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CHARTS OF PRESSURE, DENSITY, AND TEMPERATURE CHANGES

AT AN ABRUPT INCREASE IN CROSS-SECTIONAL AREA

OF FLOW OF COMPRESSIBLE AIR

By Upshur T. Joyner

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## NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

## ADVANCE RESTRICTED REPORT

CHARTS OF PRESSURE, DENSITY, AND TEMPERATURE CHANGES  
AT AN ABRUPT INCREASE IN CROSS-SECTIONAL AREA  
OF FLOW OF COMPRESSIBLE AIR

By Upshur T. Joyner

## SUMMARY

Equations have been derived for the change in the quantities that define the thermodynamic state of air - pressure, density, and temperature - at an abrupt increase in cross-sectional area of flow of compressible air. Results calculated from these equations are given in a table and are plotted as curves showing the variation of the calculated quantities with the area expansion ratio in terms of the initial Mach number as parameter. Only the subsonic region of flow is considered.

## INTRODUCTION

The well-known Borda-Carnot expression for change in pressure when an incompressible fluid passes an abrupt area expansion has long been used for estimating the pressure changes in compressible air flow at an abrupt expansion. This method is simple but not exact.

The expressions for pressure, density, and temperature ratios given herein are for subsonic flow and are in precise agreement with the exact expression for the velocity ratio in a compressible flow at an abrupt area expansion developed in reference 1. In the present report, Mach number, or the ratio of air-flow velocity to existing sound velocity, is used as a parameter; whereas in reference 1 the parameter was the ratio of existing air-flow velocity to the velocity that the air would possess if accelerated isentropically until its velocity was equal to the then existing local

sound velocity. This difference in parameters must be considered when equations from the two papers are compared.

By use of the same three fundamental equations used herein, a somewhat similar equation showing pressure and density changes across a shock loss in a pipe of uniform cross section was developed by Hugoniot and is given in reference 2.

The expressions obtained herein for pressure, density, and temperature changes in a compressible flow at an abrupt expansion are too involved to be of practical use. The present computations have therefore been made and are presented in tabular and graphical form.

#### SYMBOLS

A	cross-sectional area of flow, square feet
a	velocity of sound, feet per second
f	area ratio ( $A_1/A_2$ )
M	Mach number ( $V/a$ )
p	static pressure, pounds per square foot
V	velocity of flow, feet per second
$\gamma$	ratio of specific heat at constant pressure to specific heat at constant volume, dimensionless
R	universal gas constant, Btu per slug per $^{\circ}F$
T	absolute temperature, $460 + ^{\circ}F$
$\rho$	density of air, slug per cubic foot
Subscripts:	
1	before abrupt expansion
2	after abrupt expansion

## ANALYSIS

From the fundamental equations for the conservation of energy, of continuity, and for the conservation of momentum, equations are obtained that give the variation of the pressure, density, and temperature ratios with the area expansion ratio  $f$  in terms of the initial Mach number as parameter.

Figure 1 shows the conditions of flow assumed for the present calculations. The static pressure at  $a$  (fig. 1) is taken to be the same as at  $b$  for subsonic flow, as has been proved experimentally by Nusselt (reference 3). Uniform velocity distribution before and after the expansion is assumed. The ratio of specific heats  $\gamma$  is taken as 1.405.

The fundamental equations are the equation for the conservation of energy

$$\frac{V_1^2}{2} + \frac{\gamma}{\gamma - 1} \frac{p_1}{\rho_1} = \frac{V_2^2}{2} + \frac{\gamma}{\gamma - 1} \frac{p_2}{\rho_2} \quad (1)$$

the equation of continuity,

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad (2)$$

and the equation for the conservation of momentum

$$\rho_2 A_2 V_2^2 - \rho_1 A_1 V_1^2 = -A_2 (p_2 - p_1) \quad (3)$$

From equation (1)

$$\frac{\gamma - 1}{2} V_1^2 + a_1^2 = \frac{\gamma - 1}{2} \left( \frac{V_2}{V_1} \right)^2 V_1^2 + \frac{\gamma p_2}{\rho_2}$$

or

$$\frac{\gamma - 1}{2} M_1^2 + 1 = \frac{\gamma - 1}{2} \left( \frac{V_2}{V_1} \right)^2 M_1^2 + \frac{p_2}{p_1} \frac{\rho_1}{\rho_2} \quad (4)$$

