

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WARTIME REPORT

ORIGINALLY ISSUED

November 1944 as
Advance Restricted Report L4I23

SECONDARY STRESSES IN OPEN BOX BEAMS

SUBJECTED TO TORSION

By Paul Kuhn, S. B. Batdorf, and
Harold G. Brillmyer

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

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NACA ARR No. D4123

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ADVANCE RESTRICTED REPORT

SECONDARY STRESSES IN OPEN BOX BEAMS

SUBJECTED TO TORSION

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SUMMARY

In open box beams subjected to torsion, secondary stresses arise owing to lateral bending of the spar caps. The present paper outlines a simple method for estimating the magnitude of these stresses and gives the results of tests of an open box beam in the neighborhood of a discontinuity where the cover changed from the top to the bottom of the box.

INTRODUCTION

The main strength element of a wing is frequently a closed box of approximately rectangular cross section. In some designs, large portions of the box cover are removable and therefore nonstructural; the closed box is thus converted into an open box, or channel, for some distance along the span. The stress analysis of such an open box under torsional loads is commonly based on the assumption that the side walls of the box act as independent spars. Such an analysis is essentially correct for computing the average stresses in the spar caps but fails to disclose the existence of secondary stresses caused by lateral bending of these caps. The present paper discusses the nature of these stresses and shows how their magnitude can be estimated.

THEORETICAL ANALYSIS

Open Box without Bulkheads

If an open box without bulkheads is subjected to torque loads, the deformation will be essentially as

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shown in figure 1. The previously rectangular cross section distorts into a parallelogram. The side walls or spars develop the resisting torque by bending - one spar bending up; the other spar, down. The cover, being connected to the lower spar caps, is stretched along one edge and compressed along the other edge. The cover stresses may be assumed to vary linearly from the maximum tensile stress along one edge to the maximum compressive stress along the other edge. On this assumption, the cover may be replaced for purposes of computation by two equivalent concentrated flanges attached to the lower spar caps as indicated in figure 1(b) - each equivalent flange having a cross-sectional area equal to one-sixth the area of the cover. The open box is thereby converted into a structure consisting of two independent spars that can be analyzed by the elementary bending theory. In a practical structure, the cross-sectional area of the cover should be understood to mean the area effective in resisting longitudinal stresses (stringers and effective width of skin). The results obtained by this simple procedure are identical with those obtained by Cambilargiu (reference 1) in a rather elaborate analysis employing the calculus of variations.

At a station where the cover is interrupted by a chordwise cut, the normal stresses in the cover must be zero and, consequently, the equivalent cross-sectional area of the cover is zero. With increasing distance from the cut, the equivalent area increases. The rate of this increase may be calculated by the methods of the shear-lag theory (reference 2); in many practical cases it will be sufficiently accurate, however, to assume that the rate of increase is uniform and that the theoretical value of one-sixth the effective area is applicable at some estimated distance from the cut. If the cover is not buckled and has no stringers attached to it, this distance may be taken as one-half the width of the box. If the cover is buckled, or if stringers are attached to the cover sheet, greater distances must be assumed.

Open Box with One Bulkhead

If an open box with a bulkhead is subjected to torque loads (fig. 2), the major part of the resisting torque will be developed by bending of the two spars as in a box without bulkheads. The bulkhead, however, now keeps the cross section rectangular and, as a result,

the vertical deflections of the spars must be accompanied by lateral deflections of the upper and lower faces of the box. The internal forces associated with these lateral deflections furnish the minor part of the resisting torque.

The forces acting on the parts of the structure are shown in figure 3. The statically indeterminate unknown is the force X acting between each vertical edge of the bulkhead and the adjacent spar. The internal forces and moments acting on the structure can be expressed without difficulty in terms of the applied torque and of the unknown force X . The force X can then be calculated by any standard method such as internal work, dummy unit load, consistent deformations, or least work. When the expressions for deformation or internal work are being written, it is generally sufficient to confine attention to bending of the spars caused by the applied torque and lateral bending of the free spar caps caused by the horizontal force Y exerted by the upper edge of the bulkhead. In some cases, of course, it may be advisable to include terms for shear deformation of the walls and for lateral bending of the horizontal beam, which consists of the cover and the spar caps attached to it.

Open Box with Many Bulkheads

The open box with many bulkheads can be treated by the same principles as the box with one bulkhead. Because an analysis with many statically indeterminate unknowns is laborious, it is advisable to simplify the calculation by assuming that the lateral bending of the spar caps is confined to the root bay; that is, only the bulkhead next to the root is assumed to be active. Comparative calculations indicate that the maximum values of the lateral bending stresses at the root are not changed appreciably by changing from one active bulkhead to two, provided the bulkheads are equally and not too closely spaced.

Open Box with Wandering Cover

In some wing structures of the open-box type the cover wanders from top to bottom. In some wings on fighter airplanes, for instance, the root bay is covered only at the top and the bottom is left open to permit retraction

of the landing gear. In the adjacent bay, the bottom is covered and the top has a large opening to give access to the wing guns. In such a structure, the bulkhead at the discontinuity, where the cover changes from top to bottom, is analogous to the root bulkhead of the simple open box discussed in the preceding paragraphs. The analysis of the stresses in the vicinity of the discontinuity may therefore be made by the same methods as outlined for the simple open box; the only difference is that all four spar caps appear in the calculation instead of only two. The simple analysis given in the appendix is of this type.

EXPERIMENTAL INVESTIGATION

Test Specimen

The test specimen built to verify the theory presented in the present paper was an open box of 24S-T aluminum alloy with wandering cover. (See fig. 4.) The specimen was symmetrical about the plane of the root bulkhead, at which it was supported. The load was applied in the form of equal couples at the ends of the box. Because the structure and the loading were symmetrical about the root bulkhead, the spars could be considered to be perfectly fixed at the root bulkhead.

The box was tested first with bulkheads spaced 28 inches, as shown in figure 4. In these tests, the bulkhead at the discontinuity, where the cover changed from top to bottom, was of 0.040-inch-thick 24S-T aluminum alloy like the cover and the angles A (fig. 4) were not attached. The second series of tests was made after intermediate bulkheads had been added and the bulkhead spacing thus reduced to 14 inches. The maximum lateral bending stresses at the discontinuity measured in this second series of tests were lower than the calculated values. It was believed that the discrepancy might be caused by elastic deformation of the bulkhead at the discontinuity; this factor becomes more important as the bulkhead spacing decreases. The aluminum-alloy bulkhead was therefore replaced by the $\frac{1}{8}$ -inch-thick steel bulkhead shown in figure 4, and the steel angles A were added to give the spar caps firm support against lateral bending. The third and fourth series of tests were run with the

bulkhead at the discontinuity modified as described and with bulkhead spacings of 14 and 28 inches, respectively.

