frac{H}{E}} \quad 10b$$

In a proportional system, where

$$K = -\frac{1}{E} (V + T\dot{V})$$

fast sinusoidal hunting, for zero missile error, creates the command

$$K \approx \frac{TH\omega}{E}$$

In order to smooth this value K to unity, an RC-smoothing circuit, with the time constant

$$T_s \approx T \frac{H}{E}$$

would have to be applied. This value must be increased for large missile errors. This shows that the smoothing time lag T_h (See Equation 10b), which the computer develops, is comparable to that of a proportional system. The computer system has the advantage in that this time lag builds up automatically and vanishes with the hunting motion.

- (e) The reduction of the command value during hunting to the portion $\frac{1}{1 + \frac{\pi}{2} \frac{H}{E}}$ (Compare Equation C9 with Equation 5a),

also reduces the guidance stiffness in the same proportion. This helps in maintaining a stable guidance loop during hunting despite the smoothing time lag T_h . Figure 10g shows an input circuit for a proportional-command guidance computer that would cause the same relationship between hunting amplitude, smoothing time

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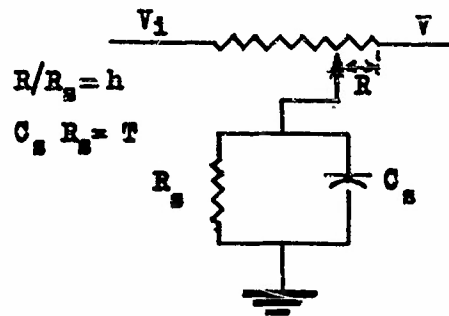


Fig. 10g. An input circuit for a proportional guidance computer that would cause the same relationship between hunting amplitude, smoothing time lag, and guidance stiffness as the on-off computer. However, in addition it would be necessary to vary automatically the resistor R in this Figure 10g with the hunting amplitude.

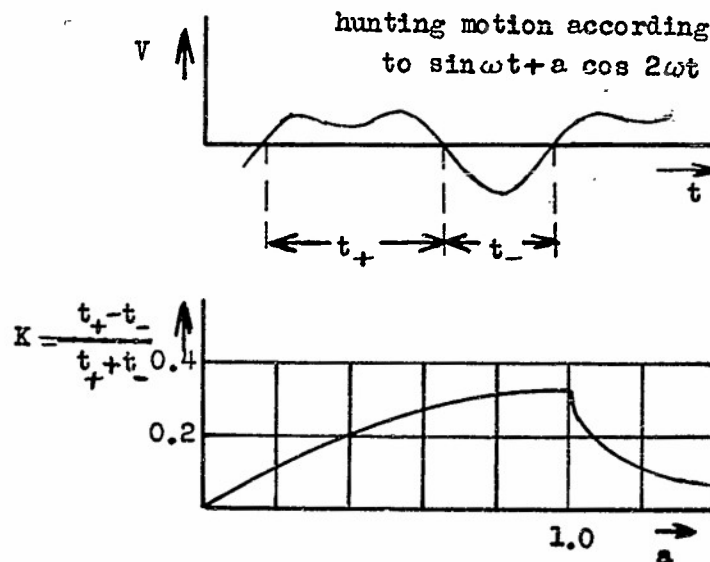


Fig. 10h. The command value caused by nonsinusoidal hunting, particularly by the second harmonic of the hunting motion.

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lag, and guidance stiffness as the on-off computer. In addition, however, it would be necessary to vary automatically the resistor R in Figure 10g with the hunting amplitude.

C. Nonsinusoidal Hunting

Hunting is caused primarily by the irregular noise of the radar output; oscillations are caused only insofar as critical frequencies of the radar antenna servomechanism are met. As a rule, the hunting motion is far from being sinusoidal and the intervals during which V deviates from the average value in plus and in minus direction are not equal. Figure 10h shows, for instance, the command value caused by a hunting motion of the kind $\sin \omega t + a \cos \omega t$; the command $\frac{t_+ - t_-}{t_+ + t_-}$ may

reach up to 30 per cent of full command. This command value is independent of the hunting amplitude, but cannot continue over a longer period of time. Even if the hunting motion would continue in the same manner, the difference between t_+ and t_- would cause the condenser voltage to change by such a part of the hunting amplitude that $t_+ \approx t_-$ is approximately attained. The time constant of this selfadjustment is comparable to that given by Equation 10b; however, the hunting motion usually will change earlier, thus creating an additional noise. The amplitude of this noise is approximately 30 per cent of full command, and the frequency somewhat less than the hunting frequency. This noise is of minor importance when compared with the effects of a sinusoidal hunting motion of large amplitude previously discussed. The application of a smoothing circuit with a small time constant would appreciably decrease the higher harmonics and diminish this noise.

II. SINGLE-COMMAND CONDITIONS

In the LOON guidance, one command of a duration t_c changes the course angle of the missile by k_1 :

$$k_1 = t_c \dot{\kappa}_0$$

11a

Should the missile cross the gap with a course angle κ , which is equal to or smaller than k_1 , and then reach the limit of the gap, only one command is needed to bring the missile back into the gap. If it is decided that only one command be given, the three conditions subsequently derived have to be fulfilled.

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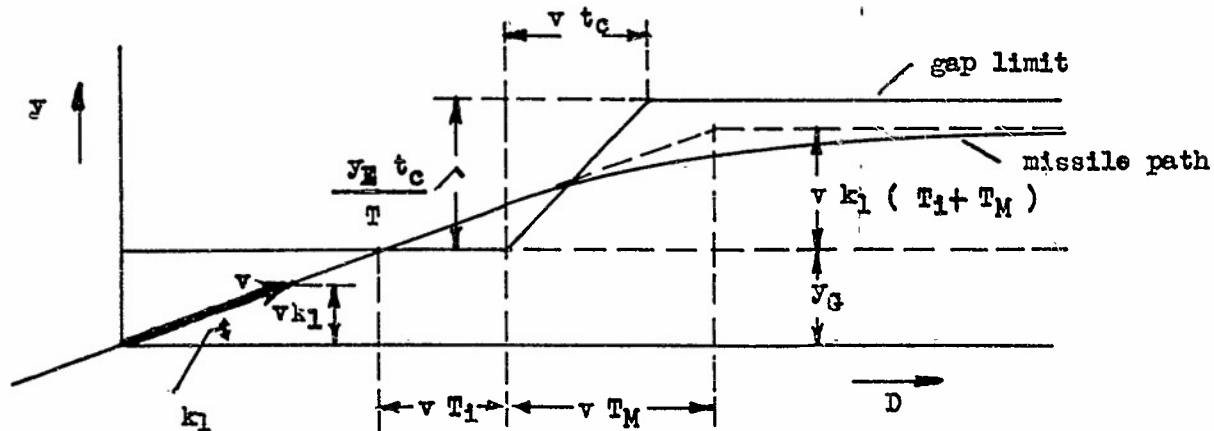


Fig. 11a. The transition $t_c y_E/T$ of the gap limit as caused by the change of the condenser voltage during the computing time t_c for one command, and the amount $v k_1 (T_i + T_M)$ by which the missile exceeds the original gap limit where k_1 designates the original course angle.

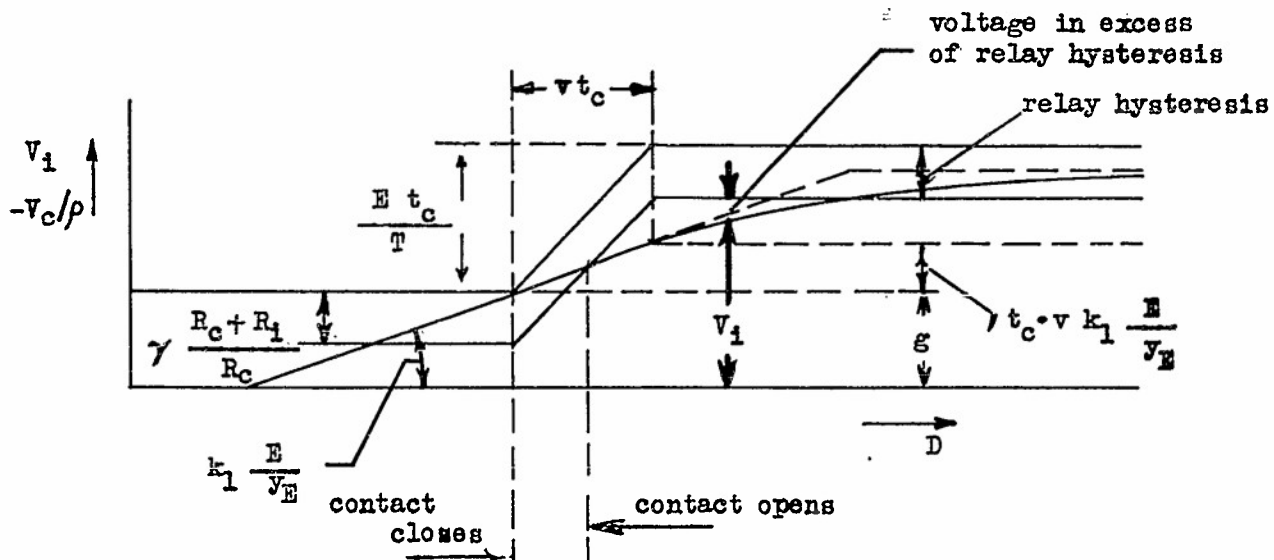


Fig. 11b. The transition t_c E/T of the gap limit and the change $t_c \cdot vk_1 \cdot E/y_E$ of the voltage V_i during t_c . This latter value must be smaller than the transition by the relay hysteresis $\gamma \frac{R_c + R_i}{R_c}$ so that the relay contact opens after one command had been transmitted.

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Figure 11a shows the missile crossing the gap with $K=k_1$. The total time lag between the missile error and the input voltage, as a result of lagging radar operation and the input circuit, is designated by T_i . The input voltage V_i effects a command T_i seconds after the missile has reached the limits of the gap. The missile responds to this command with the time constant T_M . The missile will finally overshoot the original gap limit by $vk_1(T_i+T_M)$ (See Figure 11a). To avoid a second command, this value must be smaller than the transition $\frac{t_c}{T} y_E$ of the limit of the gap as caused by the change of the condenser voltage during the computing time t_c :

$$\frac{t_c}{T} y_E > vk_1(T_i+T_M)$$

Because of Equations 11a and 5c:

$$1 > sT(T_i+T_M) \quad 11b$$

The investigation of the guidance motion shows one of the necessary stability conditions as

$$\frac{\pi}{2} > sT(T_i+T_M)$$

which is not quite as severe.