From equation (3)

$$\begin{aligned} \frac{p_2}{p_1} &= 1 + \gamma f M_1^2 \left( 1 - \frac{V_2}{V_1} \right) \\ &= 1 + \gamma f M_1^2 \left( 1 - \frac{\rho_1 f}{\rho_2} \right) \end{aligned} \quad (5)$$

From equation (2)

$$\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2} \frac{A_1}{A_2} = \frac{\rho_1}{\rho_2} f \quad (6)$$

When equations (5) and (6) are substituted in equation (4),

$$\frac{\gamma - 1}{2} M_1^2 + 1 = \frac{\gamma - 1}{2} \left( \frac{\rho_1}{\rho_2} \right)^2 f^2 M_1^2 + \frac{\rho_1}{\rho_2} + \frac{\rho_1}{\rho_2} \gamma f M_1^2 - \gamma f^2 M_1^2 \left( \frac{\rho_1}{\rho_2} \right)^2$$

or

$$\left( \frac{\rho_1}{\rho_2} \right)^2 f^2 M_1^2 \frac{\gamma + 1}{2} - \frac{\rho_1}{\rho_2} \left( 1 + \gamma f M_1^2 \right) + \frac{\gamma M_1^2 - M_1^2 + 2}{2} = 0 \quad (7)$$

When equation (7) is solved for  $\rho_1/\rho_2$  and the resulting equation is inverted,

$$\frac{\rho_2}{\rho_1} = \frac{f^2 M_1^2 (\gamma + 1)}{1 + \gamma f M_1^2 - \sqrt{2\gamma f M_1^2 (1 - f) + 1 - 2f^2 M_1^2 + f^2 M_1^4}} \quad (8)$$

In order to obtain an expression for the pressure ratio, equation (8) is substituted in equation (5) and the following equation results:

$$\frac{p_2}{p_1} = \frac{1 + \gamma f M_1^2 + \gamma \sqrt{2\gamma f M_1^2 (1 - f) + 1 - 2f^2 M_1^2 + f^2 M_1^4}}{\gamma + 1} \quad (9)$$

By differentiating  $p_2/p_1$  with respect to  $f$ , it can be shown that the maximum static-pressure recovery for any value of  $M_1$  is obtained when

$$f = \frac{\gamma + \sqrt{\frac{\gamma - 1}{2} M_1^2 + 1}}{2(\gamma + 1) - M_1^2} \quad (10)$$

The locus of maximum pressure ratio is shown in figure 2.

In equations (8), (9), and (10), the sign of the radical has been chosen so that the results obtained are in the region where the assumptions are valid.

The temperature ratio is obtained by use of the general gas law and the computed values of pressure and density ratio as

$$\begin{aligned} \frac{p_1}{\rho_1} &= RT_1 \\ \frac{p_2}{\rho_2} &= RT_2 \\ \frac{T_2}{T_1} &= \frac{p_2/p_1}{\rho_2/\rho_1} \end{aligned} \quad (11)$$

Figures 3 and 4 show the variation of density ratio and temperature ratio with area expansion ratio.

In order to make the results shown in figures 2 to 4 usable in cases for which only the conditions after the expansion are known, the value of  $M_2$  in terms of  $M_1$  and  $f$  is given in figure 5. If  $M_2$  and  $f$  are known,  $M_1$  can be determined from this figure. The relation plotted in figure 5 is developed as follows:

$$\begin{aligned}
 \left(\frac{M_2}{M_1}\right)^2 &= \frac{(v_2/a_2)^2}{(v_1/a_1)^2} \\
 &= \frac{v_2^2}{\frac{\gamma p_2 / \rho_2}{v_1^2}} \\
 &= \left(\frac{v_2}{v_1}\right)^2 \frac{p_1}{p_2} \frac{\rho_2}{\rho_1}
 \end{aligned}$$

By use of equation (6),

$$\left(\frac{M_2}{M_1}\right)^2 = \frac{f^2}{\frac{p_2}{p_1} \frac{\rho_2}{\rho_1}}$$

or

$$M_2 = \frac{f M_1}{\sqrt{\frac{p_2}{p_1} \frac{\rho_2}{\rho_1}}}$$

## RESULTS AND DISCUSSION

The calculated values of pressure ratio, density ratio, and temperature ratio are given in table I. The values have been computed to 8 decimal places because of the form of the equations, in which small differences in large quantities are involved. In the region where  $M$  and  $f$  were both small, that is, 0.1 or 0.2, it was necessary to carry some of the calculations to 12 decimal places in order to obtain smooth curves for the quantities calculated.