Test Procedure

Strain measurements were made on the spar caps in the vicinity of the discontinuity with 2-inch Tuckerman optical strain gages. The load was applied in three equal increments. The strain readings were plotted against load and a straight line was drawn through the points. If the line did not pass through the origin of the plot, a parallel line was drawn to fulfill this condition and a new value of strain obtained from the new line at maximum load was used for the computation of the corresponding stress. Check runs were made if the straight line drawn through the experimental points in the load-strain plot missed the origin by more than 0.2 ksi.

Because it would be difficult to specify the location of the strain gages on the individual final plots, figure 5 was prepared to show schematically the location of the gages for all tests and the numbering system adopted for the flanges or spar caps. For convenience, most measurements were made on the outermost fibers of the outer angles. For these fibers, the stresses caused by lateral bending of the flanges increase the stresses caused by bending of the spars (the principal action) on flanges 1 and 2 and decrease the stresses on flanges 3 and 4. Some measurements were taken on the innermost fibers of the inner angles of flanges 3 and 4 as a check.

Results and Discussion

The main test results are shown in figures 6 to 9. The results are not shown in the test sequence because this sequence was dictated simply by convenience of altering the specimen. Two sets of calculated curves are shown: one based on the assumption that only the first bulkhead was active on either side of the discontinuity, the other one based on the assumption that the first two bulkheads on either side of the discontinuity were active. For design purposes, only the peak stresses that occur at the discontinuity need be known. The discussion is therefore confined mainly to these peak stresses.

The experimental stresses are lower than the calculated peak stresses in some cases and higher in other cases. A study of the results led to the conclusion that there are at least two counteracting factors influencing the stresses caused by lateral bending. One factor was mentioned in the section "Test specimen": namely, the relief of the lateral bending by elastic deformation of the bulkhead - in particular of the parts of the bulkhead and adjacent cover that support the spar caps directly against lateral bending. This relief becomes more pronounced as the bulkhead spacing decreases because the angle of twist of the first bulkhead with respect to the discontinuity bulkhead decreases and, consequently, a given amount of deformation will result in relatively larger relief.

The factor that tends to increase the lateral bending stresses is lack of integral action of the riveted spar cap. In an effort to estimate the magnitude of this factor, a few measurements were made to establish the variation of lateral bending stress over the width of the spar caps. The results of these measurements are shown in figure 10. It will be seen that the stresses in flanges 1 and 2 agree quite well with the calculated stresses. In flanges 3 and 4, the experimental points are displaced from the calculated straight line in such a way that they lie approximately on two lines parallel to the calculated line, one above it and one below it. The nature of the displacement tends to indicate that there is lack of integral action more in the primary two-spar action than in the lateral bending action. (This lack of integral action in lateral bending would result in steepening the slopes of the two lines to produce a saw-tooth diagram.) The data are insufficient to draw definite conclusions, but in any case it may be noted that the largest discrepancies shown in figures 6 and 8 occurred on flanges 3 and 4.

The simplified theory assumes that the stresses caused by lateral bending do not extend beyond the plane of the discontinuity into the region where the spar cap is not free. The stresses actually do extend beyond this point, of course, but they rapidly approach zero with increasing distance from the discontinuity. A study of figures 6 to 9 indicates that the lateral bending stresses

become negligible at a distance of about 8 inches from the discontinuity; that is, at a distance equal to about four times the width of the spar cap.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va.

APPENDIX

ANALYSIS OF OPEN BOX WITH WANDERING COVER

As an example of the method of analysis, the stresses are analyzed near the discontinuity of an open box with wandering cover like the test specimen discussed in the main part of the present paper. Only the first bulkhead on either side of the discontinuity is assumed to be active in relieving the spars by converting some of the torque into lateral bending of the spar caps; it is also assumed that the two active bulkheads are equidistant from the discontinuity.

The vertical forces and bending moments acting on the spar caps are shown in figure 11. If W denotes the internal work, the unknown X can be obtained by the theorem of least work from the expression

$$\frac{\partial W}{\partial X} = 0 \quad (1)$$

If only bending terms are considered, as is sufficiently accurate in most practical cases, this expression may be written

$$\frac{\partial W}{\partial X} = \int \frac{M}{EI} \frac{\partial M}{\partial X} dx = 0 \quad (2)$$

where

M bending moment

E Young's modulus

I moment of inertia associated with bending moment M

The coordinate x increases from the tip toward the root. The origin of x is chosen for each bay in such a way that a convenient expression for the bending moment is obtained.

Inspection of the expressions used shows the origins chosen for each bay.

The necessary expressions for M and $\frac{\partial M}{\partial X}$ are:

From B to D (fig. 11) (vertical moment in spar)

$$M = \frac{cT}{b}L_1 + \frac{T}{b}x - Xx \quad (3)$$

$$\frac{\partial M}{\partial X} = -x \quad (4)$$

From D to E (vertical moment in spar)

$$M = \frac{T}{b}x - 2aX \quad (5)$$

$$\frac{\partial M}{\partial X} = -2a \quad (6)$$

From B to C or from C to D (lateral moment in spar caps)

$$M = X \frac{bx}{2c} \quad (7)$$

$$\frac{\partial M}{\partial X} = \frac{bx}{2c} \quad (8)$$

where

T applied torque (fig. 3)

c depth of box (fig. 3)

b width of box (fig. 3)

The meanings of a , L_0 , L_1 , and L_2 are explained by figure 11. The expressions (3) to (8) are inserted in equation (2). Care must be taken to associate the

moment of inertia of the spar I_S with the bending moments in the spars, and the lateral moment of inertia of the spar cap I_F with the lateral bending moments in the spar caps; it must also be remembered that there are two spars and four spar caps. The integrations indicated in equation (2) lead to the formula