For the same missile motion, Figure 11b shows the condition where the transition of the gap limit $\frac{t_c}{T} E$, is sufficiently larger than the change $t_c \cdot vk_1 \cdot \frac{E}{y_E}$ of the error voltage during t_c , so that after the contact of the computing relay has been closed, the relay hysteresis is overcome and the contact is opened:

$$t_c \left(\frac{E}{T} - vk_1 \frac{E}{y_E} \right) > \frac{R_c + R_i}{R_c} \gamma$$

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With Equations 11a and 5c:

$$1 > \frac{\gamma}{E} \frac{R_c + R_i}{R_c} \frac{T}{t_c} \frac{1}{1 - s t_c T} = \gamma_1 \quad 11c$$

This condition gives the upper limit for the relay hysteresis γ . Because of Equation 11c, it is necessary that

$$1 > s T t_c \quad 11d$$

12. EFFECTS OF SLOW OPERATION

In the LARK computer, the duration of a command t_c is determined by the time necessary for the condenser voltage to cross the relay hysteresis and by the relay time lag (See Figure 7c). For small and medium command values, t_c is 0.1 second or less. In the LOON computer, the timing circuit effects a computing time $t_c = 3$ seconds. Thus, for small command values, the interval between commands is long compared to the time lag of the guidance loop and is comparable to the computer time constant T .

The derivation of Equation 6d has been repeated on the assumption that the time lag for closing the relay contact is still negligibly small; however, greater accuracy was observed when the opening time lag t_c , was considered. The result, given in detail in Appendix B, is as follows: The command value K does not correspond accurately to the error voltage, i.e., to $V-g$, but to the average condenser voltage, $-V_c/\rho$, which, because of t_c , is larger than $V-g$ by the quantity Δ (See Figure 12a). This difference Δ vanishes for full command, but is noticeable at small command values. As long as V_r exceeds G by a very small amount, a finite command value $K = \frac{t_c}{2T}$ is created (See Figure 12b).

If amplitudes of the motion of a missile are not too small, the computer characteristic, Figure 12b, may be approximated by a linear characteristic with a gap, $\bar{g} = g - \frac{t_c}{2T} E$. Because of Equation 7a, $\bar{g} > 0$.

To investigate the effect of the comparatively large value of t_c/T on the time lag of the computer, it is necessary to define first the operation of an on-off system without a time lag,

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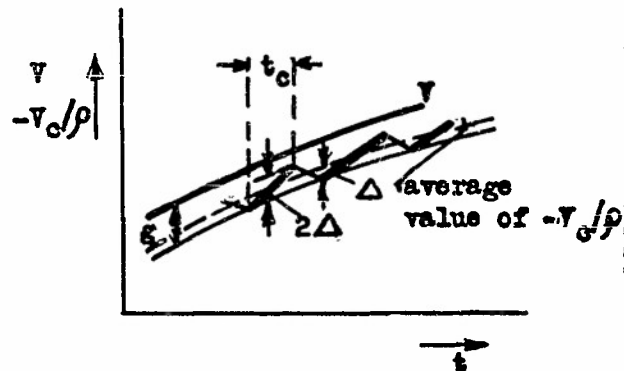


Fig. 12a. The variation of the average value of $-V_c/p$ exceeds $V - g$ by Δ . Δ is caused by the finite value of the relay closing time t_c . The command K corresponds to the average value of V_c/p .

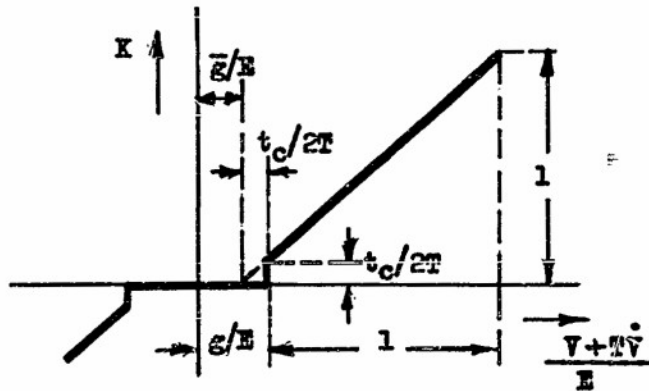


Fig. 12b. Command value K as a function of $V + \dot{V}T$ when the finite value of t_c/T is considered.

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During a period of time t_t , the error voltage $\dot{V}$ changes from V_1 to V_2 (See Figure 12c).

Therefore, the average rate of V is $\dot{V}_{av}$:

$$\dot{V}_{av} = \frac{V_2 - V_1}{t_t}$$

Let an average value of V be defined by:

$$V_{av} = \frac{V_1 + V_2}{2}$$

A system that gives a constant but proportional command during t_t (See Figure 12c) shall, therefore, be considered as time-lag-free if, during t_t , it transmits the constant command

$$-K = \frac{1}{E} (V_{av} + T\dot{V}_{av})$$

An on-off system will be designated as time-lag-free if it transmits a full command having the duration

$$t_c = t_t \left| \frac{V_{av} + T\dot{V}_{av}}{E} \right|$$

in such a way that t_c is symmetrical about the center of t_t (See Figure 12c).

Within the period t_t , the computer actually gives a full command, the duration of which corresponds to this value t_c (more accurately given in Figure 12b; see Appendix B6), but this command starts at the beginning of t_t . Therefore, the computer gives this command with a time lead τ (See Figure 12c, lower):

$$\tau = \frac{1}{2} (t_t - t_c) = \frac{t_c}{2} \left(\frac{1}{|K|} - 1 \right)$$

If the commands are small, this is a considerable time lead. For example: if $t_c = 3$ seconds, and $K = 0.1$, then $\tau = 13.5$ seconds.

*
If V_{av} is defined as the arithmetical mean of V during t_t , an additional computer time lead appears.

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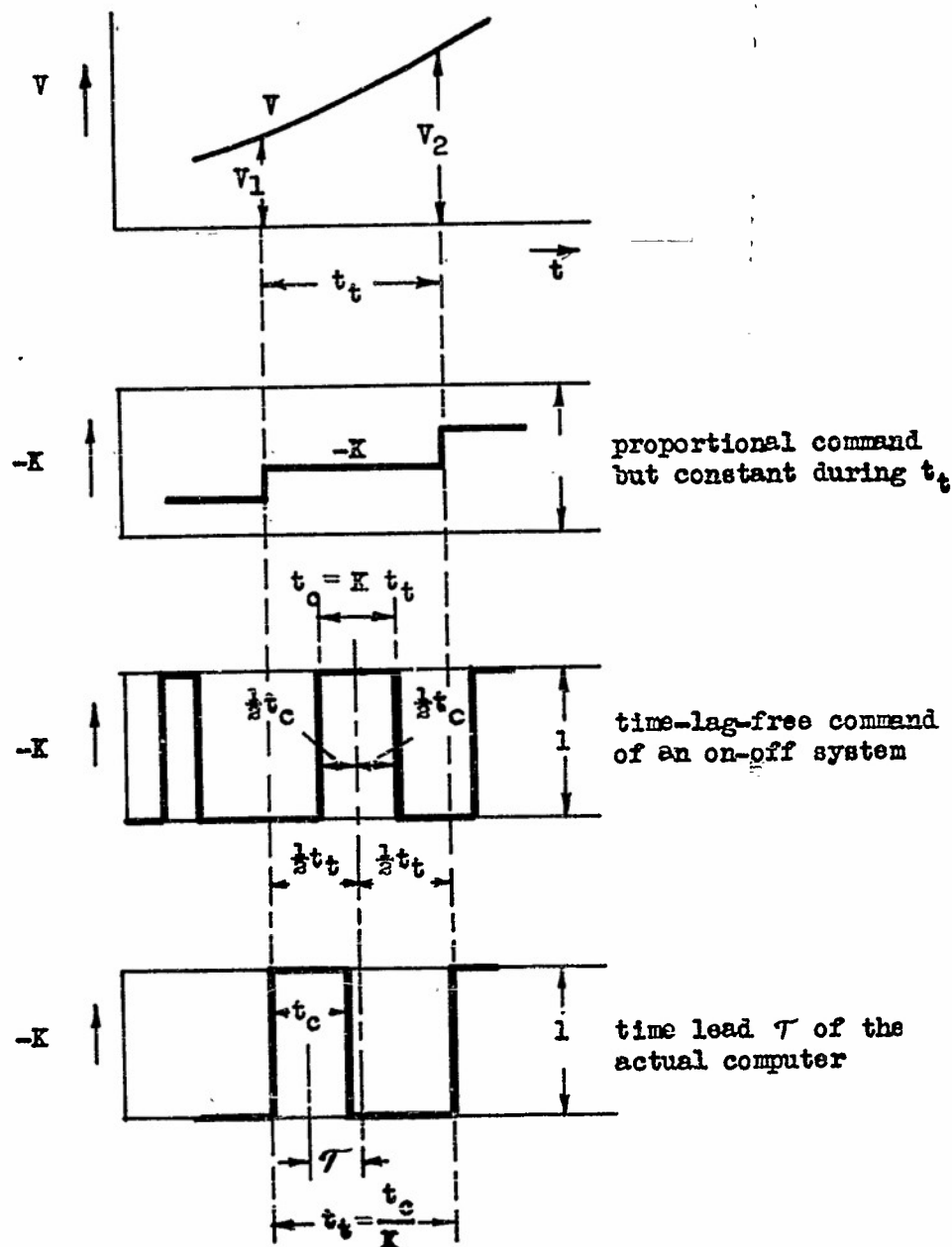


Fig. 12c. The command that corresponds to the average value of $V + TV$ during t_t is given by the computer at the beginning of t_t , and therefore, has a time lead τ .

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It is interesting to compare this value of τ with the time $\Delta t = \frac{E}{\dot{V}} \frac{t_c}{2T}$, which the missile needs to cross the part $\frac{t_c}{2T} E$ of

the gap g (See Figure 12b). The time lead of the computer for small command values is $\tau = \frac{t_c}{2|K|}$. The command that the computer gives in order to correct the error rate $\dot{V}$ is $K = \frac{T\dot{V}}{E}$; therefore, the time lead equals Δt . This is another indication that only the part $\bar{g}$ of the gap (See Figure 12b) is to be considered for missile motions outside the gap.