As in the case of incompressible flow, the calculated changes occur gradually after the abrupt increase in cross-sectional area of flow, and the calculated and measured results are in best agreement at a distance the order of 6 to 10 diameters of the large cross section downstream from the abrupt area increase.

The comparison of pressure ratios for compressible and incompressible flow is shown in figure 2, in which a long-dash line gives the pressure ratio calculated on the basis of incompressible flow for the same initial conditions that are assumed for compressible flow at an initial Mach number of unity. It is evident that the effect of compressibility is vanishingly small for values of the area ratio of expansion below about 0.25. The short-dash line in figure 2 shows the pressure ratio to be obtained with isentropic expansion and an initial Mach number of unity.

The experimental points from reference 3 shown in figure 2 were obtained from the only experiments known to the author in which pressure ratio has been measured at an abrupt expansion with compressible gas flow at high Mach number. These data were obtained for an area expansion ratio of 0.246, however, for which the difference between compressible and incompressible flow is insignificant. These experimental results agree well with the calculated results but are by no means conclusive. Agreement of experimental with calculated values at an area expansion ratio of 0.7 or 0.8 would be conclusive evidence of the difference in pressure ratio obtained with compressible flow from that calculated by the Borda-Carnot formula for incompressible flow.

An experimental investigation of the changes in pressure, density, and temperature at an abrupt increase in cross-sectional area with compressible flow would serve to determine corrections for the effect of nonuniform velocity distribution and friction on the idealized results obtained from the present calculations.

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TABLE I.- PRESSURE RATIO, DENSITY RATIO, AND ABSOLUTE-TEMPERATURE RATIO FOR VARIOUS VALUES OF INITIAL MACH NUMBER AND AREA EXPANSION RATIO