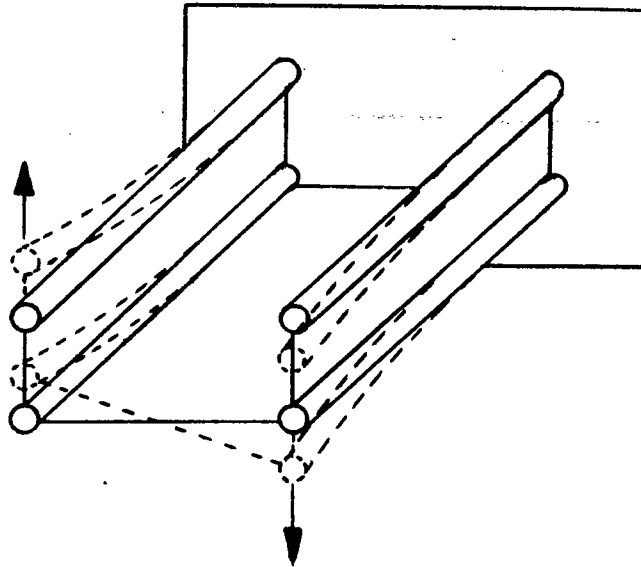
$$X = \frac{T}{ba} \frac{\frac{8}{3}a^2 + 2aL_1 + L_0^2 - L_3^2}{\frac{8}{3}a + 4(L_0 - L_3) \frac{b^2a}{6c^2} \frac{I_S}{I_F}} \quad (9)$$

This formula was used in the analysis of the test specimen under the assumption that only one bulkhead was active at either side of the discontinuity. When two bulkheads are assumed to be active, it is not practicable to write final formulas corresponding to formula (9); it is more convenient to write and solve the equations replacing expression (1) numerically for each individual problem.

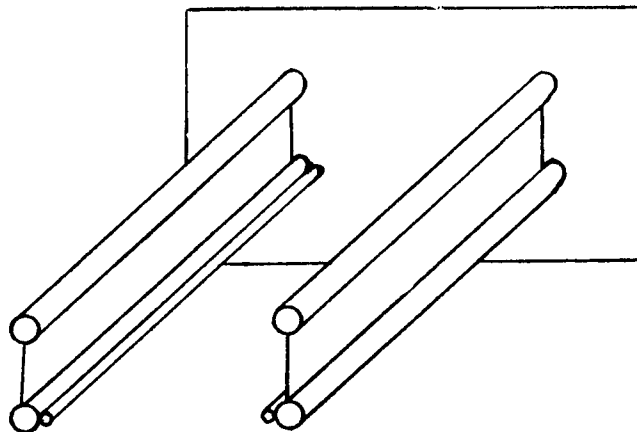
In the calculations for the test specimen, the equivalent area of the cover sheet was taken as zero at the tip and at the discontinuity. At stations 28 inches from these locations and at the root of the beam, the equivalent area was taken as one-sixth the actual area. Between the values defined in this manner, linear variation was assumed.

REFERENCES

1. Cambilargiu, E.: The Torsion of Box Beams with One Side Lacking. NACA TM No. 939, 1940.
2. Kuhn, Paul, and Chiarito, Patrick T.: Shear Lag in Box Beams - Methods of Analysis and Experimental Investigations. NACA Rep. No. 739, 1942.



(a) Idealized structure.



(b) Substitute structure.

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Figure 1.- Open box without bulkheads subjected to torsion.

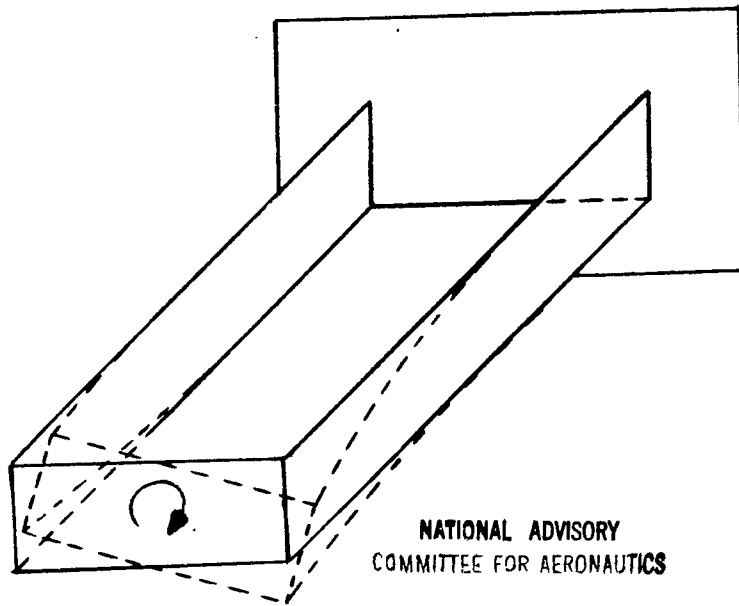


Figure 2.- Open box with a bulkhead subjected to torsion.

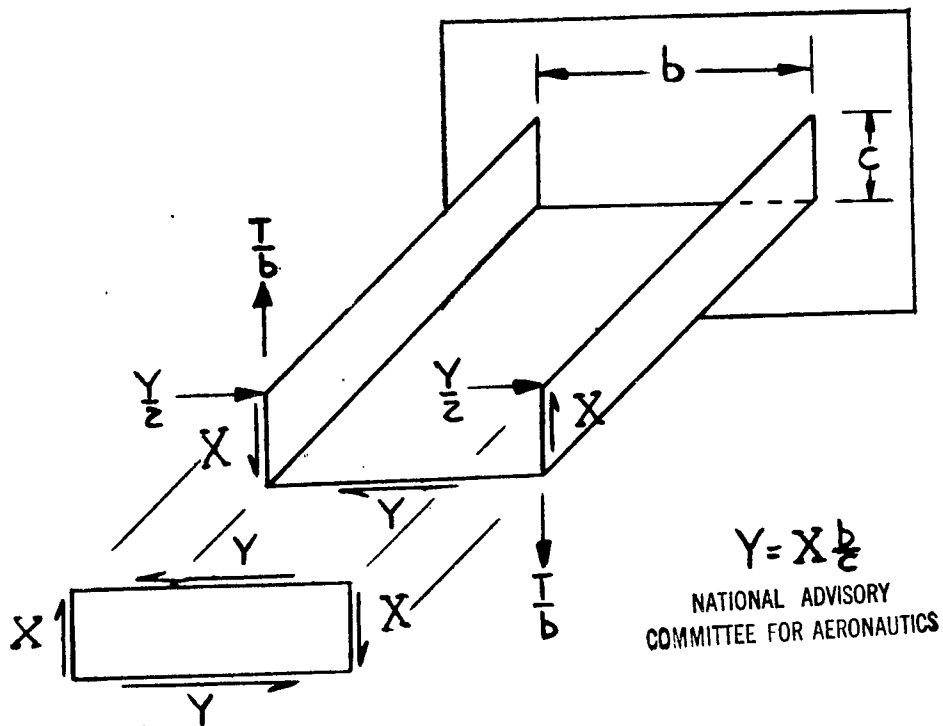


Figure 3.- Free-body diagrams of open box with bulkhead.