13. NUMBER OF COMMANDS (LOON)

The "command volume" k is the sum of the magnitudes of all course angle changes that are caused by radio commands during a flight. A change of missile course of $k_1 = 1.5^\circ = 0.0262$ is coordinated to one command; the number n of commands during a flight of the LOON is given approximately $n = k/k_1$

A. Launching Dispersion

The motion of the missile resulting from the computer action approximates a critically damped oscillation. By this assumption, the command volume k_L , which is necessary to correct the launching deviation depends upon the error y_L and the course angle κ_L of the missile after launching when guidance begins (See Figure 13a). A computation of k_L yields the following:

$$k_L = \kappa_L \left(1 + 2j e^{\frac{-(1+j)}{j}} \right) \quad \text{with } j = 1 + \frac{y_L \sqrt{s}}{\kappa_L v} \quad 13b$$

E.g., for $y_L = 0$, i.e., $j = 1$: $k_L = 1.27 \kappa_L$

for $\kappa_L = 0$, i.e., $j \rightarrow \infty$: $k_L = 0.74 \frac{\sqrt{s}}{v} y_L$

In Table 13a, numerical values are given for

$v = 115$ yds per second

$s = 0.0009$ per second²

$\kappa_0 = 0.008$ radians per second

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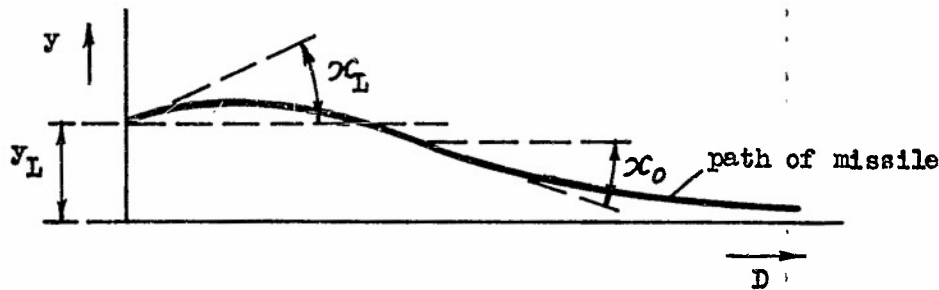


Fig. 13a. For a critically damped missile motion, the command volume k_L , which is necessary to correct the launching dispersion, equals $K_L + 2K_0$ where K_0 is the magnitude of the slope at the inflection point of the flight path. For $0 > \frac{K_L}{y_L} > -\frac{\sqrt{5}}{v}$, no inflection occurs; then $k_L = |K_L|$.

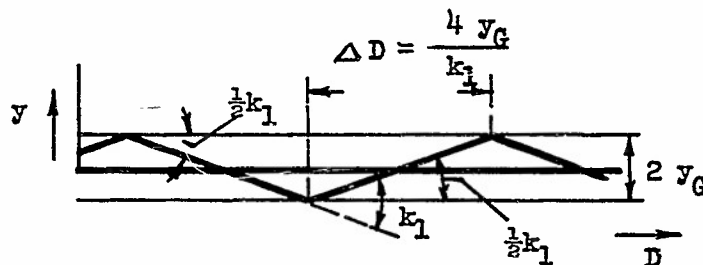


Fig. 13b. The average interval between two successive commands that is required to keep the missile within the gap is shortest for a course angle $1/2 k_1$: the missile travels a distance $\Delta D = 4y_E/k_1$ between two commands.

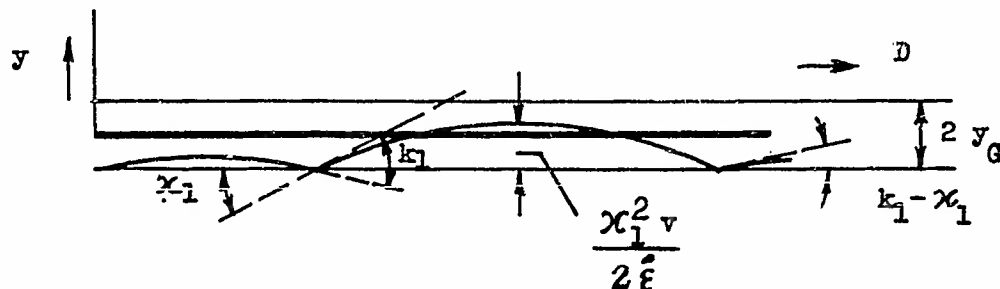


Fig. 13c. Agyrodrift causes a parabolic flight path. If $2y_G > \frac{vK_1^2}{2\epsilon}$ the missile is reflected from the same limit of the gap alternately with a course angle K_1 and $k_1 - K_1$; therefore, in two successive reflections, this course angle must reach a value between $1/2 k_1$ and k_1 .

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and for various probable launching deviations. The approximate period of time that the computer needs to correct this dispersion, (that is, until it gives the last corrective command $k_1=1.5^\circ$) and the distance the missile travels during this period, are also given in this table.

TABLE 13a

Launching Deviation (y_L in yds)	k_L	n_L	Last signal is transmitted after approximately	
			Seconds	Yards of Travel
$y_L=0, \kappa_L=5^\circ$	0.111	4.2	70	8,000
$y_L=1,000, \kappa_L=0^\circ$	0.194	7.4	120	14,000
$y_L=500, \kappa_L=2.5^\circ$	0.149	5.7	90	10,000

B. Finite Value of k_1

After the launching deviation has been corrected, the missile will be flying within the gap but not exactly parallel to the line of sight; its course angle error will be smaller than k_1 . When it arrives at a gap limit it will receive a corrective command k_1 . It will then cross the gap in the opposite direction. The largest number of commands to keep the missile within the gap occurs when it zigzags with a course angle error of $0.5 k_1$ (See Figure 13b). The number of commands, n_f , per distance of travel, is then given by:

$$\frac{dn_f}{dD} = \frac{k_1}{4y_G}$$

C. Unbalanced Gyro

An erroneous gyro precession rate, $\dot{\epsilon}$, imparts a lateral acceleration $\ddot{y}=v\dot{\epsilon}$ to the missile. If y is small, the missile will continue to zigzag between the two limits of the gap and the number of commands will not differ materially from Equation 13c; however, if the gap is large, the missile will always return to the same limit

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of the gap without reaching the other limit (See Figure 13c). Then the number of commands is given by

$$\frac{dn_e}{dD} = \frac{1}{v} \frac{dn_e}{dt} = \frac{\dot{\epsilon}}{vk_1} \quad * \quad 13d$$

Since the gap increases with the missile range (compare Figure 16b), a transition will take place from zigzagging to this latter performance.

In order that the missile return to the same gap limit (See Figure 13c):

$$2y_G > \frac{\kappa_1^2 v}{2\dot{\epsilon}}$$

In this, κ_1 is the course angle of the missile when it leaves the gap (See Figure 13c). For $\dot{\epsilon} = 0.5^\circ/\text{min}$, and $\dot{\epsilon} = 1^\circ/\text{min}$, the values of the gap y_G and the corresponding distance D at which the transition is to be expected are given in Table 13b.

TABLE 13b

		$\kappa_1/k_1 = 1/2 \quad 3/4 \quad 1$			unit
$\dot{\epsilon} = 0.5^\circ/\text{min}$	y_G	51	116	206	yards
	D (for $G=0.3$ volts)	16	74	147	1,000
	D (for $G=0.4$ volts)	5	48	106	1,000
$\dot{\epsilon} = 1^\circ/\text{min}$	y_G	26	58	103	
	D (for $G=0.3$ volts)	0	22	62	1,000
	D (for $G=0.4$ volts)	0	9	40	1,000

*This value is decreased slightly by the time lag T_M and slightly increased by the shift of the gap limits caused by the change of the condenser voltage that follows a command. An investigation of these influences showed that their sum is negligible.

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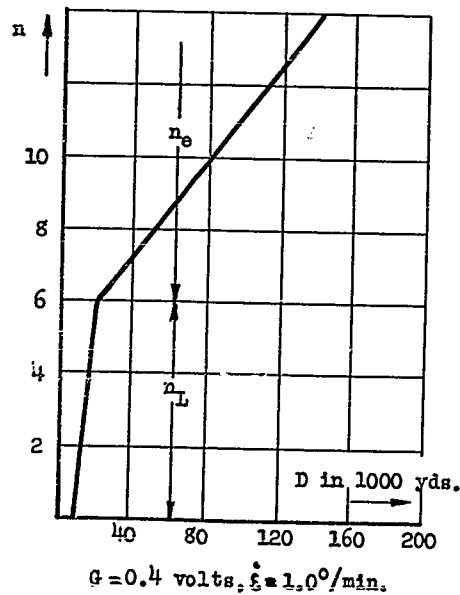
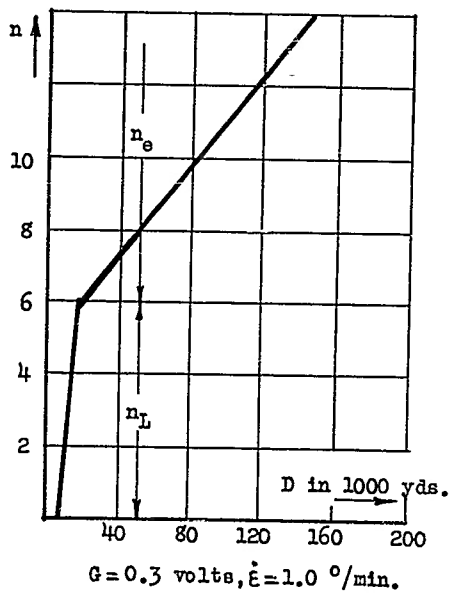
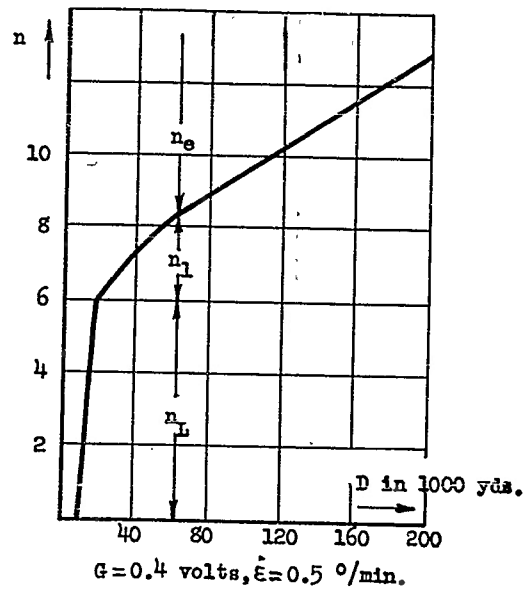
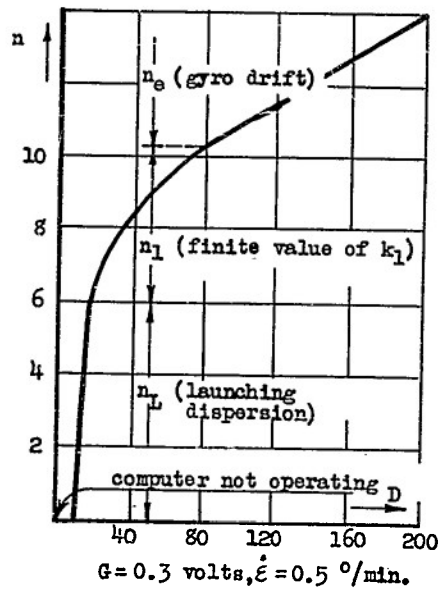


Fig. 13d. Number n of commands, during a LOON flight, for different settings of the gap voltage G and different gyro drifts $\dot{\epsilon}$. The cause for commands are indicated by the subscript of n .