$M_1$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$P_2/P_1$											
0	1	1	1	1	1	1	1	1	1	1	1
.1	1	1.00126439	1.00224817	1.00295190	1.00337575	1.00351948	1.00338250	1.00296373	1.00226171	1.00127458	1
.2	1	1.00505636	1.00505636	1.01182432	1.01354770	1.01416152	1.01365613	1.01201487	1.00921419	1.00522267	1
.3	1	1.01137213	1.02024561	1.02666591	1.03064746	1.03217434	1.03119965	1.02761199	1.02138014	1.01224395	1
.4	1	1.02020584	1.03601048	1.04755507	1.05490899	1.05798385	1.05664372	1.05068603	1.03970071	1.02310190	1
.5	1	1.03154961	1.05624622	1.07459475	1.08654401	1.09206623	1.09086329	1.08237032	1.06564042	1.03910182	1
.6	1	1.04543323	1.08113557	1.10790631	1.12595163	1.13514433	1.13495441	1.12431869	1.10132226	1.06240899	1
.7	1	1.06172422	1.11052805	1.14762338	1.17347305	1.18792807	1.19018519	1.17966963	1.14964704	1.09656476	1
.8	1	1.08053144	1.14449750	1.19389263	1.22948329	1.25117950	1.25791464	1.24727908	1.21428610	1.14715606	1
.9	1	1.10180063	1.18306256	1.24682673	1.29434745	1.32563900	1.33948264	1.33289501	1.29937517	1.22208512	1
1.0	1	1.12551775	1.22624052	1.30657959	1.36840187	1.41197583	1.43608912	1.43748166	1.40880987	1.33036914	1
$\rho_2/\rho_1$											
0	1	1	1	1	1	1	1	1	1	1	1
.1	1	.99797909	.99926105	1.00030258	1.00110672	1.00167088	1.00199105	1.00208043	1.00192483	1.00152701	1.00088642
.2	1	.99196508	.99708680	1.00120830	1.00443235	1.00667901	1.00800080	1.00838024	1.00779062	1.00621577	1.00363511
.3	1	.98210121	.99342431	1.00269844	1.00988628	1.01511640	1.01809428	1.01906528	1.01786526	1.01439136	1.00850658
.4	1	.96861682	.98849859	1.00474651	1.01745422	1.02733396	1.03278162	1.03440503	1.03260480	1.02663573	1.01601861
.5	1	.95181440	.98241031	1.00731349	1.02703842	1.04158974	1.05090698	1.05474573	1.05264609	1.04383560	1.02704072
.6	1	.93205331	.97505678	1.01036557	1.03851628	1.05970572	1.07383357	1.08081923	1.07878775	1.06723726	1.04299667
.7	1	.90973186	.96681714	1.01384469	1.05175342	1.08090755	1.10118915	1.11197350	1.11192689	1.09846363	1.06616762
.8	1	.88526912	.95776126	1.01769829	1.06670364	1.10503550	1.13294741	1.14952359	1.15293896	1.13941346	1.10003332
.9	1	.85908808	.94802960	1.02187692	1.08284570	1.13187728	1.16897634	1.19324636	1.20249440	1.19193899	1.14918560
1.0	1	.83160083	.93777649	1.02630629	1.10032668	1.16116816	1.20902157	1.24302399	1.26085565	1.25728804	1.21829825
$T_2/T_1$											
0	1	1	1	1	1	1	1	1	1	1	1
.1	1	1.00202500	1.00200483	1.00194500	1.00184313	1.00170202	1.00152539	1.00129936	1.00103690	1.00073357	1.00038781
.2	1	1.00810000	1.00799284	1.00777701	1.00735934	1.00682312	1.00611182	1.00523204	1.00419159	1.00297989	1.00158180
.3	1	1.01822500	1.01806662	1.01749995	1.01661536	1.01529978	1.01382982	1.01190735	1.00960513	1.00688963	1.00370584
.4	1	1.03240000	1.03207618	1.03111627	1.02958448	1.02784134	1.02440228	1.02149901	1.01751031	1.01272607	1.00697161
.5	1	1.05062500	1.05001912	1.04857745	1.04630432	1.04315929	1.03916545	1.03424291	1.02823763	1.02088913	1.01171354
.6	1	1.07290000	1.07217677	1.07004395	1.06681650	1.06251349	1.05709521	1.05008717	1.04220565	1.03193760	1.01861206
.7	1	1.09922500	1.09816336	1.09536308	1.09115250	1.08563683	1.07878841	1.07033592	1.06092373	1.04659544	1.02851065
.8	1	1.12960000	1.12818453	1.12459410	1.11923554	1.11261881	1.10435797	1.09429215	1.08182576	1.06571156	1.04283755
.9	1	1.16402500	1.16220066	1.15773488	1.15143526	1.14354044	1.13401695	1.12255329	1.10844175	1.09013563	1.06343580
1.0	1	1.20250000	1.20019830	1.19480951	1.18744697	1.17847001	1.16786653	1.15531890	1.14008424	1.12051481	1.09198969