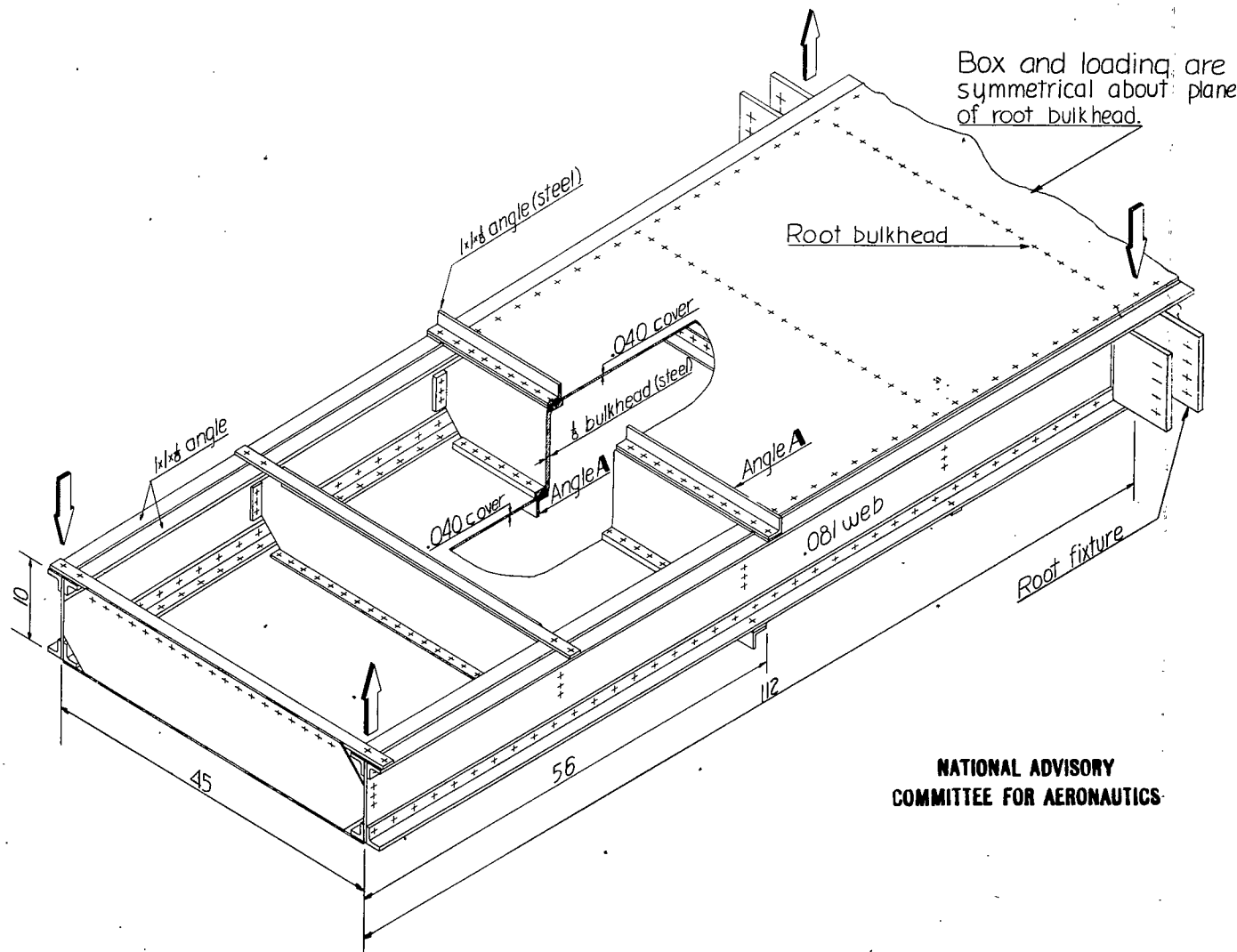


Figure 4.-Open box with wandering cover used as test specimen. All material 24S-Taluminum alloy unless otherwise specified.