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D. Total Number of Commands

The launching course angle error K_L (See Equation 13b) includes the effect of a lateral wind velocity, W , at the launching site. If W is changed later by ΔW , a command volume $\Delta W/v$ results. A lateral velocity of the target has approximately the same effect as a changing wind. These influences are neglected in Figure 13d.

Figure 13d shows the number of commands transmitted to the missile during flight and the causes for these commands. Six commands are assumed to be necessary during the first 10,000 yards of missile guidance in order to correct the launching dispersion. When the effect of ϵ is considered, $K_1/k_1 = 3/4$ is assumed (See Table 13b). Effects of radar hunting are neglected throughout.

If k_1 is increased, less commands are necessary to correct the launching dispersion and an erroneous gyro precession rate; in this, however, n_f would increase. The chosen value of 1.5 degrees for k_1 gives approximately the least number of commands, particularly when the additional effect of radar hunting on gap effectiveness is considered (See Section 10, A).

Table 13c shows the command volumes that are necessary to correct some typical causes of error for line-of-sight control and also for proportional navigation (Navigation ratio $N=4$). A constant missile speed, $V = 175$ yards per second, is assumed during the entire flight. The distance from the launcher to the target is assumed as $D = 150,000$ yards. Furthermore, it is assumed that common motion exists between launcher and guidance site. Table 13c shows that, for LOON guidance, the total command volumes of the two guidance methods differ but slightly. The least number of commands would be achieved if a trajectory selector of the type shown in Figure 8c were added to the computer.

14. CIRCUIT INACCURACIES

The interaction of the different computer components causes deviations from the operation, as described by Equation 7b. These influences have been investigated with the results given in Equation B1: They are discussed in the following:

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TABLE 13c. COMMAND VOLUME NEEDED TO CORRECT SOME TYPICAL DISTURBANCES

The numerical values are: $v = 175$ yds/sec, $D = 150,000$ yds, $s = 0.00091/\text{sec}^2$

Disturbance			Command Volume			
	Symbol	Value	For Line-of-sight Navig.		For Prop. Navigation (N=4)	
			Formula	Value	Formula	Value
Launching error	y_L	500 yds	$\frac{0.75 \sqrt{s}}{v} y_L$	0.097	$\frac{N}{N-1} y_L/D$	0.004
Launching course angle error	κ_L	3°	$1.27 \kappa_L$	0.066	$\frac{N}{N-1} \kappa_L$	0.069
Lateral wind at launcher	W	10 mph	$1.27 W/v$ *	0.041	$\frac{N}{N-1} W/v$ *	0.043
Change of lateral wind during flight	ΔW	10 mph	$\Delta W/v$	0.032	$\frac{N}{N-1} \Delta W/v$	0.043
Lateral launching site velocity	W_L	15 mph	$2W_L/v$	0.096	$\frac{N}{N-1} W_L/v$ *	0.064
Lateral target constant during flight	$\dot{Y}$	20 mph	$2\dot{Y}/v$	0.128	$\frac{N}{N-1} \dot{Y}/v$ *	0.085
Lateral target assumed when missile is at ΔD from target	$\dot{Y}$ ΔD	20 mph 1/2 D	$(1+\Delta D/D) \dot{Y}/v$	0.096	$\frac{N}{N-1} \dot{Y}/v$	0.085
Erroneous gyro precession rate	$\dot{\epsilon}$	1°/min	$\dot{\epsilon}/v$	0.250	$\frac{N}{N-1} \dot{\epsilon}/v$	0.333

*Command volume may be avoided by suitable launching direction.

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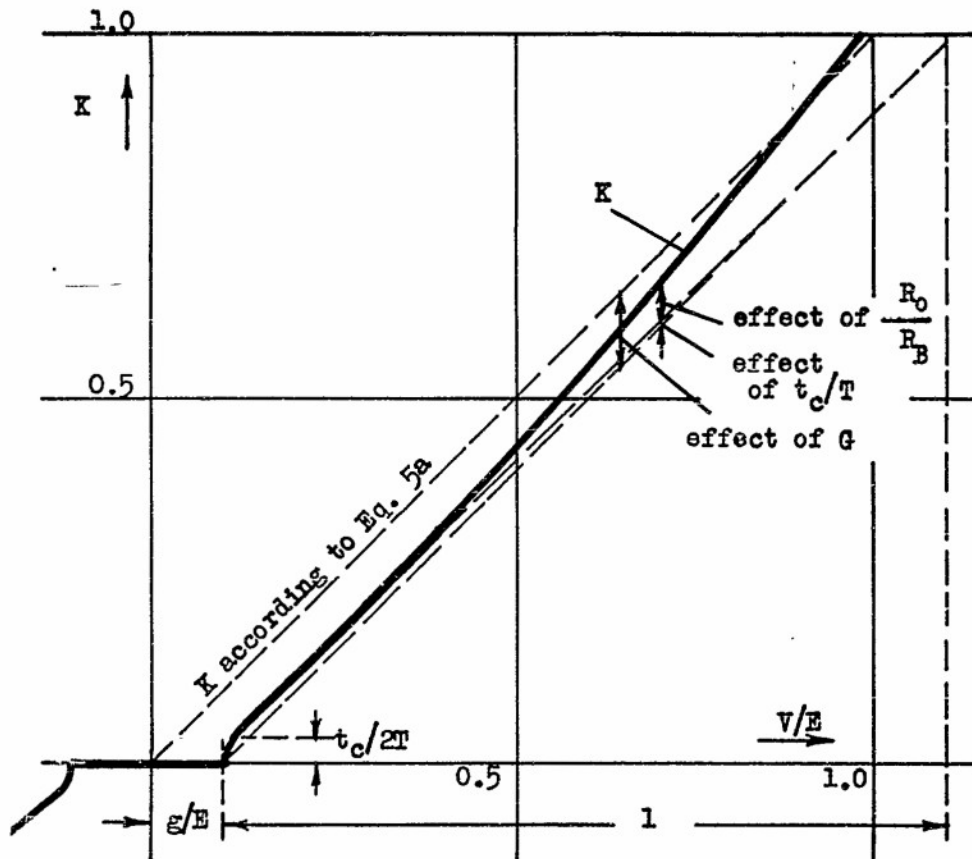


Fig. 14a. Effect of the gap G , of the finite computing time t_c and of R_0/R_B on the command value of the LOON computer for $\dot{V}=0$ and at $D=100,000$ yards.

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The fact that the operation of the computing relay maintains the voltage V_r between the computing and the input circuit approximately at zero eliminates any undesirable interference between these two circuits.

The finite value of the ratio R_o/R_B causes a smaller charging than discharging time constant of the computing circuit. This results in a nonlinear relationship between error voltage and commands (See Equation B1). This effect, which is of minor importance, is shown in Figure 14a.

The finite ratio R_o/R_c is of some interest in the LOON computer where R_c is being increased with increasing range between missile and launcher; hence, the computer time constant increases with range. This is a desirable effect that partly counteracts the destabilizing effect of the increasing time lags with range of the radar system.

results

Sections 15 and 16 give the data originally chosen of the computers. These data are changed occasionally to correspond with changes of missile properties or test conditions.

15. LARK COMPUTER

Figure 15a is a schematic of the LARK computer and gives the symbols and data for its components.

The angular range of the radar potentiometer is ± 10 degrees. A voltage that is increased proportionally to the missile range is applied to the ends of the potentiometer; thus, V/y is maintained constant ($V/y = 0.016$ volts per yard). The full command error is $y_E = 400$ yards. The computing time constant $T = 7$ seconds. The gap is 5 per cent of y_E , that is, 20 yards. The guidance stiffness is chosen as $s = 0.05$ second⁻².

16. LOON COMPUTER

Design Data

Figure 16a is a schematic of the LOON computer giving symbols and data for its components. The electrical components that have been chosen for the final design will be described in a

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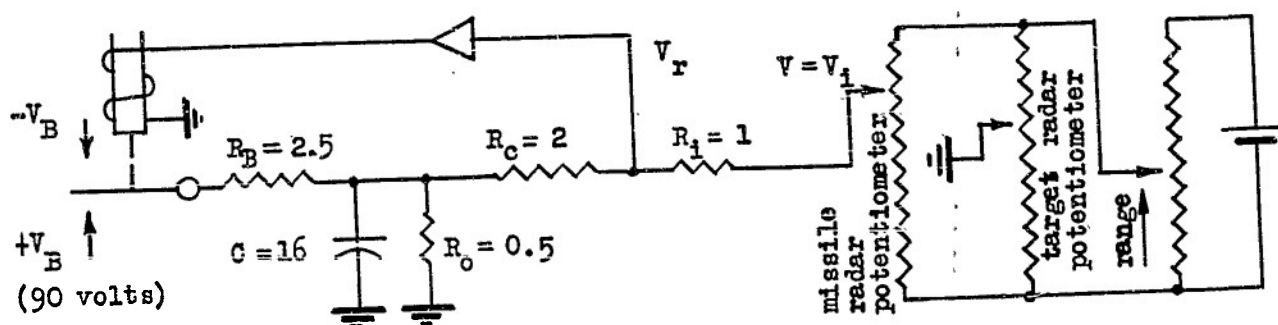


Fig. 15a. Schematic of the LARK computer and data for its components. Resistances are given in megohms, and capacitances in microfarads. The range is introduced by varying the voltage applied across the two radar potentiometers.

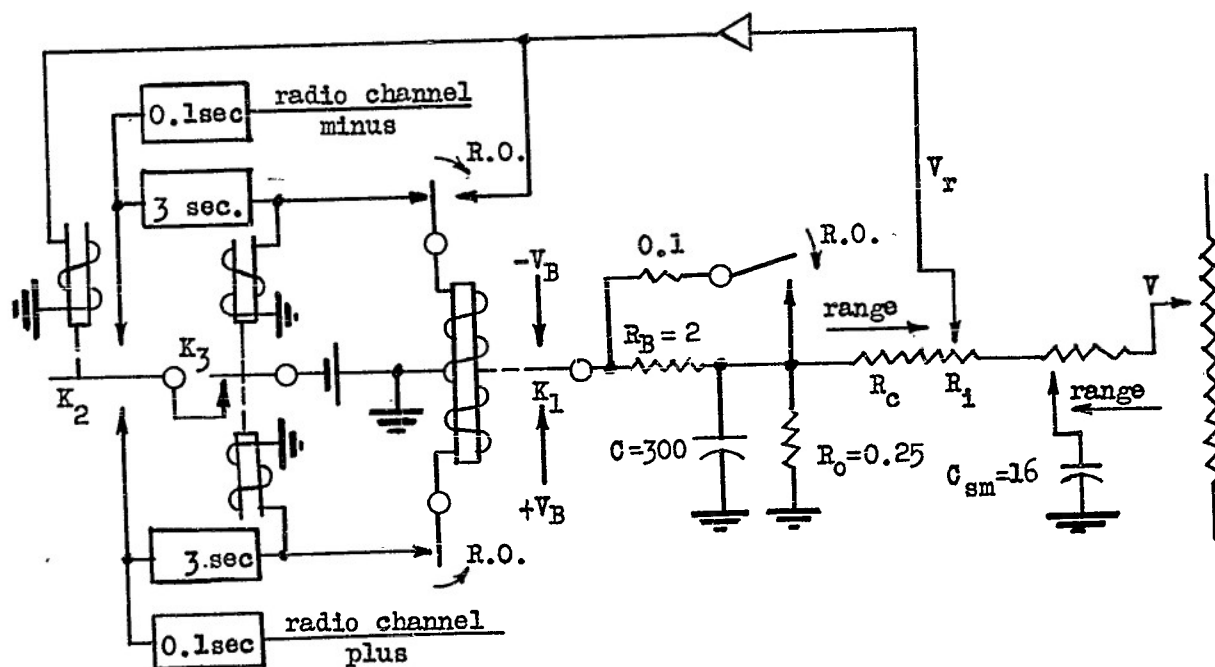


Fig. 16a. Schematic of the LOON computer and data for its components. Resistances are given in megohms, and capacitances in microfarads. The range is introduced by varying R_C/R_1 . The timing circuits achieve a distinct computing time ($t_c = 3$ seconds) and a short radio transmission time. The radar-off switches (R.O.) are used for continuous self-adjustment of the condenser voltage V_c to the error voltage V and for preventing radio transmission as long as the radar has lost track of the missile.

