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## THE HYDRODYNAMIC LUBRICATION OF NEAR-INFINITE SLIDERS SUCH AS PISTON RINGS

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ADVANCE RESTRICTED REPORT

# THE HYDRODYNAMIC LUBRICATION OF NEAR-INFINITE SLIDERS SUCH AS PISTON RINGS

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## SUMMARY

The lubrication of a piston ring is treated as though the ring were a rigid slider of finite width. An expression is obtained for pressure distribution which takes into account the effect of oil leakage from the ends of the ring and which has the convenient form of an end correction to the pressure-distribution formula for an infinite slider. The important formulas are summarized for convenience and two examples are included of their use in finding the pressure pattern over wide sliders. The method used in this paper is shown to be applicable to sliders of length-to-width ratios up to unity.

## INTRODUCTION

Although the hydrodynamic aspect of piston-ring lubrication has been investigated at some length, the investigations have considered the ring as a rigid infinite slider. Such a treatment is justified for the greater part of the ring circumference, that is, the slider width, because by virtue of the large ratio of ring circumference to face width, that is, slider length, the ring does behave almost as an infinite slider. The assumption of infinite width fails at each end of the ring where, as a consequence of the side leakage of oil, the actual load carried by the oil film is less than that indicated by the infinite-slider equation.

An investigation of the hydrodynamic lubrication existing at the ends of the ring has been conducted at the NACA Aircraft Engine Research Laboratory. The first part of the investigation consisted of an analysis by means of the rigorous classical method of Michell, which may be found in many sources as, for example, the text by Boswall (reference 1, p. 108). The calculations, normally of great length, became so intricate for a piston ring that they were not completed. The investigation was continued using the approximate method of analysis proposed by Stodola (referencd 2) and developed by Boswall (reference 1, p. 144). This method is based on the assumption that the pressure at any point on a slider can be represented by a product of two functions, one of the coordinate in the direction of the slider length and the other of the coordinate in the direction of the slider width. In the cases that were analyzed by both methods, the results obtained by means of this approximate method checked closely with the results obtained by the exact classical method.

The approximate method was also found to be too intricate for investigating the oil-pressure pattern at the ends of the ring. A simplification of Stodola's approximate method was therefore made for the special case of piston rings by assuming that the pressure at any point in the oil film is not only a product of functions of the width and length of the slider, but also that the pressure distribution at the midpoint of the width is that of a slider of infinite width. This second assumption makes possible the evaluation of the pressure functions as simple equations involving the geometry of the slider. The complete simplified method is a new procedure for pressure calculations applicable to near-infinite sliders, which by definition fulfill the condition that at the midpoint of the width, the pressure distribution is the same as that for an infinite slider.

Following the derivation of the method, a summary of the important equations with an example of their use in finding pressure distributions is presented; the equations are easily used without reference to the theoretical work. The simplified method can be used as a guide in the design of piston rings to achieve the maximum load capacity without excessive wear at the ends. A discussion of the use of those computations in ring design is given at the end of the paper.

It is shown in the appendix that the method described in this paper is not limited only to sliders of length-to-width ratios of the order of 0.003, but is applicable for ratios up to unity. The work of Muskat, Morgan, and Meres (reference 3) on the characteristics of finite sliders is at least as simple as the present method for determinations of load-carrying capacity of finite sliders, but the variation of pressure over the sliding area is not obtained as in this paper by simple calculations.

## ANALYSIS

The starting point for the analysis of wide-slider lubrication is the general differential equation of Reynolds:

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial P}{\partial x}h^3\right)+\frac{\partial}{\partial z}\left(\frac{1}{\mu}\frac{\partial P}{\partial z}h^3\right)=6\frac{d}{dx}Uh \qquad (1)$$

In this equation μ, h, U, and P are the oil viscosity, the oil-film thickness, the slider velocity, and the oil pressure at any point (x,z) on the slider surface. The usual assumptions of constant oil viscosity and flat slider surfaces will now be made, and the dimensional variables can be replaced with dimensionless variables by making the substitutions

$$x = x_o + Lx_1$$

$$z = Bz_1$$

$$h = h_o\ (1 + bx_1)$$

$$\mu = \mu_o$$

where L and B are the length and width of the slider and $h_o$ and $\mu_o$ represent conditions at the outlet edge. (See fig. 1.)