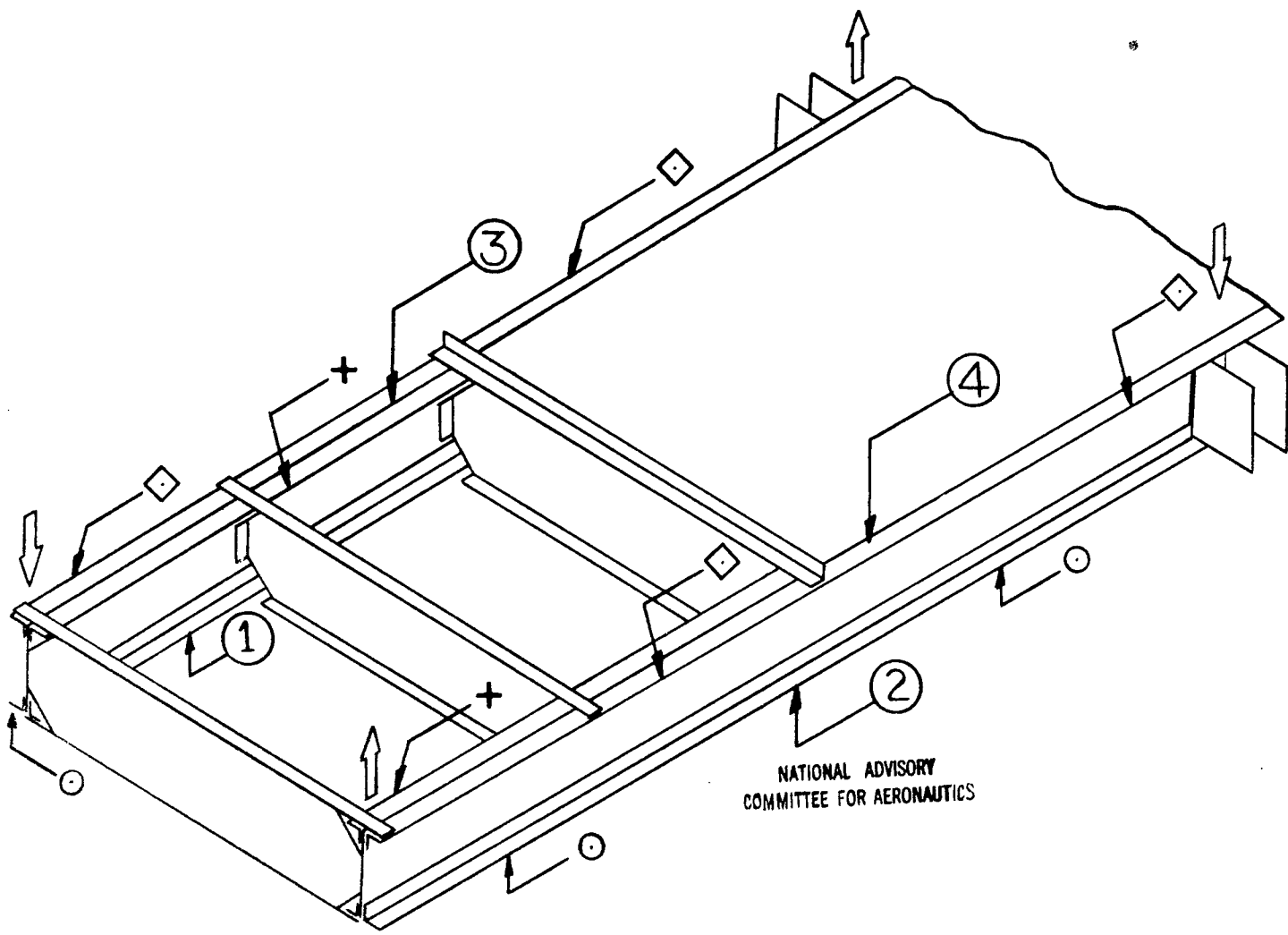


Figure 5.- Location of strain gages and designations of spar caps. Symbols indicate strain gages; numbers, spar caps.

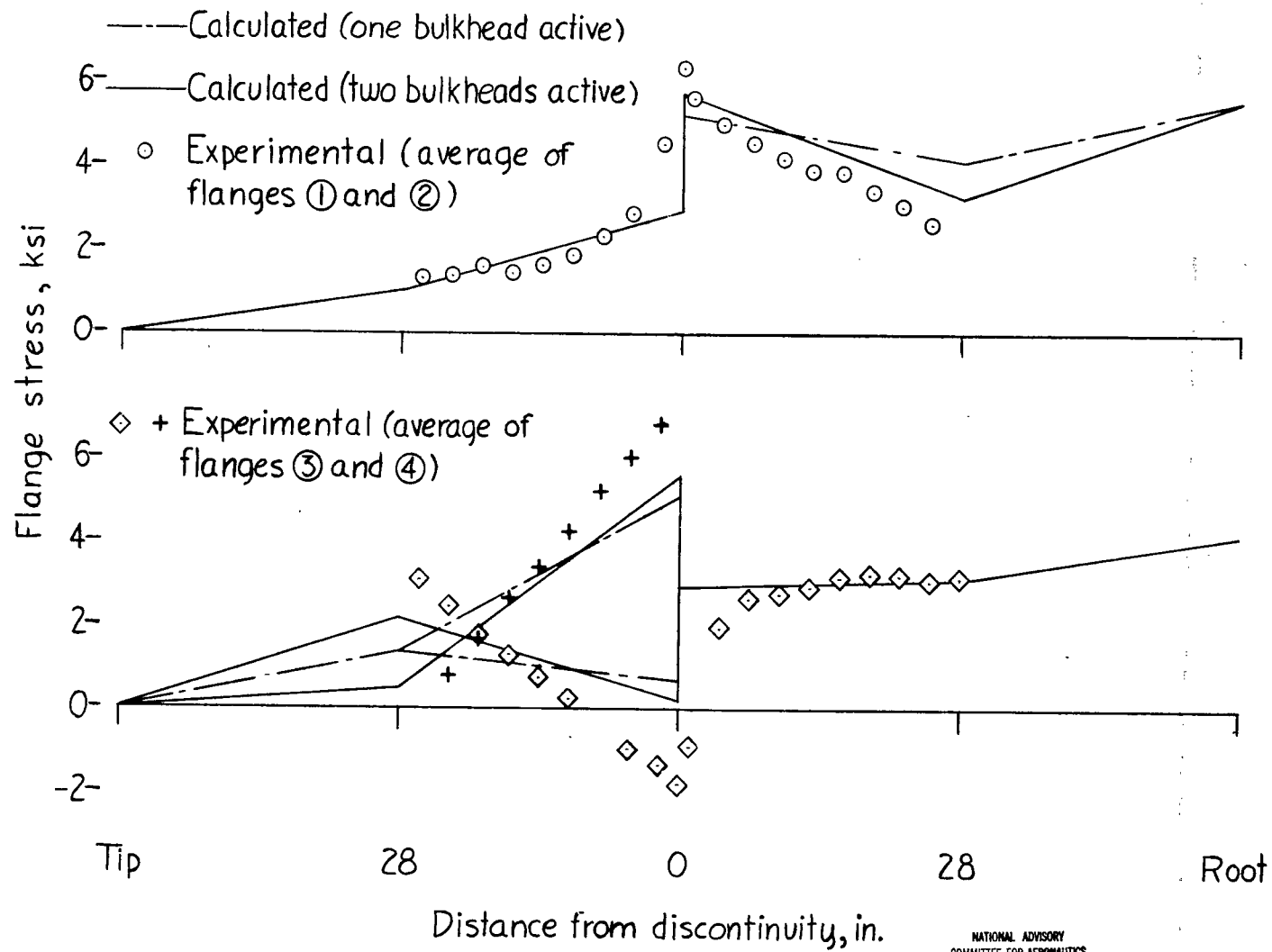


Figure 6.—Stresses in spar caps of open box with steel bulkheads spaced 28 inches.