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separate report.

The angular range of the radar potentiometer is ± 27 degrees. The voltage across the potentiometer is 270 volts; therefore:

$$E_0 = 287 \text{ volts per radian}$$

The battery voltage, which is derived actually from a voltage regulator is:

$$V_B = 180 \text{ volts}$$

The relay gap $G = 0.3$ volts and the relay hysteresis $\gamma = 0.03$ volts.

The missile range varies from $D = 10$ to $D = 200$ (in 1,000 yards). The ratio R_i/R_c is varied, according to:

$$R_i/R_c = 32.5 \cdot \frac{1 + (D/200)^2}{D}$$

between 3.26 and 0.325. The smoothing resistance R_{sm} increases linearly with range:

$$R_{sm} = 0.125 \left(1 + \frac{D}{100} \right) \text{ megohms}$$

Timing Circuit

When the voltage V_r exceeds the gap, one of the contacts of the relay K_2 closes. This causes the following operations:

One of the contacts of the transmitter relay closes for the duration of 0.1 second and a command is transmitted to the missile on this channel, effecting a course change of 1.5 degrees.

One of the contacts, K_1 of the computing relay, closes for $t_c = 3$ seconds. The contact K_3 opens for 3 seconds, thus disconnecting the computing and timing circuits from V_r ; when K_1 opens and K_3 closes again, the computer is ready for a new cycle.

Radar off Switch (RO)

If, by some means outside of the computer, the radar information is found to be erroneous, the switches RO in Figure 16a are switched in the direction indicated by the arrows. This stops any further command transmission, the timing circuit is by-passed, and the computing relay K_1 is operated directly from V_r ; furthermore, the resistor R_p of the computing circuit is by-passed.

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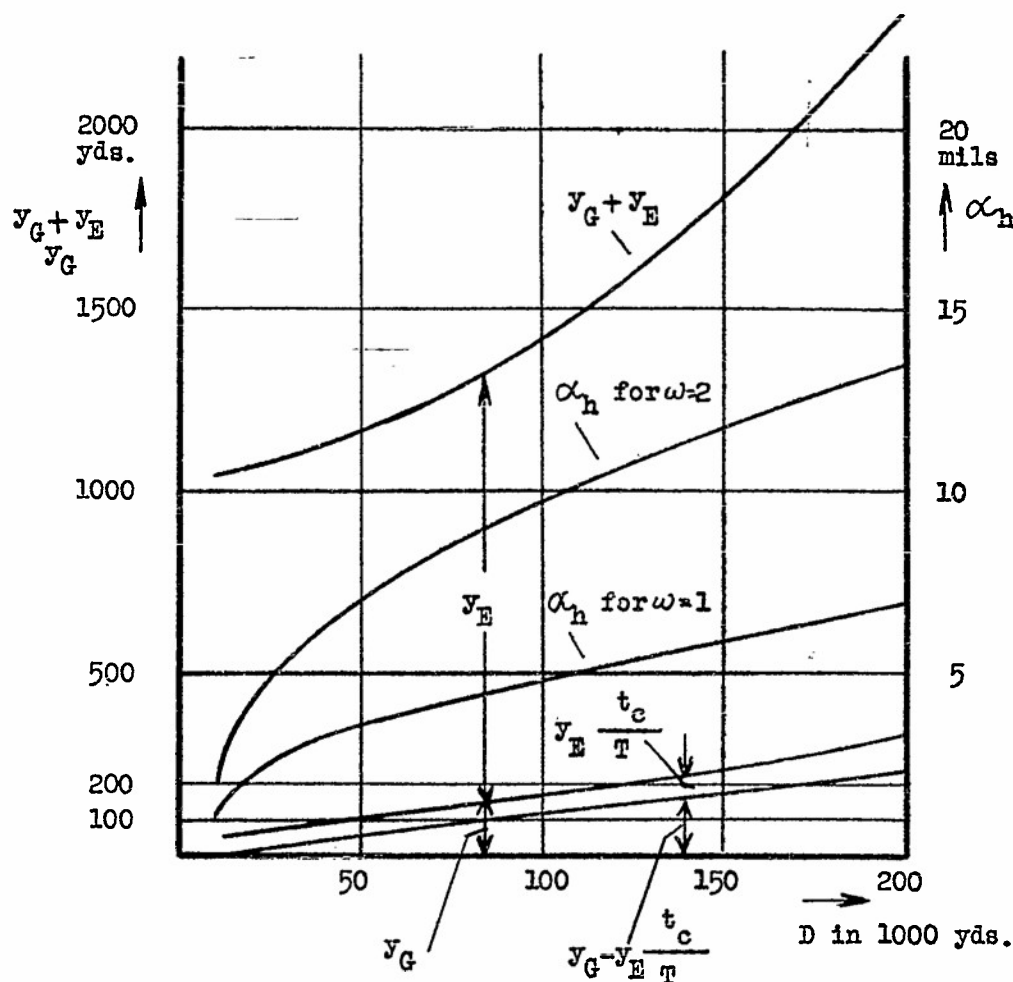


Fig. 16b. Performance data of the LOON computer. Error gap y_G and full command error, $y_G + y_E$, vs missile range D . The permitted average hunting angle amplitude, $\alpha_h = (y_G - y_E \frac{t_c}{2T}) \sqrt{1 + \omega^2 T_i^2} / D$ is given for $\omega=1$, i.e., for a hunting frequency of 0.16 cps, and for $\omega=2=0.32$ cps. This figure is plotted for $G=0.3$ volts.

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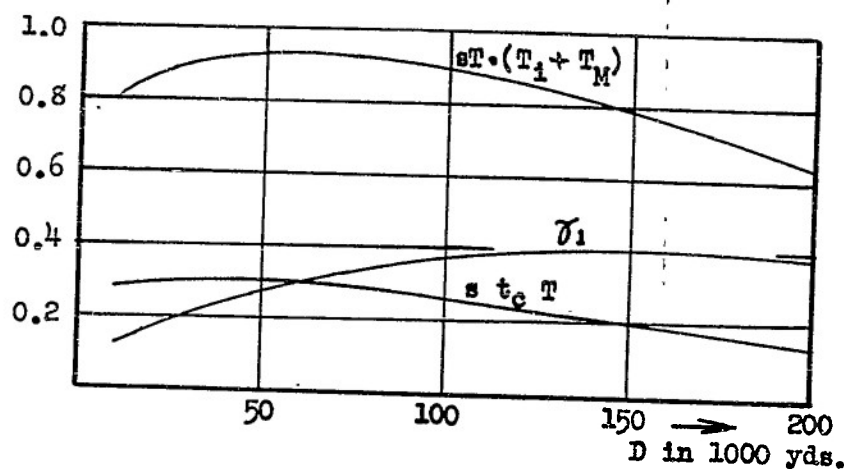


Fig. 16c. LOON computer data. The three constants which, according to Equations 11b, 11c, and 11d, must be smaller than one, are plotted vs missile range D .

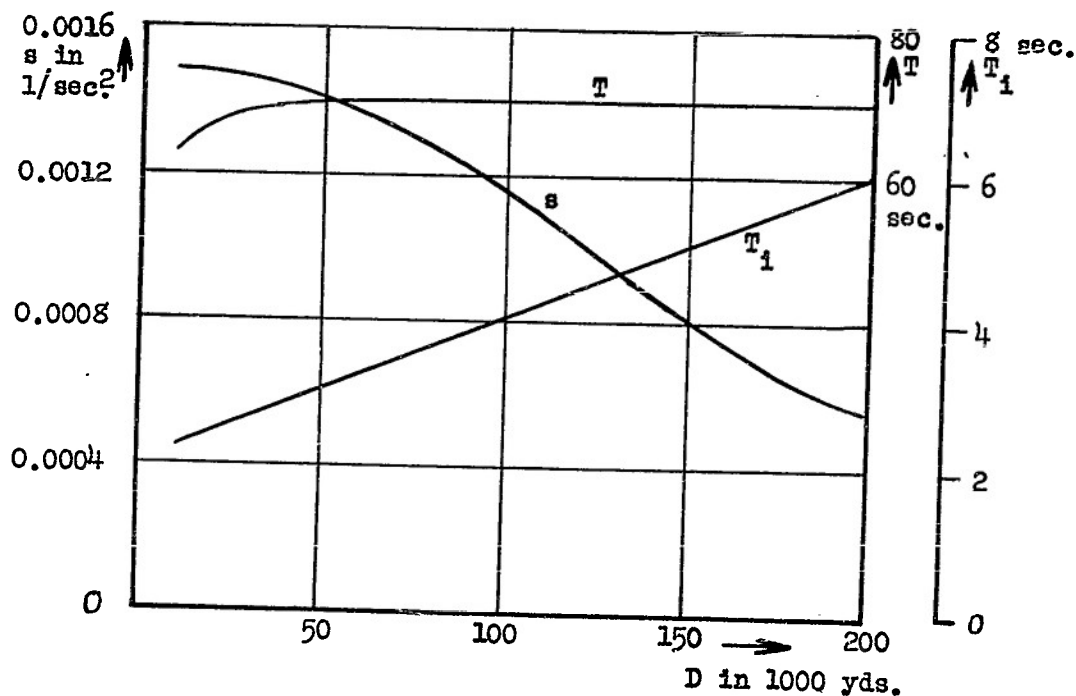


Fig. 16d. LOON computer constants T and T_1 vs missile range D . The stiffness constant s is given for $v=175$ yards per second and $K_0=0.008$ radians per second.

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thus diminishing the charging time constant. As a result, the condenser voltage V_c remains adjusted continuously to the error voltage so that the computer starts with the correct command value when the radar is again working properly.