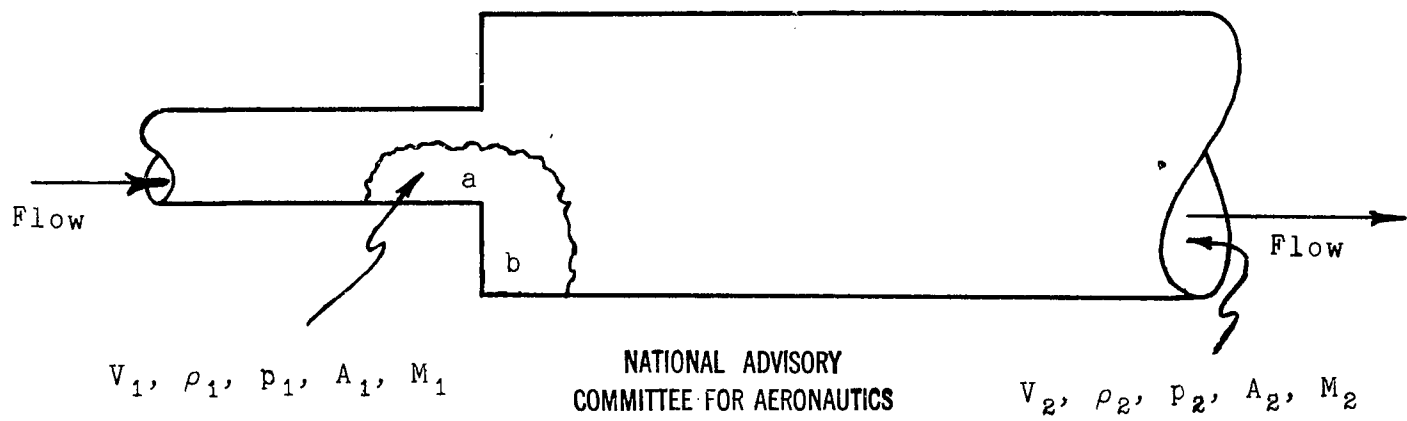


Figure 1.- Flow conditions assumed for calculations.

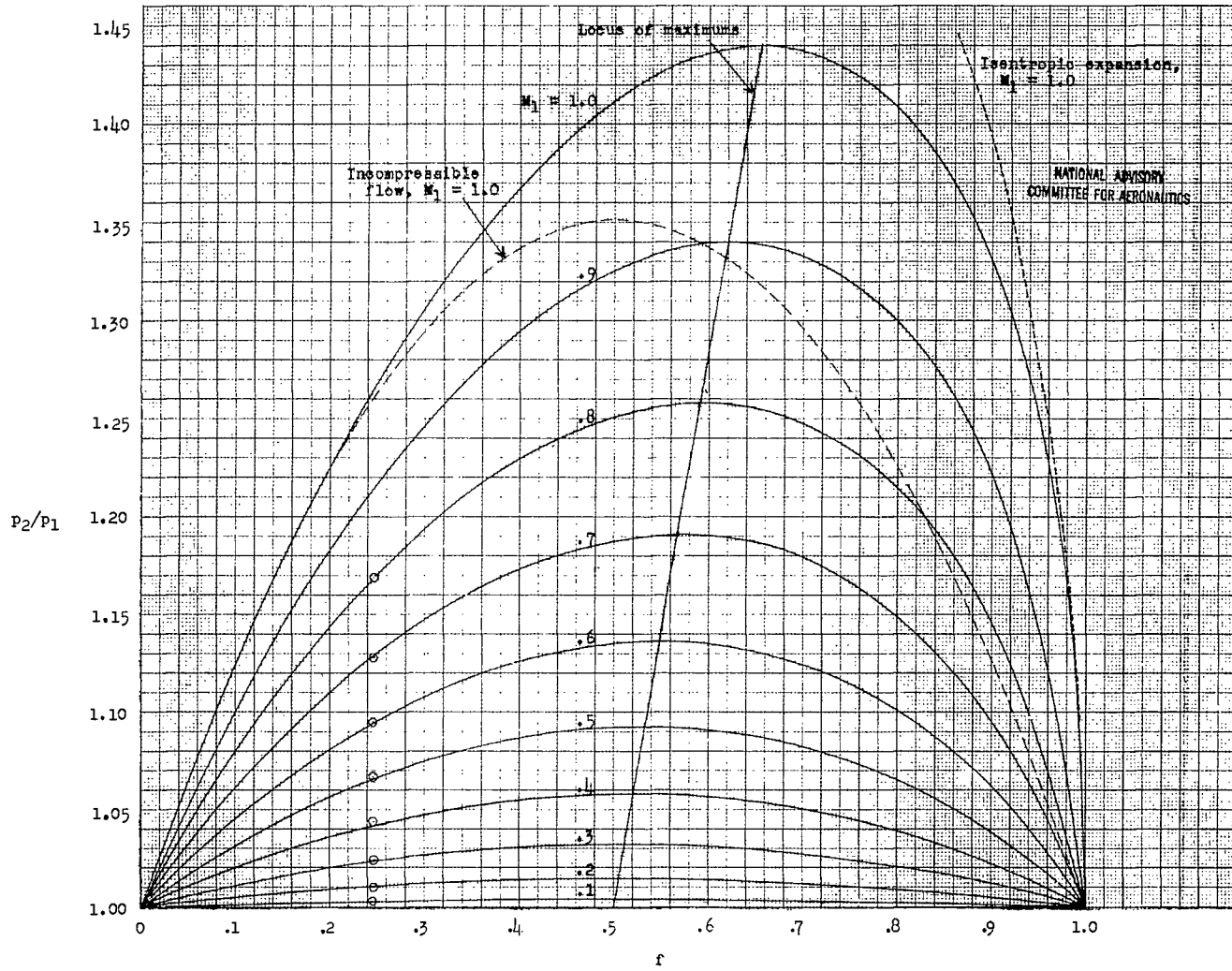


Figure 2.- Variation of pressure ratio with area ratio and initial Mach number at an abrupt expansion of compressible air flow. (Experimental points from reference 3.)