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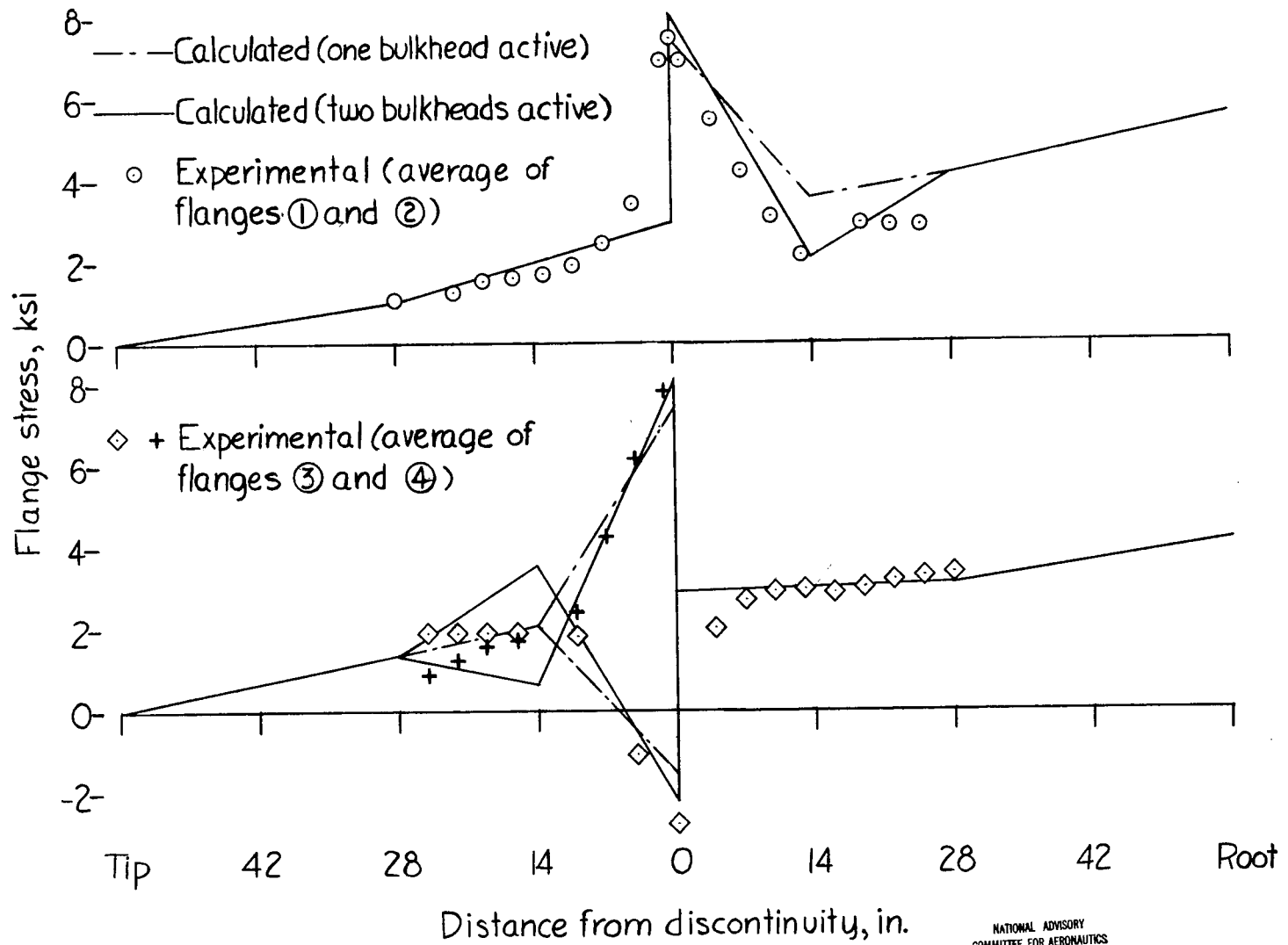


Figure 7.-Stresses in spar caps of open box with steel bulkheads spaced 14 inches.

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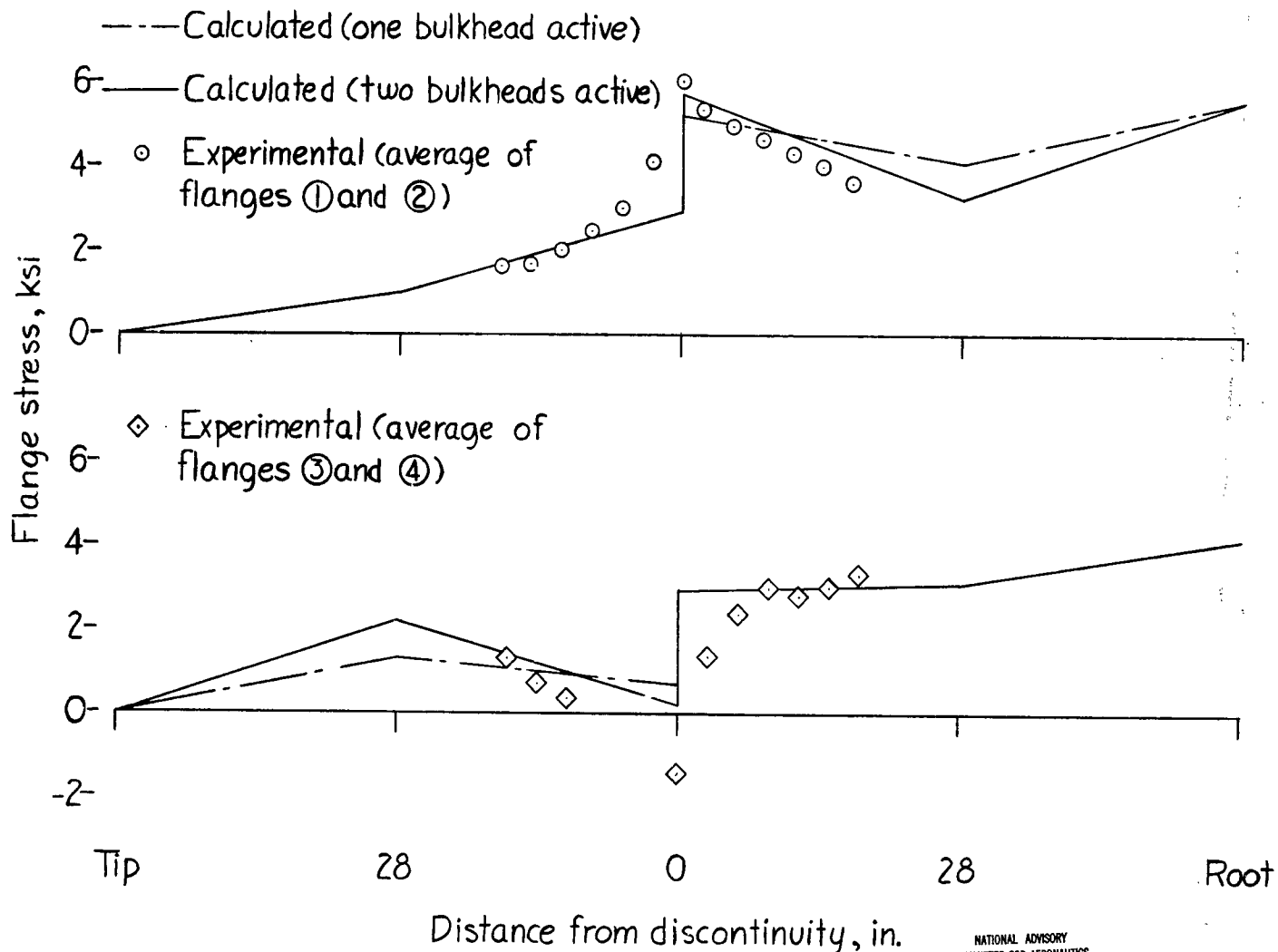


Figure 8.—Stresses in spar caps of open box with dural bulkheads spaced 28 inches.