Operational Data

In Figure 16b to 16d, the operational data are summarized. Figure 16b shows the "permitted" average hunting angle amplitude α_h , for frequencies ω of 1 and 2 radians per second. Should higher amplitudes occur for such relatively fast hunting oscillations, the computer, because of the computing time $t_c=3$ seconds would create a rather high command noise; however, larger amplitudes are not expected at these or at higher frequencies. At lower frequencies, the computer will develop the smoothing properties discussed in Section 10. Figure 16b shows also that the part

$$y_E \frac{t_c}{2T}$$

of the gap y_G , which, in regard to hunting, is lost because of the long computing time of 3 seconds, is only of consequence at close ranges where the hunting motions will remain within the permitted range.

Figure 16c shows the three constants that must be smaller than 1 (See Equations 11b, 11c, and 11d) for the computer to give only one command when one only is needed to correct the course of the missile. The constant $sT(T_i+T_M)$ is very close to unity. If difficulties arise, the guidance stiffness s would have to be reduced; however, radar hunting will assist in avoiding difficulties. The constant γ is appreciably smaller than unity; this is necessary, however, because changes of the relay hysteresis are to be expected.

Figure 16d shows the computer time constant T , the smoothing time constant T_i , and the guidance stiffness constant s .

The characteristics of the computer can be changed easily by varying G and/or R_i/R_c with range.

17. SUMMARY OF THEORETICAL RESULTS

The LARK and the LOON computers are designed to guide the missile on the line of sight between the ground radar station and the target or on another line, if desired. The deviation of the

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missile from this line is the "error" that the radar output voltage indicates as the "error voltage."

In consideration of the radar noise and the requirement for a smooth flight path of the missile, second derivatives (e.g., the rate of the error rate) were not used in either computer. Furthermore, radar noise, slow response of the missile, and short time of flight prevented the use of an integrating circuit in the LARK computer; therefore, only the error and its rate are formed in the computers. It is intended, however, to incorporate an integrating circuit in the LOON computer at a later date.

$\bar{V}$ is a dimensionless value for the sum of error and error rate. This sum is derived and transformed into intermittent on-off command signals by means of a simple circuit (See Section 6). The average value of this command K equals $-\bar{V}$ as long as $|\bar{V}| < 1$. In the LARK guidance, the signals follow each other at such short intervals that the missile control system can act as an averaging circuit. Attempts have been made to reduce the number of commands (and the radar transmission time for each command) in the LOON computer to a minimum.

Continuous full command, that is, $K=1$, corresponds to the maximum missile acceleration. As a rule, radar hunting and a large launching deviation may lead to $|\bar{V}| > 1$. When this occurs, the computer must lose its linear properties or co-ordination of the computer to the motion of the missile would be lost. The designed computers are examples of a device adjusting itself automatically to such conditions. They have the following properties:

A "gap", or insensitive range, is provided, the center of which follows the average error voltage. As long as the difference between the instantaneous error voltage and the average value is smaller than a certain value g , no command is given whatever the error rate might be, that is, differentiation takes place only when the difference exceeds this gap. Thus, the usual hunting amplitudes, being smaller than the gap, do not lead to superfluous commands. Hunting amplitudes that are somewhat larger than the gap lead to relatively short commands; these are averaged by the missile control pilot without much missile motion. The sacrifice for these advantages is an insensitivity of the computer to missile errors that are smaller than the gap minus the average hunting amplitude; therefore, the gap should be widened with increasing distance between missile and radar station. This is done in the LOON computer but not in the LARK because of the latter's use exclusively at close ranges.

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Hunting motions having appreciably larger amplitudes than the gap, and for which $|\bar{V}| > 1$, cause the command K to fluctuate with the hunting frequency between $+1$ and -1 . Thus, as $|\bar{V}|$ is reduced to 1, the computer acts as a smoothing device, causing an amplitude ratio $\frac{1}{|\bar{V}|}$. In a proportional system, the sacrifice for smoothing is a time lag. In the computers under discussion, a higher price is paid for smoothing. For hunting amplitudes of medium magnitude, the sum of missile error and error rate is converted by the computer into a command with a time lag almost twice as large as that of a proportional system with the same amplitude ratio for the same frequency band. At larger hunting amplitudes, the time lag approximately equals that of a proportional system and the average command value is increasingly reduced. Variations of the hunting amplitude cause additional noise of an amplitude that may reach the average command value. Furthermore, higher harmonics of the radar hunting motion also introduce noise into the command which, because of its higher frequency, is of minor importance. The usual proportional systems do not have these unfavorable properties. The advantage of the nonlinear computer system over such a proportional system is that it does not introduce any time lag as long as hunting does not occur and that, with increasing or varying hunting amplitude, it always introduces automatically the time lag that is necessary to smooth the command to a value with which the missile can still comply. The question whether the price that is paid for this automatic performance is justified will be answered only after more general investigation of such automatic systems; an attempt to introduce a conventional smoothing circuit into the LOON computer, with additional means for increasing automatically the smoothing effect with the hunting amplitude, shows that this cannot be done without causing undesirable effects. As a rule, a combination of conventional smoothing and amplitude dependent smoothing as used in the LOON computer, should be considered.

If a launching deviation is so great that the error itself leads to full command, the computer continues to give full command as long as this condition exists, even after $|\bar{V}|$ (or error plus error rate) has dropped below 1. This undesirable effect may be avoided by adding a "trajectory selector" to the computer (See Appendix E).

The best protection against enemy interference is offered by a command system in which a small number of very short commands are transmitted and in which the number of different types of commands are kept to a minimum. To reduce the number of commands and their transmission time in the LOON computer, a change of missile course of a certain magnitude k_1 was co-ordinated to each single

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command. Thus, for small command values K , the intervals between the commands exceed the time lags in the guidance loop. The investigation shows that the computer delivers the command with a time lead of approximately one-half the command interval. At first it seems that the total sacrifice for a small number of commands is very low. To avoid commands being created by a certain average radar hunting amplitude, the gap is to be widened somewhat because of the finite command value. If hunting does not occur, the missile must reach the limit of this widened gap before a corrective command is given; however, the flight path of the missile outside of the gap is then approximately the same as though the gap has not been widened. Apparently the only important deficiency of such a system is that each radar hunting motion, even if of very short duration but exceeding the gap, causes a change of missile course of the chosen magnitude k_1 . If, during the computing time t_c for this command, a reverse hunting motion occurs, the computer disregards this motion, thereby falsifying the average command value. Thus, for hunting motions that are fast compared to the computing time, the LOON computer does not develop the smoothing effects of the LARK computer. In any event, hunting motions larger than the gap would be inconsistent with a small number of commands. A smoothing circuit, therefore, is provided at the input of the LOON computer. This circuit is expected to reduce the fast hunting amplitudes of the error voltage to a magnitude smaller than the gap.

If the value of the one-command-course-angle-change k_1 is chosen large, then only a small number of commands are necessary to correct disturbances, but a large number are necessary to keep the missile zigzagging within the gap should no disturbance exist. A medium value of k_1 ($k_1=1.5$ degrees) gives the minimum number of commands. Approximately 15 commands are to be expected during the flight of a LOON.

conclusions

- (a) The limitation of the maximum command to a value that corresponds to the greatest missile acceleration appears to be a favorable means for smoothing the radar noise. The time lag connected with this smoothing effect, though comparable to that of conventional smoothing networks, adjusts itself automatically to the noise level.
- (b) An insensitive range of the error detector (a "gap"),

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which follows the error, assists in avoiding command noise and reducing the number of commands.

- (c) It appears to be possible to guide a missile with a rather small number of commands. A long interval between commands does not cause a time lag in the guidance loop.

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appendices

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appendix a

THE COMPUTING CIRCUIT: SIMPLIFIED CASE

Assumption: the resistor R_B of the computing circuit Figure 6b is very large compared to R_0 . The charging time constant is then approximately equal to the discharging time constant T . Let the condenser voltage, at a certain instant be V_c ; its rate depends upon the relay operation (See Figure 6c);

$$\left. \begin{array}{ll} \text{When the plus} & \\ \text{contact is closed,} & \left(\frac{dV_c}{dt}\right)_+ = \frac{1}{T}(-V_c + \rho E) \\ \\ \text{When both con-} & \\ \text{tacts are open,} & \left(\frac{dV_c}{dt}\right)_d = \frac{1}{T}(-V_c) \\ \\ \text{When the minus} & \\ \text{contact is closed,} & \left(\frac{dV_c}{dt}\right)_- = \frac{1}{T}(-V_c - \rho E) \end{array} \right\} \quad A1$$

In this, ρE according to 6c is the saturating condenser voltage if the plus contact remains permanently closed.

If, within the period of time t_t , which is infinitely short compared to T , the relay arm is at plus during t_+ , in middle position during t_d , at minus during t_- , then the condenser voltage changes during $t_t = t_+ + t_d + t_-$ by:

$$\Delta V_c = \left(\frac{dV_c}{dt}\right)_+ t_+ + \left(\frac{dV_c}{dt}\right)_d t_d + \left(\frac{dV_c}{dt}\right)_- t_- \quad A2$$

Let the average rate of change of V_c be designated by $\dot{V}_c$:

$$\frac{\Delta V_c}{t_t} = \dot{V}_c$$

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All of these equations finally result in:

$$K = \frac{t_+ - t_-}{t_t} = \frac{1}{\rho E} (V_c + T \dot{V}_c)$$

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appendix b

CIRCUIT INVESTIGATION

The network of the combined computing and input circuit, Figure 6a, 15a, and 16a, has been investigated with greater accuracy than in Appendix A, taking into account:

(a) That the voltage V_r is not maintained accurately at zero but at $\pm G$ and that, therefore, the error voltage V across the resistors R_i and R_c influences directly the condenser voltage V_c .

(b) That the charging and discharging time constants of the condenser of the computing circuit, because of the finite ratio of R_o/R_B , are different.

The result of this investigation for $t_c/T \rightarrow 0$, but without any other neglect, is as follows:

$$K = -\rho \frac{R_B}{V_B} \frac{V(1/R_o + 1/R_c) + \dot{V}C - g[1/R_o + 1/(R_c + R_i)]}{1 - \rho(\dot{V} - g)/V_B}$$

and after some minor neglects:

$$K = -\frac{\bar{V} \operatorname{sign}(V + T\dot{V})}{1 - \frac{V}{E} \frac{R_o}{R_B} \operatorname{sign}(V + TV)}$$

$$\bar{V} = \left| \frac{V + T\dot{V}}{E} \right| - \frac{g}{E}$$

$$T = \frac{C}{1/R_o + 1/R_c}$$

$$E = \frac{V_B R_i}{R_B(1 + R_c/R_o)}$$

B1

This equation B1 gives the correct signs for all values of V and $\dot{V}$; however, should $\bar{V}$ be negative, then $K=0$.