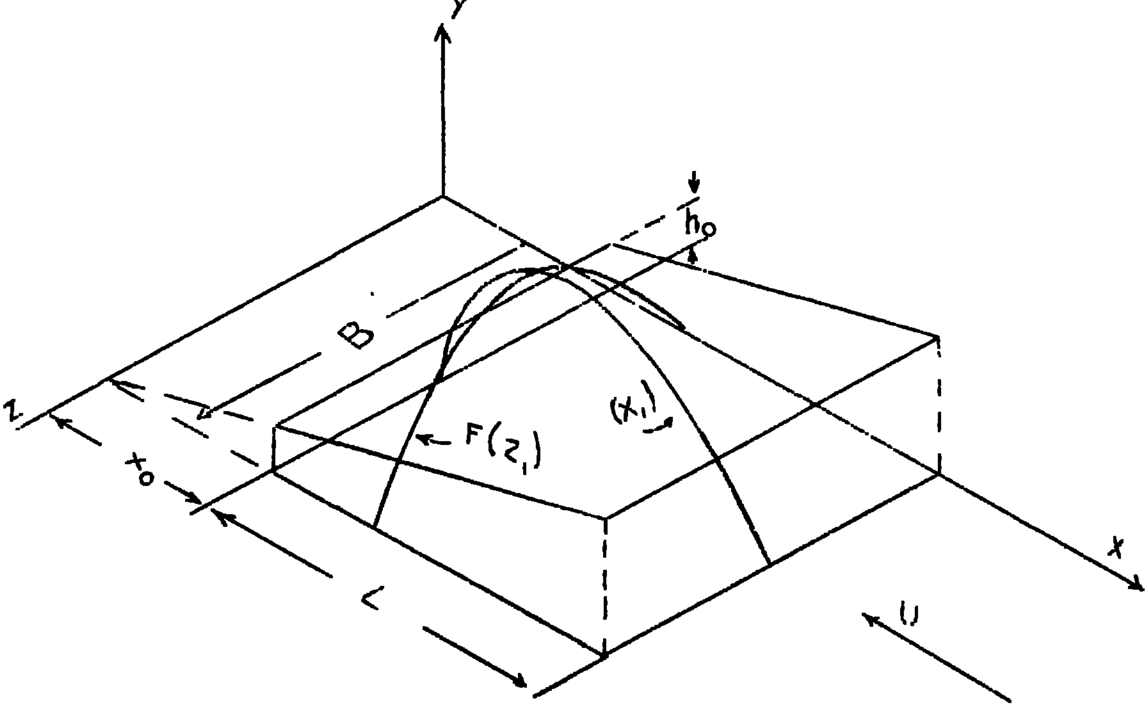


Figure 1

The variables $x_1$ and $z_1$ are the fractional distances across the length and width of the slider. Equation (1) becomes

$$\frac{\partial}{\partial x_1}\left[(1 + bx_1)^3\ \frac{\partial P}{\partial x_1}\right] + N^2\ (1 + bx_1)^3\ \frac{\partial^2 P}{\partial {z_1}^2} = Ab \qquad (2)$$

in which $N$ is the ratio of the slider length to the slider width and $A$ is the product $\frac{6\mu_o UL}{h_o^2}$, which contains all the dimensional parameters.

It will now be assumed that the pressure at any point can be expressed as the product of two functions

$$P = A \cdot F(x_1) \cdot f(z_1) \qquad (3)$$

The value of $P$ from equation (3) is substituted in equation (2), with the result

$$\frac{\partial}{\partial x_1}\left[(1 + bx_1)^3 f(z_1) \frac{dF(x_1)}{dx_1}\right] + N^2 (1 + bx_1) F(x_1) \frac{d^2 f(z_1)}{dz_1^2} = b \qquad (4)$$

The next step is to find the forms of the two functions in equation (3). By the definition of a near-infinite slider the pressure distribution along $z_1 = 0.5$ is the same as that for an infinite slider. The form of $F(x_1)$ is consequently the usual form of this function for an infinite slider (reference 1, p. 44),

$$F(x_1) = \frac{bx_1 (x_1 - 1)}{(b + 2) (bx_1 + 1)^2} \qquad (5)$$

In order to find the form of $f(z_1)$, equation (4) is solved for the condition where $x_1 = x_m$; $x_m$ is the value at which $F(x_1)$ has its maximum value. Since $\frac{dF(x_1)}{dx_1}$ is zero along this line,

$$\frac{d^2 f(z_1)}{dz_1^2} + \frac{M}{N^2} f(z_1) = \frac{J}{N^2} \qquad (6)$$

where $M$ designates the value of $\frac{1}{F(x_m)}\left[\frac{d^2F(x_1)}{dx_1^2}\right]_{x_1=x_m}$ and