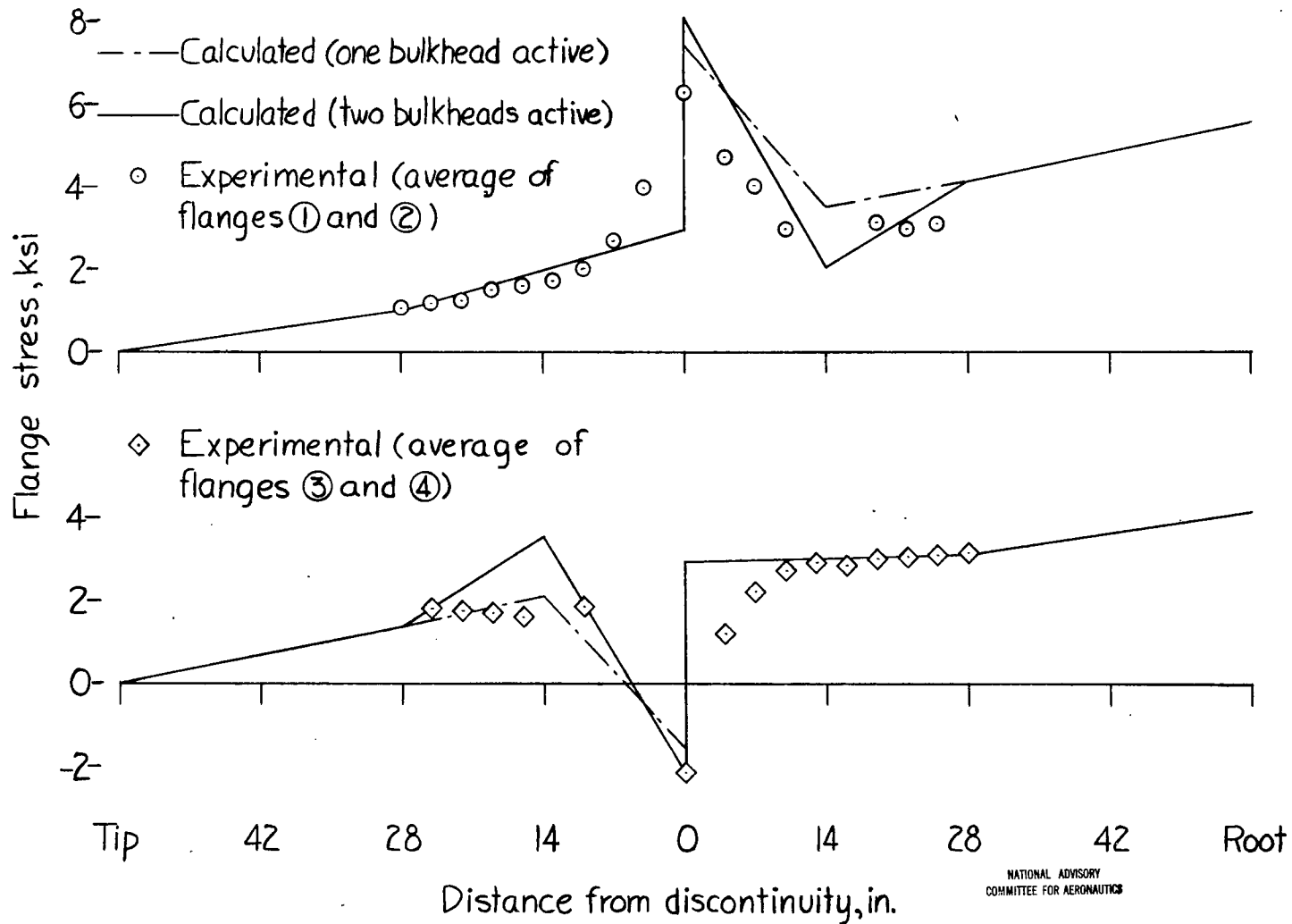


Figure 9.-Stresses in spar caps of open box with dural bulkheads spaced 14 inches.