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Equation B1 shows that the computer operation is independent of R_i (with the exception that R_i/R_c determines the relative influence of error voltage and battery voltage and the effective relay gap). In other words; by the relay operation, the voltage V_r between the computing and the input circuit is maintained close enough to zero for separating these two circuits.

Let the voltage that is applied to the radar potentiometer per unit of azimuth angle be E_o ; therefore, $V = E_o y/D$. Equate the denominator in the upper equation B1 to unity and, neglecting some other inaccuracies, derive:

$$K = -\frac{1}{y_E} (|y + Ty| - y_G) \text{ sign } (y + Ty)$$

with

B2

$$y_E = \frac{E_1}{E_o} \frac{R_i}{R_c} D ; E_1 = \frac{R_o}{R_B} \frac{V_B}{1 + R_o/R_c} ; y_G = \frac{G}{E_o} D + \frac{G}{E_1} y_E$$

In the LOON computer, the times t_c and t_d cannot be considered as being infinitely short when compared with T . Instead of introducing the derivatives in Equation A2, the e-power functions, representing the solutions of Equation A1, have been developed into MacLaurin's series and only terms that are small, of the third order of t_c/T , have been neglected. The following equation results for $T_d = T_c = T$:

$$K = -\left[\frac{t_c}{2T} + \left(1 - \frac{t_c}{2T}\right) \frac{|V_{av} + TV_{av}| - g}{E}\right] \text{ sign } (V_{av} + TV_{av})$$

B3

V_{av} and $\dot{V}_{av}$ are defined by Equations 12a and 12b. If $|V_{av} + TV_{av}| - g < 0$ then $K = 0$. This equation, without change of its inaccuracy, may be transformed into

$$K = -\frac{1}{E} (|V_{av} + TV_{av}| - g + \frac{\Delta}{\rho}) \text{ sign } (V_{av} + TV_{av})$$

B4

with

$$\Delta = \frac{t_c}{2} \left| \left(\frac{dV}{dt} \right)_c + \rho \dot{V}_{av} \right|$$

B5

The first term in Equation B5 is the rate of change of the condenser voltage during t_c . The second term is minus the average rate of change of the condenser voltage during t_t (See Figure 12a); therefore, the curve raised by Δ over the curve $\rho(V-g)$ follows the average value of the condenser voltage. Equation B4

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shows that the command corresponds to this average condenser voltage and its average rate.

Equation B3 shows that, if a command is created at all, it has at least the finite value Δ (See also Figure 14a). A more accurate investigation of small command values, with use of the complete e-power function, shows that there is no step in the command value when the gap is exceeded: the step, however, approximates well the actual performance (See Figure 14a).

The Equations B1 and B3 may be combined into

$$K = - \left[\frac{t_c}{2T} + \left(1 - \frac{t_c}{2T} \right) \frac{\bar{V}}{1 - \frac{V}{E} R_o/R_B \operatorname{sign}(V + T\dot{V})} \right] \operatorname{sign}(V + T\dot{V}) \quad B6$$

with E and T according to Equation B1 (See Figure 14a).

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appendix c

FAST HUNTING AND LIMITATION OF THE COMMAND VALUE

In the following investigations of large hunting amplitudes, the relay gap is neglected and it is assumed that the time

$$t_h = \frac{2\pi}{\omega} \quad (1)$$

of a sinusoidal hunting oscillation is short compared to the computer time constant T , but long compared with t_c .

If an input smoothing circuit is not used,

$$V_i = V = V_y + H \sin \omega t$$

H is the amplitude of hunting. V_y is the part of the input voltage that corresponds to the missile error y . The arm of the computing relay switches when $V + V_c/\rho$ crosses through zero, that is, (See Figure 10b) when

$$V_y + H \sin \omega t + V_c/\rho = 0 \quad (2)$$

This equation may be solved with respect to t and with $t_t = t_h = \frac{2\pi}{\omega}$ gives the command value K_h in case of hunting:

$$K_h = \frac{t_+ - t_-}{t_t} = -\frac{2}{\pi} \sin^{-1} \frac{V_y + V_c/\rho}{H}, \text{ with } |\sin^{-1}| < \frac{\pi}{2} \quad (3)$$

The two unknowns V_c/ρ and K_h can be computed from the two Equations C2 and 6d (with K_h instead of K in Equation 6d).

For steady-state conditions, or for the condition that V_y and H remain constant and that $V_c = 0$, the result of this computation is plotted in Figure 10c and Figure 10d. If the arc is substituted for the sine in Equation C2, that is, if the equation

$$K_h = -\frac{V_y + V_c/\rho}{Eh} \quad (4)$$

with

$$h = \frac{\pi}{2} \frac{H}{E} \quad (5)$$

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is used instead of Equation C2, the equation

$$-V_c/\rho = -EK_h = \frac{V_y}{1+h} \quad C5$$

is derived with Equation 6d. Figures 10c and 10d show that Equation C3 gives quite accurate results as long as $y/y_E \leq 0.5$.

The approximation Equation C3 is now used for investigating the effects of varying values of the missile error voltage V_y and of the hunting amplitude H . Equations C3 and 6d yield:

$$(1+h)K_h + T \frac{d(hK_h)}{dt} = - \frac{V_y + TV_y}{E} \quad C6$$

and

$$(1+T_h \frac{d}{dt})V_c = -\rho \frac{V_y}{1+h} \quad C7$$

with

$$T_h = T \frac{h}{1+h} \quad C8$$

In these equations, h is to be considered as a given function of time. These equations have not been generally solved.

For constant hunting amplitude H , the Equations C7 and C8 remain formally unchanged but $h = \text{const}$. The Equation C6 yields:

$$(1+T_h \frac{d}{dt}) K_h = - \frac{1}{E} \frac{V_y + TV_y}{1+h} \quad C9$$

These equations may be used to investigate a step function of H . Assume, for example, a constant missile error V_y . As long as hunting does not occur, $K = V_y/E$ and $V_y + V_c/\rho = 0$. Should hunting with a constant amplitude then begin, the condenser voltage does not change immediately; therefore, $V_y + V_c/\rho$ is still zero and Equations C2 or C3 yield $K_h = 0$, or, the command value drops immediately to zero (See Figure 10f). From then on, Equation C7 holds, that is, the condenser voltage reaches the value $-V_c/\rho = \frac{V_y}{1+h}$ with the time constant T_h and the command

value simultaneously recovers up to K_h as given by Equation B5. Thus, steady-state hunting with amplitude H causes a continuous decrement of command value (See Equation C5) and an additional loss of command volume (See Figure 10e):

$$\Delta K = \int_0^{\infty} \Delta K_h dt = \frac{V_y T}{E} \frac{h}{(1+h)^2}$$

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immediately after $t = 0$, or, after hunting has begun. When hunting ceases, the command value returns to $-\frac{V_y}{E}$ after a gain of command volume:

$$\Delta K \approx -\frac{V_y T}{E} \frac{h}{1+h}$$

Less pronounced changes occur when the hunting amplitude increases or decreases (See Figure 10f):

$$\Delta K = \frac{V_y T}{E} \frac{h_2 - h_1}{(1+h_1)(1+h_2)^2} \quad C10$$

The derivation of Equation C3 has been repeated, with the assumption of an infinitely small ratio t_h/T dropped. If t_h becomes comparable to T , the change of the condenser voltage $-V_c/\rho$ during a half-cycle of the hunting oscillation becomes comparable to the hunting amplitude; however, the assumption that in computing the zeros of Equation C1, the sine can be replaced by the arc, was maintained. Therefore, the following Equation C11 holds only as long as the amplitude of V_c/ρ is equal or smaller than approximately $1/2H$, or as long as $t_h/T < \frac{2H}{E}$.

Furthermore, it was assumed that the e-power functions that describe the change of the condenser voltage between two zeros of Equation C1, are still approximated well enough by the MacLaurin series if terms of the order of $(t_c/T)^3$ or smaller, are neglected. Therefore, with $t_c \approx 1/2 t_h$, Equation C11 is only an acceptable approximation as long as $t_h/T < 1$. The computation results in

$$K_h = -\frac{V_y + V_c/\rho}{Eh} \cdot \frac{1 + \frac{V_y + V_c/\rho}{H\omega}}{1 - \frac{V_y + V_c/\rho}{H\omega}} \quad C11$$

Equation C11 holds simultaneously with Equation 6a with K_h instead of K . In Equation C11, V_c is the average value of the condenser voltage, $\dot{V}_c$ is the rate of the so-defined value V_c . $\dot{V}_c/\rho$ is of the order of $\dot{V}_y$ or smaller. Thus, Equation C3 holds as long as the error rate $\dot{V}_y$ is small compared with $H\omega$ (the rate of the hunting motion) and as long as $t_h/T < 1$,

$$t_h/T < \frac{2H}{E}, \quad y/y_E = V/E < \frac{1}{2}.$$

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appendix d

EFFECT OF MISSILE VELOCITY ON GUIDANCE CONSTANTS

Comparison of Equations 5a and 6d yields:

$$-\frac{V_c}{\rho E} = \frac{y}{y_E} \quad D1$$

Hence, if y_E is variable but ρE is constant (See Equation 6c):

$$\frac{\dot{V}_c}{V_c} = \frac{\dot{y}}{y} - \frac{\dot{y}_E}{y_E} \quad D2$$

Equations D1 and D2, introduced into 6d yield:

$$K = -\frac{1}{y_{Ee}} (y + T_e \dot{y}) \quad D3$$

with

$$y_{Ee} = \frac{y_E}{1 - T \dot{y}_E / y_E}; \quad T_e = \frac{T}{1 - T \dot{y}_E / y_E} \quad D4$$

If a certain variation of y_E with D is chosen,

$$\dot{y}_E = \frac{dy_E}{dD} v$$

may be introduced into Equation D4.

The Equations D3 and D4 show that the "effective" full-command-error y_{Ee} , and the "effective" computer time constant T_e , are influenced by the speed of the missile.