$J$ designates the value of $b\left(\frac{1}{F(x_m)}\right)\left(\frac{1}{(1 + bx_m)^3}\right)$. The solution of equation (6) is

$$f(z_1) = C_1 e^{nz_1} + C_2 e^{-nz_1} + C_3 \qquad (7)$$

$$n^2 = -\frac{M}{N^2}$$

$$C_3 = \frac{J}{M}$$

The integration constants $C_1$ and $C_2$ are determined from the boundary conditions that $f(z_1)$ disappears at $z_1 = 0$ and 1. This condition occurs when

$$C_1 = \frac{J}{M}\left(\frac{e^{-n} - 1}{e^n - e^{-n}}\right)$$

$$C_2 = \frac{J}{M}\left(\frac{1 - e^n}{e^n - e^{-n}}\right)$$

For the special case where $x_1 = x_m$ and $z_1 = z_m$, that is, where $P$ is a maximum, equation (6) is simplified to

$$M + N^2\left[\frac{d^2f(z_1)}{dz_1^{\,2}}\right]_{z_1=z_m} = J \tag{8}$$

A number of approximations in equations (7) and (8) are now in order. As the ratio of width to length of the slider approaches infinity, the value of $N^2$ rapidly decreases, approaching zero. The value of $\frac{d^2f(z_1)}{dz_1^{\,2}}$, which is also zero for an infinitely wide slider, does not differ much from zero for a near-infinite slider. The second term in equation (8) may consequently be completely neglected:

$$J \approx M$$

and

$$\frac{J}{M} \approx 1$$

Then by equation (7)

$$n^2 = -\frac{M}{N^2} \approx -\frac{J}{N^2}$$

$$C_1 = \frac{J(e^{-n} - 1)}{M(e^n - e^{-n})} \approx \frac{(0 - 1)}{(e^n - 0)} = -\frac{1}{e^n}$$

$$C_2 = \frac{J(1 - e^n)}{M(e^n - e^{-n})} \approx \frac{(-e^n)}{(e^n - 0)} = -1$$

$$C_3 = \frac{J}{M} \approx 1$$

The form of the $f(z_1)$ then becomes

$$f(z_1) = 1 - e^{-n(1 - z_1)} - e^{-nz_1} \tag{9}$$

and it only remains to evaluate $J$ in terms of known quantities so that $n$ may be found. The derivative of equation (5) is set equal to zero, resulting in

$$x_m = \frac{1}{b + 2}$$

When this quantity is substituted in the expression for $J$ given after equation (6), it is possible to compute $J$ directly from the known value of $b$

$$J = -\frac{(b + 2)^4}{2(b + 1)^2}$$

In this exposition the boundary pressures have always been considered zero. The resulting equations give the pressure above the boundary pressure at any point on the slider, if in Stodola's method the actual pressure is considered to be:

$$P_2 = P + P_1$$

where

$P_1$ boundary pressure assumed constant over all boundaries of the slider

$P_2$ actual pressure

and P is given by equation (3).

For any fixed value of $z_1$ each value of the pressure coefficient calculated for an infinite slider from equation (5) is multiplied by a factor obtained from equation (9); it is therefore obvious that the integral of equation (3) also contains the same factor. Thus, the load per unit width carried by the slider varies with $z_1$ in proportion to the value of function (9).

The important equations of the preceding analysis are summarized for convenience in use:

$$P = A \cdot F(x_1) \cdot f(z_1) \tag{3}$$

$$F(x_1) = \frac{bx_1(x_1 - 1)}{(b + 2)\ (bx_1 + 1)^2} \tag{5}$$

$$f(z_1) = 1 - e^{-n(1 - z_1)} - e^{-nz_1} \tag{9}$$

The values of the constants in these equations are obtained from

$$A = \frac{6\mu_o UL}{h_o^2}$$

$$n^2 = -\frac{J}{N^2}$$

where

$$J = -\frac{(b + 2)^4}{2(b + 1)^2}$$

$$N = L/B$$

## EXAMPLES OF APPLICATION

The pressure distribution over a slider of length-to-width ratio of 0.003255 and ratio of inlet-film thickness to outlet-film thickness of 2.20, that is, $b = 1.20$, will now be investigated. The value of 0.003255 is chosen because it represents the proportions of one face of $6\frac{1}{8}$-inch biconically faced piston ring. The value of 2.20 is approximately the ratio that gives the maximum load-carrying capacity for an infinite flat slider, if the pressure and temperature effects on viscosity are ignored (determined from reference 1, p. 42). For such a slider, equation (5) becomes