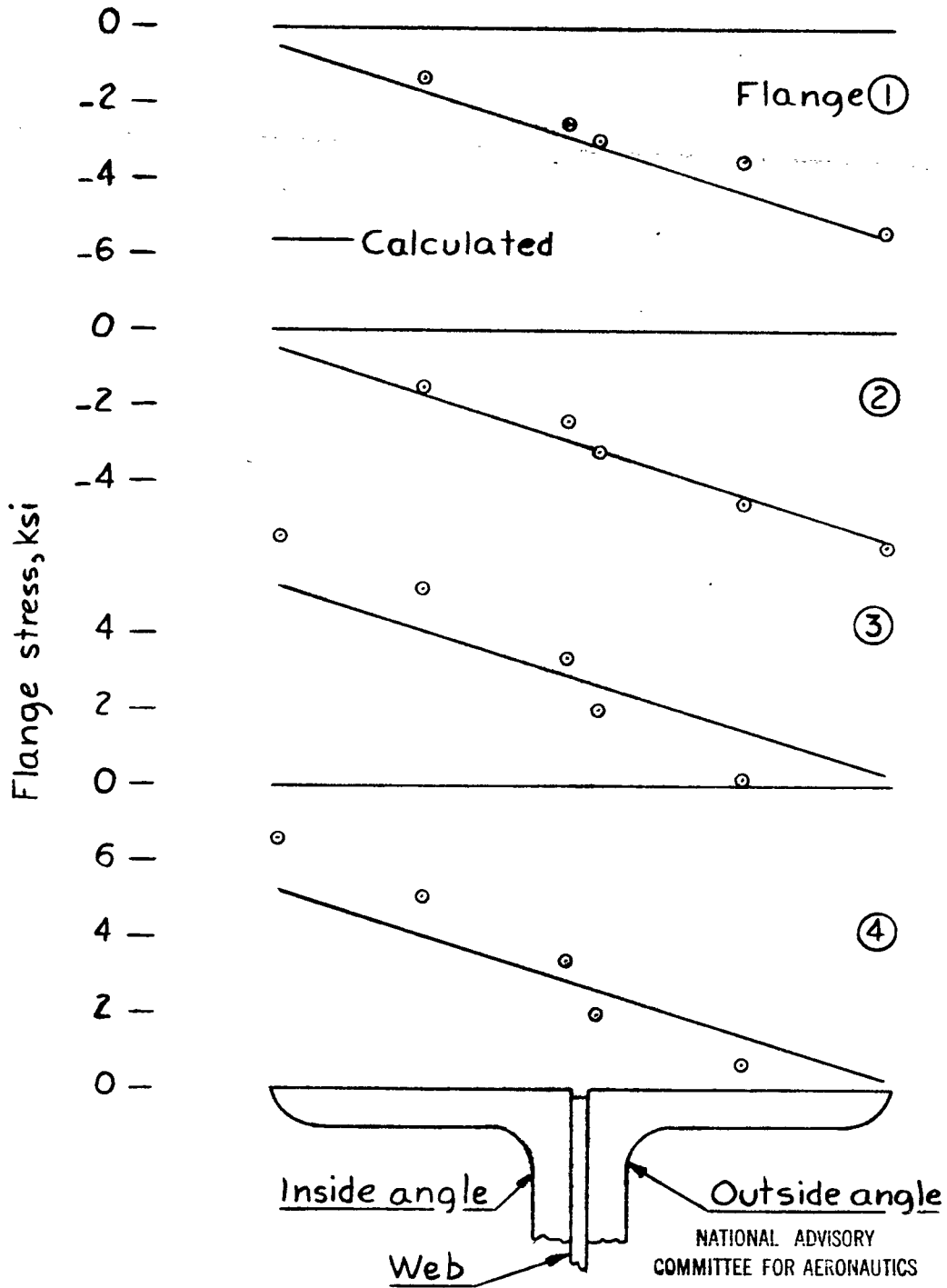


Figure 10.- Stress distribution across free spar caps 2 inches from the discontinuity. Dural bulkheads spaced 28 inches.

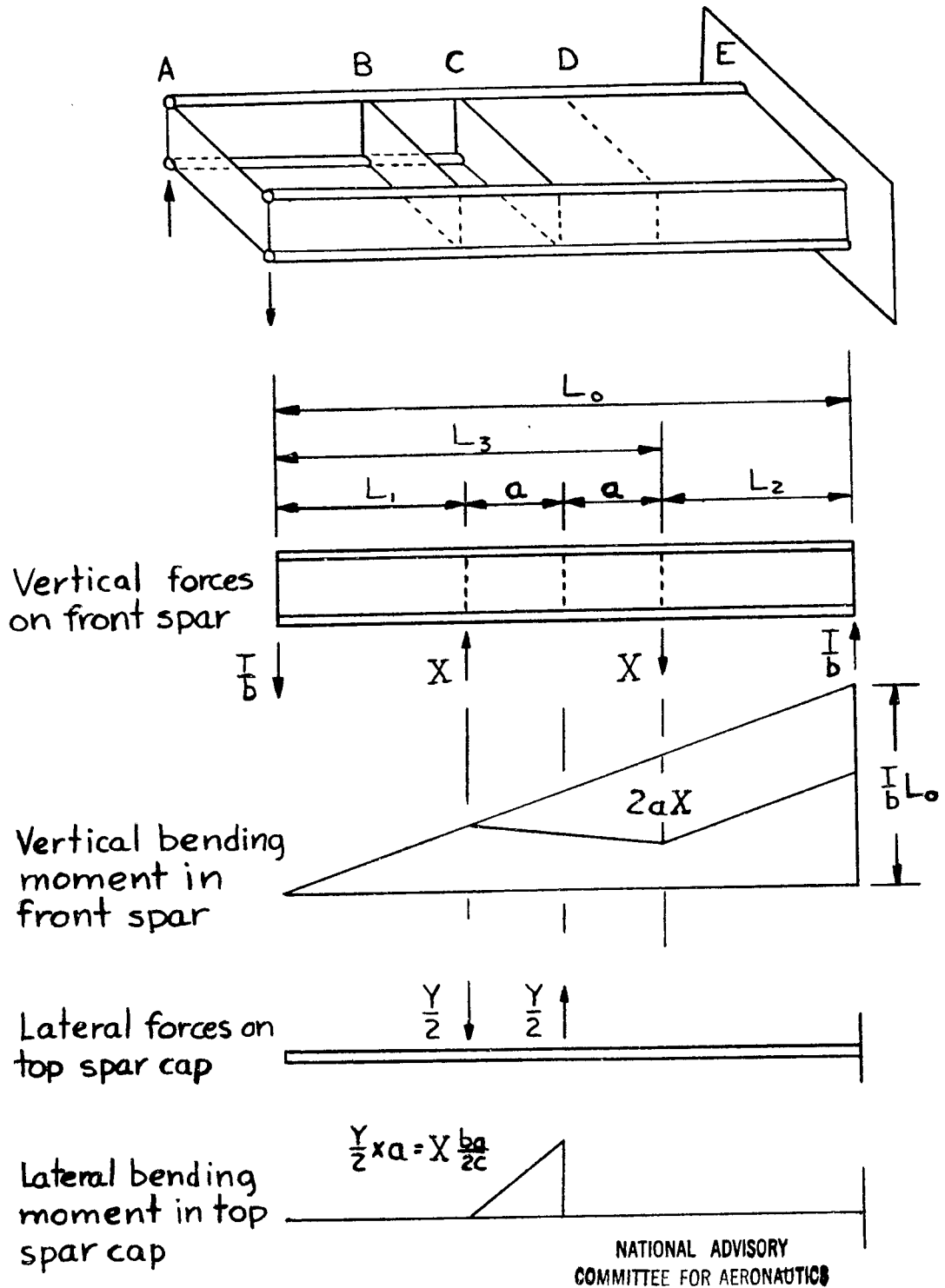


Figure 11.-Force and bending-moment diagrams.

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Kuhn, Paul
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R-7-2-17

DIVISION: Strees Analysis and Structuree (?)
SECTION: Structural Theory and Analysis Methods (2)
CROSS REFERENCES: Beams, Box - Strees analysis (15985)

ATI-7886

ORIG. AGENCY NUMBER

ARR-L4123

REVISION

AUTHOR(S)

AMER. TITLE: Secondary stresses in open box beams subjected to torsion

FORG'N. TITLE:

ORIGINATING AGENCY: National Advisory Committee for Aeronautics, Washington, D. C.

TRANSLATION:

COUNTRY	LANGUAGE	FORG'N. CLASS	U. S. CLASS.	DATE	PAGES	ILLUS.	FEATURES
U.S.	Eng.		Unclass.	Nov'44	20	12	diags, drwgs

ABSTRACT

In open box beams subjected to torsion, secondary stresses arise because of lateral bending of the spar caps. An outline is presented of a simple method for estimating the magnitude of these stresses and results of tests of an open box beam in the neighborhood of a discontinuity where the cover changed from the top to the bottom of the box.

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