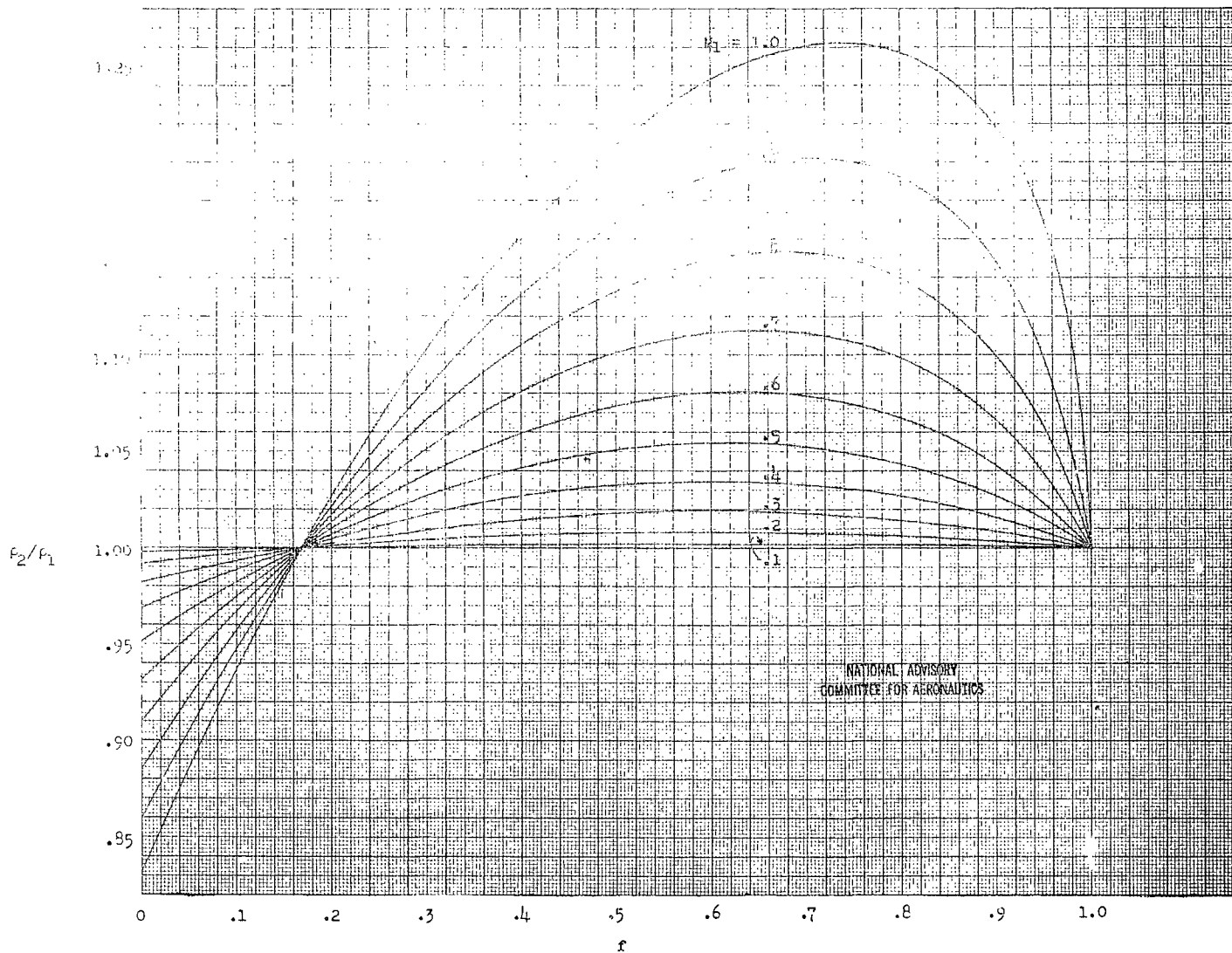


Figure 3.- Variation of density ratio with area ratio and initial Mach number at an abrupt expansion of compressible air flow.