$$F(x_1) = \frac{1.20x_1(x_1 - 1)}{(3.20)(1.20x_1 + 1)^2}$$

The value of $J$ is

$$-\frac{(3.20)^4}{2(2.20)^2} = -10.336$$

and $n$ may be found from $N$ and $J$

$$n = \frac{\sqrt{10.836}}{0.003255} = 1011$$

The equation for the pressure distribution in the $z$ direction from equation (9) is

$$f(z_1) = 1 - e^{-1011(1 - z_1)} - e^{-1011z_1}$$

The product of the values of $F(x_1)$ and $f(z_1)$ for any selected values of $x_1$ and $z_1$ gives, when multiplied by $A$, the actual magnitude of the pressure at the point $(x_1, z_1)$. It is convenient, however, to plot the values of $F(x_1) \cdot f(z_1)$ since this type of graph illustrates the shape of the pressure pattern over the slider without involving circumstances that limit the generality of the solution.

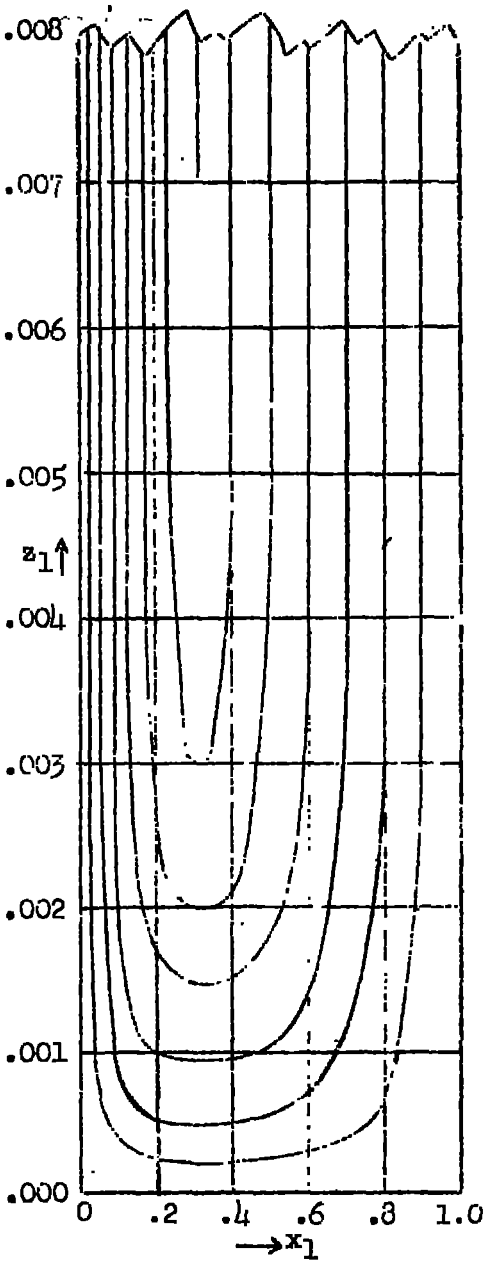


Figure 2

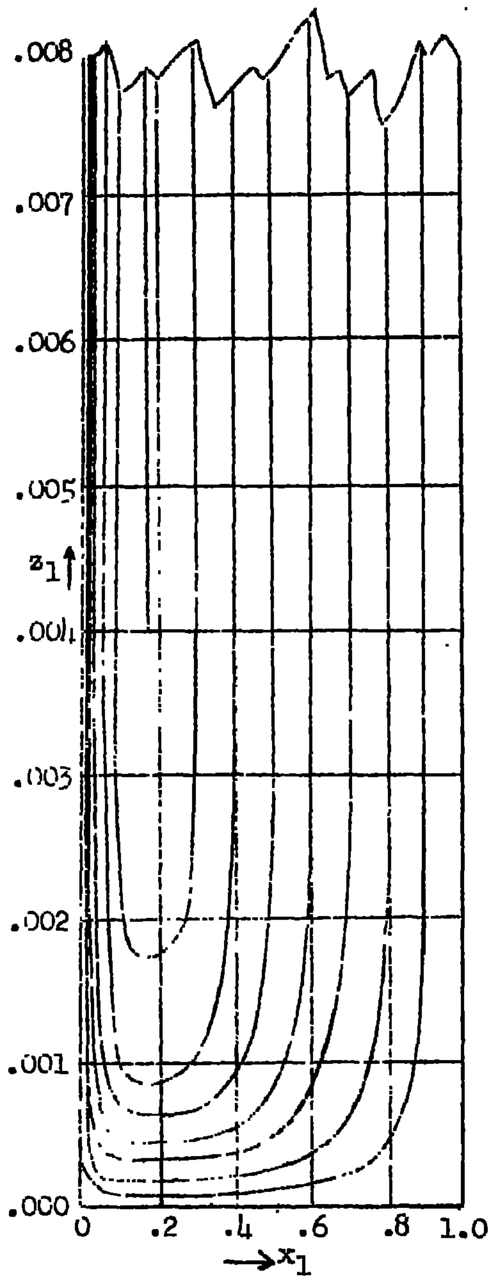


Figure 3

Figure 2 shows the locations of isobars at the end of the slider. The scales for $x_1$ and $z_1$ have been so chosen that the figure exhibits the proportions of a piston ring. For comparison with figure 2, figure 3 gives the same information on an identical slider for which, however, b = 4.00. It is clear in what manner the pressure decreases to zero at the end of the slider: at 0.175 inch from the end of the slider for which b = 1.20 the pressure distribution is within 0.01 percent of that for an infinite slider.

## APPLICATION OF METHOD TO PISTON-RING DESIGN

This method of pressure-distribution calculation should be applicable to piston rings, which are certainly as near infinite as any practical slider, if they are hydrodynamically lubricated. There is no doubt that hydrodynamic lubrication does exist, at least part of the time, between piston ring and cylinder (reference 4) and that under film lubrication the ends of a piston ring cannot support any load. Equation (9), however, shows exactly how the ring pressure must decrease in the vicinity of the ends and should therefore be of use in the design of rings. Figures 2 and 3 make it evident that the effects of end leakage can be completely ignored except within about 0.2 inch of the end. If the pressure exerted along the ring were adjusted in accordance with equation (9), the entire alteration would be within 0.2 inch of the end. The application of this equation, therefore, does not imply the abandonment of plus-circularity, which is desirable in rings.