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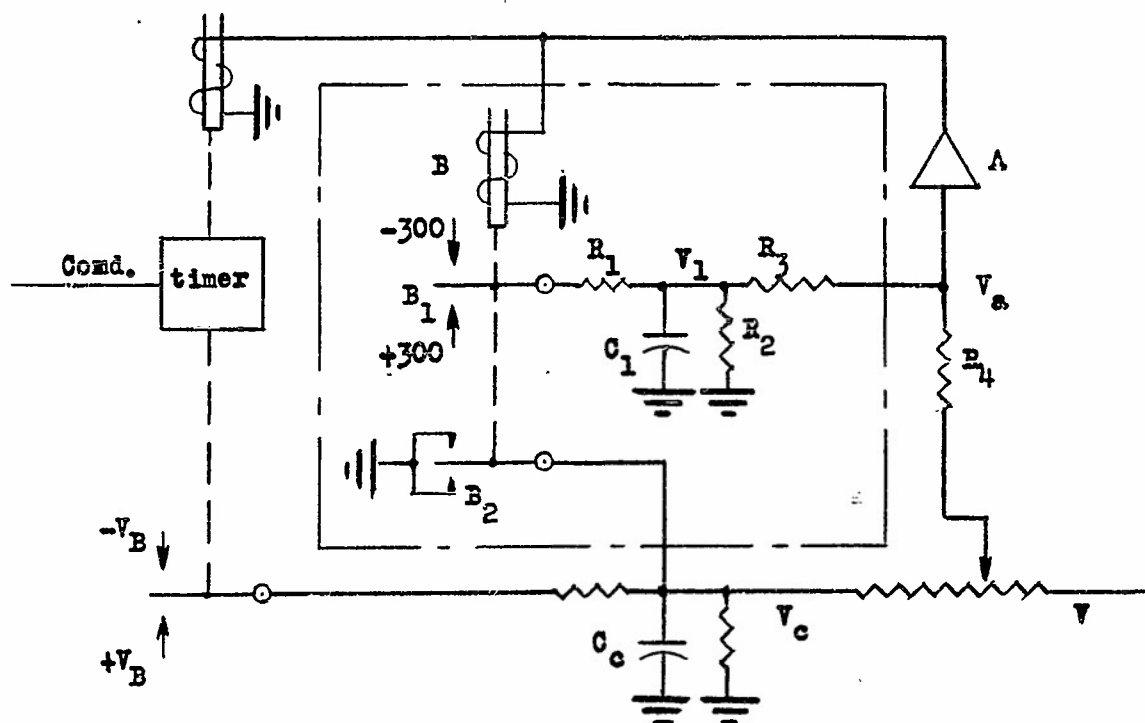


Fig. E1. The original computer is shown outside, and the TS inside, of the dash-dotted rectangle. This type of TS yields the family of trajectories shown in Figure 8b. $R_1=1$, $R_2=4$, $R_3=15$, $R_4=5$, $C_1=20$.

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appendix e

OPERATION OF THE TRAJECTORY SELECTOR (TS)

The TS (see Figure E1) is operated by the output voltage of the error detector amplifier A of the computer. The output of the TS is fed to the input of A (or any other point of the network suitable for changing the error reference).

As long as proportional control exists, the computer maintains the magnitude $|V_a|$ of the voltage at the input of A approximately at the gap limit G . The TS is insensitive to such small voltages. Only in event that the voltage V_c of the computing circuit condenser is unable to follow the error, does $|V_a|$ become appreciably larger than G . The TS must then operate, and, therefore, $G + \Delta G$ is chosen as the insensitivity of the relay B at the input of the TS wherein ΔG is so large that radar hunting does not cause operation of the TS, for example $\Delta G = 1.5G$. The relay B is a polarized, double-pole, double-throw relay with middle position (or any substitute relay set). If $|V_a| > G + \Delta G$, the pair of contacts B_2 causes the computing condenser C_c to discharge immediately. The pair of contacts B_1 causes charging of the TS condenser C_1 to such a voltage V_1 that $|V_a|$ is reduced to $G + \Delta G$. Then the relay B falls back to middle position. In short, during nonproportional control, the TS alone assumes the task of maintaining $|V_a|$ close to $G + \Delta G$ and, therefore, during this operation, full command is continuously given.

Figure E2 shows these cycles of TS operation in greater detail. (All voltage ratios are equated to unity.) It shows particularly the change of the condenser voltage V_c when the contacts of B are open and that the contact of B remains open after the rate of the difference of the error voltage V and the condenser voltage $-V_1$ has become smaller than the rate of change of the condenser voltage $-V_c$, that is, after the difference of the course angle of the missile and the new trajectory has become smaller than y_E/vT . The new trajectory is then definitely established and full command maintained only until the full missile acceleration has reduced the difference between missile error and condenser voltage V_c from $G + \Delta G$ to G . During this time, the flight path of the missile will have become almost parallel to the new trajectory. The missile starts to follow the new trajectory with an initial error that is well within the proportional range of the computer.

The type of the trajectory is determined by the means used for achieving the now following decay of the effect of V_1 . If

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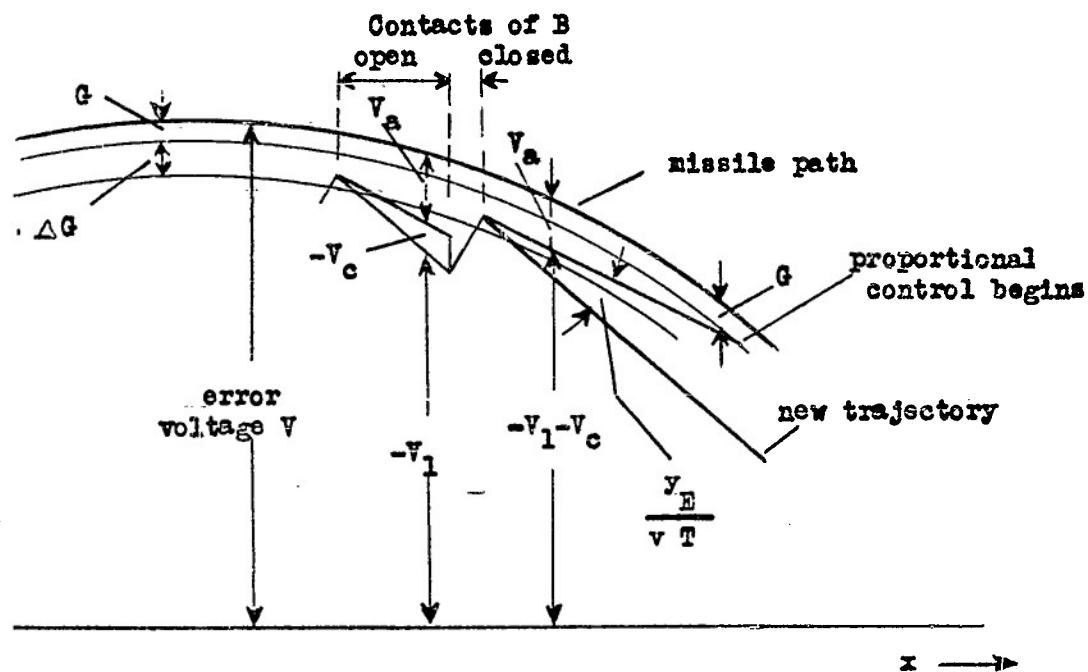


Fig. E2. Operation of the TS in detail. The operation of the relay B maintains $-V_1 - V_c$ close to $V - (G + \Delta G)$. The operation of the relay B ceases when the rate of V no longer exceeds the rate $-V_1 - V_c$.

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only the condenser C_1 is used (See Figure E1), its discharge must be fast enough that, when the missile reaches the target, the trajectory does not differ noticeably from the line of sight, and that the curvature of the trajectory is small*.

Figure E3 shows a simple design of the TS for a straight approach to the target. In the LOON computer, a voltage proportional to the missile error does not exist. The effect of a missile error on the voltage V_a decreases with missile range; therefore, the output of the TS must not only decrease proportionally to the difference $x_T - r_M$ of target range x_T and missile range r_M (this is achieved by the variable resistor R_3), but faster. This is achieved by discharging the condenser C_1 with a time constant of 240 seconds. This decay of the voltage V_1 also assists in attaining an approach of the trajectory to the line of sight when the launching angle has been large and when, therefore, the missile range r_M is almost constant at the transition from the full command circle to the new trajectory (See Figure 8c). The shunt resistor R_4 assists in attaining a small curvature of the trajectory at target cross-over for all possible target ranges.

*If z designates the deviation of the trajectory from the line of sight, the miss y is given approximately by the following equation:

$$y = z - \frac{d^2 z}{dt^2} \frac{1}{g}$$

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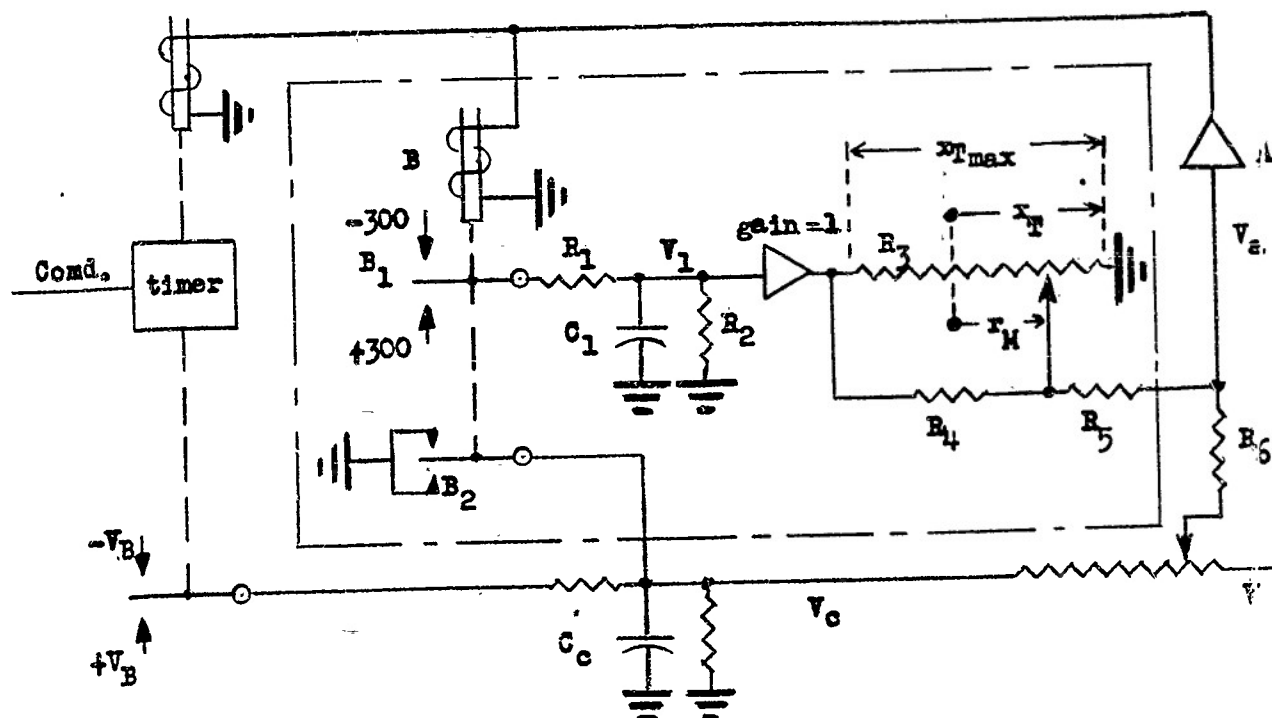


Fig. E3. The design of the TS that yields the trajectories in Figure 8c is shown inside the dash-dotted rectangle. $R_1=1$, $R_2=12$, $R_3=0.1$, $R_4=0.05$, $R_5=5$, $R_6=5$, $C_1=20$.