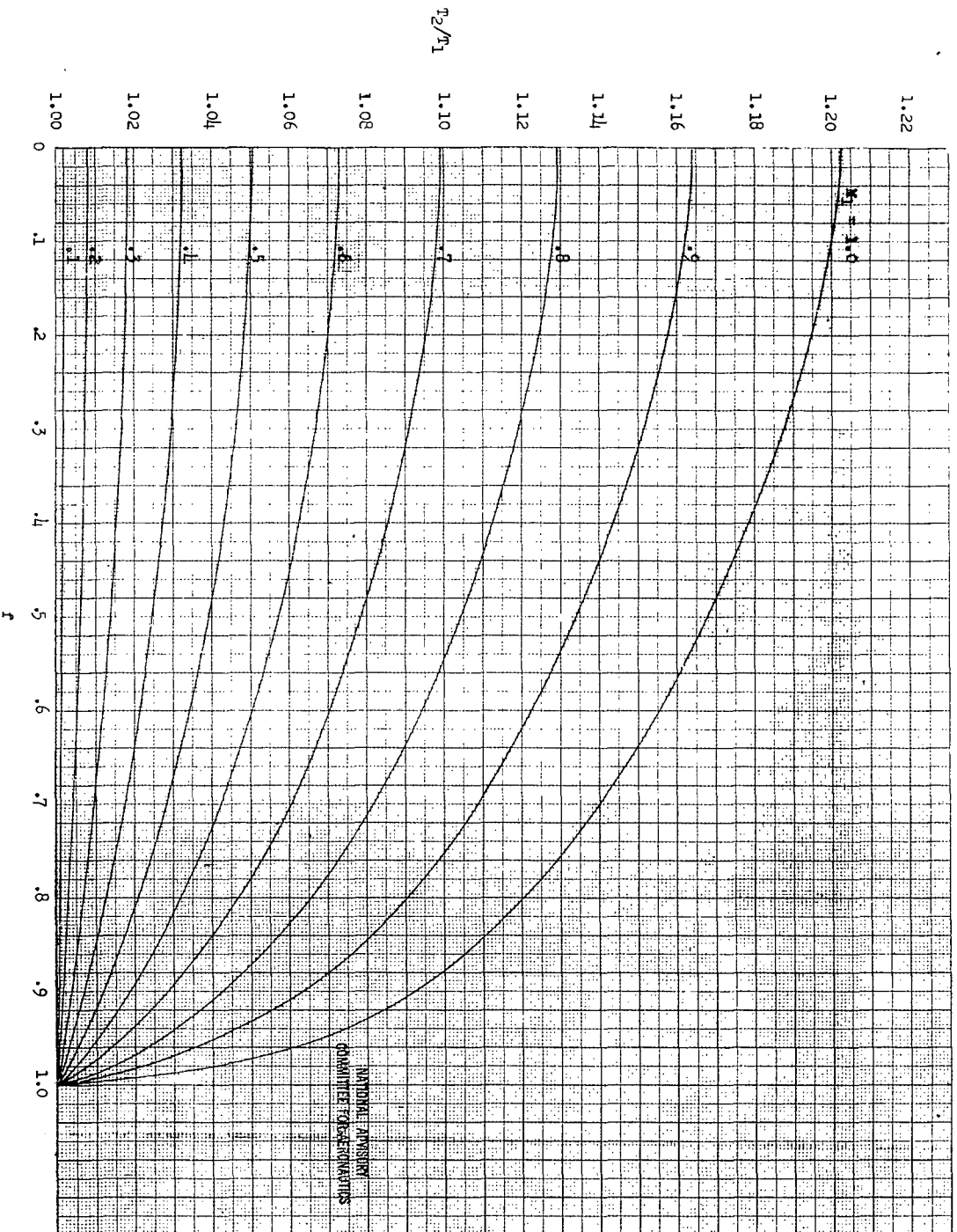


Figure 4.- Variation of absolute-temperature ratio with area ratio and initial Mach number at an abrupt expansion of compressible air flow.