Strictly speaking, the method described in this report is applicable only to sliders for which the film thickness is constant along the entire width; this condition is not fulfilled in the case of piston rings. If the effect of a change from a finite to an infinite slider at the end of the ring extends inward no further than 0.2 inch, then it is unlikely that a small change in film thickness will have any other than a strictly local effect. Each portion of the ring consequently behaves as though it were infinitely long and the method should be applicable to the ends without considering conditions further in than about 0.3 inch from the end.

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## APPENDIX

### APPLICABILITY OF SIMPLIFIED METHOD

In an effort to determine the range of applicability of the method described in the report, the $W/W_\infty$ ratio (the ratio of the actual load carried to the total load supported by an equal segment of an infinite slider) has been related to the length-to-width ratios of a series of sliders for which $b = 1.5$. This value of b results in the minimum coefficient of friction. The comparison of the $W/W_\infty$ ratio thus determined to the actual $W/W_\infty$ ratio found by Kingbury's experimental method (reference 5) gives an indication of the range over which the simplified method is usable.

The theoretical $W/W_\infty$ ratio is immediately obtained by integrating $f(z_1)$, equation (9)

$$\frac{W}{W_\infty} = \int_0^1 \left[1 - e^{-nz_1} - e^{-n(1 - z_1)}\right] dz_1$$

$$= \left[1 - \frac{2}{n}\left(1 - e^{-n}\right)\right]$$

For each N the value of n is first calculated, and the $W/W_\infty$ ratio can then be found. The comparison of the $W/W_\infty$ ratio thus computed with the actual $W/W_\infty$ ratio is shown in figure 4.

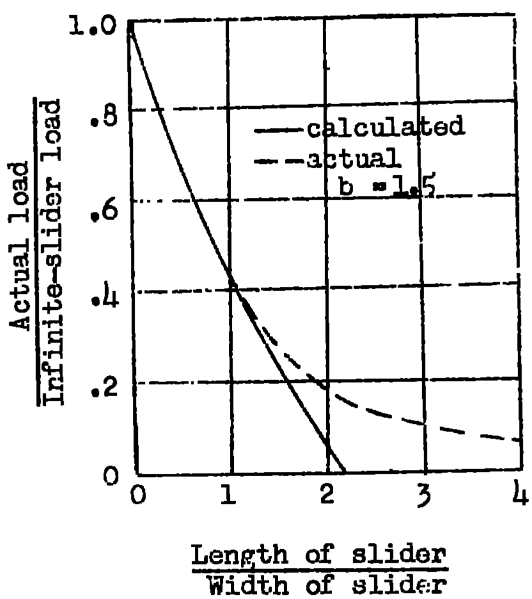


Figure 4

These preliminary calculations seem to indicate that the range of applicability of the new method is wider than was originally anticipated. Although much more verification will have to be made, especially for other values of b, it appears that the method is usable at least for total load determinations in the range of N = 0 to 1.

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ABSTRACT

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