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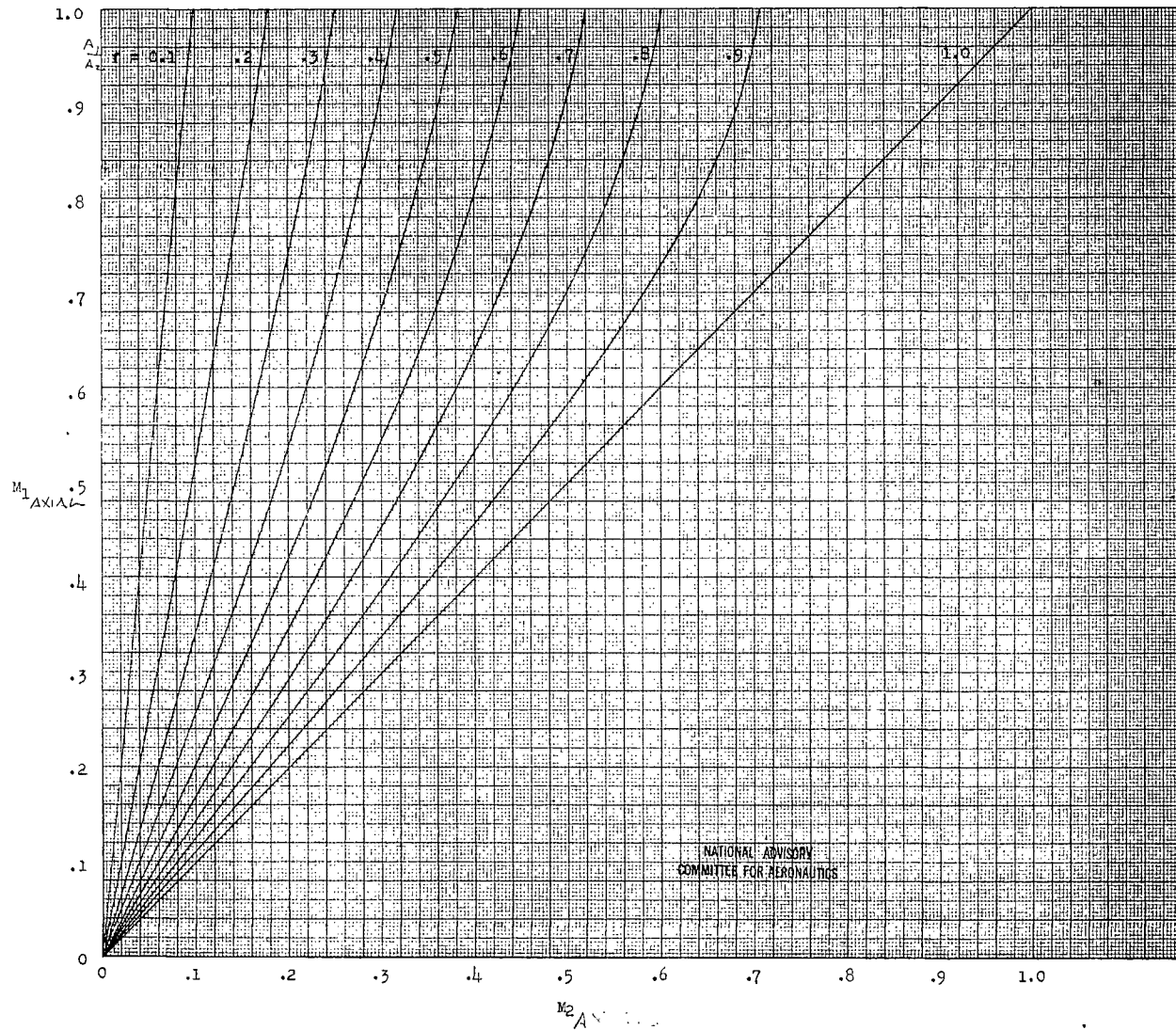


Figure 5.- Initial Mach number in terms of area ratio and final Mach number at an abrupt expansion of compressible air flow.

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**TITLE:** Charts of Pressure, Density, and Temperature Changes at an Abrupt Increase in Cross-sectional Area of flow of Compressible Air

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